Simulating Starlings Murmuration

Deepanshu Jindal
Arpan Mangal

May 1, 2018

Contents

Preface

This documents details the mathematical modelling and functional design for an application for simulating the flocking behaviour shown by Starlings: often known as Starlings murmuration.

Figure 1: Starlings murmuration

Starlings murmuration is a type of swarm behaviour. From an abstract point of view, swarm behaviour is the collective motion of a large number of self-propelled entities. From the perspective of the mathematical modelling, it is an emergent behaviour arising from simple rules that are followed by individuals and does not involve any central coordination.

To model this we would use the boids program created by Craig Reynolds as an inspiration.

1

Mathematical Model

1.1 Intuition behind the model

From the general observation of swarm behaviour we can identify some basic rules which we can assume that each member of the swarm is following:

1. Alignment : Move in the same direction as your neighbours
2. Cohesion : Remain close to your neighbours
3. Separation : Avoid collisions with your neighbours

Apart from these basic rules, we can add in several assumptions such as they try to stay above a sleeping site, and when they happen to move outwards from the sleeping site, they return to it by turning.

1.2 Extension to the rules

We bring in obstacle avoidance and wall avoidance properties so as to confine the boids to a certain section of the space.
We add in random noise to the motion of each bird to bring in the effect of constant stirring motion.

1.3 Assumptions

1. The birds fly with a fixed speed, lets call it v_{max}.

2. The birds can change the direction vector of velocity by a fixed maximum amount say Δv_{max}.

3. A bird's motion is influenced by its n_c closest neighbours only.

1.4 Quantification of the rules

We mathematically describe the computation for a particular bird. Let there be n birds in the flock.
Let the velocities and position vectors of birds be represented by $\vec{v_1}, \vec{v_2}, \vec{v_3}...\vec{v_n}$ and $\vec{r_1}, \vec{r_2}, \vec{r_3}...\vec{r_n}$ respectively.

We define some predicates that would be useful for further computations
Let us define a predicate p as

$$p(i,k) = \begin{cases} 1 & \mid \vec{r_{ik}} \mid < \epsilon \\ 0 & otherwise \end{cases}$$

Let us define a predicate n as

$$n(i,k) = \begin{cases} 1 & bird\ i\ \epsilon\ the\ set\ of\ n_c\ closest\ neighbours\ of\ bird\ k \\ 0 & otherwise \end{cases}$$

Now, for k^{th} bird,
First, we initialise $\Delta\vec{v_k} = 0$.

Change in velocity due to rule of separation,

$$\Delta\vec{v_k} + = -\alpha\, p(i,k) \frac{\vec{r_{ik}}}{\mid \vec{r_{ik}} \mid^2} \tag{1.3}$$

where α is the hyper-parameter controlling separation.

Change in velocity due to rule of alignment,

$$a\vec{v_k} = \sum_{i=1}^{n} n(i,k)\vec{v_i} \tag{1.4}$$

$$\Delta\vec{v_k} + = \beta\, \frac{a\vec{v_{ik}}}{\mid a\vec{v_k} \mid} \tag{1.5}$$

where β is the hyper-parameter controlling alignment.

Change in velocity due to rule of cohesion,

$$a\vec{r}_k = \sum_{i=1}^{n} n(i,k)\vec{r_i} \tag{1.6}$$

$$\Delta\vec{v}_k + = \gamma \frac{a\vec{r}_k - \vec{r_k}}{\mid a\vec{r}_k - \vec{r_k} \mid} \tag{1.7}$$

where γ is the hyper-parameter controlling cohesion.

Introduction of noise

$$\Delta\vec{v}_k + = \delta \frac{random(\vec{a})}{\mid random(\vec{a}) \mid} \tag{1.8}$$

where δ is the hyper-parameter controlling noise.

Introduction of wall avoidance We model our confinement area as a sphere of radius R centred at origin

$$\Delta\vec{v}_k + = -\mu \frac{\vec{r_k}}{R - \mid \vec{r_k} \mid} \tag{1.9}$$

where μ is the hyper-parameter controlling noise.

