Prework 2.2b: Building a PDA for a CFG

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Ben, Grace, Micah] Use the procedure in the proof of Lemma 2.21 to build a state diagram for a PDA that recognizes the language described by the following CFG. The start variable is E and the terminals are $\Sigma = \{\mathtt{a}, +, *,), (\}$.

$$E \to E + T \mid T$$
$$T \to T * F \mid F$$
$$F \to (E) \mid \mathtt{a}$$

2. [Curtis, Ky, Todd] Use the procedure in the proof of Lemma 2.21 to build state diagram for a PDA that recognizes the language described by the following CFG. The start variable is R.

$$R \to XRX \mid S$$
$$S \to \mathtt{a}T\mathtt{b} \mid \mathtt{b}T\mathtt{a}$$
$$T \to XTX \mid X \mid \varepsilon$$
$$X \to \mathtt{a} \mid \mathtt{b}$$

3. [David, Meghan, Joshua] In this problem you will show that the set of context-free languages is closed under the regular operations. Let $G_1 = (V_1, \Sigma, R_1, S_1)$ and $G_2 = (V_2, \Sigma, R_2, S_2)$ be CFG's that describe the languages A_1 and A_2, respectively, where $V_1 \cap V_2 = \emptyset$. (Hint: For each part (a)-(c), add a new start state S and a new first rule.)
   a. Give a CFG that describes $A_1 \cup A_2$.
   b. Give a CFG that describes $A_1 \circ A_2$.
   c. Give a CFG that describes A_1^*.

4. [Andrew, Connor, Allie, Levi] Consider the following grammar. The start state is S.

$$S \to \varepsilon \mid 0S1 \mid SAS \mid 111$$
$$A \to 1A0 \mid 0 \mid 010 \mid ASA$$

   Recall that the *height* of a tree is the length of the longest path from the root to a leaf. What is the length of the largest string that a parse tree of height 5 can derive?

BEGIN YOUR SOLUTIONS BELOW THIS LINE