Prework 4.1b: Countable Sets

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Curtis, Grace, Joshua] Determine whether each of the following functions is one-to-one, and explain why/why not.
   a. $f : \mathbb{R} \longrightarrow \mathbb{N}$ given by $f(x) = \lfloor x \rfloor$ (the floor function)
   b. $f : \mathbb{Z} \longrightarrow \mathbb{N}$ given by $f(x) = x^2 + 1$.
   c. $f : \mathbb{N} \longrightarrow \mathbb{N}$ given by $f(x) = x^2 + 1$.
2. [Ky, Micah, Levi] Prove that the integers $\mathbb{Z}$ are countable.
3. [Connor, Todd, Meghan] Recall the argument that shows that $\mathbb{Q}$ is countable. Generalize the argument to show that $X \times Y$ is countable, for any countable X and Y. Conclude that $X_1 \times X_2 \times \cdots \times X_n$ is countable for a finite collection of countable sets X_i. (Recall that $X \times Y$ is the set of all ordered pairs (x, y) with $x \in X$ and $y \in Y$.)
4. [Allie, Ben, Andrew, David] Is the set of all polynomials $a_0 + a_1 x + \cdots + a_n x^n$ with integer coefficients countable? Justify your answer.

BEGIN YOUR SOLUTIONS BELOW THIS LINE