

# Notes on this SPAC implementation

Daniel Kennedy - [djk2120@columbia.edu](mailto:djk2120@columbia.edu)

January 22, 2020

## **1 Introduction**

## 2 Model description

### 2.1 Plant Water Supply Equations

The basic Darcy flux is described by:

$$q = - \int_{\psi_{soil}}^{\psi_{leaf}} k(\psi) d\psi \quad (1)$$

And we adopt a simple linear conductance attenuation parameterization:

$$k(\psi) = \frac{\psi - p_2}{p_1 - p_2} \cdot k_{\max} \quad (2)$$

Such that equation (1) can be rewritten as:

$$q = -k_{\max} \cdot f_k(\Psi_L, \Psi_s) \cdot (\psi_L - \psi_s) \quad (3)$$

Where  $f_k$  is the fraction of maximal soil-to-leaf hydraulic conductance, and is a function of both  $\Psi_s$  and  $\Psi_L$ :

$$f_k = \frac{\frac{1}{2}(\psi_L + \psi_s) - p_2}{p_1 - p_2} \in [0, 1] \quad (4)$$

## 2.2 Plant Water Demand Equations

Here we implement hydraulic limitations to transpiration. As  $\Psi_L$  becomes more negative, transpiration is reduced relative to some maximal value, using a simple linear attenuation function. This function uses  $p_3$  and  $p_4$  as parameters to define the onset of transpiration reduction  $p_3$  and the point at which there is no transpiration  $p_4$ .

$$T = T_{\max} \cdot f_T \quad (5)$$

$$f_T = \frac{\psi_L - p_4}{p_3 - p_4} \in [0, 1] \quad (6)$$

Right now we have to produce some reasonable values for  $T_{\max}$  and force the model with that data, directly. Instead we could implement a stomatal conductance model, if we would rather force with micro-met.

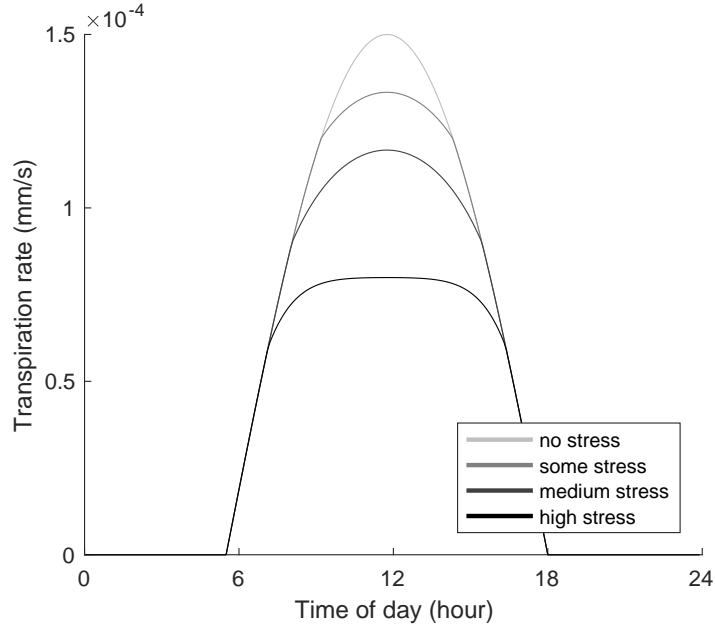


Figure 1: We are forcing the model with the lightest gray curve. And then based on  $\Psi_L$ , we attenuate the transpiration accordingly. In this case, I am varying  $\Psi_s$  to achieve different levels of stress.

## 2.3 Solution

We solve for leaf water potential and transpiration by requiring that:

$$T = q \quad (7)$$

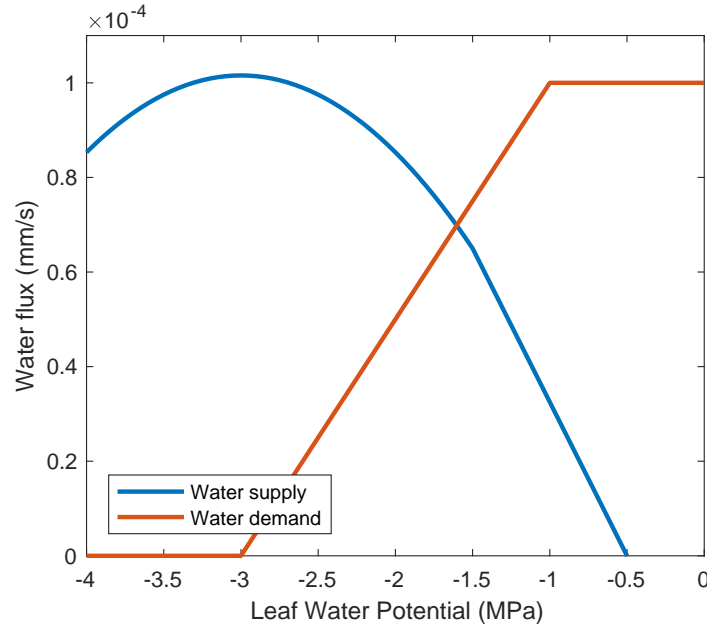


Figure 2: The solution for  $\Psi_L$  occurs where the two curves intersect.

