

1. Aditya, Bhaskar and Chandra took the Logic exam Using three propositions- A “Aditya passed the exam”, B “Bhaskaro passed the exam”, and C “Chandra passed the exam” represent each of the following sentences as a propositional logic formula.
  - (a) Chandra is the only one who passed.
  - (b) Aditya is the only one who failed.
  - (c) Only one of the three passed the exam.
  - (d) At least one of the three passed.
  - (e) At least two passed.
  - (f) At most two passed.
  - (g) Exactly two passed.
2. Use mathematical induction to show that  $11^n - 4^n$  is divisible by 7 for all natural numbers  $n \geq 1$ .
3. Consider the following argument:

If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If he exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

Is this argument valid? If yes, prove by formulating (using suitable propositions) and using propositional resolution.
4. Show how to implement the 3-input *majority* function (*true* when two or more inputs is *true*) using 2-input *NOR* (complement of *OR*) gates.
5. Consider the propositional formula  $(p \wedge q) \vee (r \wedge s)$  with 4 propositional symbols.
  - Construct a BDD for the formula using the ordering  $p < q < r < s$ .
  - Find a different variable ordering that gives the biggest size BDD. Show this BDD.
6. Banta Singh has been gifted a colourful globe model with all countries named, coloured and names of continents and oceans marked. Banta needs your help to use the following predicates (e.g.  $B(x, y)$  means country  $x$  shares some border with country  $y$ ,  $In(x, y)$  means country  $x$  is in continent  $y$ ,  $C(x, y)$  means “country  $x$  is coloured  $y$ ”) to write the following sentences in formal first order logic.
  - (a) There is a country that borders both Iraq and Pakistan.
  - (b) No country in South America borders any country in Asia.
  - (c) No two countries sharing a border have the same colour.

7. In Prolog, assume that you have a number of facts of the form:

```
mother(M, C).          /* M is the mother of C */
father(F, C).          /* F is the father of C */
male(X).               /* X is male */
female(X).             /* X is female */
```

Write Prolog rules for

```
grandmother(G, X).     /* G is a grandmother of X */
sister(S, X).          /* S is a sister of X */
half_brothers(B1, B2). /* B1 and B2 are male and have the
                        same mother or the same father, but not both. */
```

8. Given the following Prolog program (clauses numbered for convenience), what is the result of the given queries?

```
(1) p2(0).              (2) p3(s(0)).
(3) p2(s(s(X))) :- p2(X). (4) p3(s(X)) :- p3(X).
(5) p1(X) :- p3(X), p2(X). (6) p1(X) :- p2(X).
```

(a) p1(Ans). (give first 4 answers)

(b) p1(0).