

### Homework #3

$$1) \text{Sleep}_i = \underset{(200)}{3639} - \underset{(.05)}{.15} \times \text{Work}_i - \underset{(3.3)}{11} \times \text{Educ}_i + \underset{(.8)}{2.2} \times \text{Age}_i$$

Holding all other  
variables constant

- a. On average, adults lose .15 minutes of sleep each one unit increase they work a week.  
On average, adults lose 11 minutes of sleep each additional year they spend towards educational attainment.  
On average, adults gain 2.2 minutes of sleep each week for every year they age.

b.  $\beta_1 = -.2$  @ .05 level

$$H_0: \beta_1 \geq -.2; H_a: \beta_1 = -.2$$

$$t = \frac{-.15 + .2}{.05} = \frac{.05}{.05} = 1.0$$

Reject  $H_0$  if  $|t| > 1.96$  ← .05 significance level with  $n > 100$ .

$$|1.0| > 1.96 \approx \boxed{1.0 < 1.96}$$

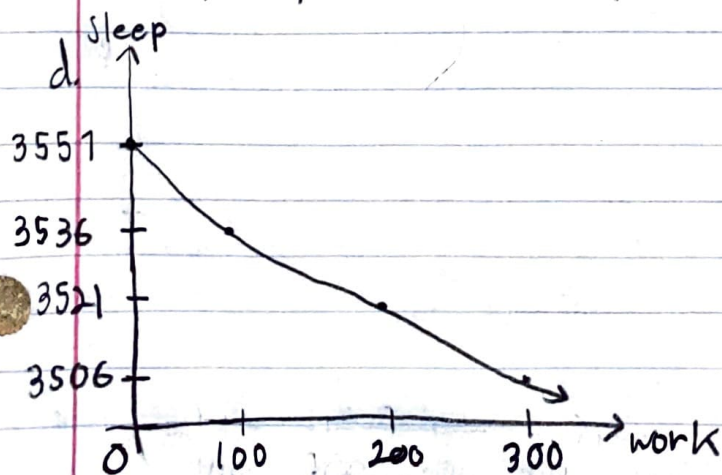
In this case, since  $1.0 < 1.96$ , we reject  $H_0$ . In other words, we do have evidence that  $\beta_1$  does  $\neq -.2$ .

c.  $\text{Sleep}_i = 3639 - .15(1) - 11(1) + 2.2(25) = 3682.85 \approx \text{age } 25$

$$\text{Sleep}_i = 3639 - .15(1) - 11(1) + 2.2(65) = 3770.85 \approx \text{age } 65$$

$$3770.85 - 3682.85 = 88 \text{ minutes per week}$$

- Holding other factors constant, the predicted difference in time sleeping per week for a person age 25 and age 65 is 88 minutes.



$$\begin{aligned} \text{Sleep}_i &= 3639 - .15 \times \text{work} - 11 \times 16 + 2.2 \times 40 \\ &= 3551 - .15 \times \text{work} \\ &= 3551 - .15 \times 100 \\ &= 3551 - .15 \times 200 \\ &= 3551 - .15 \times 300 \end{aligned}$$

$$2. \overbrace{\% \text{ Yes}} = 10 + 0.5 \times \text{Income} + 0.5 \times \text{Coll} + 0.1 \times \text{Children} - 0.8 \times \text{Private}$$

(2.0)
(0.7)
(0.2)
(0.2)
(0.3)

a. - A 1 unit increase in Income leads to a 0.5 increase in the percent of voters in the school district  $i$  who vote in favor of a bond issue (% Yes), holding all other factors constant.

- A 1 unit increase in % of voters w/ a college degree or higher in the district (Coll) leads to a 0.5 increase in % Yes, holding all other factors constant.

- A 1 unit increase in the % of voters w/ school-age children (Children) leads to a 0.1 increase in % Yes, holding all other factors constant.

- A 1 unit increase in the % of voters w/ children enrolled in private school (Private $_i$ ) leads to a 0.8 decrease in % Yes, holding all other factors constant.

b. Yes, I think the signs of the coefficients make sense. There is a couple of reasons why I think this. Those that make a higher income and send their kids to public school are willing to pay more in taxes w/ the bond issue because their kids will get renovated school facilities. This same thought process applies to coefficients Coll and Children, as those who are more educated and have more children are willing to vote in favor of % Yes. Lastly, it makes sense that parents that send their kids to private school will less likely vote in favor of the bond issue b/c they would have to pay higher taxes and not have their kids benefit from using the new facilities b/c they go to private school.

$$c. ESS = 10,000 ; RSS = 40,000 ; TSS = \underset{RSS}{40,000} + \underset{ESS}{10,000} = 50,000$$

$$R^2 = \frac{ESS}{TSS} = \frac{10,000}{50,000} = \boxed{.2}$$

Generally,  $R^2$  is interpreted as how well the regression model fits the observed data. In this model, 20% (0.2) of the data fits the regression.



d. Because the coefficient on children is not statistically different from zero, I would recommend dropping this variable from the reg. model. Keeping variables within a model that are not statistically significant can reduce the model's precision. This is because a p-value that is greater than the significance level indicates that there is insufficient evidence in your sample to conclude that a non-zero correlation exists. With this, the variable should be removed from the regression model.

e.  $\widehat{\% \text{ Yes}} = 10 + 0.5(40) + 0.5(0.3) + 0.1(0.4) - .8(0.1)$   
 $\widehat{\% \text{ Yes}} = 30.11$

- With the average statistics provided, it does not seem the bond initiative would pass. Only 30.11% of the voters would vote yes in the average district.

