



UIT

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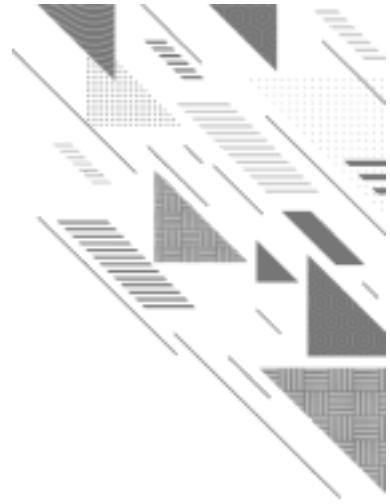
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B.Tech. Computer Science & Engineering

Semester-3

Data Structures and Algorithms

Course Code: 71203002002



Analyzing Control Statements



For Loop # Input : int A[n], array of n integers # Output : Sum of all numbers in array A

Algorithm: int Sum(int A[], int n)

{

1

}

```
for (int  
    i=0; i<n;  
    i++) s = s  
    + A[i];  
return s;
```

n

n+1

```
int s=0;
```



Running Time of Algorithm

- The time complexity of the algorithm is : $\text{steps} + \text{steps} = \text{steps}$

$\text{steps} + \text{steps}$

- Estimated running time for different values of n :

$$n = 4 \text{ steps}$$

$$n = 13 \text{ steps}$$

$$n = 100 \text{ steps}$$

$$n = 1000 \text{ steps}$$

- As n grows, the number of steps grow **in linear proportion** to n for the given algorithm Sum.
- The dominating term in the function of time complexity is n : As n gets large, the $+3$ becomes insignificant.

□ The time is linear in proportion to $\diamond\diamond\diamond$.

Analyzing Control Statements

Example 1:

- Statement is executed once only
 - So, The execution time $\Theta(\Theta)$ is some constant c

- ## ■ Analysis

Example 3:

Example 2:

$$\begin{aligned}
 & \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?} * \text{?} \diamond \text{?} \diamond \text{?} + \text{?} \diamond \text{?} \diamond \text{?} \\
 & \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?} = \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?}_1 + 1 + \\
 & \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?}_2 + 1 + \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?}_3 \\
 & \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?} (\text{?} \diamond \text{?}) = \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?}_1 + \\
 & \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?}_1 + \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?}_2 \text{?} \diamond \text{?}^2 + \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?}_2 \text{?} \diamond \text{?} + \\
 & \text{?} \diamond \text{?} \diamond \text{?} \diamond \text{?}_3 \text{?} \diamond \text{?}^2 \text{?} \diamond \text{?} (\text{?} \diamond \text{?}) = \text{?} \diamond \text{?}^2 (\text{?} \diamond \text{?})
 \end{aligned}$$

$+ \text{Total time is denoted as, } T$

T_1

T_2

$$T = T_1 + T_2 + T_3$$

\vdots

$$\begin{aligned} T &= T_1 + T_2 + T_3 \\ T_1 &= * \\ T_2 &= * \\ T_3 &= * \\ T &= * \end{aligned}$$

$$T = T_1 + T_2 + T_3 \approx T$$

$$T = T_1 + T_2 + T_3$$

$$T = T + 1$$

Analyzing Control Statements Example 4:

$$i = 0$$

$$i = i + 1$$

Example 5:

$$i = 0$$

$$i = i + 1$$

$$i = i + 1$$

$$i = i + 1$$

Example 6:

$$i = i + 1$$

??????

????????? = 1 ??????

??????

????????? = ?????? + ?? *

??

????????? = 1 ??????

??????

????????? = ?????? - ??

+ 1

printf("sum is now
%d", ??????)

????? = ??? []
+ ??? ???? = ??? []



Sorting Algorithms

Bubble Sort, Selection Sort, Insertion Sort

Introduction

- Sorting is any process of arranging items systematically or arranging items in a sequence ordered by some criterion.
- Applications of Sorting
 1. Phone Bill: the calls made are date wise sorted.
 2. Bank statement or Credit card Bill: transactions made are date wise sorted.
 3. Filling forms online: “select country” drop down box will have the name of countries sorted in Alphabetical order.
 4. Online shopping: the items can be sorted price wise, date wise or relevance wise.
 5. Files or folders on your desktop are sorted date wise.

