1) $\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{15^8} = 0.1012369$

$\approx 10\%$ chance

2)

In 1-99, 0 even integers with 2 odd starting digits

In 100-999, $5 \cdot 4 \cdot 5 = 100$

In 1000-9999, $5 \cdot 4 \cdot 3 \cdot 5 + 5 \cdot 4 \cdot 5 \cdot 4 = 700$

In 10000-99999, $5 \cdot 4 \cdot 3 \cdot 2 \cdot 5 + 5 \cdot 4 \cdot 3 \cdot 5 \cdot 4 + 5 \cdot 4 \cdot 5 \cdot 3 \cdot 4 + 5 \cdot 4 \cdot 5 \cdot 4 \cdot 3$

$= 3240$

$\frac{3240 + 700 + 100}{10^5} = 0.0404$

$P(X=5) = {}_8C_5 (0.0404)^5 (0.9596)^3 = 5.3256 \times 10^{-6}$

3) Clearly not independent. (4, 4, 4), (5, 5, 5), (6, 6, 6) are all examples of when A and B happen simultaneously

4) $\frac{{}_4C_1 \times {}_{13}C_5}{{}_{52}C_5} = \frac{\frac{4!}{3!} \cdot \frac{13!}{8!5!}}{\frac{52!}{47!5!}} = 0.0019808$

expected trials until success $= \frac{1}{p}$ where $p = 0.0019808$

$\frac{1}{p} = 504.8465 \approx 505$ hands

5) Superstar plays: $p(4/5\ W) = {}_5C_4 \times 0.7^4 \times 0.3 = 0.36015$

Superstar sits: $p(4/5\ W) = {}_5C_4 \times 0.5^4 \times 0.5 = 0.15625$

Probability of 4/5 W overall: $0.36015 \times 0.75 + 0.15625 \times 0.25$
$= 0.309175$

using Bayes: $\dfrac{p(4/5\ W \text{ with superstar}) \cdot p(\text{superstar plays})}{p(4/5\ W \text{ overall})}$

$$= \frac{0.36015 \times 0.75}{0.309175} \approx 0.873656$$