Simple Plant Hydraulics
model development and implementation

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1 Model development

1.1 Recipe

1. Solve for maximum stomatal conductance based on the Medlyn model (involves iterating for intercellular CO_2).

2. Solve for the vegetation water potential (and associated stomatal conductance) that matches plant water supply with Penman-Monteith demand.

3. Calculate the photosynthesis based on the stomatal conductance solution (involves iterating for intercellular CO_2).

1.2 Plant Water Supply Equations

$$q = \int_{\psi_{soil}}^{\psi_{leaf}} \frac{K(\psi)}{z} d\psi \tag{1}$$

$$K(\psi) = \frac{\psi - p_2}{p_1 - p_2} \cdot K_{max} \tag{2}$$

$$q = \frac{K_{max}}{z} \cdot f(\psi) \cdot (\psi_{soil} - \psi_{leaf} - \rho g z) \tag{3}$$

$$\begin{aligned} f(\psi) &= \frac{\frac{1}{2}(\psi_{soil} + \psi_{leaf}) - p_2}{p_1 - p_2} \\ 0 &\le f \le 1 \end{aligned} \tag{4}$$

1.3 Plant Water Demand Equations

$$g_{c,max} = g_0 + \left(1 + \frac{g_1}{\sqrt{D}}\right) \frac{A}{C_a} \tag{5}$$

$$C_i = C_a - \frac{A}{g_c} \tag{6}$$

$$A = \frac{j/4\left(C_i - \Gamma\right)}{C_i + 2\Gamma} \tag{7}$$

$$g_c = g_{c,max} \cdot h\left(\psi_{leaf}\right) \tag{8}$$

$$h\left(\psi_{leaf}\right) = \frac{1}{1 + \left(\frac{\psi_{leaf}}{p_{50}}\right)^a} \tag{9}$$

$$g_w = 1.6 g_c \tag{10}$$

$$E = \frac{\frac{\Delta}{\gamma}\left(R_{net} - G\right) + \rho L_v g_a dq}{L_v\left(1 + \frac{\Delta}{\gamma} + \frac{g_a}{g_w}\right)} \tag{11}$$

$$E = q \tag{12}$$

1.4 Bucket

$$\theta_1 = \theta_0 - \frac{q\Delta t}{z_r} \tag{13}$$

2 Why are tall Amazonian forests more resistant to precipitation variability?

2.1 Thread 1

$$q = k\Delta\psi \tag{14}$$

$$\Delta\psi = \psi_{soil} - \psi_{leaf} \tag{15}$$

Assume $\dfrac{dk}{d\psi} = 0$

$$q = k\left(\psi_{soil} - \psi_{leaf}\right) \tag{16}$$

$$\frac{dq}{d\psi_{soil}} = k\left(1 - \frac{d\psi_{leaf}}{d\psi_{soil}}\right) \tag{17}$$

$$\sigma = \frac{d\psi_{leaf}}{d\psi_{soil}} \tag{18}$$

$$\frac{dq}{d\psi_{soil}} = k\left(1 - \sigma\right) \tag{19}$$

Use initial pressure drop and water flux to infer k:

$$\begin{aligned} k &= \frac{q}{\Delta\psi} \\ k &= \frac{q_0}{\psi_{soil,0} - \psi_{leaf,0}} \end{aligned} \tag{20}$$

Then we have another way to express $\dfrac{dq}{d\psi}$

$$\frac{dq}{d\psi_{soil}} = q_0 \frac{(1-\sigma)}{\psi_{soil,0} - \psi_{leaf,0}} \tag{21}$$

2.2 Thread 2

Given the same precipitation sequence, soil potential variability is smaller for taller trees due to deeper roots,