Prework 3.2a: Turing Machine Variants

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Micah, Curtis, Todd] Suppose that M_1 is a TM that recognizes a language A_1 and M_2 is a TM that recognizes a language A_2. How can the following TM fail to recognize the concatenation $A_1 \circ A_2$? How can you fix it? (Hint: It works fine if M_1 and M_2 are deciders.)

   On input w,
   1. For each way of splitting w into two parts w_1, w_2: Do steps 2–4.
      2. Run M_1 on w_1.
      3. Run M_2 on w_2.
      4. If both 2 and 3 accept, then accept w.
   5. Reject w if no acceptance is made in step 4 for any split.

2. [Connor, Levi, Andrew] How many different ways can you divide the string `abcd` into nonempty substrings $w_1, w_2, \ldots, w_k$, for $k \geq 1$, such that $w_1 w_2 \cdots w_k = w$? How about `abcde`? Can you describe an organized method for running through each possible division?

3. [Joshua, Allie, Ky] Let M_3 be a TM that recognizes a language B. Explain how to build a TM that recognizes B^*.

4. [David, Meghan, Ben, Grace] Suppose that M_4 is a TM that decides a language C. Describe a simple way to construct a TM that decides $\overline{C}$, the complement of C. Explain why this construction would not be so simple if M were just a recognizer, and not a decider.

BEGIN YOUR SOLUTIONS BELOW THIS LINE