Accelerated Life Testing Modeling

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***Abstract*—*Numerous papers and texts have been written in the reliability literature regarding the determination of the optimum test duration for a production stress or a burn-in test. Testing time is often the only factor affecting this decision. A model that brings together the impact of production test failures and field performance, including their respective costs, achieves the goal of determining the duration of production tests that minimizes the total cost. This model is composed of two Weibull models combined into a single model that helps companies find the best test duration for testing. This paper further optimizes the above model, because some products that are damaged in the extreme environment of the test, will not be damaged in the real application, and we will consider the impact of this part of the product on the optimal test duration. The advantage of the proposed improvement is that the optimal test duration is reduced, reducing the necessary cost and helping the company to build a realistic EST model with changes.*

Keywords—life testing model, environmental stress testing, Weibull model, test optimization, model improvement

I. INTRODUCTION

Early field failures of products may reduce customer satisfaction and increase warranty and total costs. Therefore, in order to reduce total costs and improve customer satisfaction, products need to undergo environmental stress testing (EST) before being put on the market. This is a costly and energy-intensive process, and in order to control the cost, it is necessary to shorten the time and optimize the test scheme. In the paper by Bahman [1], a method is proposed to integrate production testing and field reliability models. This method determines the optimal test time by establishing the relationship between production testing and field reliability. However, this method has drawbacks during the experiment. Stress testing is often a way to assess how well a product can withstand extreme conditions. When performing a pressure test, various adverse conditions that a product may encounter during use, such as high temperature, high humidity, high pressure, etc., are simulated to determine whether the product can work properly under these conditions. However, this situation almost never arises in the context of actual use. Therefore, in the experiment, we should consider the maintenance cost of products damaged by the extreme environment of the laboratory during the test process, which may have an impact on the total cost and has the value of analysis. This experiment can help companies analyze and simulate the cost of warranty or compensation that is more in line with the actual situation and reduce the budget.

In this paper, we aim to investigate how to accelerate the life test modeling. Section 2 provides an overview of previous research in the field, as well as some theoretical background behind our approach. Section 3 then elaborates on our experimental methodology and the formulation concepts involved. The contents of Section 4 show the various data obtained in the experiment and analysis of results. In Sections 5, we give some conclusions according to the results of experiments. Finally, Section 6 contains some ideas about future improvements and additions that can be made to the simulation.

II. RELATED WORK

Environmental stress testing plays a vital role in the development and manufacturing of various products, materials, and systems. It is a crucial step to ensure that these products can withstand the range of environmental conditions they will encounter during their lifecycle. This essay will explore the purpose of environmental stress testing, common types of tests, and the benefits it provides in improving product quality, safety, and reliability.

The primary objective of environmental stress testing is to identify any potential weaknesses or failure points in a product, material, or system before it reaches the end-user. By simulating real-world conditions and subjecting the product to various stresses, manufacturers can evaluate its performance, durability, and reliability. This allows them to address any design or manufacturing issues early in the development process, reducing the risk of costly recalls, product failures, or damage to the company's reputation.

Temperature testing is one of the most common environmental life testing. Products are exposed to a range of temperature extremes, such as high heat, freezing conditions, or rapid temperature fluctuations, to evaluate their performance and stability under various conditions.

In the context of EST experiments, this thesis will discuss the problem of bias in experimental results caused by EST experiments themselves. Environmental stress testing (EST) improves product quality and reliability by detecting potential or marginal defects in products. Typically, products with hard defects will be tested as part of production testing. However, products with latent or marginal defects may not be detected and may be delivered to customers. The result is that those weak products may fail in the field environment and under certain stress conditions. Early product failures in the field not only increase the cost of product warranties and field repairs, but also affect the customer's perception of product quality. Results such as "Environmental Stress Testing with Boundary-Scan" [2] are clearly not what the company requesting the EST wants to see, and at the same time, those products that are damaged or fail due to the nature of the EST experiment itself can increase the total cost of the EST experiment and affect the final outcome of the experiment.

In the paper "Environmental Stress Testing Experiment Using the Taguchi Method" [3], which also uses EST, additional stress models and measurement criteria are added to the experimental results in order to determine whether the damage caused by the EST experiment is significant in revealing potential failures, which also greatly increases the complexity of the experiment and the cost of calculating the experimental data.

From an industrial point of view, cost is an important factor, and "Environmental Stress Screening for electronic equipment by random vibration: a critical approach to reliability estimation and planning" [4] deals with the cost analysis of the experiment. In particular, the change in cost ratio between the cost of screening undetected weak (or "bad") equipment and the cost of good equipment at its expense is the most sensitive factor affecting the final experimental cost results, and needs to be estimated by the producer.

III. METHODOLOGY

The first step in developing an integration test and field cost model is to derive a statistical model to explain the performance of the product during EST. We use the Weibull model as a statistical model with a cumulative probability of failure at time tp of response 32. Thus we have

$$ln(t_p) = \alpha + \frac{1}{\beta} ln[- ln(1 - p)] \quad (1)$$

where t_p is the p-quintile of the failure time distribution. Alpha and beta denote the scale and shape parameters, respectively, and can be estimated using the maximum likelihood method or other methods described in McCullagh and Nelder [5] and Prabhakar Murthy et al [6] Methods for estimation.

The Weibull model in Equation (1) can be extended to a Weibull regression model when any regression variables are available, such as other production tests or the state of the production conditions (2).

$$ln(t_p) = \alpha_0 + \sum_i \alpha_i X_i + \frac{1}{\beta} ln[- ln(1 - p)] \quad (2)$$

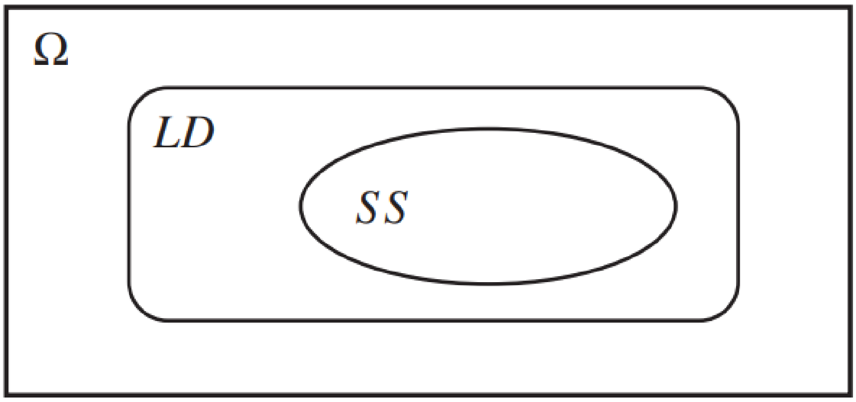
where X_i denotes the model regression variable. Rearranging equation (1) will give the expression (3) for the reliability of the unit at time t_p.

$$r(t_p) = 1 - p = exp\left[-\left(\frac{t_p}{\eta}\right)^{\beta}\right], \eta = e^{\alpha} \quad (3)$$

