CS 621 Artificial Intelligence

Final Exam: Nov 10, 2016

Do rough work (using small font) on back sides only. Then plan and write *concise clear* answers.
No doubts allowed. Closed book. Only 1 handwritten page notes allowed.
There are 6 questions on 5 pages. Each question carries 17 marks.

1. Consider the 8-puzzle whose goal state is

1	2	3
4	5	6
7	8	b

where "b" is the blank or empty square. Starting from

1	2	3
4	8	5
7	6	b

use $A*$ search to find a solution. Use evaluation function $f(n) = g(n) + h(n)$ where $g(n)$ is number of steps made from start position and heuristic cost $h(n)$ is the number of misplaced tiles. When generating children nodes (without repeating previous nodes) move the blank square (if possible) in the following order- left, right, up, down. Give each child node a global sequence number which is incremented when any new node is generated. Split ties (if needed) for which node to open by using the one with lower sequence number. Show the search tree until first solution is found. Write clearly the sequence number and the value of $f(n)$ near each node. For example, the first node generated by moving the blank to the left will be written as-

1	2	3
4	8	5
7	b	6

$\langle 1, 5 \rangle$ - where 1 is the sequence number and 5 is the evaluation function for this node.

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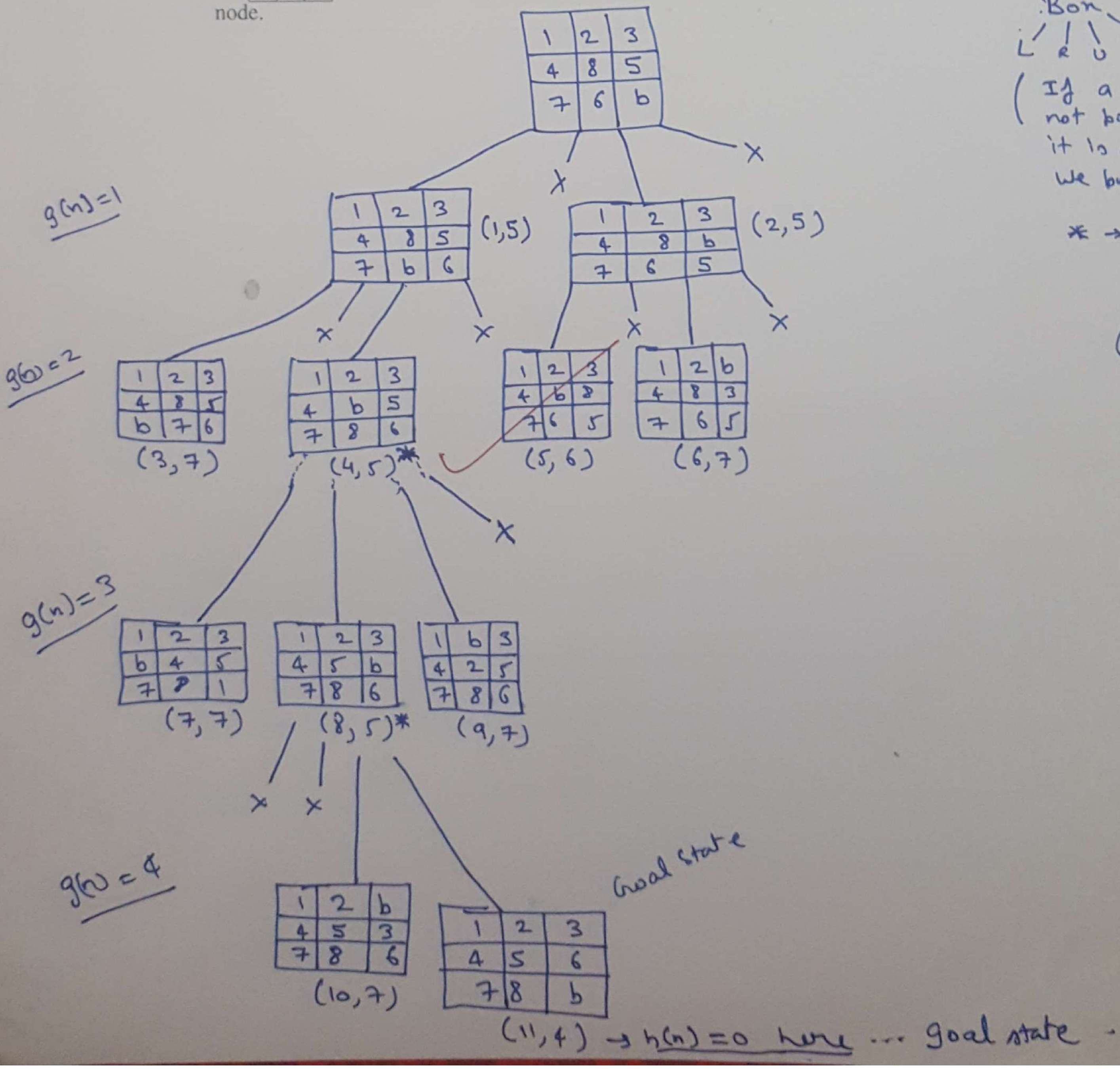

