Prework 5.3a: Mapping Reducibility

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Micah, David, Todd] Let $A = \{a^n b^n \mid n \geq 1\}$ and $B = \{a, b\}$.

   a. Show tht $A \leq_m B$ by defining a computable function $f : \Sigma^* \longrightarrow \Sigma^*$ such that $w \in A$ exactly when $f(w) \in B$.
   b. Similarly, show that $B \leq_m A$.
   c. Parts (a) and (b) show that $\leq_m$ is not *antisymmetric*, so it is not a partial order. Is $\leq_m$ *transitive*? (See p. 9.)

2. [Curtis, Andrew, Connor] Recall that a language is *closed under reversal* if, for every w in the language, its reverse w^R is also in the language. Let $CUR_{TM} = \{\langle M \rangle \mid M$ is a TM and $L(M)$ is closed under reversal$\}$. Show that CUR_{TM} is undecidable by showing that $A_{TM} \leq_m CUR_{TM}$. (Hint: Map the string $\langle M, w \rangle$ to the string $\langle M_w \rangle$, where M_w is a TM that recognizes a simple closed-under-reversal language exactly when M accepts w.)

3. [Ky, Joshua, Grace] In Prework 4.2b we showed that CUR_{DFA} is decidable by using a decider for EQ_{DFA}. Following this argument, give an alternative proof that EQ_{TM} is undecidable by showing that CUR_{TM} reduces to EQ_{TM}.

4. [Allie, Ben, Levi, Meghan] Let $\Sigma = \{0, 1\}$. Show that a language A is decidable if and only if $A \leq_m \{0^* 1^*\}$.

─────────────BEGIN YOUR SOLUTIONS BELOW THIS LINE─────────────