MIT ELECTRICAIRCRAFT INITIATIVE

May 30th, 2017

Euler Equation Modeling of Ion Thruster Pressure Change

Dongjoon Lee

Project Goals

- Analyze the pressure, density, and velocity change across an ion thruster from emitter to collector
- Simulate the Euler equations to model compressible flow with electrostatic body forces
- Compare results with incompressible analytical solutions

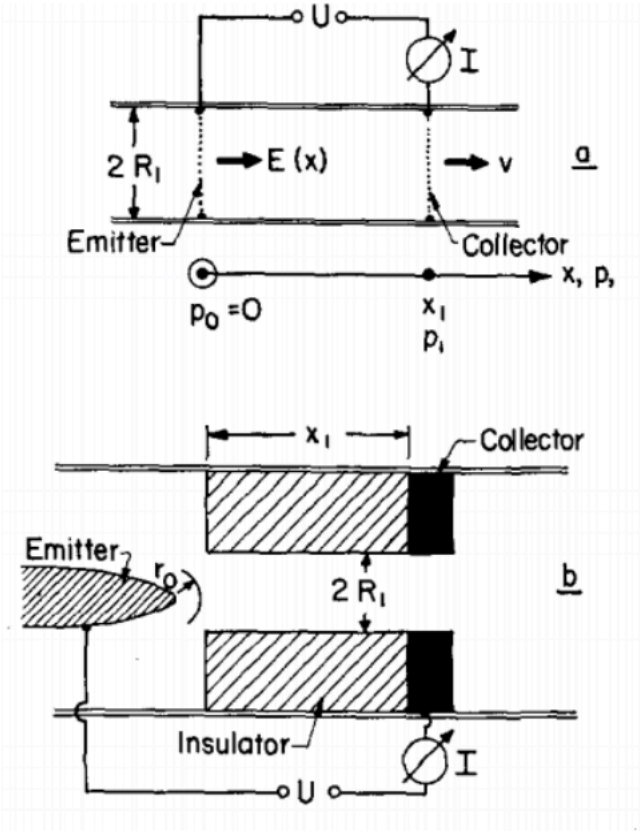


Figure from ”Ion Drag Pumps” Stuetzer (1969)

Assumptions

- Electrostatic forces and quantities don't change from the incompressible solution
  - Small change in drift velocity
  - Results in analytical solutions for current density and electric field
- Steady state flow

Conservation of Mass

- $\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0$

Conservation of Momentum

- $\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} = -\frac{\partial p}{\partial x} + \rho_c E$

Conservation of Energy

- $\frac{\partial(\rho e)}{\partial t} + \frac{\partial(\rho u e)}{\partial x} = -\frac{\partial(p u)}{\partial x} - \frac{\partial Q}{\partial x} + \rho_c E u$

