

LOGIC

PROPOSITIONAL LOGIC (Deals with propositions)

1

Proposition! A declarative statement that is either true or false

but not both.

Example J Ahmedabad is the capital of Gujarat. F
| Proposition

$$\begin{array}{c} 2 \\ 5+2 \\ - \\ \hline 4 \\ F \\ =2 \quad T \end{array}$$

© 1+1

pass the paper. (not a proposition as not declaring anything)

4 Please

$x+5=7$ (not a proposition but a predicate logic) not propositions and expressions

- Requests, orders

that

are

cannot be evaluated as

T or F are

not
propositions
propositio
ns.

Propositional variable

or statement variables

are denoted
by

are denoted by

T or F.

\$, q, r, s,
Truth values

propositions, compound propositions

can be formed from simple

or

operators using logical connectives

© Let $\&$ be a proposition.

negation of denoted

The p

by $Tp"$, is the

statement that,

The

a It is not the case that

$p"$. truth value of $+p$ is the

opposite

F

the

@ $P \wedge q$ (conjunction/ and) is the statement "p and q". P: Today is Friday
9! It is raining today

Example

\$19: Today is Friday and truth value of p. it is raining

raining today.

The statement is T on

rainy Example : Today is friday.idays and is F on
any day

tp

I that

not the

case To: "It is not

Friday" or "Today is is
Friday

today is

not friday"

Truth Table

friday & on that is not a fridays when it does not
rain. Table (T.T.)

Truth Table

P

A

q

▶ 19

가..

\$

T

T

T

14

T

F

T

F

T

F

FT

F

F

FF

F

a

3 Disjunction p \vee q : "P or q

"

T.T

P

9

p \vee q

T T

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T

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F
F
FT

Exclusive OR]

pa
T.T
@q
O q
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T
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T
f
T
FIT
T
F

Example

(Inclusive
OR)

2

Same as the previous
it is raining
today.

Today is Friday or

True

on any day that is either friday or a rainy
day (including the rainy fridays),
false on

on days
days that are not fridays when it also
does not rain.

\$@q: " either & or q

but not both"

True when exactly one of them is true and is
false otherwise

G

ff

Conditional
Statements

pq is a false when

Conclusio

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consequ
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is

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a

a

\$ → q!" If & then q"

\$ → 9:

p is T & q is F & true
otherwise.

3

←

4

"If p then q
" If p, q

q

q

a
n
y

2

"

is sufficient for q
if

when p

3

necessary condition for p is q" unless + p

imp

lies

q"

q"

only if

q

n

a sufficient condition for q is p"

P

a

b

a

a

q

9

a

whenever p"

p"

is necessary for p.

q follows from p"

T. T.
Ion verse

2

Inverse & Contra positive

9

p
is the

79
is the

+

p

+ q → +p is the
7q

Biconditional

s

bqpq

Converse of $p \rightarrow q$

inverse

7 4→q Contra positive of
 $p \rightarrow q$.

449

и

1 if and only if q"

3

T
T
T
TF
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7

4

4

T. T.

Example

T

T

q

T

F

True only when both are true or both are false & false otherwise.