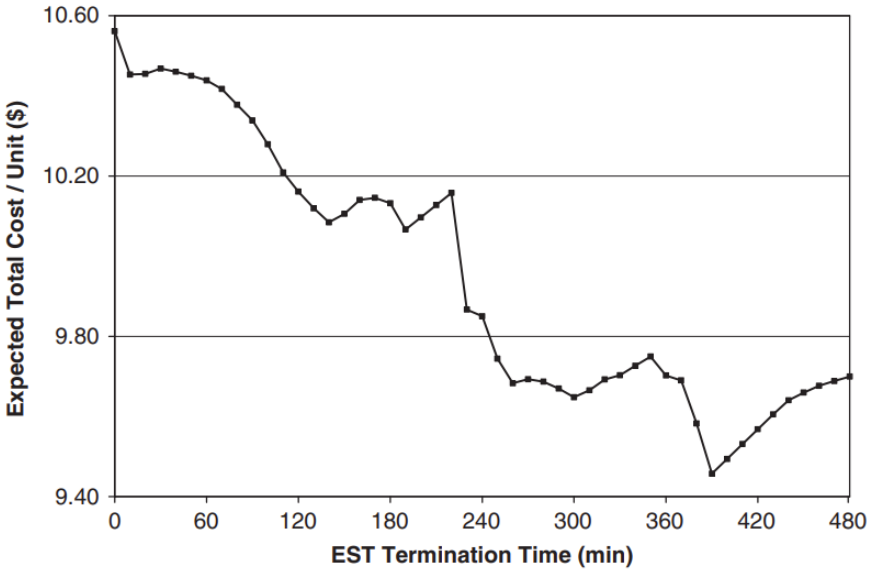
To calculate the reliability function for each segment using Equation (3), the following general equation (4) applicable to each segment needs to be used:

$$R(t) = \begin{cases} r(t), & t \in segment\ 1 \\ R(t_{j-1} \cdot r(t - t_{j-1}) & t \in segment\ j,\ j > 1 \end{cases} \quad (4)$$

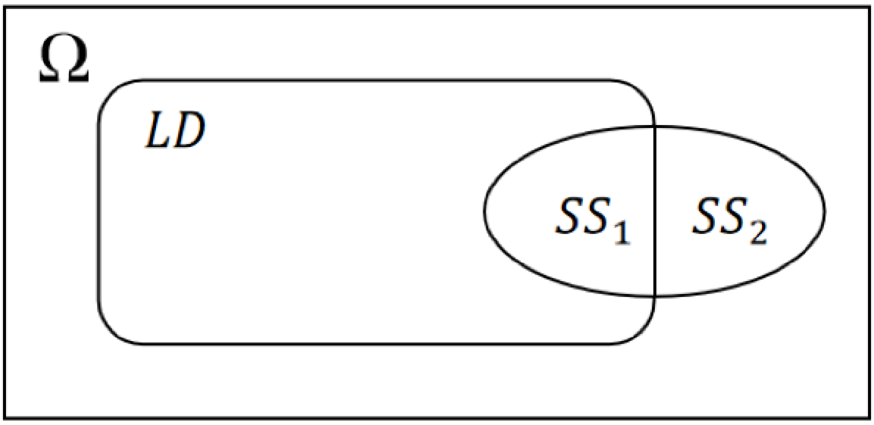
Up to this step, the analytical methods and formulas used for the experiments were no different from those used in the original paper [1]. However, for the next analysis, we need to consider completely new data composition and variables.



This figure shows the different components of the overall experimental data in the original paper. The SS in the original paper represents the detected corrupted samples. The total cost graph obtained in the original paper is as follows:



In our paper, since it is necessary to consider objects that would not have been damaged in the real application environment due to the EST experimental environment, the overall experimental data structure should be represented as follows:



SS_1: Equivalent to the original SS (Units that fail the EST.) i.e., the bad ones are detected

SS_2: The good ones were damaged during testing, which means that this product would not be damaged within the warranty period in actual use but was damaged due to the extreme environment of EST.

$SS = SS_1 + SS_2$

With the SS_1 and SS_2 we got:

$$E[Total\ Cost] = E[Test\ Cost\ SS_1] + E[Test\ Cost\ SS_2] + E[Warranty\ Cost] \qquad (5)$$

In this equation, part $E[Test\ Cost\ SS_2]$ of it is the resources that are wasted in the experiment

$$E[Test\ Cost] = C_f + C_V(\frac{\tau}{T}) + C_R P(SS_1 + SS_2|\theta) \qquad (6)$$

$$C_R P(SS_1 + SS_2|\theta) = C_R P(SS_1|\theta) + C_R P(SS_2|\theta)$$

θ: θ denote the vector of parameters, such as temperatures, ramp rates, number of cycles, etc., that define a particular EST.

C_f: represent the fixed cost of conducting the test

C_V: represent the variable cost for conducting the test for the current duration of 8 h

C_R: represent the repair cost of each unit that fails during the EST

$C_R P(SS_1 + SS_2|\theta)$: same as the original $C_R P(SS|\theta)$, since $SS = SS_1 + SS_2$, And the damage time is constant (damage time is fixed in FIGURE 3, TABLE VII [1]: P-value is related to damage time), so the P-value is constant.

In (6) C_f denotes the fixed cost of performing the test and C_V denotes the variable cost of performing the test within the current time during the 8 hour. Finally, C_R denotes the repair cost for each unit that fails during EST. $P(SS|\theta)$ in Equation (6) can be determined directly from Equation (4). In addition, the expected cost of warranty is expressed as (7).

$$E[Warranty\ Cost] = C_W P(LD - SS_1|\ \Omega - SS_1; \theta, \varphi) \qquad (7)$$

For the field conditions involved in the experiments calculate:

$$C_W P(LD - SS_1|\ \Omega - SS_1; \theta, \varphi) = \frac{P(LD - SS_1 \Omega - SS_1|\theta,\varphi)}{P(\Omega - SS_1|\theta,\varphi)} = \frac{P(LD - SS_1|\theta,\varphi)}{1 - P(SS_1|\theta)} \qquad (8)$$

φ: represent the conditions in the field, such as geographical location, humidity, etc.

$LD - SS_1$: represent the potential fail in the field during the warranty period.

$\Omega - SS_1$: represent units are tested and only those that pass the EST are released into the field.

$P(LD - SS_1|\theta, \varphi)$: represents the probability that a product has a fault but is not detected under fixed conditions.

$1 - P(SS_1|\theta)$: The probability that the device does not have a fault plus the probability that the device has a fault but is not detected is the probability that it passes the test.

P is a function that varies according to time, and all values are time-dependent

$$1 - P(SS_1|\ \theta) = R(\tau) \qquad (9)$$

τ: represent the reduced duration of the EST

$R(\tau)$: applied to the EST data in Table III, it was possible to estimate (estimated by applying Equation (3) to the EST data in Table III).

These probabilities will be replaced by the denominator of (8), and the final calculation of:

$P(LD - SS_1|\Omega - SS_1; \theta, \varphi)$

If the EST experiment terminated on time, Then for all the damaged units in the test after the time will be added to the total field failure probability using equation (10).

$$P_{EST\to F} = P(SS_1|\ \tau_0; \theta) - P(SS_1|\ \tau; \theta), \quad \tau < \tau_0 \qquad (10)$$

$P_{EST\to F}$: presents the portion of failures that move from the EST to the field for an EST of duration $\tau < \tau_0$.

$\tau_0 = 480$: Represent the initial EST duration for the particular product under evaluation.

Fitting the field failure time with the Weibull model for an EST of duration $\tau_0 = 480$ min yields the updated Eq:

$$P(LD - SS_1|\ \tau_0, \theta, \varphi) = P(Failure\ time\ in\ the\ field\ \leq Warranty\ Period\ |\ \tau_0, \theta, \varphi)$$

$$= 1 - exp(-(\frac{Warranty\ Period}{e^{\beta}})^{\beta}) \qquad (11)$$

$P(LD - SS_1 | \tau_0, \theta, \varphi)$ has a value of 0.016, because the number of faulty but undetected products is constant and the damage time in actual use does not change.
the portion of the units explained by Equation (10) has to be added to the value obtained in Equation (11),when the EST duration time τ_0 reduce to τ, which will get the formula (12) after the update.

$$P(LD - SS_1|\tau, \theta, \varphi) = P(LD - SS_1|\tau_0, \theta, \varphi) + P_{EST\rightarrow F}, \tau < \tau_0 \qquad (12)$$

In the following section will analysis different simulation result with multiple SS_2 part estimation.