Equation of State

- $P = \rho R T$

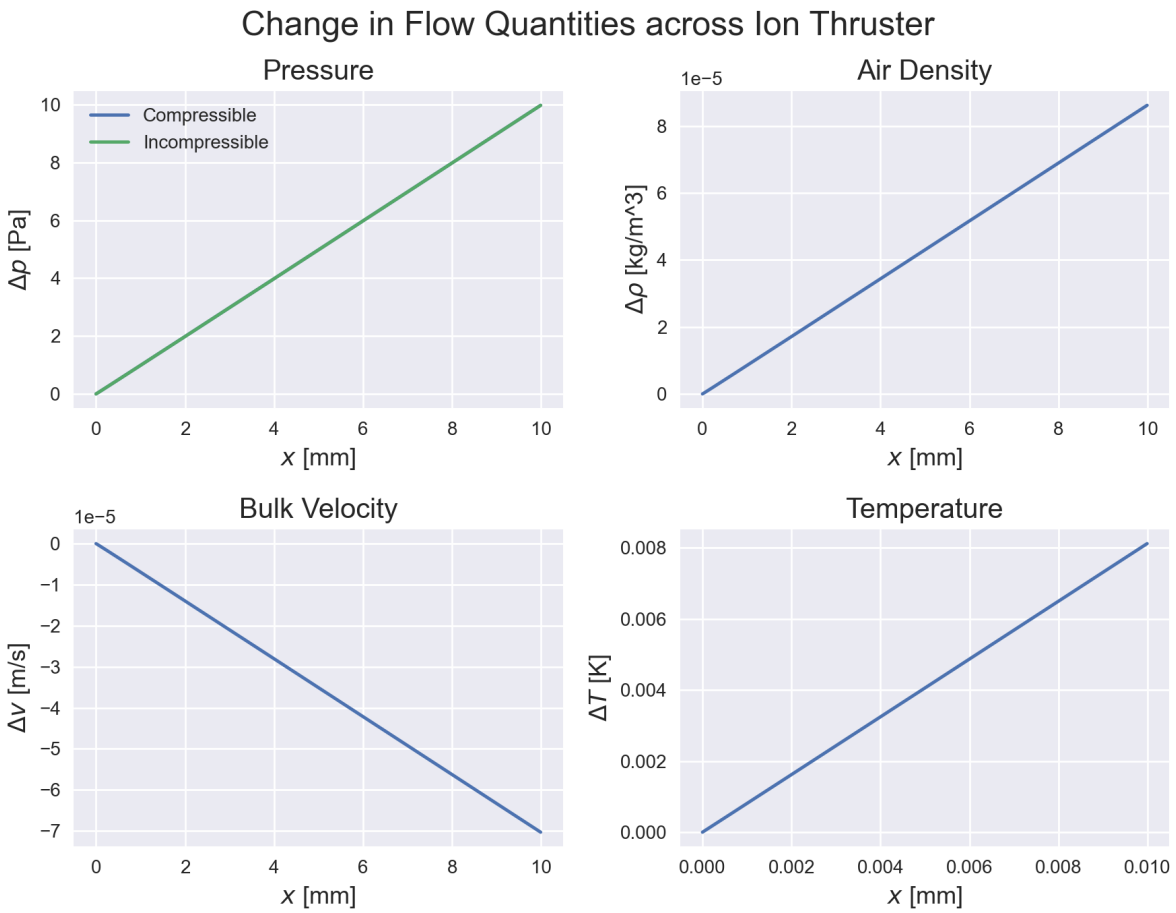
Energy

- $e = c_V T + \frac{1}{2} u^2$

Heat

- $Q = -\kappa(\frac{\partial T}{\partial x})$

Results

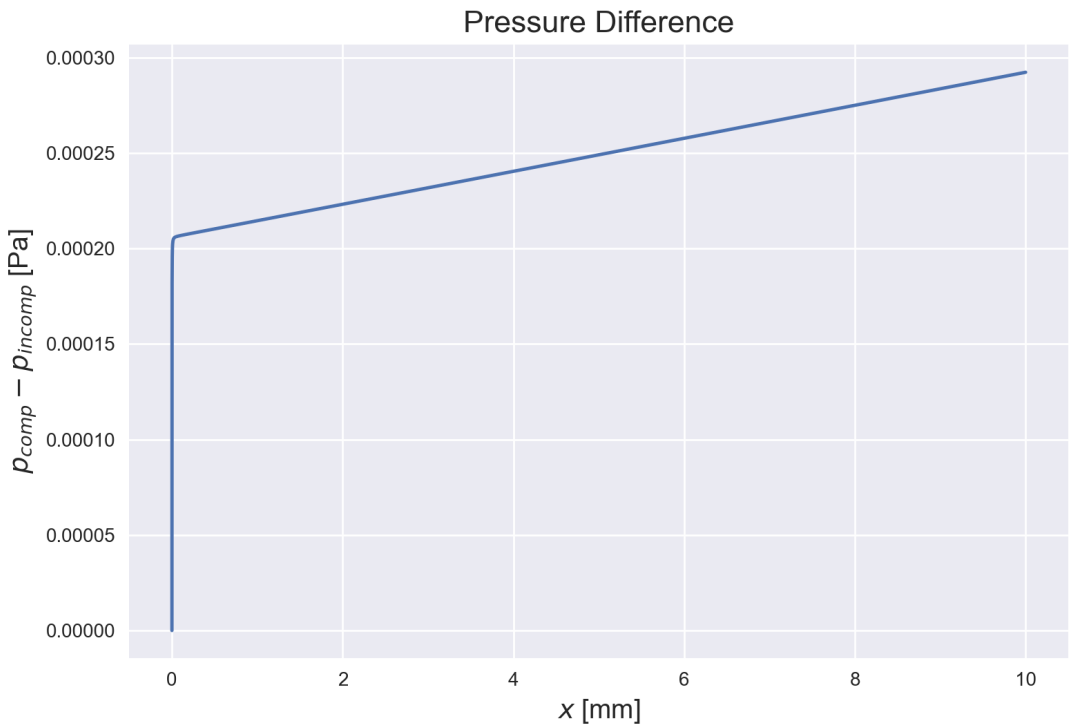


Boundary Parameter	Value @ x=0
Pressure	101,325 Pa
Air Density	1.225 kg / m^3
Bulk Velocity	1 m/s
Electric Field	0 V/m
Thruster Parameter	**Value**
Potential Difference	10,000 V
Gap Spacing	1 cm

