Practice Problems for CS 621

1. Define a relation called twice, where twice(Elem,List) is true if List is a list and Elem appears in List at least twice. For example, the following are true: twice(2,[1,2,3,2]), twice(a,[a,b,a,c,a,d,a]) but the following are not true: twice(2,[1,3,5]), twice(b,[a,b,c]) The query twice(X,[1,2,3,2,1]) should generate the answers X = 1 and X = 2.

2. Consider the following code:

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mystery([H|T]) :- mys2(H, T).

mys2(X, [X]).
mys2(X, [_|T]) :- mys2(X, T).
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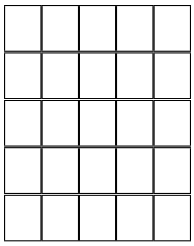
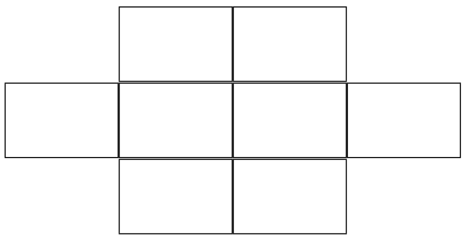
   On what kind of lists will the mystery predicate succeed? What will be the result of the call mys2([1, 2, 3, 4]) ?

3. Consider solving the 5-Queens puzzle (placing 5 queens Q1 to Q5 on a 5x5 chess board with no two queens attacking each other) using Constraint Satisfaction techniques as follows. Place Q1 first in the top row, second column. Do forward checking to reduce the domains of other queens (cross out in diagram below). Backtrack if any remaining queen's domain becomes empty. Pick the least constrained queen (split ties by lower variable number) and place it in the first available (from left) square and proceed as before. Show the search tree below (root node drawn for your convenience) until you find the first feasible solution.



   Forward checking is defined as a single iteration of constraint propagation only on those edges that terminate at the variable whose value was just set, and that do not originate from variables which have already been set.

4. Fill the 8 boxes in the figure shown on the right with different numbers from 4 to 11 (both inclusive) so that any two adjacent boxes (horizontal, vertical or diagonal) differ by at least 2.



   How many different solutions are possible? _______________

5. IIT Bombay has introduced a new scheme where cycles can be freely borrowed every morning from any one of three cycle stands (H1, MB, YP) and returned to any one of the three in the night. Planners estimate that cycles borrowed from H1 are returned to H1 itself 80% of the time, MB 10% of the time and YP the remaining 10% of the time. Cycles borrowed from MB are returned at H1 30% of the time, MB itself 20% of the time and at YP 50% of the time. Cycles borrowed from YP are returned at H1 20% of the time, MB 60% of the time and YP itself 20% of the time.

   If IIT's total fleet is 610 bicycles, the parking space provided should be for ____________ bicycles at H1, ____________ bicycles at MB, and ____________ bicycles at YP.

   You can assume demand for bicycles always exceeds the numbers available.

6. HMMs can be used to decode simple DNA sequences. A DNA sequence is a series of symbols from {A, C, G, T }. Assume there is one hidden variable S that controls the generation of DNA sequence. S takes 2 possible states {S1 , S2 }. Assume the following transition probabilities for HMM M

   P (S1 |S1) = 0.8, P (S2 |S1) = 0.2, P (S1 |S2) = 0.2,
   P (S2 |S2) = 0.8

   emission probabilities as following

   P (A|S1) = 0.4, P (C|S1) = 0.1, P (G|S1) = 0.4, P (T |S1) = 0.1
   P (A|S2) = 0.1, P (C|S2) = 0.4, P (G|S2) = 0.1, P (T |S2) = 0.4

   and start probabilities as following P (S1) = 0.5, P (S2) = 0.5

   Assuming the observed sequence is x = CGT CAG, calculate (24 marks)

   (a) The probability of observing this sequence given the HMM model.
   (b) The probability of being in each state (S1, S2) after seeing each symbol.
   (c) The most likely sequence of states that produced this observation.