IV. IMPLEMENTATION AND RESULT

The temperature cycling profile that was actually used on the units examined in the paper is shown in the figure 1 below [1]. The Environmental Stress Test that is used most frequently involves temperature cycling. The temperature in an environmental chamber, where the units are placed, is gradually and periodically varied between low and high temperatures. While the rate at which the temperature varies is referred to as ramps, the high and low temperature extremes are referred to as dwells. A cycle is the name for each successive section of the test that consists of a reducing ramp, a cold dwell, an increasing ramp, and a hot dwell. These cycles are included in the EST test in the publication [1], which lasts for 6 hours.

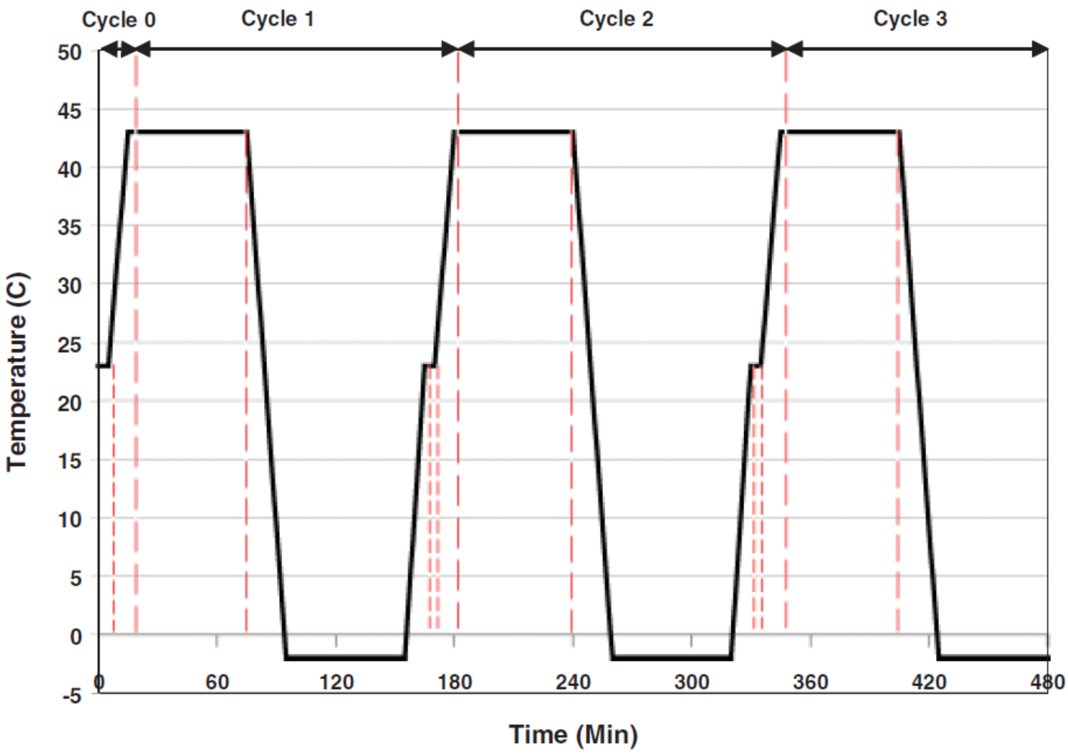


Fig. 1. Environmental Stress Test profile. [1]

The original dataset is from the paper [1].
The organization of the EST database for the real product used to illustrate the suggested technique is shown in Table I. Each EST segment, such as the ramp or dwell, is treated separately for analytical reasons. These segments are displayed in Table I's first column. The start time of the segment in which the breakdown occurred is shown in the second column. The unit's failure time relative to the

test's start is shown in the third column. [1] For a segment, a unit's lifetime that is unaffected by failure is taken into account as censored.

Table I. The EST failure database structure

Segment	Segment start point	Time in the EST	Segment	Segment start point	Time in the EST	Segment	Segment start point	Time in the EST
1	0	2	4	95	111	7	240	255
1	0	2	4	95	114	8	260	273
1	0	5	4	95	127	8	260	279
1	0	7	4	95	136	8	260	283
1	0	12	4	95	139	8	260	283
2	15	25	4	95	142	8	260	288
2	15	29	5	155	160	8	260	293
2	15	32	5	155	178	8	260	300
2	15	59	6	180	218	8	260	302
2	15	63	6	180	223	9	320	324
2	15	73	6	180	224	9	320	329
2	15	74	6	180	224	10	345	351
3	75	77	6	180	224	10	345	352
3	75	77	6	180	228	10	345	354
3	75	87	6	180	231	10	345	356
3	75	92	6	180	233	10	345	358
4	95	99	7	240	242	10	345	366
4	95	99	7	240	244	10	345	372
4	95	104	7	240	247	12	420	439
4	95	105	7	240	249	12	420	455
4	95	107	7	240	249	12	420	462
4	95	108	7	240	250	12		
4	95	110	7	240	252	12		

Figure 2[1], which shows the distribution of failures during the EST segments, can be connected to Table I. The dots denote the temperature and time at which each failure was noticed. Only a few problems were noticed during the ramps, while the majority of failures happened during the dwells.

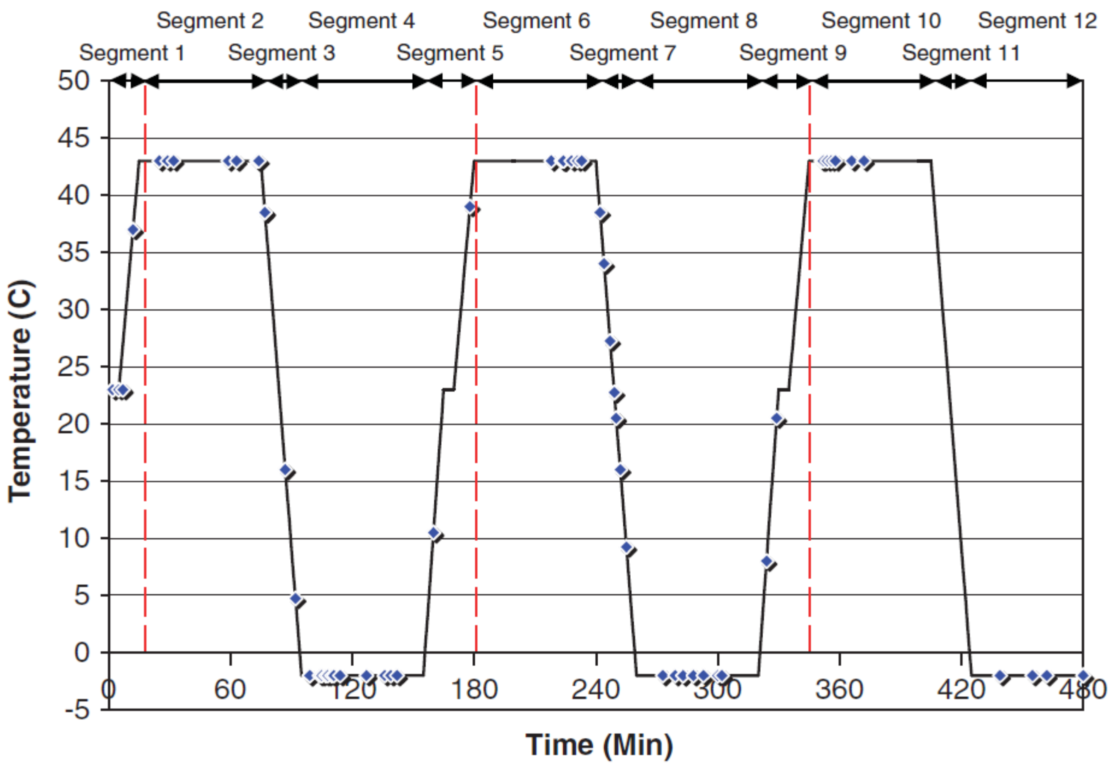


Fig. 2. Distribution of failures during the EST cycles. [1]

