Daniel Lesh
09/25/2020

HW #1 - ECON 2317Q

1.

(a) $Y = 6x + 10$

$E(x) = 6(6) + 10$ $\quad$ $Var(x) = 6^2(16)$

$= 36 + 10$ $\quad$ $= 36(16)$

$\boxed{E(x) = 46}$ $\quad$ $\boxed{Var(x) = 576}$

(b) $Y = 0.5x - 8$

$E(x) = 0.5(6) - 8$ $\quad$ $var(x) = .5^2(16)$

$= 3 - 8$ $\quad$ $= .25(16)$

$\boxed{E(x) = -5}$ $\quad$ $\boxed{Var(x) = 4}$

(c) $y = x/10$

$E(x) = 6/10$ $\quad$ $var(x) = 1^2\left(\frac{16}{10}\right)$

$\boxed{E(x) = .6}$ $\quad$ $\boxed{var(x) = .6}$

2. $\mu_x = 4{,}000$, $\sigma_x = 3{,}500$

(a) $Pr(x \geq 6000) \rightarrow$ standardize x

$$z = \frac{x - \mu_x}{\sigma_x} = \frac{6000 - 4000}{3500} = \frac{2000}{3500} = .571$$

$z = .571 \approx \underline{.7157}$

$Pr(x \geq 6000) = Pr(z \geq .571) = Pr(z \leq .7157) = 1 - .7157 = \boxed{.2843}$

- There is a 28.43% probability observing a value of $x \geq 6000$.

(b) $Pr(3{,}000 \leq x \leq 5{,}500) \rightarrow$ standardize x

$$z = \frac{x - \mu_x}{\sigma_x} = \frac{3{,}000 - 4{,}000}{3500} = \frac{-1000}{3500} = -.2857 \qquad z = -.2857 \approx .3897$$

$$z = \frac{x - \mu_x}{\sigma_x} = \frac{5500 - 4{,}000}{3500} = \frac{1500}{3500} = .4285 \qquad z = .4285 \approx .6628$$

$Pr(z \leq .4285) - Pr(z \leq -.2857) = .6628 - .3897 = .2731$

- There is a 27.31% probability observing a value between $3{,}000 \leq x \leq 5{,}500$.

(c) $P(Z > X) = 1 - .05 = .95$

$3500 \cdot .8289 = \frac{x - 4{,}000}{3{,}500} \cdot 3500 = 2{,}901.15 = x - 4000$
$+4{,}000$ $\quad$ $+4000$

$\boxed{6901.15 = x}$

3. $u_x = 35$, $\sigma = 10$

(a) $Pr(x \geq 50) \longrightarrow$ standardize x

$z = \frac{x - u_x}{\sigma} = \frac{50 - 35}{10} = \frac{15}{10} = 1.5 \quad z = 1.5 \approx .9332$

$Pr(x \geq 50) = Pr(z \geq 1.5) = Pr(z \leq .9332) = 1 - .9332 = \boxed{.0668}$

- There is a 6.68% chance that customers will wait in line 50 minutes or longer.

(b) $P(z = x) = 1 - .05 = .95$

$10 \cdot .8289 = \frac{x - 35}{10} \cdot 10 \quad 8.289 = x - 35$
$+35$ $\quad$ $+35$ $\quad$ $\boxed{x = 43.29}$

4. Random Sample = 453, $\bar{x} = 1013$, $s = 108$, 95% CI

(a) $CI = \bar{x} \pm t_{\frac{\alpha}{2}, (n-1)} \frac{s}{\sqrt{n}}$

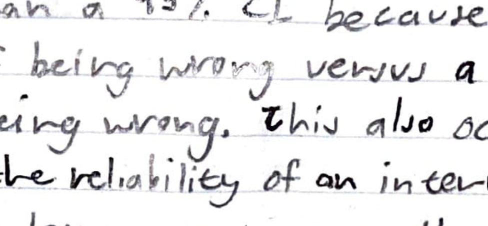
$CI = \bar{x} \pm t_{\frac{.05}{2}, 452} \frac{s}{\sqrt{n}}$

$CI = \bar{x} \pm 1.96 \left(\frac{s}{\sqrt{n}}\right)$

$= 1{,}013 \pm 1.96 \left(\frac{108}{\sqrt{453}}\right)$

$= 1{,}013 \pm 9.9456$

- so the CI is between 1,003.0545 to 1,022.9455



(b) a 90% CI would be narrower than a 95% CI because in a 90% CI you have a 10% chance of being wrong versus a 95% CI, you only have a 5% chance of being wrong. This also occurs b/c as the precision of the CI increases, the reliability of an interval containg the actual mean decreases. There is less range to cover the mean.

