# Notes on this SPAC implementation

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January 22, 2020

# 1 Introduction

# 2 Model description

#### 2.1 Plant Water Supply Equations

The basic Darcy flux is described by:

$$q = -\int_{\psi_{soil}}^{\psi_{leaf}} k(\psi) d\psi \tag{1}$$

And we adopt a simple linear conductance attenuation parameterization:

$$k(\psi) = \frac{\psi - p_2}{p_1 - p_2} \cdot k_{\text{max}} \tag{2}$$

Such that equation (1) can be rewritten as:

$$q = -k_{\text{max}} \cdot f_k \left( \Psi_L, \Psi_s \right) \cdot \left( \psi_L - \psi_s \right) \tag{3}$$

Where  $f_k$  is the fraction of maximal soil-to-leaf hydraulic conductance, and is a function of both  $\Psi_s$  and  $\Psi_L$ :

$$f_k = \frac{\frac{1}{2} (\psi_L + \psi_s) - p_2}{p_1 - p_2} \in [0, 1]$$
(4)

#### 2.2 Plant Water Demand Equations

Here we implement hydraulic limitations to transpiration. As  $\Psi_L$  becomes more negative, transpiration is reduced relative to some maximal value, using a simple linear attenuation function. This function uses  $p_3$  and  $p_4$  as parameters to define the onset of transpiration reduction  $p_3$  and the point at which there is no transpiration  $p_4$ .

$$T = T_{\text{max}} \cdot f_T \tag{5}$$

$$f_T = \frac{\psi_L - p_4}{p_3 - p_4} \in [0, 1] \tag{6}$$

Right now we have to produce some reasonable values for  $T_{\text{max}}$  and force the model with that data, directly. Instead we could implement a stomatal conductance model, if we would rather force with micro-met.

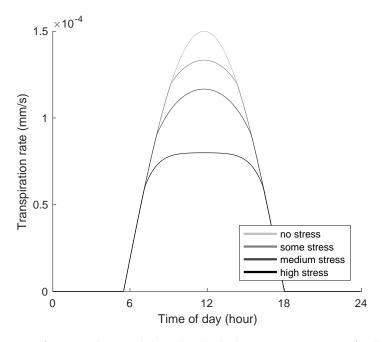


Figure 1: We are forcing the model with the lightest gray curve. And then based on  $\Psi_L$ , we attenuate the transpiration accordingly. In this case, I am varying  $\Psi_s$  to achieve different levels of stress.

### 2.3 Solution

We solve for leaf water potential and transpiration by requiring that:

$$T = q \tag{7}$$

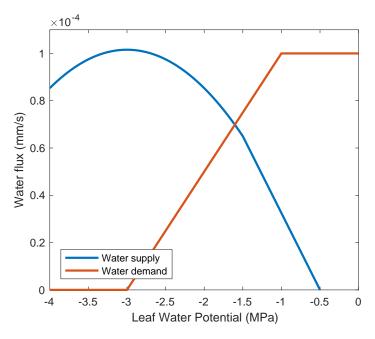


Figure 2: The solution for  $\Psi_L$  occurs where the two curves intersect.

Because transpiration decreases with decreasing  $\Psi_L$  and sap flux increases, we can usually find a satisfactory solution.

### 2.4 Bucket

Right now the bucket is the simplest it can be. Remove the transpiration each timestep. The bucket is sized according to the effective rooting depth  $Z_r$ .

$$\theta_1 = \theta_0 - \frac{q\Delta t}{Z_r} \tag{8}$$

$$\psi_{soil}\left(\theta\right) = \psi_{soil,sat}\left(\frac{\theta}{\theta_{sat}}\right)^{-b}$$
(9)

Do not currently have implementations of:

- $\bullet$  Rain
- $\bullet$  Runoff
- Drainage

## 3 Experiments

#### 3.1 Experiment 1

No drainage, no rain. Looking at a 30-day drydown. I'm forcing the model with a diurnal course  $T_{\rm max}$ , which with no stress produces approximately 4.3 mm/d of transpiration. All five runs use the same  $k_{\rm max}=6\text{e-}5$  mm/s/MPa;  $Z_r$  varies from 0.5 to 2.5m.

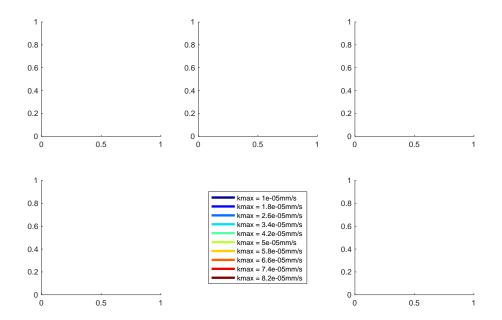


Figure 3: Varying  $Z_r$ .

#### 3.2 Experiment 2

No drainage, no rain. Looking at a 30-day drydown. I'm forcing the model with a diurnal course  $T_{\rm max}$ , which with no stress produces approximately 4.3 mm/d of transpiration. All five runs use the same  $Z_r=1$ m;  $k_{\rm max}$  varies from 4 to 8e-5.

- We should consider whether any other traits should be coordinated with  $k_{\text{max}}$  (e.g.  $p_{50}$ ).
- Here a drydown doesn't seem as appropriate...
- Maybe it's better to have some rain?

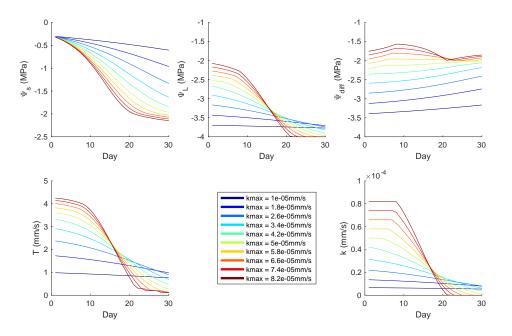


Figure 4: Varying  $k_{\text{max}}$ .

### References

J. S. Sperry, F. R. Adler, G. S. Campbell, and J. P. Comstock. Limitation of plant water use by rhizosphere and xylem conductance: results from a model. *Plant Cell Environment*, 21(4):347–359, 1998. ISSN 1365-3040. doi: 10.1046/j.1365-3040.1998.00287.x. URL http://dx.doi.org/10.1046/j.1365-3040.1998.00287.x.