Notes on this SPAC implementation

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1 Introduction

2 Model description

2.1 Plant Water Supply Equations

The basic Darcy flux is described by:

$$q = -\int_{\psi_{soil}}^{\psi_{leaf}} k(\psi)\, d\psi \tag{1}$$

And we adopt a simple linear conductance attenuation parameterization:

$$k(\psi) = \frac{\psi - p_2}{p_1 - p_2} \cdot k_{\max} \tag{2}$$

Solve the integral and define f_k:

$$q = -k_{\max} \cdot f_k(\Psi_L, \Psi_s) \cdot (\psi_L - \psi_s) \tag{3}$$

$$f_k = \frac{\frac{1}{2}(\psi_L + \psi_s) - p_2}{p_1 - p_2} \in [0, 1] \tag{4}$$

2.2 Plant Water Demand Equations

$$T = T_{\max} \cdot f_T \tag{5}$$

$$f_T = \frac{\psi_L - p_4}{p_3 - p_4} \in [0, 1] \tag{6}$$

2.3 Solution

We solve for leaf water potential and transpiration by requiring that:

$$T = q \tag{7}$$

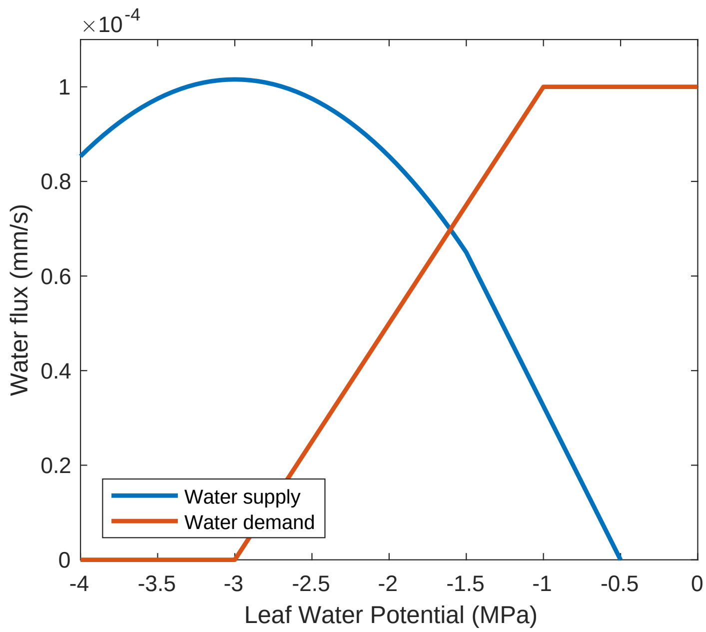

Figure 1: The solution for Ψ_L occurs where the two curves intersect.

Because transpiration decreases with decreasing Ψ_L and sap flux increases, we can usually find a satisfactory solution. There are two potential issues:

1. The curves intersect twice.
   - This is possible due to the parabolic shape of the sap flux curve.
   - We want to choose the less negative value of Ψ_L as our solution.
   - The numerical implementation is designed to minimize the chances that we find the alternate solution.
2. The curves never intersect.
   - The sap flux curve can potentially peak and return to zero before there is enough transpiration attenuation to cause the lines to intersect.
   - This functionality does not comport well with our expectation of vegetation function, but could arise if we pick 'bad' parameter values.
   - In this case, the model will crash, and provide a message that the parameter values may be unrealistic.

2.4 Bucket

Right now the bucket is the simplest it can be. Remove the transpiration each timestep. The bucket is sized according to the effective rooting depth Z_r.

$$\theta_1 = \theta_0 - \frac{q \Delta t}{Z_r} \tag{8}$$

$$\psi_{soil}(\theta) = \psi_{soil,sat} \left(\frac{\theta}{\theta_{sat}} \right)^{-b} \tag{9}$$

Do not currently have implementations of:

- Rain
- Runoff
- Drainage

3 Experiments

3.1 Experiment 1

No drainage, no rain. Looking at a 30-day drydown. I'm forcing the model with a constant $T_{\max}$ of 1e-4 mm/s, which is approximately 4.3 mm/d.

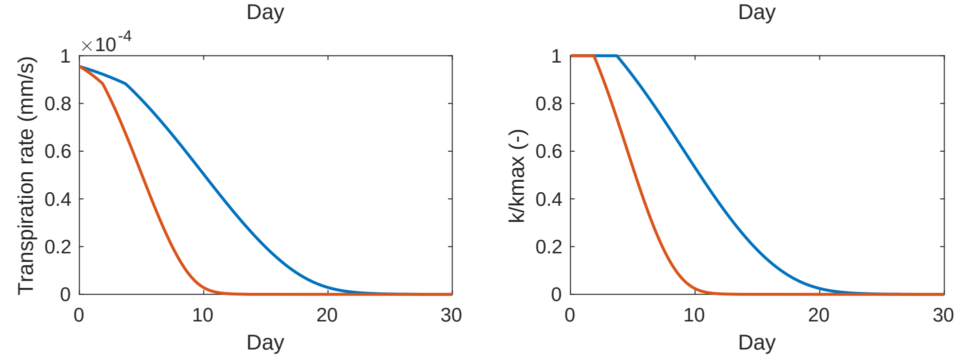

Figure 2: As expected, deeper rooting leads to a longer decay period.

References

J. S. Sperry, F. R. Adler, G. S. Campbell, and J. P. Comstock. Limitation of plant water use by rhizosphere and xylem conductance: results from a model. *Plant Cell Environment*, 21(4):347–359, 1998. ISSN 1365-3040. doi: 10.1046/j.1365-3040.1998.00287.x. URL `http://dx.doi.org/10.1046/j.1365-3040.1998.00287.x`.