Prework 5.3a: Mapping Reducibility

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Allie, Micah, Levi] Let $A = \{\mathtt{a}^n\mathtt{b}^n \mid n \geq 1\}$ and $B = \{a, b\}$.
   a. Show tht $A \leq_{\mathrm{m}} B$ by defining a computable function $f : \Sigma^* \longrightarrow \Sigma^*$ such that $w \in A$ exactly when $f(w) \in B$.
   b. Similarly, show that $B \leq_{\mathrm{m}} A$.
   c. Parts (a) and (b) show that $\leq_{\mathrm{m}}$ is not *antisymmetric*, so it is not a partial order. Is $\leq_{\mathrm{m}}$ *transitive*? (See p. 9.)
2. [Todd, Ben, Grace] Recall that a language is *closed under reversal* if, for every w in the language, its reverse w^R is also in the language. Let $CUR_{\mathrm{TM}} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) \text{ is closed under reversal}\}$. Show that CUR_{TM} is undecidable by showing that $A_{\mathrm{TM}} \leq_{\mathrm{m}} CUR_{\mathrm{TM}}$. (Hint: Map the string $\langle M, w\rangle$ to the string $\langle M_w\rangle$, where M_w is a TM that recognizes a simple closed-under-reversal language exactly when M accepts w.)
3. [Connor, Andrew, Ky] In Prework 4.2b we showed that CUR_{DFA} is decidable by using a decider for EQ_{DFA}. Following this argument, give an alternative proof that EQ_{TM} is undecidable by showing that CUR_{TM} reduces to EQ_{TM}.
4. [Joshua, Meghan, David, Curtis] Let $\Sigma = \{\mathtt{0}, \mathtt{1}\}$. Show that a language A is decidable if and only if $A \leq_{\mathrm{m}} \{\mathtt{0}^*\mathtt{1}^*\}$.

BEGIN YOUR SOLUTIONS BELOW THIS LINE