Subsequently, we bring into account the fact that maximum change in velocity is limited.

$$scale = min(\Delta v_{max}, \mid \Delta\vec{v_k} \mid) \tag{1.10}$$

We do this for all the birds and then in one pass update the direction vector for velocity for each bird.

$$\vec{v_k} + = scale \frac{\Delta\vec{v_k}}{\mid \Delta\vec{v_k} \mid} \tag{1.11}$$

Finally, we update the position vectors for all the birds as

$$\vec{r_k} + = v_{max} \frac{\vec{v_k}}{\mid \vec{v_k} \mid} \tag{1.12}$$

1.5 Measurements

The main objective of our realistic simulation was to measure typical values like power used by each bird, angular momentum and force that each bird has to withstand in a flight ritual. We describe below the mathematics pertaining to some measurements:

Average Power and Energy

Consider a bird i. Given its mass m_i, its acceleration vector $\vec{a_i}$, and velocity vector $\vec{v_i}$, the power exerted, P by it is given by:

$$P_i = \vec{F_i} \cdot \vec{v_i} \tag{1.13}$$

or,

$$P_i = m_i.\vec{a_i} \cdot \vec{v_i} \tag{1.14}$$

The average power[1] exerted by a typical bird of the flock is given by:

$$P_{avg} = \frac{\sum_{i=1}^{n} P_i}{n} \tag{1.15}$$

To get the average energy spent by each bird, we will simply integrate power over the time of simulation, T.

$$E_{avg} = \int_0^T P_{avg}(t)dt \tag{1.16}$$

Assuming calculation of average power, P_{avg} at discreate intervals of time, Δt in our simulation, average energy E_{avg} is given by:

$$E_{avg} = \sum_{t=0}^{T/\Delta t} P_{avg}(t)\Delta t \tag{1.17}$$

Average Angular Momentum

Angular momentum of a typical bird i, having mass m_i, position vector r_i (w.r.t. origin), and velocity vector v_i is given by:

$$\vec{L_i} = m_i.\vec{r_i} \times \vec{v_i} \tag{1.18}$$

The average magnitude of angular momentum of a bird is given by:

$$|\vec{L_{avg}}| = \sum_{i=0}^{n} |\vec{L_i}|/n \tag{1.19}$$

[1]This is the power exerted over and above the power exerted in wing flapping to keep afloat

Average Force

The force[2] being withstood by a typical bird, i given its mass m_i and acceleration vector a_i is given by:

$$\vec{F_i} = m_i.\vec{a_i} \tag{1.20}$$

The average magnitude of force being experienced by a typical bird is given by:

$$|\vec{F_{avg}}| = \sum_{i=0}^{n} |\vec{F_i}|/n \tag{1.21}$$

[2]This force is the extra force exerted by bird apart from the force expended against gravity and drag

2

Functional Specification

The boids will be simulated in 3D, according to the mathematical model as described in the previous chapter. The simulation is based on the real flocking behaviour of Starlings. The user will be shown the various scientific results of the simulation.

2.1 Features

The simulation starts with display of an initial number of boids. These are simulated according to some default value of hyper-parameters. There is a spherical wall enclosing the simulation space, which acts like confinement area for the boids.

The user can interact with the simulation in multiple ways:

- Add Boid(s) into the simulation with a mouse click
- Tweak various hyper-parameters. These include:
  - Separation Factor
  - Cohesion Factor
  - Alignment Factor
  - Noise Factor
- Rotate the simulation view to be able to see the simulation from various angles.

2.2 Measurements

The user will be constantly shown the following results. He / she can tweak the hyper-parameters to see how the simulation will appear and see the corresponding changes in the result values.

- **Power**: The power being consumed by an average bird as well as the power consumed by the complete flock at the particular instant of time.
- **Acceleration**: The average magnitude of acceleration being produced by each bird.
- **Dispersion**: The dispersion of the flock from the centre of mass.
- **Angular Momentum**: The average magnitude of angular momentum of a typical bird.