Table II contains the appropriate figures for genuine and censored failure times during the EST. The EST started at 1912 units. Only 1907 units will enter segment 2 because 5 units in segment 1 failed. The failure time for each segment is shown in Table II which could connect with Table I. For instance, in segment 1, the unit that Table I indicates failed after 25 minutes is taken as censored. Segment 2, which begins at 15 and ends at 75 minutes, will have a failure time of 25 - 15 = 10 minutes. Additionally, for segment 2, failures after 75 minutes are treated as censored at 60 minutes.

Table II. The EST failure and the censored data applied to the Weibull model

Segment 1	Segment 2	Segment 3	Segment 4	Segment 5	Segment 6	Segment 7	Segment 8	Segment 9	Segment 10	Segment 11	Segment 12
2	10	2	4	5	38	2	13	4	6	15	19
2	14	2	4	23	43	4	19	9	7	15	35
5	17	12	9	25	44	7	23	25	9	15	42
7	44	17	10	25	44	9	23	25	11	15	60
12	48	20	12	25	44	9	28	25	13	15	60
15	58	20	13	25	48	10	33	25	21	15	60
15	59	20	15	25	51	12	40	25	27	15	60
15	60	20	16	25	53	15	42	25	60	15	60
15	60	20	19	25	60	20	60	25	60	15	60
15	60	20	32	25	60	20	60	25	60	15	60
15	60	20	41	25	60	20	60	25	60	15	60
15	60	20	44	25	60	20	60	25	60	15	60
15	60	20	47	25	60	20	60	25	60	15	60
15	60	20	60	25	60	20	60	25	60	15	60
15	60	20	60	25	60	20	60	25	60	15	60
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
15	60	20	60	25	60	20	60	25	60	15	60
1912 *units*	*1907 units*	*1900 units*	*1896 units*	*1883 units*	*1881 units*	*1873 units*	*1865 units*	*1857 units*	*1855 units*	*1848 units*	*1848 units*

The failure distribution times for the various EST segments are displayed in Figure 3. The dots on the horizontal axis for each segment depicted in the vertical axis represent the failure times for that segment. Additionally, each EST segment's censoring time is indicated by a cross [1].

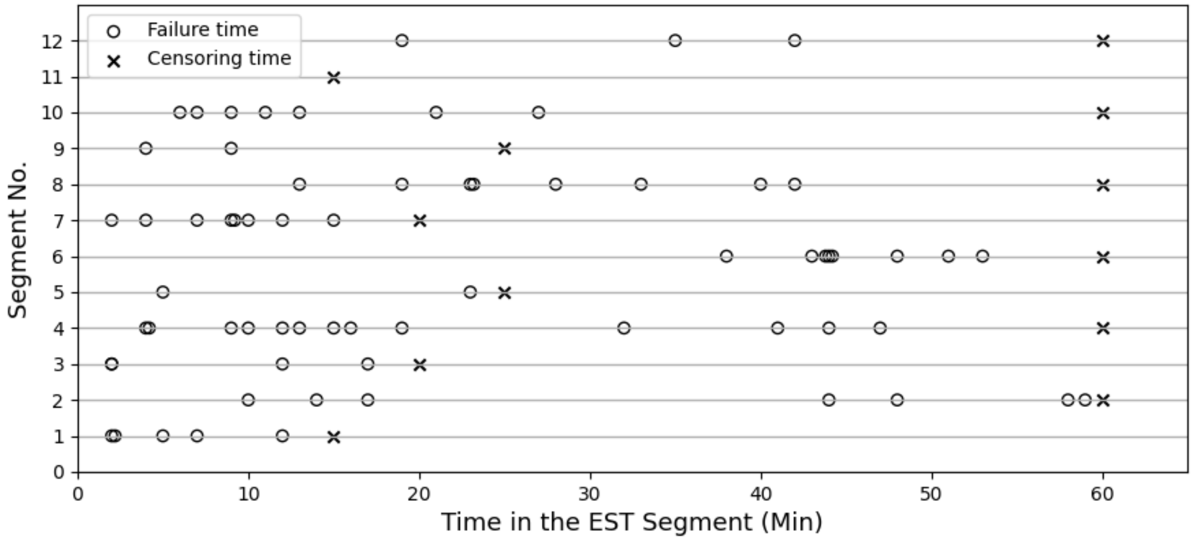


Fig. 3. Distribution of failures during the EST segments. Dots represent the failures and the crosses show the censoring time for each cycle segment. [1]

The experiment aims to combine the performance model of the unit during the EST and the lifetime model of the unit in the field to form an integrated cost model. During the EST testing process, it is difficult to determine whether a failed device is due to a defect in the device itself or the extreme environment of the EST. In addition, we are unable to do further experiments to find out the true effect of the EST testing environment due to the lack of devices and products. Therefore, in this EST testing, we are unsure how many of the 67 failed devices are good devices that were damaged by the testing. In order to simulate the actual situation with the greatest probability, we assume that the number of good devices that are damaged by the testing ranges from 10 to 25. This range is already quite wide, and we believe it is sufficient to cover most possible situations. Finally, we selected 10/15/20/25 good devices that were tested to be damaged, accounting for 15%/22%/30%/37% of the total damaged devices. We will assume that in all products that were originally broken in the real test , 10/15/20/25 products are corrupted because of the experimental environment, that is, they would not be corrupted in real use, we will randomly pick 10/15/20/25 data and assume that these data are not corrupted. We will discuss the effect of different quantities on the total cost separately. At the same time, in order to eliminate the influence of randomness on the experiment, we will run the experiment for 1000 times. All the data in the following are the average values after 1000 experiments.

Let us assume that the total cost of testing a unit has both fixed and variable cost elements. We assume a fixed cost of $C_f = \$6$ per unit for performing EST regardless of test duration, and a variable cost of $C_V = \$2$ per unit for a 480-minute test. Assume that the maintenance cost of each faulty unit is $C_R = \$6$ during testing. We also assume that the warranty and total costs $C_W = \$90$ for each device that fails in the field during the warranty period. Equipment that fails after the warranty period will have no cost imposed on the manufacturer. Next, we will analyze and discuss the impact of the number of failed products on the total cost in detail.

In the following experiments, ***PF*** will represent the number of units that fail in EST, but work properly in the real environment, that is, the SS2 part.

A. 10 Points

When it is assumed that 10 of the units that fail in the original test are damaged due to testing (***PF=10***), but do not fail in actual use, its impact on the comprehensive cost is calculated as follows. First estimate the probability of passing the test when the EST test lasts for different times, we will use equations (6)-(9) to calculate the probability of passing the EST.