Bon
L R U D

(If a move is not possible or it is a repetition we put a cross)

* → chosen due to low cost (greedy)

2. Assuming the following definitions are loaded into Prolog. Note that *var* and *nonvar* are built-in predicates that check if the argument variable is already bound to a value or not.

```
sNs([], 0, Lst, Lst).
sNs([X1|R1], Tot, L2, Rst) :- var(X1), pckA(X1, L2, Rem1),
                    T1 is Tot - X1, sNs(R1, T1, Rem1, Rst).
sNs([X1|R1], Tot, L2, Rst) :- nonvar(X1), T1 is Tot - X1,
                    sNs(R1, T1, L2, Rst).

pckA(X1, [X1 | Rest], Rest).
pckA(X1, [X2 | R2], [X2 | Rest]) :- pckA(X1, R2, Rest).
```

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How many solutions will the goal `sNs([X1,3,X3,X4,5], 20, [1,2,3,4,2,5], Ans)` produce? List the first and last solution.

Total 6 solutions

X_1	X_2	X_3
3	4	5
3	5	4
4	3	5
4	5	3
5	3	4
5	4	3

The various assignments to X_1, X_2 & X_3 are in this table.

First Solution → [1, 2, 2]

Last Solution → [1, 2, 2].

The function returns the elements left in the list [1, 2, 3, 4, 2, 5] when we get a solution. It will always be [1, 2, 2] I think.

3. Consider the integer variables X, Y, Z. X is known to be between 1 and 10 (both inclusive), Y between 5 and 15 (both inclusive) and Z between 5 and 20 (both inclusive). The following constraints also hold $C_1 : X > Y, C_2 : Y + Z = 12, C_3 : X + Z = 16$. Propogating C_1 on X restricts value of X to be between 6 and 10 (both inclusive). Let us use the notation $C_1(X)$ to denote this propogation. The notation $C_1(X), C_1(Y), C_2(Y)$... means first propogate C_1 on X then C_1 on Y, then C_2 on Y and so on. We can make the constraints arc consistent by repeatedly propogating different constraints until no variable's domain changes. Fill in the following blanks after working out the problem carefully.

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(a) There are __2__ feasible solutions to this problem.
One of them is X = __9__, Y = __5__, Z = __7__.

(b) The minimum number of constraint propogations needed to make the graph arc consistent is __4__.
One such sequence is $C_2(Z), C_3(Z), C_2(Y), C_3(X)$.

