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Homework 6

Q1. Long-horizon forecasts are less likely to be as accurate as short-horizon forecasts because as t grows larger away from original forecasts point (i.e. y_{t-12}^{+}), the forecast becomes less accurate. This is due to a variety of factors like seasonal adjustments, better performing drugs entering the marketplace in the future, economic decline, etc. Even if the forecast is behaving optimally, short-horizon forecasts are always more likely to be accurate than short-horizon forecasts.

Q2. $y_t = \phi y_{t-1} + \varepsilon_t$

1. $\hat{y_t}(h) = E[y_{t+h} \mid y_t, y_{t-1}, \dots]$

$= E[\phi y_{t-1} + \varepsilon_{t+h} \mid y_t, y_{t-1}, \dots]$

$= \phi E[y_{t+h-1} \mid y_t, y_{t-1}, \dots] + E[\varepsilon_{t+h} \mid y_t, y_{t-1}, \dots]$

$= \phi \hat{y_t}(h-1) + 0$

$= \phi \hat{y_t}(h-1)$

$= \phi \cdot \phi \hat{y_t}(h-2) = \phi^2 \hat{y_t}(h-2)$

$= \phi^2 \cdot \phi \hat{y_t}(h-3) = \phi^3 \hat{y_t}(h-3)$

$= \phi^h \hat{y_t}(0)$

$= \phi^h y_t$, i.e. $y_{t+h,t} = \phi^h y_t$

2. Forecast error $= \varepsilon_{t+h} = (y_{t+h} - y_{t+h,t})$

$= (\phi y_{t+h-1} + \varepsilon_{t+h} - \phi^h y_t)$

$= \varepsilon_{t+h} + \phi[\phi y_{t+h-2} + \varepsilon_{t+h-1}] - \phi^h y_t = \varepsilon_{t+h} + \phi \varepsilon_{t+t-1} + \phi^2[y_{t+h-2}] - \phi^h y_{t-h-1}$

$= \varepsilon_{t+h} + \phi \varepsilon_{t-h-1} + \phi^2[\phi y_{t+h-3} + \varepsilon_{t+h-2}] - \phi^h y_t \dots = \varepsilon_{t+h} + \phi \varepsilon_{t+h-1} + \phi^2 \varepsilon_{t+h-2} + \dots + \phi \varepsilon_{t+1}$

Forecast Errors $= \varepsilon_{t+1,t} = y_{t+1} - y_{t+1,t} = \phi y_t + \varepsilon_{t+1} - \phi y_t = \varepsilon_{t+1}$

$\varepsilon_{t+2,t} = y_{t+2} - y_{t+2,t} = \phi y_{t+1} + \varepsilon_{t+2} - \phi^2 y_t$

$= \phi(y_{t+1} - \phi y_t) + \varepsilon_{t+2}$

$= \phi \varepsilon_{t+1} + \varepsilon_{t+2}$

$y_{t+1} = \phi y_t + \varepsilon_t$

$y_{t+1} - \phi y_t = \varepsilon_{t+1}$

$\varepsilon_{t+3,t} = y_{t+3} - y_{t+3,t} = \phi y_{t+2} + \varepsilon_{t+3} - \phi^3 y_t$
$= \varepsilon_{t+3} + \phi[\phi y_{t+1} + \varepsilon_{t+2}] - \phi^3 y_t$
$= \varepsilon_{t+3} + \phi \varepsilon_{t+2} + \phi^2[y_{t+1} - \phi y_t]$
$= \varepsilon_{t+3} + \phi \varepsilon_{t+2} + \phi^2 \varepsilon_{t+1}$

$y_{t+1} - \phi y_t = \varepsilon_{t+1}$
$\varepsilon_{t+h,t} = y_{t+h} - y_{t+h,t} = \varepsilon_{t+h}, \phi \varepsilon_{t+h-1} + \dots + \phi^{h-1} \varepsilon_{t+1}$

3. $\varepsilon_{t+1,t} = \varepsilon_{t+1}$
$\sigma_1^2 = var(\varepsilon_{t+1,t})$
$= \sigma^2$

ε_t var σ^2; ε_t; uncorrelated
$\sigma^2 = var(\varepsilon_{t+2,t}) = var(\phi \varepsilon_{t+1} + \varepsilon_{t+2})$
$= \phi^2 \sigma^2 + \sigma^2$
$= (1+\phi^2)\sigma^2$

$\sigma^2 = [1 + \phi^2 + \dots + \phi^{\phi(h-1)}]\sigma^2$
$= var[\varepsilon_{t+h} + \phi \varepsilon_{t+h-1} + \phi^2 \varepsilon_{t+h-2} \dots + \phi^{h-1} \varepsilon_{t+1}]$
$= \sigma^2 + \phi^2 \sigma^2 + (\phi^2)^2 \sigma^2 + \dots + (\phi^{h-1})^2 \sigma^2$
$= [1 + \phi^2 + \dots + \phi^{2(h-1)}]\sigma^2$
$= \sigma^2 \sum_{i=0}^{h-1} \phi^{2i}$

4. $\lim_{h \to \infty} \sigma_h^2 = \frac{\sigma^2}{1-\varphi^2}$
$= \sum_1^{\infty} \frac{1}{(1-\varphi^2)}, \quad |\varphi| < 1$

5. $\hat{y}_{t+1,t} = \rho_1 y_t$
$\sigma_1^2 = \sigma^2(1+e)$
$y_{t+1,t} \pm 1.96\sigma$

Q3. $y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \rho_3 y_{t-3} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$

1. $y_{t+1,t} = \rho_1 y_t + \rho_2 y_{t-1} + \rho_3 y_{t-2} + \theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-1}$
$y_t(1) = \rho_1 y_t + \rho_2 y_{t-1} + \rho_3 y_{t-2} + 0 + \theta_1 \varepsilon_t + \theta_2 \varepsilon_{t-1}$
2. $y_{t+2,t} = \rho_1 y_{t+1} + \rho_2 y_t + \rho_3 y_{t-1} + \theta_2 \varepsilon_t$
$y_t(2) = \rho_1 y_t(1) + \rho_2 y_t + \rho_3 y_{t-1} + 0 + 0 + \theta_2 \varepsilon_t$
3. $y_{t+3,t} = \rho_1 y_{t+2} + \rho_2 y_{t+1} + \rho_3 y_t + \varepsilon_{t+3} + \theta_1 \varepsilon_{t+2} + \theta_2 \varepsilon_{t+1}$
$y_t(3) = \rho_1 y_t(2) + \rho_2 y_t(1) + \rho_3 y_t + 0$
4. $h > 3$
$y_{t+h} = \rho_1 y_{t+h-1} + \rho_2 y_{t+h-2} + \rho_3 y_{t+h-3} + \varepsilon_{t+h} + \theta_1 \varepsilon_{t+h-1} + \theta_2 \varepsilon_{t+h-2}$
$y_t(h) = \rho_1 y_t(h-1) + \rho_2 y_t(h-2) + \rho_3 y_t(h-3) + 0$

Q4. Correlation measures the linear association between 2 variables. Partial autocorrelation measures linear association between 2 variables controlling the effects of 1 or more additional variables. Autocorrelation represents the degree of similarity between a given time-series and a lagged version of itself over successive time intervals. It measures the relationship between a variable's current value and it's past values. The reason autocorrelations can be positive and partial autocorrelations be negative at certain displacements is because of the amount of variables they are looking to account for.