Prework 4.2b: More decidability and recognizability problems

Write your preliminary solutions to each problem and submit a PDF on Canvas. The names in brackets indicate the subset responsible for presenting the problem.

1. [Allie, Curtis, Connor] Suppose that A and its complement $\overline{A}$ are both recognizable. Prove that A is decidable.

2. [Levi, Grace, Todd] Sometimes pushdown automata have states that are never entered on any input. Construct a TM decider that will input a PDA description $\langle P \rangle$ and accept if P has such a state and reject if P doesn't have such a state. (Hint: We know that E_{CFG} is decidable, so there is a TM decider $M_\emptyset$ that decides $E_{\mathrm{PDA}} = \{\langle P \rangle \mid P \text{ is a PDA and } L(P) = \emptyset\}$. Use $M_\emptyset$ in your construction.)

3. [Andrew, Joshua, Meghan] We say that a language is *closed under reversal* if, for every w in the language, its reverse w^R is also in the language. Construct a TM decider that inputs a DFA description $\langle D \rangle$ and accepts iff $L(D)$ is closed under reversal. (Hint: We have a decider for EQ_{DFA}.)

4. [Micah, Ben, David, Ky] Let $Q = \{\langle D \rangle \mid D \text{ is a DFA and } L(D) \text{ contains a string with } 01110 \text{ as a substring}\}$. Show that Q is decidable by building a TM decider for it. (Hint: We have a decider for E_{DFA}.)

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