Prework 7.1b: Polynomial and Exponential Time

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Joshua, Ben, Andrew] Which is bigger (asymptotically), $n!$ or n^n? How about $\log(n!)$ or $\log(n^n)$?

2. [Andrew, Todd, Allie] Refresh your memory about *graphs* by reading pages 10–13, if necessary. By default, when we say "graph," we mean an undirected graph. A *complete graph* is a graph where every two vertices have an edge between them. A *Hamilton path* is a path in a graph that visits every vertex. How many Hamilton paths are there in a complete graph on n vertices?

3. [David, Ky, Levi] Give an asymptotic upper bound for the number of edges in a graph with n vertices. If the n vertices are represented by binary numbers, and the edges are represented by pairs of binary numbers (representing pairs of vertices), give an asymptotic upper bound for the length of a binary representation of a graph with n vertices.

4. [Micah, Curtis, Grace] The *greatest common divisor* of two natural numbers j and k is the largest natural number g such that j and k are both multiples of g. We write $g = \gcd(j, k)$. Give an asymptotic estimate in terms of n of the running time of the following TM that computes $\gcd(j, k)$, where n is the number of digits in both j and k combined. On input $\langle j, k \rangle$,

   1: For every number i from 1 up to the largest of j and k: Check to see if i divides both j and k. If it does save it by writing it at the front of the tape.

   2: Erase the tape except for the last number saved. Accept.

5. [Connor, Meghan] Two natural numbers are *relatively prime* if their greatest common divisor is 1. Let *RELPRIME* = $\{\langle x, y \rangle \mid \gcd(x, y) = 1\}$. Explain how you could modify the TM in #4 to decide the language *RELPRIME*.

———————————————————BEGIN YOUR SOLUTIONS BELOW THIS LINE———————————————————