Prework 5.3b: Mapping reducibility and recognizability proofs

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Joshua, Ky] Show that $\overline{E_{\text{TM}}}$ is recognizable by describing a TM that accepts $\langle M \rangle$ whenever $L(M) \neq \emptyset$. What does your TM do if $L(M) = \emptyset$?
2. [Curtis, David, Micah] Show that a language A is recognizable if and only if $A \leq_{\text{m}} A_{\text{TM}}$.
3. [Levi, Andrew, Todd] Let $J = \{w \mid w = 0x \text{ for some } x \in A_{\text{TM}}, \text{ or } w = 1y \text{ for some } y \in \overline{A_{\text{TM}}}\}$.
   a. Show that J is not recognizable by showing that $\overline{A_{\text{TM}}} \leq_{\text{m}} J$.
   b. Show that $\overline{J}$ is not recognizable by showing that $A_{\text{TM}} \leq_{\text{m}} J$.
4. [Ben, Connor, Meghan] Let J be as given in #3. Show that $J \leq_{\text{m}} \overline{J}$.
5. [Allie, Grace] Let $X = \{\langle M, w \rangle \mid M \text{ is a single-tape TM that never writes over its input when computing on } w\}$. Suppose (to the contrary) that D is a TM that decides X. Describe how to use D to build a TM E that decides A_{TM}. What does this prove about X?

BEGIN YOUR SOLUTIONS BELOW THIS LINE