(c) Without doing any calculations, the 95% CI of a sample of 1,000 students would be narrower than a sample of 453 students b/c the part you subtract from the sample mean would be smaller w/ n = 1,000 vs. n = 453.

5. $n = 503, \bar{x} = 1019, s = 95$

(a) construct 95% CI:

$CI = \bar{x} \pm t_{\frac{\alpha}{2}, (n-1)} \frac{s}{\sqrt{n}}$

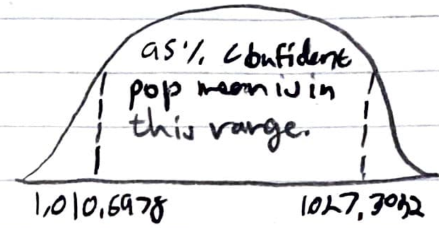
$CI = \bar{x} \pm t_{\frac{.05}{2}, (502)} \frac{s}{\sqrt{n}}$

$CI = \bar{x} \pm 1.96 \left(\frac{s}{\sqrt{n}}\right)$

$= 1{,}019 \pm 1.96 \left(\frac{95}{\sqrt{503}}\right)$

$= 1{,}019 \pm 8.3022$

- so the CI is between 1,010.6978 to 1,027.3022.



(b) 2 types of schools: testing

more on back!

testing	avg. score ($\bar{x}$)	std. dev (s)	obs (n)
no prep-course	1,013	108	453
prep-course	1,019	95	503

step 1: $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

more on back →

6. sample: 79; 73; 68; 77; 86; 71; 69; n = 7; $\bar{x} = 74.7$; $s = 6.4$ (calculated in Excel)

a) construct a 95% CI:

$CI = \bar{x} \pm t_{\frac{\alpha}{2}, (n-1)} \frac{s}{\sqrt{n}}$

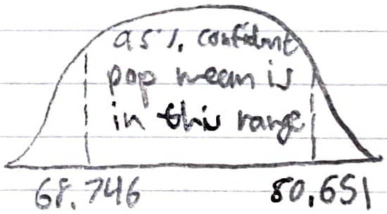
$= \bar{x} \pm t_{\frac{.05}{2}, (6)} \frac{s}{\sqrt{n}}$

$= x \pm 2.46 \left(\frac{s}{\sqrt{n}}\right)$

$= 74.7 \pm 2.46 \left(\frac{6.4}{\sqrt{7}}\right)$

$= 74.7 \pm 5.951$

- so the CI is between 68.746 and 80.651



b) 4 steps:

step 1: $H_0: \mu \leq 70$

$H_1: \mu > 70$

step 2: $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{74.7 - 70}{6.4/\sqrt{7}} = \frac{4.7}{6.4/\sqrt{7}} = 1.82$

step 3: significance level: $\alpha = .05$

step 4: Reject H_0 if $|t| > t_{\alpha, n-1}$

- So because $1.82 < 1.94$, we fail to reject the H_0. We do not have enough evidence to suggest that pop mean is significantly greater than 70.

5(b) cont. step 2: compute t-stat:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}}} = \frac{1013 - 1019}{\sqrt{\frac{108^2}{453} + \frac{95^2}{503}}} = -0.9077$$

step 3: significance level: $\alpha = 0.05$

Step 4: Reject H_o if $|t| > t_{a}\ n-1$

@ 5% significance level, $t_{.05, 502} = 1.96$

- so because $-.9077 < 1.96$, we fail to reject H_o that there is a significant difference between the average test scores of students that received the prep course and those that did not receive the prep course.