3.

a.

i.  $H_0: \beta_1 = 0$ ;  $H_a: \beta_1 \neq 0$

$$t = \frac{10.47}{.29} = 36.1034$$

Reject  $H_0$  if  $|t| > t_{.025, 7176}$

$$- |36.104| > t_{1.96} \approx \boxed{36.1034 > 1.96}$$

$$CI = \beta_1 \pm t_{n-2, \alpha/2} \times SE(\beta_1)$$

$$= 10.47 \pm 1.96(.29)$$

$$= \boxed{(9.9016, 11.0384)}$$

ii.  $H_0: \beta_2 = 0$ ;  $H_a: \beta_2 \neq 0$

$$t = \frac{-4.69}{.29} = -16.1724$$

$$- |-16.1724| > t_{1.96} \approx \boxed{16.1724 > 1.96}$$

$$CI = -4.69 \pm 1.96(.29)$$

$$= \boxed{(-5.2584, -4.1216)}$$

b.

i. age is important determinant @ 1% level

$$H_0: \beta_3 = 0; H_a: \beta_3 \neq 0$$

$$t = \frac{.61}{.05} = 12.2$$

yes - age is an important determinant for earnings

Reject  $H_0$  if  $|t| > t_{1,96}$

$$|12.2| > t_{1,96} \approx 12.2 > 1.96$$

$$CI = .61 \pm 1.96(.05)$$

$$= (.512, .708)$$

ii.  $\Delta \overset{.61}{\text{Age}} \times (\text{lower limit}, \text{upper limit}) \leftarrow CI \text{ for } \beta_3$   
 $\quad \quad \quad |1.31| \quad \quad \quad |1.43|$

c.

i. Yes, there appears to be important regional differences at the 1% level. Since the F-stat is  $9.32 > 3.78$ , the regional effects are significant @ the 1% level.

ii.

aa. Juanita - South; Molly - West

$$CI \text{ for } \beta_6: \beta_6 \approx .33, 1.47$$

$$.33 \pm 1.96(1.47)$$

$$= (-2.5512, 3.2112)$$

ab. The exp. difference between Juanita <sup>(south)</sup> & Jennifer <sup>(Midwest)</sup> is:  $\beta_6 - \beta_5$   
 - a 95% CI could be constructed by omitting MW from the regression and replacing it w/  $X_5 = \text{West}$ . With this new regression, the coefficient of South measures South - MW earnings difference & the 95% CI is computed directly.

d.  $t = (\hat{\beta}_{\text{coll}, 2015} - \hat{\beta}_{\text{coll}, 1992}) / SE(\hat{\beta}_{\text{coll}, 2015} - \hat{\beta}_{\text{coll}, 1992})$   
 $(8.94 - 10.44) / (.48 - .41) = -21.43$

$$SE(\text{diff in means}) = \sqrt{se(\bar{x}_{\text{coll}})^2 + se(\bar{x}_{\text{coll}})^2}$$



- e. In isolation, these results do point to sexual discrimination in the U.S. However, sex discrimination means 2 workers, identical in every way but gender, are paid different wages. It is important to control for other factors that could show statistically different significance between men & women other than sex. There are potential omitted variables in the regression that will lead to bias in the OLS coefficient estimator for Female. Since these characteristics were not controlled for stat analysis, it is premature to reach a conclusion about sex discrimination.
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4.

- a. If  $\text{Corr}(\text{Educ}; \text{Age}) > 0$  and you estimate model (1) when the true model is model (2), your  $\beta_1$  will be biased.  $\beta_1$  will be biased b/c it leaves out key factors explaining for the correlation between wage and educ. Since wages tend to generally increase with age, leaving this key factor out of the model leaves a lot up to interpretation within the regression itself.
- b. If  $\text{Corr}(\text{Educ}; \text{Age}) > 0$  and you est. for model (1) when the true model is model (2), your  $\beta_1$  will overstate the impact of educ. on earnings. This is b/c with Age being statistically different from zero, it will be a good estimator of wage increase and should be included in the model to control for wage. It is considered a potential omitted variable.
- c. If  $\text{Corr}(\text{Educ}; \text{Age}) = 0$  and you est. for model (1) when the true model is model (2), your estimate will be consistent and potentially unbiased. Since  $\text{Corr}(\text{Educ}; \text{Age}) = 0$ , it is not statistically significant. With this, the variation in which age controls for <sup>wage</sup> would not necessarily be needed to be included in the model. It theoretically should not make a difference being present or not in the regression.

- d. If you est. for model (1) instead of model (2) when  $\text{Corr}(\text{Educ}; \text{Age}) = 0$ , then you could expect a variation of different things between the estimators of  $R^2$ ,  $\beta_1$ , and std. errors. The  $R^2$  in Model (2) would be larger than Model (1), as Model (2) accounts for larger variation of the model being explained by the  $\beta$  coefficients. The  $\beta_1$  would be smaller in Model (2), as some of the effect Educ plays on wage would be accounted for in age. The SEs of  $\beta_1$  between the 2 models would stay relatively the same, as the SE would account for  $\beta_1$  in both regressions regardless of the correlatory effects between educ and age on wage.

5. On Stata \*\*\*