$\boxed{1}$

A] Probability that kth symbol is received as 0 is $p(1-\delta) + (1-p)\delta$

B] 0 can be correctly decoded if 000 is sent and 001, 000, 100, 010 is [one of them] is received.

$Pr \rightarrow \cancel{ppp} \cdot (1-\delta)(1-\delta)(1-\delta) +$

$\cancel{ppp} \cdot (1-\delta)(1-\delta)\delta +$

$\cancel{ppp} \cdot \delta(1-\delta)(1-\delta) +$

$\cancel{ppp} \; (1-\delta)\,\delta(1-\delta)$

$= \cancel{p}\,(1-\delta)^3 + 3\,\cancel{p}\,\delta(1-\delta)^2$

C] Here, suppose decoder gives 0 with probability x, 1 with probability y and $\perp$ with probability z.

$\therefore x + y + z = 1$ — (1)

→ Now, if we want to ~~maximize~~ minimize the penalty then we should create objective function which with we apply maximization

- probability of correctly getting 0 at receiver → $a = p(1-\delta) + (1-p)\delta$

for getting 1 → $b = (1-p)(1-\delta) + p\delta$.

$= f(x,y,z) = -2x\cdot a - 2yb - 1\cdot z\,[a+b]$

$= -2xa - 2yb - \cancel{\phi}\, z$

~~Now~~ we have $x+y+z=1$

Applying Lagrange minimization,

$L(x,y,z) = -2xa - 2yb - z + \lambda(x+y+z-1)$

$\frac{\partial L}{\partial x} = -2a + \lambda = 0 \Rightarrow \lambda = 2a$

$\frac{\partial L}{\partial y} = -2\cancel{b} + \lambda = 0 \Rightarrow \lambda = 2b$

$\frac{\partial L}{\partial z} = -1 + \lambda = 0 \Rightarrow \lambda = 1$

→ Here, ~~a~~ $\lambda = 1, 2a, 2b$

we can interprete that if we take $p = \frac{1}{2}$ than ~~p~~ ~~to~~ probabilistic cost → 1

if we take $p < \frac{1}{2}$ then cost ~~f~~ ~~then~~ for penalty will be less for outputting 1~~a~~ then outputting 0. if we take $p > \frac{1}{2}$ then penalty will be less for outputting 0 then outputting 1.

→ So, if $p < \frac{1}{2}$ then $x < y$ [~~p~~ ~~x~~ y increases as p decreases]

if $p > \frac{1}{2}$ then $x > y$

~~a~~ [x increases as p increases