Table V. Estimated probability of passing the EST of duration τ< τ0

τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)
0	1.0000	70	0.9955	140	0.9875	210	0.9862	280	0.9782	350	0.9746	420	0.9715
10	0.9982	80	0.9938	150	0.9871	220	0.9858	290	0.9768	360	0.9724	430	0.9715
10	0.9978	90	0.9933	160	0.9866	230	0.9835	300	0.9759	370	0.9720	440	0.9710
10	0.9969	100	0.9920	170	0.9866	240	0.9826	310	0.9755	380	0.9715	450	0.9710
10	0.9964	110	0.9897	180	0.9862	250	0.9800	320	0.9755	390	0.9715	460	0.9706
10	0.9964	120	0.9889	190	0.9862	260	0.9791	330	0.9746	400	0.9715	470	0.9702
10	0.9960	130	0.9884	200	0.9862	270	0.9791	340	0.9746	410	0.9715	480	0.9702

We will use Equation (10) to calculate the partial faults transferred from the test to the field during the test.

Table VI. Portion of failures that move from the EST to the field for EST duration of τ< τ0

τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F
0	0.0298	70	0.0254	140	0.0174	210	0.0160	280	0.0080	350	0.0044	420	0.0013
10	0.0280	80	0.0236	150	0.0169	220	0.0156	290	0.0067	360	0.0022	430	0.0013
20	0.0276	90	0.0231	160	0.0165	230	0.0133	300	0.0058	370	0.0018	440	0.0008
30	0.0267	100	0.0218	170	0.0165	240	0.0125	310	0.0053	380	0.0013	450	0.0008
40	0.0263	110	0.0196	180	0.0160	250	0.0098	320	0.0053	390	0.0013	460	0.0004
50	0.0263	120	0.0187	190	0.0160	260	0.0089	330	0.0044	400	0.0013	470	0.0000
60	0.0258	130	0.0183	200	0.0160	270	0.0089	340	0.0044	410	0.0013	480	0.0000

Having obtained the probability of passing and the probability of stopping the experiment before 480 minutes and not being detected as a failure, we can calculate the probability of a failure occurring within the warranty period.

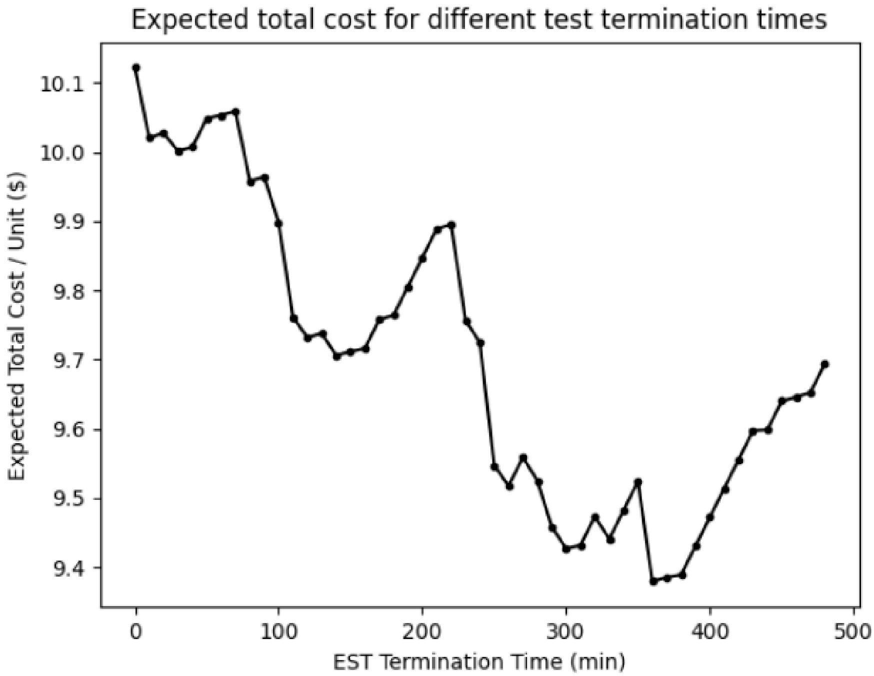
Table VII. Probability of failure during the warranty period for an EST with duration of τ< τ0

τ	p	τ	p	τ	p	τ	p	τ	p	τ	p	τ	p
0	0.0458	70	0.0414	140	0.0334	210	0.0320	280	0.0240	350	0.0204	420	0.0173
10	0.0440	80	0.0396	150	0.0329	220	0.0316	290	0.0227	360	0.0182	430	0.0173
20	0.0436	90	0.0391	160	0.0325	230	0.0293	300	0.0218	370	0.0178	440	0.0168
30	0.0427	100	0.0378	170	0.0325	240	0.0285	310	0.0213	380	0.0173	450	0.0168
40	0.0423	110	0.0356	180	0.0320	250	0.0258	320	0.0213	390	0.0173	460	0.0164
50	0.0423	120	0.0347	190	0.0320	260	0.0249	330	0.0204	400	0.0173	470	0.0160
60	0.0418	130	0.0343	200	0.0320	270	0.0249	340	0.0204	410	0.0173	480	0.0160

Finally, we can get the estimated test warranty total cost.

Table VIII. Expected total test–warranty costs for the warranty period of 1 year and the EST duration τ< τ0

τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]
0	10.12	70	10.06	140	9.71	210	9.89	280	9.53	350	9.52	420	9.56
10	10.02	80	9.96	150	9.72	220	9.90	290	9.46	360	9.38	430	9.60
20	10.03	90	9.97	160	9.72	230	9.76	300	9.43	370	9.38	440	9.60
30	10.00	100	9.90	170	9.76	240	9.73	310	9.43	380	9.39	450	9.64
40	10.01	110	9.76	180	9.77	250	9.55	320	9.47	390	9.43	460	9.65
50	10.05	120	9.74	190	9.81	260	9.52	330	9.44	400	9.47	470	9.65
60	10.06	130	9.74	200	9.85	270	9.56	340	9.48	410	9.51	480	9.69



Compared with the results of the original experiment, it can be found that the trend and peak value of the total cost have changed. The original data generally shows a downward trend, and the lowest cost value occurs around 390 minutes. However, in our experiment, the trend of total cost is not monotone, but fluctuates. At the same time, it is obvious that the lowest value of total cost of our experiment (9.39) is less than the lowest value of the original experiment (9.44), and the lowest value has already appeared in about 360 minutes, which greatly reduces the experimental time.

The total cost of testing is the sum of the test cost and the warranty cost. The test cost increases as the testing time increases, while the warranty cost decreases as the testing time increases since from the testing time could estimate the actual warrant time range. In our hypothetical scenario, the test cost remains unchanged compared to the original paper, while the maintenance cost decreases compared to the original paper. Therefore, the result displayed in this graph shows a further reduction in the total cost, and the time at which the minimum cost occurs is earlier.

B. 15 Points

When it is assumed that 15 of the products that fail in the original test are damaged due to testing (***PF=15***), but do not fail in actual use, its impact on the total cost is calculated as follows.

