3)

a) void f1(int n)

```
{
    int i=2;            //θ(1)
    while (i<n){
        //O(1)
        i = i*i;        //O(1)
    }
}
```

i	2	4	16	256
n	3	5	17	257
K	1	2	3	4

$$n = 2^{2^{k-1}}$$

$$\log_2 n = 2^{k-1}$$

$$\log_2\log_2 n = k-1$$

$$1+\log_2\log_2 n = k$$

$$\text{while loop} = \sum_{k=1}^{1+\log\log n} \theta(1)$$

$$T(n) = \theta(1) + \sum_{k=1}^{1+\log\log n} \theta(1) = \theta(1) + \theta(\log\log n) = \boxed{\theta(\log\log n)}$$

b)
```
Void f2(int n)
{
        for (int i = 1; i <= n; i++) {
                if ((i % (int)sqrt(n)) == 0) {
                        for (int K = 0; K < pow(i,3); K++) {
                                //O(1)
                        }
                }
        }
}
```

j	1	2	3	4
n	1	4	9	16

$$\sum_{i=1}^{n} \theta(1) + \sum_{j=1}^{\sqrt{n}} \sum_{K=0}^{(j\sqrt{n})^3-1} \theta(1)$$

$j^2 = n$

$j = \sqrt{n}$

$$= \theta(n) + \sum_{j=1}^{\sqrt{n}} \theta((j\sqrt{n})^3 - 1) = \theta(n) + \theta(n^{7/2})$$

$T(n) = \theta(n^{7/2})$

c)
```
for (int i=1; i<=n; i++){
    for (int k=1; k<=n; k++){
        if (A[k] == i){
            for (int m=1; m<=n; m=m+m){
                //O(1)
                //Assume the contents of the A[] array are not changed
            }
        }
    }
}
```

j	1	2	3	4
n	2	4	8	16
m	2	4	8	16

$j = \log_2 n$

$$T(n) = \sum_{i=1}^{n}\sum_{k=1}^{n}\Theta(1) + \sum_{i=1}^{n}\sum_{j=1}^{\log n}\Theta(1)$$

$$= \sum_{i=1}^{n}\Theta(n) + \sum_{i=1}^{n}\Theta(\log n)$$

$$= \Theta(n^2) + \Theta(n\log n) = \Theta(n^2)$$

$$\boxed{T(n) = \Theta(n^2)}$$

```
d) int f(int n)
   {
      int *a = new int[10];
      int size = 10;
      for (int i = 0; i < n; i++)
      {
         if (i == size)
         {
            int newsize = 3*size/2;
            int *b = new int [newsize];
            for (int j = 0; j < size; j++) b[j] = a[j];
            delete [] a;
            a = b;
            size = newsize;
         }
         a[i] = i*i
      }
   }
```

$10 \cdot 1.5^k = n$

$1.5^k = n/10$

$k = \log_{3/2} n/10$

$$T(n) = \Theta(2) + \sum_{i=0}^{n-1} \Theta(2) + \sum_{k=0}^{\log_{3/2}(n/10)} \left(\Theta(5) + \sum_{j=0}^{(10 \cdot 1.5^k)} \Theta(1)\right)$$

$$= \Theta(2) + \Theta(2n) + \sum_{k=0}^{\log_{3/2}(n/10)} \left(\Theta(5) + \Theta(10 \cdot 1.5^k)\right)$$

$$= \Theta(2) + \Theta(2n) + \Theta\left(5\log_{3/2}(n/10)\right) + \Theta\left(10 \cdot 1.5^{\log_{3/2}(n/10)}\right)$$

$$= \Theta(2) + \Theta(2n) + \Theta\left(5\log_{3/2}(n/10)\right) + \Theta\left(10 \cdot \frac{n}{10}\right)$$

$$= \Theta(2) + \Theta(2n) + \Theta\left(5\log_{3/2}(n/10)\right) + \Theta(n) = \Theta(n)$$

$$\boxed{T(n) = \Theta(n)}$$