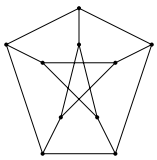
Prework 7.4a: Vertex Covers

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Ky, Grace] A *vertex cover* of an undirected graph G is a subset of vertices of G such that every edge in G touches some vertex in the cover. What is the smallest vertex cover of the Petersen graph?



2. [Curtis, Andrew] The *complement* $\overline{G}$ of a graph G is the graph with the same vertex set, but with edges between pairs of vertices that do *not* have edges in G. How many edges are in the complement of the Petersen graph?

3. [Allie, Ben, Joshua] Let G be a graph with n vertices. Prove that if G has a clique with k vertices, then $\overline{G}$ has a vertex cover with $n-k$ vertices. (Hint: First, try some examples with small values of n, e.g., $n=5$.)

4. [Meghan, Connor, Levi] Prove the converse of the statement in #3: If $\overline{G}$ has a vertex cover of size $n-k$, then G has a clique of size k.

5. [Micah, Todd, David] Let $\textit{VERTEX-COVER} = \{\langle G,k\rangle \mid G \text{ has a vertex cover of size } k\}$. Recall that $\textit{CLIQUE} = \{\langle G,k\rangle \mid G \text{ has a clique with } k \text{ vertices}\}$. Prove that $\textit{CLIQUE} \leq_{\text{P}} \textit{VERTEX-COVER}$ by describing a polynomial-time reduction function, and also explain why *VERTEX-COVER* is NP-complete.

BEGIN YOUR SOLUTIONS BELOW THIS LINE