First estimate the probability of passing the test for different EST durations:

Table V. Estimated probability of passing the EST of duration $\tau < \tau 0$

τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)
0	1.0000	70	0.9959	140	0.9886	210	0.9874	280	0.9801	350	0.9768	420	0.9740
10	0.9983	80	0.9943	150	0.9882	220	0.9869	290	0.9789	360	0.9748	430	0.9740
20	0.9979	90	0.9939	160	0.9878	230	0.9849	300	0.9780	370	0.9744	440	0.9736
30	0.9972	100	0.9927	170	0.9878	240	0.9841	310	0.9776	380	0.9740	450	0.9736
40	0.9967	110	0.9906	180	0.9874	250	0.9817	320	0.9776	390	0.9740	460	0.9732
50	0.9967	120	0.9898	190	0.9874	260	0.9809	330	0.9768	400	0.9740	470	0.9728
60	0.9963	130	0.9894	200	0.9874	270	0.9809	340	0.9768	410	0.9740	480	0.9728

When moving from EST test to field, the portion of failures under different test durations:

Table VI.	Portion of failures that move from the EST to the field for EST duration of τ< τ0												
τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F
0	0.0272	70	0.0231	140	0.0158	210	0.0146	280	0.0073	350	0.0040	420	0.0012
10	0.0255	80	0.0215	150	0.0154	220	0.0141	290	0.0061	360	0.0020	430	0.0012
20	0.0251	90	0.0211	160	0.0150	230	0.0121	300	0.0052	370	0.0016	440	0.0008
30	0.0244	100	0.0199	170	0.0150	240	0.0113	310	0.0048	380	0.0012	450	0.0008
40	0.0239	110	0.0178	180	0.0146	250	0.0089	320	0.0048	390	0.0012	460	0.0004
50	0.0239	120	0.0170	190	0.0146	260	0.0081	330	0.0040	400	0.0012	470	0.0000
60	0.0235	130	0.0166	200	0.0146	270	0.0081	340	0.0040	410	0.0012	480	0.0000

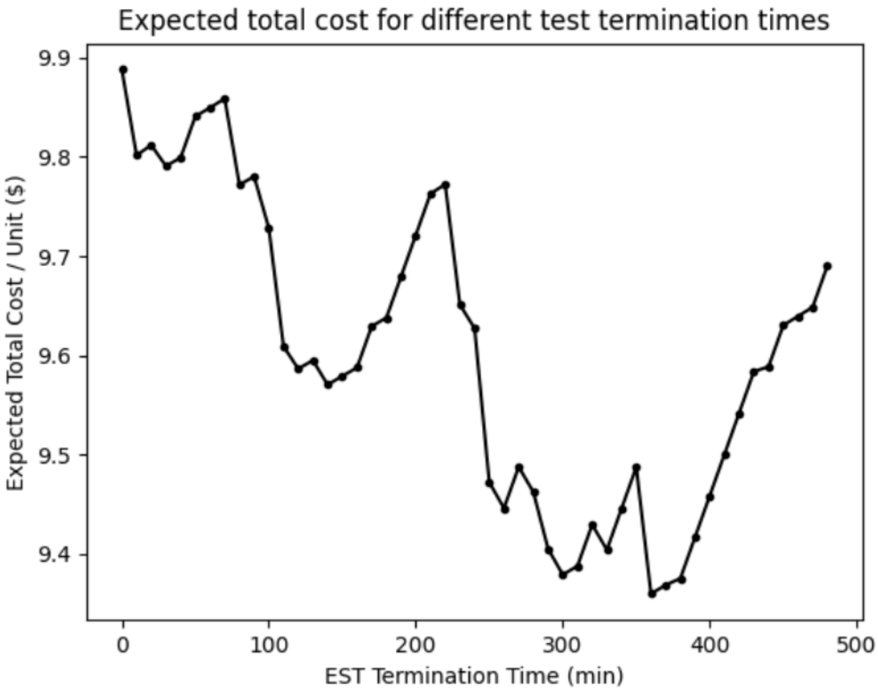
Next, calculate the probability that the test units will break during the warranty period after they pass the tests of different durations:

Table VII.	Probability of failure during the warranty period for an EST with duration of τ< τ0												
τ	p	τ	p	τ	p	τ	p	τ	p	τ	p	τ	p
0	0.0432	70	0.0391	140	0.0318	210	0.0306	280	0.0233	350	0.0200	420	0.0172
10	0.0415	80	0.0375	150	0.0314	220	0.0301	290	0.0221	360	0.0180	430	0.0172
20	0.0411	90	0.0371	160	0.0310	230	0.0281	300	0.0212	370	0.0176	440	0.0168
30	0.0404	100	0.0359	170	0.0310	240	0.0273	310	0.0208	380	0.0172	450	0.0168
40	0.0399	110	0.0338	180	0.0306	250	0.0249	320	0.0208	390	0.0172	460	0.0164
50	0.0399	120	0.0330	190	0.0306	260	0.0241	330	0.0200	400	0.0172	470	0.0160
60	0.0395	130	0.0326	200	0.0306	270	0.0241	340	0.0200	410	0.0172	480	0.0160

Finally, the total cost of testing and warranty period of 1 year with different duration is calculated:

Table VIII.	Expected total test–warranty costs for the warranty period of 1 year and the EST duration τ< τ0												
τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]
0	9.89	70	9.86	140	9.56	210	9.76	280	9.46	350	9.48	420	9.54
10	9.80	80	9.77	150	9.57	220	9.77	290	9.40	360	9.36	430	9.58
20	9.81	90	9.78	160	9.58	230	9.65	300	9.37	370	9.37	440	9.59
30	9.79	100	9.73	170	9.62	240	9.62	310	9.38	380	9.38	450	9.63
40	9.80	110	9.60	180	9.63	250	9.47	320	9.42	390	9.42	460	9.64
50	9.84	120	9.58	190	9.67	260	9.44	330	9.40	400	9.46	470	9.65
60	9.85	130	9.59	200	9.72	270	9.48	340	9.44	410	9.50	480	9.69

The above total cost and EST test termination time were plotted as the following figure for analysis. It can be observed that the trend when PF=15 is roughly similar to that at PF=10, with larger fluctuations relative to the original data. But on the whole, the total cost of PF=15 is slightly less than the cost of PF=10. At 360 minutes of the test, the lowest total cost was 9.36. In the following section, we will further explore the impact of 20/25 on the total cost to further verify it.



C. 20 Points

We assumed that 20 of the products with a test result of Fail were damaged due to the experimental environment and were normal in actual use. We randomly selected 20 samples with a Fail test result, and assuming they were not damaged, the impact on the total cost was calculated as follows:
First estimate the likelihood of passing the test when the EST test lasts for different times. The results are shown in the table below:

Table V. Estimated probability of passing the EST of duration $\tau < \tau 0$

τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)
0	1.0000	70	0.9963	140	0.9897	210	0.9886	280	0.9791	350	0.9791	420	0.9765
10	0.9985	80	0.9948	150	0.9893	220	0.9882	290	0.9772	360	0.9772	430	0.9765
10	0.9981	90	0.9944	160	0.9889	230	0.9864	300	0.9768	370	0.9768	440	0.9761
10	0.9974	100	0.9933	170	0.9889	240	0.9857	310	0.9765	380	0.9765	450	0.9761
10	0.9970	110	0.9915	180	0.9886	250	0.9835	320	0.9765	390	0.9765	460	0.9757
10	0.9970	120	0.9908	190	0.9886	260	0.9827	330	0.9765	400	0.9765	470	0.9754
10	0.9967	130	0.9884	200	0.9886	270	0.9827	340	0.9765	410	0.9765	480	0.9754

When moving from EST testing to the field, the proportion of failures for different test durations is as follows:

Table VI. Portion of failures that move from the EST to the field for EST duration of $\tau < \tau 0$

τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F
0	0.0246	70	0.0209	140	0.0150	210	0.0132	280	0.0073	350	0.0036	420	0.0011
10	0.0231	80	0.0195	150	0.0143	220	0.0132	290	0.0066	360	0.0036	430	0.0011
20	0.0227	90	0.0191	160	0.0139	230	0.0128	300	0.0055	370	0.0018	440	0.0011
30	0.0220	100	0.0190	170	0.0135	240	0.0110	310	0.0048	380	0.0014	450	0.0007
40	0.0216	110	0.0179	180	0.0135	250	0.0103	320	0.0044	390	0.0011	460	0.0007
50	0.0216	120	0.0161	190	0.0132	260	0.0081	330	0.0044	400	0.0011	470	0.0003
60	0.0213	130	0.0154	200	0.0132	270	0.0073	340	0.0037	410	0.0011	480	0.0000

Next, the probability that the test units will become damaged during the warranty period when they have been tested for different durations is calculated as follows:

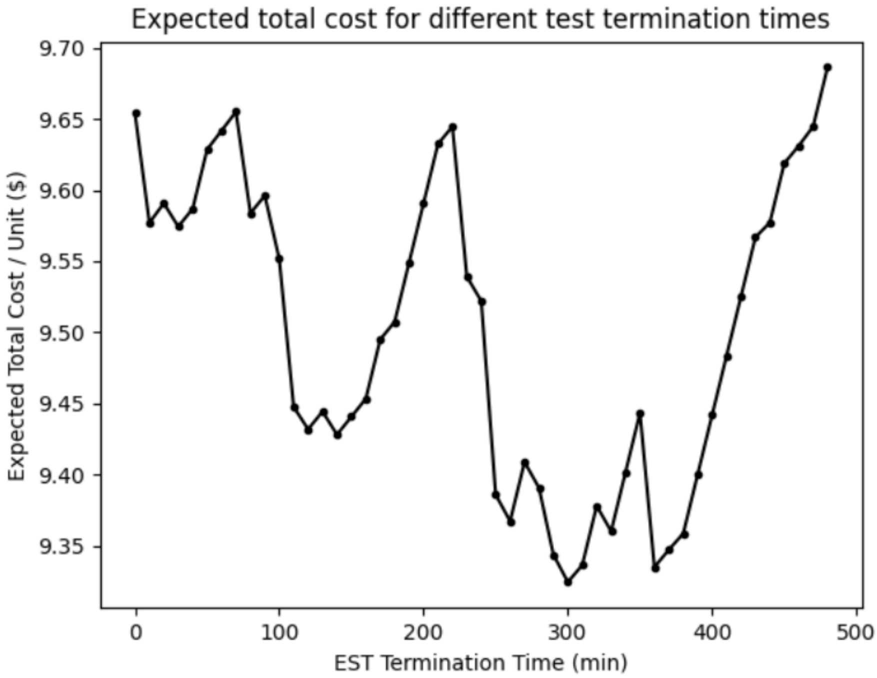
Table VII. Probability of failure during the warranty period for an EST with duration of $\tau < \tau 0$

τ	p	τ	p	τ	p	τ	p	τ	p	τ	p	τ	p
0	0.0405	70	0.0369	140	0.0303	210	0.0292	280	0.0226	350	0.0196	420	0.0171
10	0.0391	80	0.0354	150	0.0299	220	0.0288	290	0.0215	360	0.0178	430	0.0171
20	0.0387	90	0.0351	160	0.0295	230	0.0270	300	0.0208	370	0.0174	440	0.0167
30	0.0380	100	0.0339	170	0.0295	240	0.0263	310	0.0204	380	0.0171	450	0.0167
40	0.0376	110	0.0321	180	0.0292	250	0.0241	320	0.0204	390	0.0171	460	0.0163
50	0.0376	120	0.0314	190	0.0292	260	0.0233	330	0.0196	400	0.0171	470	0.0160
60	0.0373	130	0.0310	200	0.0292	270	0.0233	340	0.0196	410	0.0171	480	0.0160

Finally, we can calculate the total cost for 1 year of testing and warranty for different durations, with the results is as follows:

Table VIII. Expected total test–warranty costs for the warranty period of 1 year and the EST duration $\tau < \tau 0$

τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]
0	9.65	70	9.65	140	9.42	210	9.63	280	9.39	350	9.44	420	9.52
10	9.67	80	9.58	150	9.44	220	9.64	290	9.34	360	9.34	430	9.56
20	9.59	90	9.59	160	9.45	230	9.54	300	9.33	370	9.34	440	9.57
30	9.57	100	9.55	170	9.49	240	9.52	310	9.34	380	9.36	450	9.62
40	9.59	110	9.44	180	9.50	250	9.38	320	9.38	390	9.40	460	9.63
50	9.63	120	9.43	190	9.54	260	9.37	330	9.36	400	9.44	470	9.64
60	9.64	130	9.44	200	9.59	270	9.41	340	9.41	410	9.48	480	9.68



As shown in the figure above, there is a noticeable difference in trend between PF=20 and PF=15. However, overall, the total cost for PF=20 is slightly lower than that for PF=15. In the 300-minute test, the lowest total cost is 9.33. In the next section, we will further explore the impact of 25 on the total cost to further verify.

D. 25 Points

Subsequently, we hypothesize that among all the products initially damaged during the experiments, 25 of them were damaged as a result of the experimental environment, but would remain undamaged under actual usage conditions. We randomly selected 25 samples and assumed that these data points were not damaged, in order to investigate the implications for the total costs.

Firstly, we compute the estimated probability of passing the EST at varying duration. It is obvious that as the temperature increases, the probability of passing the EST decreases.

Table V. Estimated probability of passing the EST of duration τ< τ0

τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)	τ	1-P(SS)
0	1.0000	70	0.9967	140	0.9907	210	0.9898	280	0.9845	350	0.9813	420	0.9790
10	0.9986	80	0.9953	150	0.9904	220	0.9882	290	0.9839	360	0.9813	430	0.9790
20	0.9983	90	0.9951	160	0.9901	230	0.9894	300	0.9829	370	0.9796	440	0.9790
30	0.9976	100	0.9940	170	0.9901	240	0.9878	310	0.9822	380	0.9793	450	0.9787
40	0.9973	110	0.9924	180	0.9898	250	0.9871	320	0.9819	390	0.9790	460	0.9787
50	0.9973	120	0.9917	190	0.9898	260	0.9852	330	0.9819	400	0.9790	470	0.9783
60	0.9970	130	0.9914	200	0.9898	270	0.9845	340	0.9813	410	0.9790	480	0.9779

And calculate the portion of failures that move from the EST to the field over time.

Table VI. Portion of failures that move from the EST to the field for EST duration of τ< τ0

τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F	τ	PEST->F
0	0.0219	70	0.0186	140	0.0127	210	0.0117	280	0.0058	350	0.0032	420	0.0009
10	0.0206	80	0.0173	150	0.0124	220	0.0114	290	0.0049	360	0.0016	430	0.0009
20	0.0203	90	0.0170	160	0.0121	230	0.0097	300	0.0042	370	0.0013	440	0.0006
30	0.0196	100	0.0160	170	0.0121	240	0.0091	310	0.0039	380	0.0009	450	0.0006
40	0.0193	110	0.0143	180	0.0117	250	0.0071	320	0.0039	390	0.0009	460	0.0003
50	0.0193	120	0.0137	190	0.0117	260	0.0065	330	0.0032	400	0.0009	470	0.0000
60	0.0189	130	0.0134	200	0.0117	270	0.0065	340	0.0032	410	0.0009	480	0.0000

Then we can calculate the probability of failure during the warranty period and the results are in the following Table VII.