Init

X: 1-10
Y: 5-15
Z: 5-20

$\xrightarrow{C_2(Z)}$

X: 1-10
Y: 5-15
Z: 5-7

$\xrightarrow{C_3(Z)}$

X: 1-10
Y: 5-15
Z: 6,7

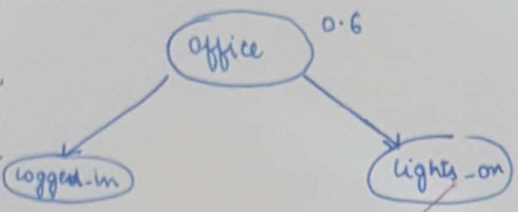
$\xrightarrow{C_2(Y)}$

X: 1-10
Y: 5,6
Z: 6,7

$\xrightarrow{C_3(X)}$

X: 9,10
Y: 5,6
Z: 6,7

4. Prof. Singh spends 60% of his time in his office, rest elsewhere. When he is in his office, he is logged in to the department server 80% of the time, but he keeps his light off 50% of the time. When he is not in his office he forgets to switch off his light only 5% of the time and logins to the department server only 10% of the time. Draw a Bayesian network that best represents the above information. If you see Prof. Singh logged in the server, what is the probability that the light is on in his office?



logged-in

O	T	F
T	0.8	0.2
F	0.1	0.9

lights-on

O	T	F
T	0.5	0.5
F	0.05	0.95

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$P(\text{lights-on} \mid \text{logged-in}) = ?$

$$P(\text{lights-on}=T \mid \text{logged-in}=T) = \frac{P(\text{lights-on}=T, \text{logged-in}=T)}{P(\text{logged-in}=T)}$$

$$= \frac{\sum_O P(\text{lights-on}=T, \text{logged-in}=T, O)}{\sum_O P(\text{logged-in}=T, O)}$$

Now, $\sum_O P(\text{lights-on}=T, \text{logged-in}=T, O) = P(\text{lights-on}=T, \text{logged-in}=T, O=T) + P(\text{lights-on}=T, \text{logged-in}=T, O=F)$

$$= P(\text{lights-on}=T \mid O=T) * P(\text{logged-in}=T \mid O=T) * P(O=T)$$
$$+ P(\text{lights-on}=T \mid O=F) * P(\text{logged-in}=T \mid O=F) * P(O=F)$$

$$= 0.5 * 0.8 * 0.6 + 0.05 * 0.1 * 0.4$$
$$= 0.24 + 0.0020$$
$$= 0.2420$$

similarly, $\sum_O P(\text{logged-in}=T, O) = P(\text{logged-in}=T \mid O=T) * P(O=T) + P(\text{logged-in}=\cancel{F} \mid O=F) * P(O=F)$

$$= 0.8 * 0.6 + 0.1 * 0.4$$
$$= 0.48 + 0.04$$
$$= 0.52$$

$\therefore P(\text{light-on}=T \mid \text{logged-in}=T) = \frac{0.2420}{0.52}$

$$\approxeq 0.465$$

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5. Brahma, Vishnu and Siva are members of the Himalayan Club. Every Himalayan Club member, who is not a skier, is a mountain climber. Mountain climbers do not like rain. Any one who does not like snow is not a skier. Vishnu dislikes whatever Brahma likes, and likes whatever Brahma dislikes. Brahma dislikes rain and snow.

   (a) Represent the above knowledge as predicate logic statements (convert to clausal form each formula). Use constants b, v, s, rn, sn to represent Brahma, Vishnu, Siva, rain and snow. Use predicates $Sk(x)$ for "x is a skier", $Mc(x)$ for "x is a Mountain climber", $L(x, y)$ for "x likes y". You can assume all people in the domain are members of Himalayan club and need not check for membership of the club in your formulae.

   (b) Answer the following question using resolution theorem proving *Is there a member of Himalaya Club who is a skier, but not a mountain climber?*. Show the resolution steps used. For full credit you will need to use at most 6 resolution steps.

Clausal form with ∀ dropped

a)

$\forall x \ \neg Sk(x) \rightarrow Mc(x) \Rightarrow$ $\quad Sk(x_1) \vee Mc(x_1)$ — ①

$\forall x \ Mc(x) \rightarrow \neg L(x, rn) \Rightarrow$ $\quad \neg Mc(x_2) \vee \neg L(x_2, rn)$ — ②

$\forall x \ \neg L(x, sn) \rightarrow \neg Sk(x) \Rightarrow$ $\quad L(x_3, sn) \vee \neg Sk(x_3)$ — ③