Compound propositions

$(p \vee q) \rightarrow (p \wedge q)$

s

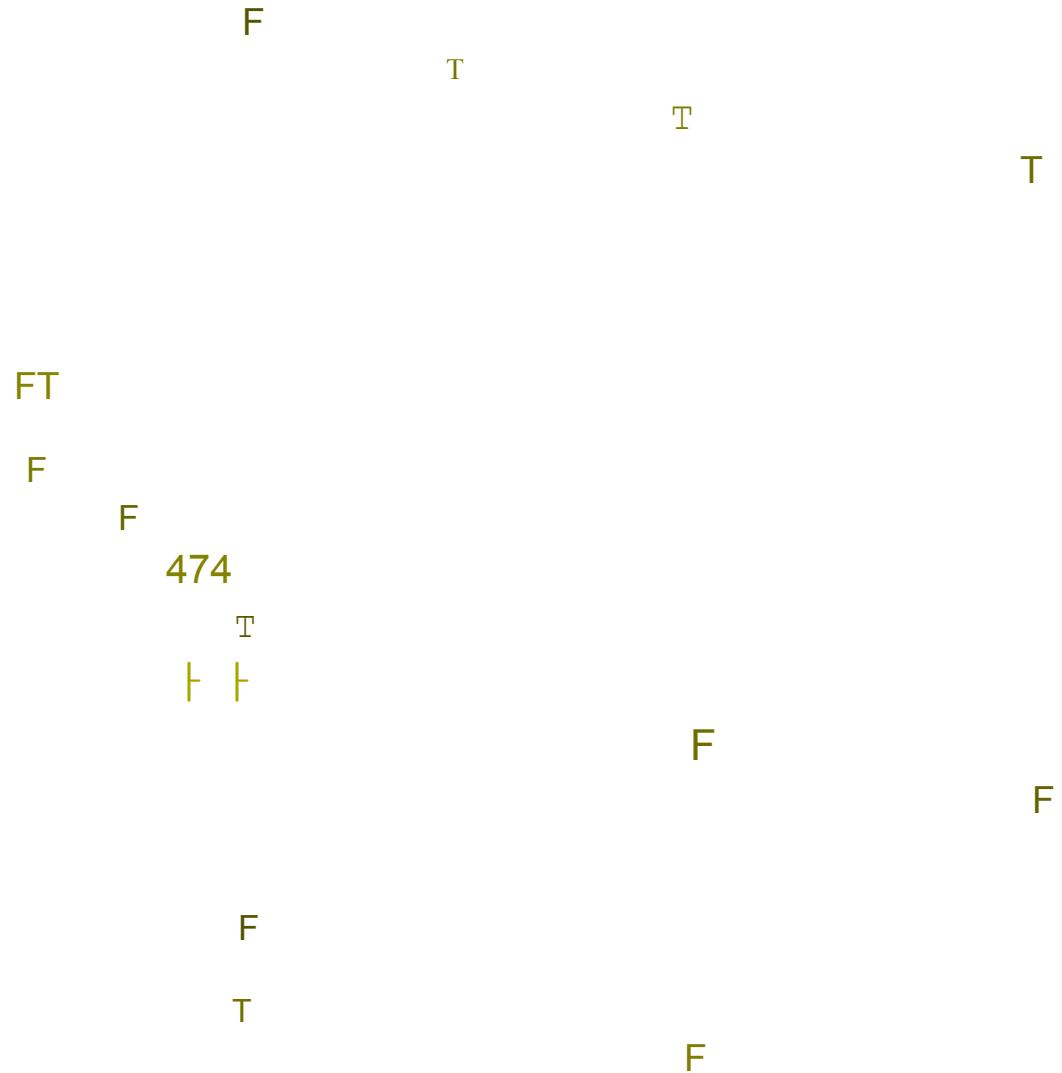
+9

PV + q

\$19

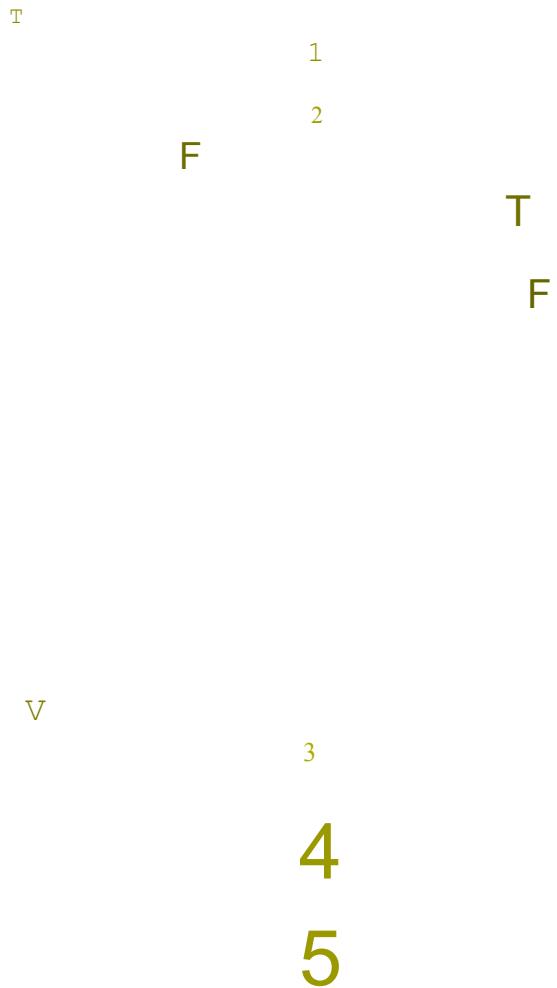
$(p \vee q)$

, (рля)



Precedence of logical operators

operator
Precedence



Propositional Equivalence

A compound proposition that is always true no matter what the truth values of the propositions that

Called

Tautolog

y

that occur in it is

A compound proposition that is always false is called contradiction.

Contingency: Compound prop. that is neither tautology

hor

a

Contradiction.

Example

e

\$ VTP is

a tautology & $\neg(\neg p \vee q)$ is a

4

Contradiction.