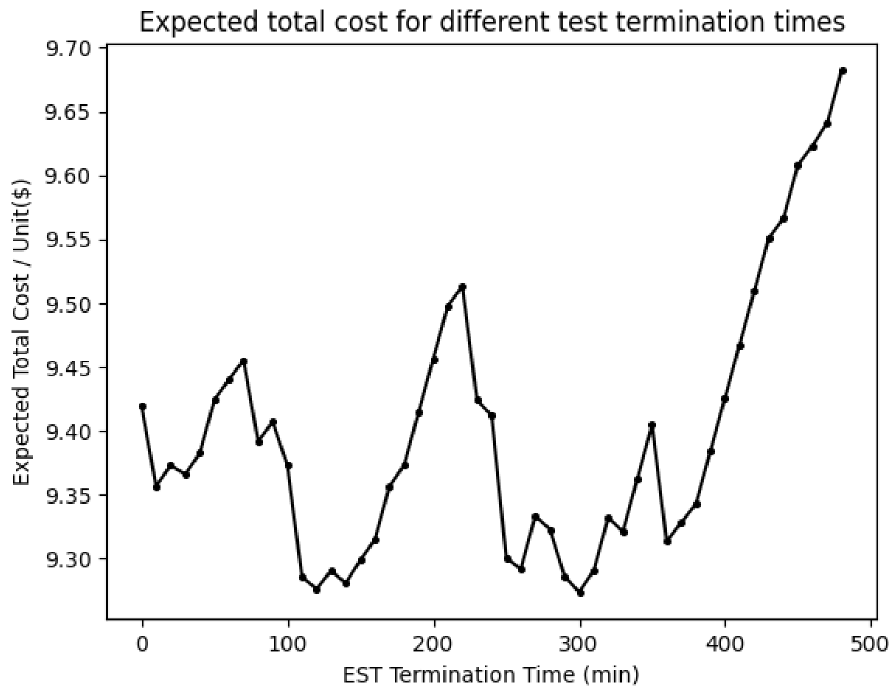
Table VII. Probability of failure during the warranty period for an EST with duration of τ< τ0

τ	p	τ	p	τ	p	τ	p	τ	p	τ	p	τ	p
0	0.0379	70	0.0346	140	0.0287	210	0.0277	280	0.0218	350	0.0192	420	0.0169
10	0.0366	80	0.0333	150	0.0284	220	0.0274	290	0.0208	360	0.0176	430	0.0169
20	0.0363	90	0.0330	160	0.0281	230	0.0257	300	0.0202	370	0.0172	440	0.0166
30	0.0356	100	0.0320	170	0.0281	240	0.0251	310	0.0199	380	0.0169	450	0.0166
40	0.0353	110	0.0303	180	0.0277	250	0.0231	320	0.0199	390	0.0169	460	0.0166
50	0.0353	120	0.0297	190	0.0277	260	0.0225	330	0.0192	400	0.0169	470	0.0160
60	0.0349	130	0.0293	200	0.0277	270	0.0225	340	0.0192	410	0.0169	480	0.0160

Finally we can get the expected total test-warranty costs per unit for the warranty period of 1 year and plot the outcome.

Table VIII. Expected total test–warranty costs for the warranty period of 1 year and the EST duration τ< τ0

τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]	τ	E[TC]
0	9.42	70	9.45	140	9.28	210	9.49	280	9.32	350	9.36	420	9.51
10	9.35	80	9.39	150	9.29	220	9.51	290	9.28	360	9.31	430	9.55
20	9.37	90	9.41	160	9.31	230	9.42	300	9.27	370	9.33	440	9.56
30	9.36	100	9.37	170	9.35	240	9.41	310	9.29	380	9.34	450	9.60
40	9.38	110	9.28	180	9.37	250	9.29	320	9.33	390	9.38	460	9.62
50	9.42	120	9.27	190	9.41	260	9.28	330	9.32	400	9.42	470	9.64
60	9.43	130	9.29	200	9.45	270	9.33	340	9.36	410	9.47	480	9.68



As shown in the above figure, the unit expected total cost generally increases with the increase of testing time. The trend of total cost is quite different from the first three sets of data (10, 15, and 20) in this study. When the test termination time is at 120, the lowest unit expected total cost is 9.27.

According to the formula, the total cost of testing is the sum of the test cost and warranty cost. The test cost increases with the increase of testing time, while the warranty cost decreases with the increase of testing time. In our assumed scenario with PF points, according to our formula (6), the test cost is unchanged compared to the original paper. According to formula (8) $\frac{P(LD-SS_1|\theta,\varphi)}{1-P(SS_1|\theta)}$, with the increase of PF points, the SS_1 decreases, the $1 - P(SS_1|\,\theta)$ will increase. As $P(LD - SS_1) = 0.016 + P_{EST\to F}$ and $P_{EST\to F} = P(SS_1|\,\tau_0;\theta) - P(SS_1|\,\tau;\theta)$, so when the SS_1 decreases, the $P_{EST\to F}$ decreases, and thereby $\frac{P(LD-SS_1|\theta,\varphi)}{1-P(SS_1|\theta)}$ decreases. So the warranty cost gradually decreases, as the PF points increase.

Through the above experiments, it can be seen that with the increase of PF points, the volatility of the total cost continues to increase. When there are only 10 PF points, the total cost result is almost consistent with the original experimental result, showing a V-shaped structure with a similar trend. However, when the number of PF points transitions to 25, the fluctuation of the total cost is very large, and it is very different from the original experiment. With the increase of PF points, the time of the lowest cost appearance is gradually decreasing, and the minimum cost is also gradually decreasing. This is consistent with the theoretical analysis above, so we can draw the conclusion that the larger the number of PF points, the smaller the total cost, and the shorter the time for the lowest cost to appear.

V. CONCLUSION

In this paper, we propose an improved environmental stress testing model based on the Weibull model. This model means that a part of the units that fail in the test because of the experimental environment can work normally in the real environment, and this part will affect the total cost of the experiment. We use the same method as in the original paper to construct the model and calculate the total cost, and change the number of SS_2 parts in the test. At the same time, in order to reduce the influence of random rows on the experiment, we repeat the experiment 1000 times and take the results after flattening the experimental results.

According to our stress testing model, we have come to the conclusion that the larger the number of PF points, the lower the overall cost and the shorter the time to achieve the minimum cost. Companies can estimate the proportion of PF devices in failed equipment based on their actual situation. By selecting suitable PF data, companies can spend less money and time on EST testing, obtain more realistic simulation results, and make better testing and maintenance plans. Our model also addresses the limitations of the original experiment, making the results more realistic, more practical, and more accurate in estimating the overall cost.

VI. FUTHER WORK

In our current experiment, we improved the data of the failed EST test based on the original paper, taking into account the units that were damaged due to the test but were normal in practice, and the units that failed the test in the first place. After the above experiments, we found that it is very meaningful to explore this part. Because the overall cost of the test and warranty does change under different assumptions, and the optimal time to stop testing also changes. Therefore, the next urgent task is to determine the number of PFs and the distribution of them based on time is possible, as it is crucial for the impact of the test. Meanwhile, our definition of the warranty part of the total cost is 1 year, and there is undoubtedly some room for improvement in this part. For example, you could try to figure out how to minimize the total cost for different warranty years, rather than just one year. At the same time, the data and results mentioned in this paper are from simulation experiments, not real experiments. Therefore, in order to make the experimental data more realistic, an experiment should be carried out according to the method mentioned in this paper and real data should be obtained. The project can be further expanded from the above three aspects.

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