Prework 4.2a: Uncountability and non-recognizable languages

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. Let $\mathscr{B}$ be the set of all infinite binary sequences. In class, we used a diagonalization argument to prove that $\mathscr{B}$ is uncountable. Let $\mathscr{B}_{\mathrm{F1}}$ be the set of all infinite binary sequences with a finite number of 1's.
   a. [Levi, David, Grace] Explain where the diagonalization argument fails when applied to $\mathscr{B}_{\mathrm{F1}}$.
   b. [Todd, Micah, Joshua] Prove that $\mathscr{B}_{\mathrm{F1}}$ is, in fact, countable.
2. [Ben, Allie, Connor] Let $A_{\mathrm{TM}} = \{\langle M, w\rangle \mid M \text{ is a TM that accepts } w\}$. Prove that A_{TM} is recognizable by giving a description of a TM that recognizes it. Is your TM a decider? Explain why or why not.
3. [Ky, Meghan, Andrew, Curtis] Find a countably infinite collection of countably infinite disjoint subsets of $\mathbb{N}$. That is, give a description of an infinite sequence of subsets $X_1, X_2, X_3, \ldots$ of natural numbers such that $X_i \cap X_j = \emptyset$ whenever $i \neq j$ and such that each X_i is countably infinite.

BEGIN YOUR SOLUTIONS BELOW THIS LINE