Because transpiration decreases with decreasing  $\Psi_L$  and sap flux increases, we can usually find a satisfactory solution.

## 2.4 Bucket

Right now the bucket is the simplest it can be. Remove the transpiration each timestep. The bucket is sized according to the effective rooting depth  $Z_r$ .

$$\theta_1 = \theta_0 - \frac{q\Delta t}{Z_r} \quad (8)$$

$$\psi_{soil}(\theta) = \psi_{soil,sat} \left( \frac{\theta}{\theta_{sat}} \right)^{-b} \quad (9)$$

Do not currently have implementations of:

- Rain
- Runoff
- Drainage

### 3 Experiments

#### 3.1 Experiment 1

No drainage, no rain. Looking at a 30-day drydown. I'm forcing the model with a diurnal course  $T_{\max}$ , which with no stress produces approximately 4.3 mm/d of transpiration. All five runs use the same  $k_{\max} = 6\text{e-}5 \text{ mm/s/MPa}$ ;  $Z_r$  varies from 0.5 to 2.5m.

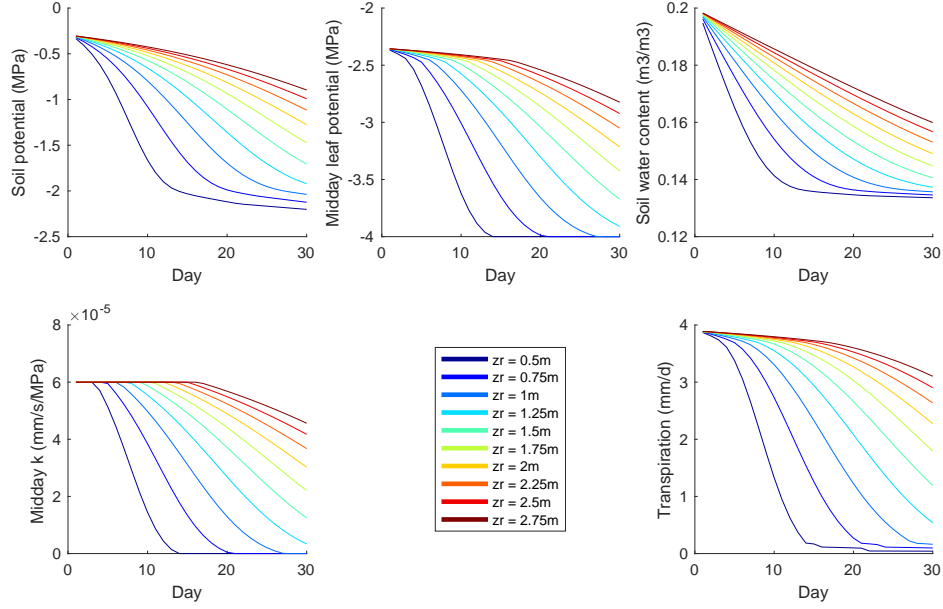


Figure 3: Varying  $Z_r$ .

## 3.2 Experiment 2

No drainage, no rain. Looking at a 30-day drydown. I'm forcing the model with a diurnal course  $T_{\max}$ , which with no stress produces approximately 4.3 mm/d of transpiration. All five runs use the same  $Z_r = 1\text{m}$ ;  $k_{\max}$  varies from 4 to  $8\text{e-}5$ .

- We should consider whether any other traits should be coordinated with  $k_{\max}$  (e.g.  $p_{50}$ ).
- Here a drydown doesn't seem as appropriate...
- Maybe it's better to have some rain?

### 3.3 Experiment 3

Factorial approach mixing the various  $Z_r$  and  $k_{\max}$ .

