Prework 5.1a: Reduction Arguments

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Ben, Ky, Todd, Connor] Let $CFL_{\text{TM}} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is context free}\}$. Show that CFL_{TM} is undecidable by showing that A_{TM} reduces to CFL_{TM}. (Hint: Mimic the reduction argument for $REGULAR_{\text{TM}}$.)

2. [David, Meghan, Andrew, Allie, Micah] During the course of a computation, we say that a TM *erases* if it ever writes a blank symbol on its tape. Let $ERASE_{\text{TM}} = \{\langle M, w \rangle \mid M \text{ erases when run on input } w\}$. Show that $ERASE_{\text{TM}}$ is undecidable by showing that A_{TM} reduces to $ERASE_{\text{TM}}$. (Hint: Look at Problem 5.10 and its solution in the text.)

3. [Curtis, Joshua, Grace, Levi] Recall that in our definition of a TM, we specified that the tape head does not move if, during the course of a computation, it is instructed to move left when it is already at the beginning of the tape. Call such an event a *thud*. Let $THUD_{\text{TM}} = \{\langle M, w \rangle \mid M \text{ thuds when run on input } w\}$. Show that $THUD_{\text{TM}}$ is undecidable by showing that A_{TM} reduces to $THUD_{\text{TM}}$.

BEGIN YOUR SOLUTIONS BELOW THIS LINE