T, T,

A

T

f

$\neg p$

f

T

Logical Equivalences

T

T

always true

БАР

F

4

4

F

always

are

false

compound propositions &

& q

pe p→ q a is a tautology

and

if

p

= q

Example

T. T.

n

or

a

+

7

6 q

и

(пла) = трина

(pvq) = p λ + q

+

pvq (pvq)

т

Logically
equivalent
is denoted by

De

Morgan 's

Lan

Б

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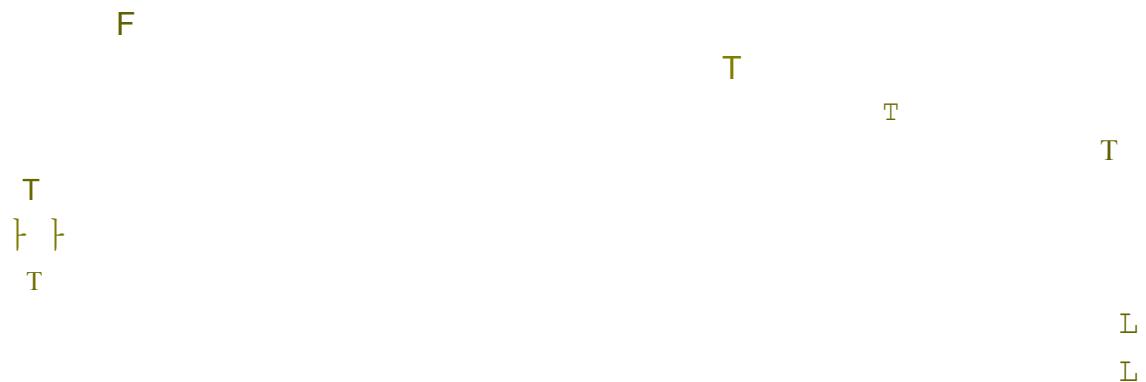
T

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Exempl

e

T, T,

F

F

B → q = +pv q

A

A

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9

+ p

трич

T

T

F

Example

$P_V(q \wedge r) = (pva) \ 1 \ (pur)$ Distributive

aar bv (918) pvq

Law
pur (puq) (pvr)

q

r

T

T
F
----4444
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F

$$+ (P1 \vee P2 \vee \dots \vee Pn) = \\ (+P_1 + P_2 + \dots + P_n)$$

Стр.алтра л....Атры)

$$\vee \neg S_n$$

$$TCP1 \wedge P2 \wedge \dots \wedge Pn) = (+P1 \wedge +P2 \wedge \dots \wedge +Pn)$$

Some

Logical Equivalences

$$PAT = P$$

Identity Laws

$$S \wedge T = T$$

Domination Laws.

$$b \vee F = p$$

$$p$$

PAFEF

Double

$$p \vee p = p$$

Idempotent Laws

$P \wedge P = P$

$p \wedge p$

$\neg(p \vee q) = \neg q \vee \neg p$

пред= аль

}

Commutative
Laws

$p \wedge (q \wedge r) = (p \wedge q) \wedge r$

$(p \vee q) \vee (p \vee r) = p \vee (q \vee r)$

$\neg(p \wedge (q \vee r)) = (\neg p) \vee (\neg q \vee \neg r)$

Distributive Laws

$p \vee (p \wedge$

$q) = p$

$(p \vee q) \wedge p = p$

Absorption

Laws

$+(+1) = P$ | Negation
Law

$(p \vee q) \vee r = p \vee (q \vee r)$ & Associative (\$19)^8 = P1

(910) Laws

$$+(p \cdot q) = +PV + q$$

$$+(p \vee q) = +P 1 + q$$

De Morgan's
Laws

Витрет PAP

F Negation

Laws

Conditional logical Equi.

$$P \rightarrow q = \neg p \vee q$$

$$P \rightarrow q = \neg q \rightarrow \neg p$$

=

$$P \vee q \models 19$$

$$\neg (\neg p \rightarrow q) =$$

$$\neg(\neg q \rightarrow \neg p)$$

$$p \wedge \neg q$$

$$(p \wedge q) \lambda(p \wedge r) = b + (918)$$

$$p \rightarrow r \wedge q \rightarrow r = p \vee q$$

$$\neg a \wedge (b \rightarrow q) \vee (b \rightarrow r) = p \rightarrow (q \vee r)$$

$$(p \rightarrow r) \vee (q \rightarrow r) = (\$19) \rightarrow r$$

Biconditional Equi

$$p \leftarrow q = (p \rightarrow q) \wedge (q \rightarrow$$

$$b) p \leftrightarrow q = \overrightarrow{\rightarrow} + q \text{ P}$$

$$\overrightarrow{\rightarrow} E = (p 19)$$

$$\sqrt{76119} + (P \leftarrow q) = P$$

79

$$+(p \neg q) = p 1 \neg q$$

$$\$ \rightarrow q = +p \vee q$$

4→

$$+ (+pva)$$

$$+ (pq) = +(\rightarrow \vee q)$$

$$+pvq$$

= Pλ+q (using De Morgan's Laws).

рата

(Example)

Show that

We

know that,

Then,

Bitwise operations

Bitwise OR

Bitwise AND, Bitwise

Bitwise XOR

Consider two

bit

x

y

bit string x

10110

and y=11011.

xly

xoy

1

1

1

and

1

1

O

1

1

1

1

1

Consider 0 as F

as T.

and computation follows bitwise

1

1

0

1

O

1