1. Let A and B be events such that $P(A) = 7/10$, $P(B) = 3/10$, and $P(B|A) = 2/7$.

Find $P(A|B)$, $P(A \cap B)$, $P(A \cup B)$, $P(\tilde{A} \cap \tilde{B})$. Draw a Venn Diagram representing this situation, and label it.

(2) [redacted]

Suppose X is a random variable with density

$$f(x) = \begin{cases} C/x & x \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

a) Determine C.

b) Determine $E(X)$

c) Find the CDF $F(x)$.

d) Evaluate $\lim_{n \to \infty} x(1 - F(x$

[redacted]

(3) Suppose X is a random variable with density

$$f(x) = \begin{cases} c/x^2 & x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Determine C.

b) Compute $E(X)$.

c) Compute $V(X)$.

[redacted]

(4) Let X be a continuous random variable on $[-1,1]$, with $E(X)=0$ and $V(X)=1/4$.

Compute $$\int_{-1}^{1} x F(x)\, dx$$

where $F(x)$ is the CDF of X.

(5) Let $P = (X, Y)$ be a random (uniformly random) point in the unit disk $\{(x,y) \mid x^2 + y^2 \leq 1\}$

(1) Give a formula for the conditional PDF $f(x \mid Y = \frac{1}{2})$. Are X and Y independent?

(2) Let $R = X^2 + Y^2$. Compute $P(R \leq \frac{1}{9})$.

(3) Find a CDF and PDF for R.

Let X be the number of wireless access points (WAPs) on campus that fail in a given month. Assuming the WAPs are independent, X can be modeled by a oisson random variable. It is known that, on average, 2 WAPs fail each month.

a) Find $P(X \geq 4)$, using the Poisson distrib.

Suppose there are 50 WAPs on campus. Use the binomial distribution to calculate $P(X \geq 4)$.

c) Repeat (b) if $n = 200$.

(7) Let X be an exponential random variable with parameter λ.

Let $Y = \sqrt{X}$

a) Find the CDF $F_Y(y)$

b) Find the PDF $f_Y(y)$.

c) (Hard?) Find $E(Y)$

8. Let $X_1, X_2, \ldots, X_n$ be iid random variables with mean μ and variance σ^2. Let

$$T = 1 \cdot X_1 + 2 \cdot X_2 + 3X_3 + \cdots + nX_n$$

a) Compute $E(T)$ and $V(T)$.

b) Compute $E(T/n)$ and $V(T/n)$.

9 Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with $\mu = 20$ and $\sigma^2 = 16$. Use Chebyshev's inequality to determine how large n needs to be guarantee that

$$P(|\bar{X} - 20| \leq 1) \geq 0.95$$

(10) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with $\mu = 20$ and $\sigma^2 = 16$ Apply the C.L.T. (i.e., normal approximation) to determine n so that

$$P(|\bar{X} - 20| \leq 1) \geq 0.95$$

GRAPHBOARD TABLET