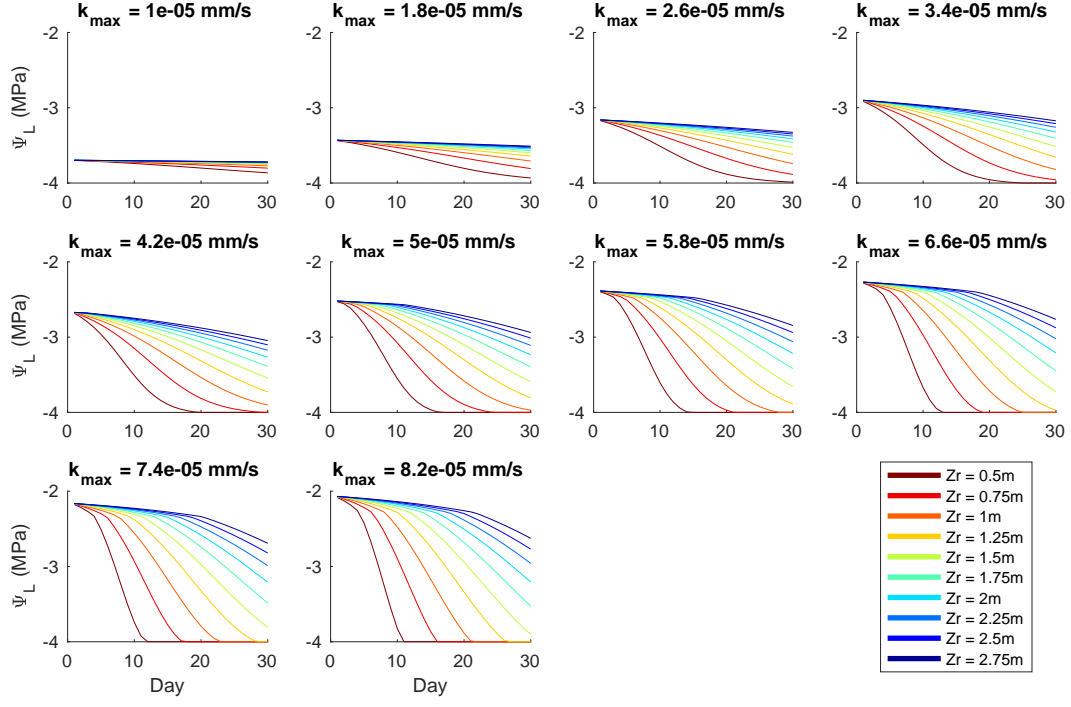


Figure 4: There are a couple of challenges here for retrieving the correct  $Z_r$



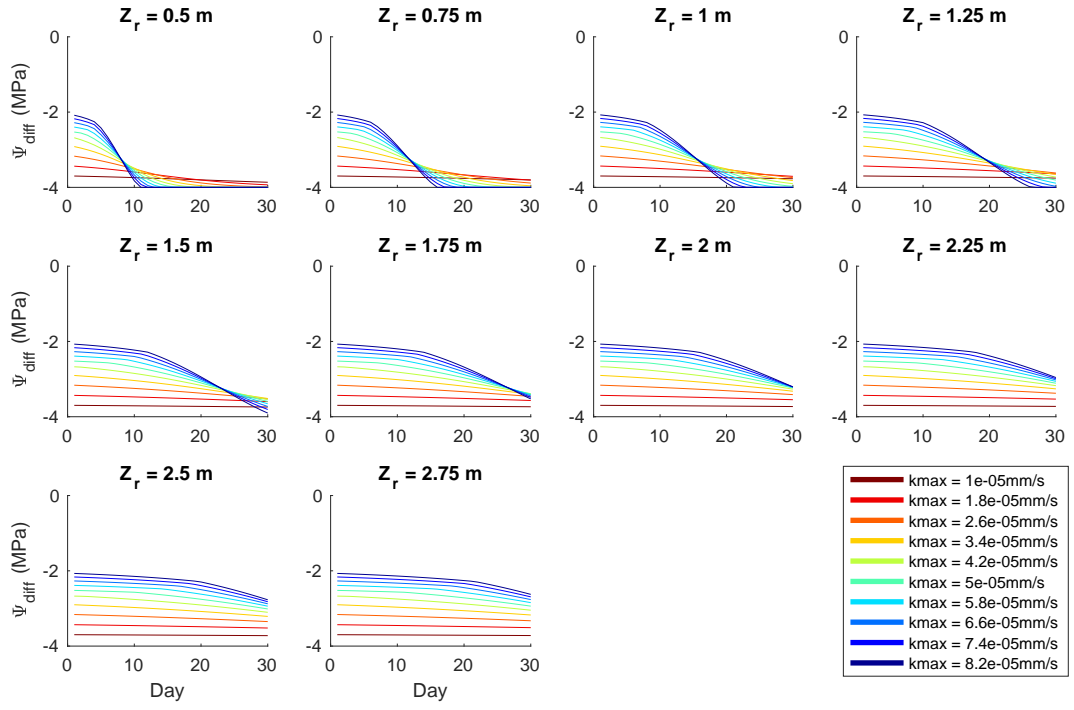


Figure 5: As  $Z_r$  gets smaller, our classification for  $k_{plant}$  becomes more challenging

## References

- J. S. Sperry, F. R. Adler, G. S. Campbell, and J. P. Comstock. Limitation of plant water use by rhizosphere and xylem conductance: results from a model. *Plant Cell Environment*, 21(4):347–359, 1998. ISSN 1365-3040. doi: 10.1046/j.1365-3040.1998.00287.x. URL <http://dx.doi.org/10.1046/j.1365-3040.1998.00287.x>.