Question Bank - Recurrence Relation

- Problems 1 − 4 are based on the topic "Basics of recurrence relation".
- Problems 5 and 6 are based on the topic "Modelling problems using recurrence relations". •

Problems 7 − 13 are based on the topic "Solving recurrence relations using characteristic equations".

- Problems 14 − 18 are based on the topic "Generating function and solving recurrence relations using generating functions".

1. Determine which of these are linear homogeneous recurrence relations with constant coefficients. Also, find the order and degree of each of the recurrence relations.

Find a_0, a_1, a_2, $a3$, and a_4.

(i) $a_n = 3a_{n-1} + 4a_{n-2} + 5a_{n-3}$ (ii) $a_n = a^2_{n-1} + a_{n-2}$

(iv) $a_n = \frac{a_{n-1}}{n}$

(iii) $a_n = a_{n-1} + 2$

(v) $a_n = a_{n-1} + a_{n-4}$ (vi) $a_n = 3$

2. Let $a_n = 2^n + 5.3^n$ for $n = 0, 1, 2, \ldots$ a)

   b) Show that $a_2 = 5a_l - 6a_0$, $a_3 = 5a_2 - 6a_1$, and $a_4 = 5a_3 - 6a_2$.

3. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if

   d) $a_n = 2(-4)^n + 3$.

   a) $a_n = 0$. b) $a_n = 1$.

   c) $a_n = (-4)^n$.

4. Is the sequence an a solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ if $a_n = 0$, $a_n = 1$, $a_n = 2^n$, $a_n = 4n$, $a_n = n4^n$, $a_n = n^2 4^n$?

5. Suppose that the number of bacteria in a colony doubles every hour. Set up a recurrence relation for the number of bacteria after n hours have elapsed. Determine whether the recurrence relation is linear, homogeneous, and with constant coefficients. Also, find the order and degree of the recurrence relation. If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?

6. A new employee at an exciting new software company starts with a salary of \$30, 000 and is promised that at the end of each year her salary will be double her salary of the previous year, with an extra increment of \$5, 000 for each year she has been with the company. Construct a recurrence relation for her salary for her nth year of employment. Determine whether the recurrence relation is linear, homogeneous, and with constant coefficients. Also, find the order and degree of the recurrence relation.

7. Find the solution to $a_n = 7a_{n-2} + 6a_{n-3}$ with $a_0 = 9$, $a_1 = 10$, and $a_2 = 32$.

8. Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a_0 = 7$, $a_1 = -4$, and $a_2 = 8$. 9.

Solve the recurrence relation $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$.

10. Solve the recurrence relation $a_n = -3a_{n-1} + 4a_{n-3}$.

11. Solve the third-order homogeneous recurrence relation $a_n = 11a_{n-1} - 39a_{n-2} + 45a_{n-3}$ with the initial conditions $a_0 = 5$, $a_1 = 11$, $a_3 = 25$.

12. Determine the characteristic equation and the characteristic roots of the recurrence relation that represents the Fibonacci sequence. Also, find the solution.

13. What is the general form of the solutions of a linear homogeneous recurrence relation if its characteristic equation has roots 1, 1, 1, 1, −2, −2, −2, 3, 3, −4?

14. Find a closed form for the generating function for each of these sequences. (For each sequence, use the most obvious choice of a sequence that follows the pattern of the initial terms listed.)

a) 0, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, . . .

b) 0, 0, 0, 1, 1, 1, 1, 1, 1, . . .

c) 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, . . .

d) 2, 4, 8, 16, 32, 64, 128, 256, . . .

e) 2, −2, 2, −2, 2, −2, 2, −2, . . .

f) 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, . . .

g) 0, 0, 0, 1, 2, 3, 4, . . .

15. Find a closed form for the generating function for the sequence $\{a_n\}$, where

a) $a_n = 5 \ \forall \ n \geq 0$

b) $a_n = 3^n \ \forall \ n \geq 0$

c) $a_n = 2$ for $n \geq 3$ and $a_0 = a_1 = a_2 = 0$

d) $a_n = 2n + 3 \ \forall \ n \geq 0$

16. Use generating functions to solve the recurrence relation $a_k = 7a_{k-1}$ with the initial condition $a_0 = 5$.

17. Use generating functions to solve the recurrence relation $a_k = 5a_{k-1} - 6a_{k-2}$ with initial conditions $a_0 = 6$ and $a_1 = 30$.

18. Solve the recurrence $a_k = 2a_{k-1} + 3a_{k-2}$, $a_0 = 3$, $a_1 = 1$, using generating function. 2