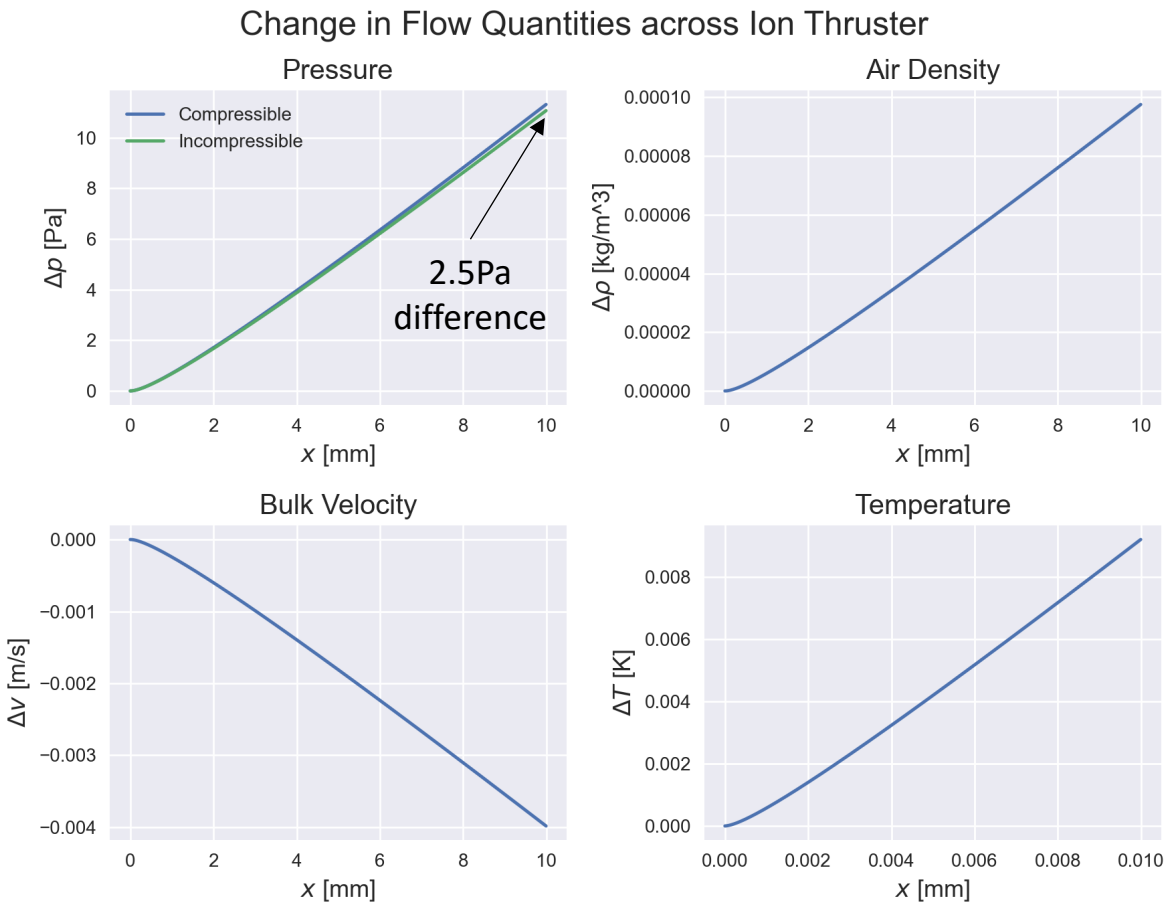
Incompressible assumption reasonable

Comparison to Incompressible Solution

- Negligible differences in all parameters even at extreme thruster values
- Spike at x=0 due to numerical resolution



