Prework 4.2a: Uncountability and non-recognizable languages

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Levi, David, Grace] Let $\mathscr{B}$ be the set of all infinite binary sequences. Use a diagonalization argument to explain why $\mathscr{B}$ is uncountable.
2. [Todd, Micah, Joshua] Recall that a *one-to-one correspondence* is a function that is both one-to-one and onto. Describe a natural one-to-one correspondence $f : \mathscr{B} \longrightarrow \mathscr{P}(\mathbb{N})$, where $\mathscr{P}(\mathbb{N})$ is the power set of $\mathbb{N}$ and $\mathscr{B}$ is the set of all infinite binary sequences. (This shows that $\mathscr{P}(\mathbb{N})$ is uncountable.)
3. [Ben, Allie, Connor] Let $A_{\mathrm{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM that accepts } w\}$. Prove that A_{TM} is recognizable by giving a description of a TM that recognizes it. Is your TM a decider? Explain why or why not.
4. [Ky, Meghan, Andrew, Curtis] Find a countably infinite collection of countably infinite disjoint subsets of $\mathbb{N}$. That is, give a description of an infinite sequence of subsets $X_1, X_2, X_3, \ldots$ of natural numbers such that $X_i \cap X_j = \emptyset$ whenever $i \neq j$ and such that each X_i is countably infinite.

BEGIN YOUR SOLUTIONS BELOW THIS LINE