$\forall y \ \cancel{L(v,y) \wedge \neg L(b,y)}$

$y \in \{rn, sn\}$ $\forall y \ L(b,y) \rightarrow \neg L(v,y) \Rightarrow$ $\quad \neg L(b, y_1) \vee \neg L(v, y_1)$ — ④

$\forall y \ \neg L(b,y) \rightarrow L(v,y) \Rightarrow$ $\quad L(b, y_2) \vee L(v, y_2)$ — ⑤

$\neg L(b, rn) \wedge \neg L(b, sn) \Rightarrow$ $\quad \neg L(b, rn)$ — ⑥

$\neg L(b, sn)$ — ⑦

b) $\exists x \ Sk(x) \wedge \neg Mk(x)$

Negation of goal $\neg(\exists x \ Sk(x) \wedge \neg Mk(x))$

$= \forall x \ \neg Sk(x) \vee Mk(x) \Rightarrow \neg Sk(x_4) \vee Mk(x_4)$ — ⑧

- $L(v, rn)$ — ⑤ & ⑥ y_2/rn — ⑨
- $\neg Mc(v)$ — ⑨ & ② x_2/v — ⑩
- $Sk(v)$ — ① & ⑩ x_1/v — ⑪
- $Mc(v)$ — ⑧ & ⑪ x_4/v — ⑫
- $\square$ — ⑩ & ⑫

∴ Statement True.

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6. **Do rough work first on back side and show the trellis cleanly in the space given below. You can keep the numbers as fractions. No need to evaluate as decimal and round off etc.**

Three boxes contain a mix of black (B) and white (W) marbles. Box 1 has 2 black, 1 white. Box 2 has 2 black, 2 white. Box 3 has 1 black, 2 white. A box is selected at random (equal probability) and one marble is drawn. Its colour is noted and the marble is put back. Suppose this is done 5 times and we see the following output sequence (O) of marbles- B, B, W, W, B.

(a) Suppose we know that the sequence of boxes chosen was 1, 1, 3, 3, 2. What is the probability of observing the given output sequence (O)?

$$P = \overset{B}{\underset{1}{\frac{2}{3}}} \times \overset{B}{\underset{1}{\frac{2}{3}}} \times \overset{W}{\underset{3}{\frac{2}{3}}} \times \overset{W}{\underset{3}{\frac{2}{3}}} \times \overset{B}{\underset{2}{\frac{2}{4}}} = \frac{8}{81}$$

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(b) What is the overall probability of observing this output sequence ($P(O \mid \lambda)$)? Show the appropriate trellis and fill in the intermediate values you computed.

	B (t_1)	B (t_2)	W (t_3)	W (t_4)	B (t_5)
3	1/9	1/18	1/18	1/36	1/144
2	1/6	1/12	1/24	1/48	1/96
1	2/9	1/9	1/36	1/72	1/72

$\alpha_t(i)$ are calculated

$\alpha_1(i) = b_i(O_1) \cdot \pi(i)$

$\alpha_{t+1}(i) = \sum_{j=1}^{N} \alpha_t(j) \cdot A_{ji} \cdot b_i(O_{t+1})$

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$$P(O|\lambda) = \sum_{i=1}^{N} \alpha_T(i) = \frac{1}{144} + \frac{1}{96} + \frac{1}{72} = \frac{9}{288} = \frac{1}{32}$$

(c) If we do not know which boxes were chosen, what is the most likely sequence of boxes chosen given this output sequence (O)? Show the appropriate trellis and fill in the intermediate values you computed.

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	B (t_1)	B (t_2)	W (t_3)	W (t_4)	B (t_5)
3	1/9	2/81	8/729	16/6561	16/59049
2	1/6	1/27	2/243	4/2187	8/19683
1	2/9	4/81	4/729	8/6561	32/59049

$\delta_t(i)$ are calculated

$\delta_1(i) = \pi(i) \cdot b_i(O_1)$

$\delta_{t+1}(i) = \max_j(\delta_t(j) \cdot a_{ji} \cdot b_i(O_{t+1}))$

The most likely sequence of boxes given this output sequence BBWWB is 1 1 3 3 1