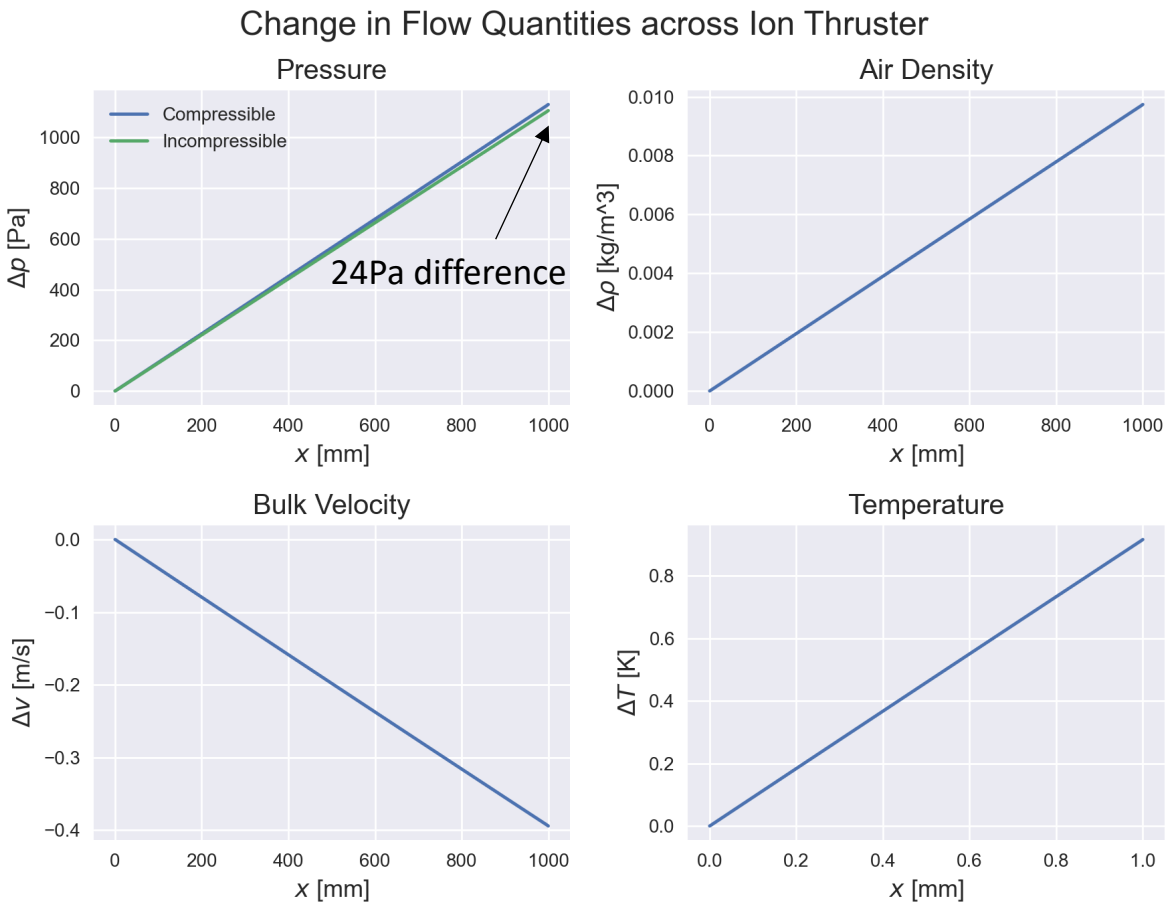
Results – High Speed



Boundary Parameter	Value @ x=0
Pressure	101,325 Pa
Air Density	1.225 kg / m^3
Bulk Velocity	**50 m/s**
Electric Field	0 V/m
Thruster Parameter	**Value**
Potential Difference	10,000 V
Gap Spacing	1 cm

Incompressible solution still reasonable

Results – Multi Stage



Boundary Parameter	Value @ x=0
Pressure	101,325 Pa
Air Density	1.225 kg / m^3
Bulk Velocity	50 m/s
Electric Field	0 V/m
Thruster Parameter	**Value**
Potential Difference	10,000 V
Gap Spacing	1 cm
Number of Stages	**100**

Future Work

- Investigate multi-stage thruster
  - Accumulation of error between compressible and incompressible case

MIT ELECTRICAIRCRAFT INITIATIVE

Dongjoon Lee
dongjoon@mit.edu

ELECTRICAIRCRAFT.MIT.EDU