Bubble Sort – Example

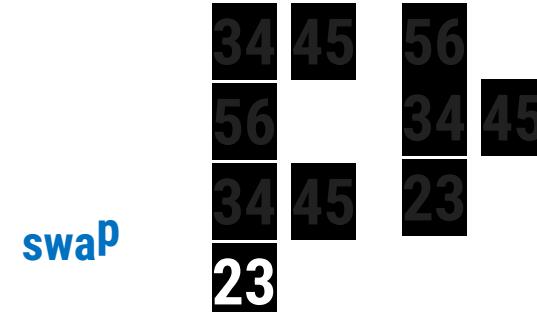
Sort the following array in Ascending order

45	34	56	23	12
----	----	----	----	----

Pass 1 :



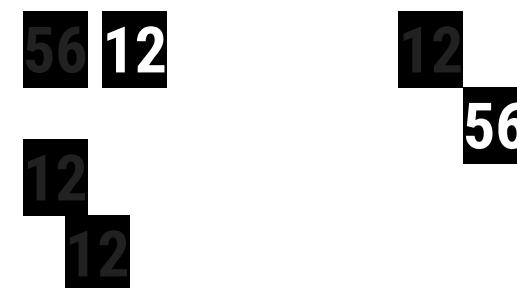
swap



swap



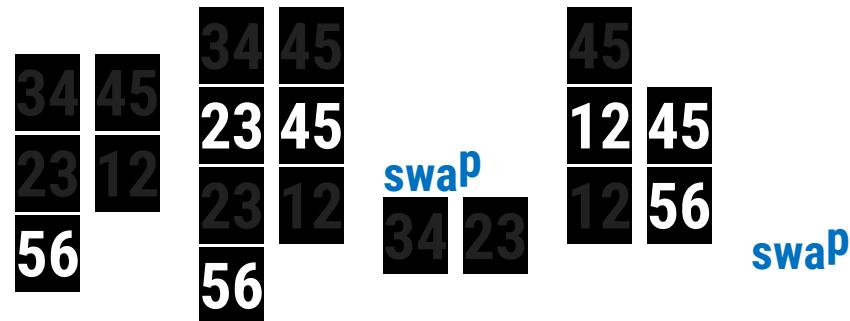
swap



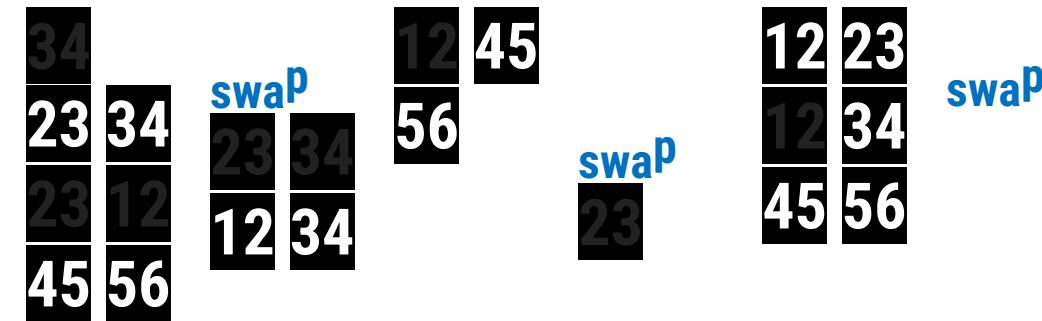
□ ? □ ? □ ? □ ? (□ ? □ ? [□ ? □ ?] > □ ? □ ? [□ ? □ ? + 1])
□ ? □ ? □ ? □ ? □ ? □ ? □ ? (□ ? □ ? [□ ? □ ?], □ ? □ ? [□ ? □ ? + 1])

Bubble Sort – Example Pass 3 :

Pass 2:



Pass 4 :



```
? ? ? ? ( ? ? [ ? ? ] >
? ? [ ? ? + 1 ])
? ? ? ? ? ? ? ? ( ? ?
[ ? ? ], ? ? [ ? ? +
1 ])
```

Bubble Sort - Algorithm # Input:
Array A

Output: Sorted array A

for $j \leftarrow 1$ to $n-i$ do if $A[j] >$
 $A[j+1]$ then

Algorithm: Bubble_Sort(A) for $i \leftarrow$ 1 to $n-1$ do

$A[j] \leftarrow$
 $A[j+1]$
 $A[j+1] \leftarrow$
temp
 \leftarrow swap($A[j]$,
 $A[j]$ $A[j+1]$)

Bubble Sort

- It is a simple sorting algorithm that works by comparing each pair of adjacent items and

swapping them if they are in the wrong order.

- The pass through the list is repeated until no swaps are needed, which indicates that the list is sorted.
- As it only uses comparisons to operate on elements, it is a comparison sort. □ Although the algorithm is simple, it is too slow for practical use.
- The time complexity of bubble sort is $\Theta(n^2)$

Algorithm:

Bubble_Sort(A)

Bubble Sort Algorithm int flag=1;

– Best Case Analysis #

Input: Array A

12

23

34

i = 1

j = 1 j = 2 j = 3

Output: Sorted array Pass 1:

A

for i ← 1 to n-1 do for j ← 1 to n-i do

true

Condition never becomes

45 59

j = 4

```
cout<<"already sorted" << endl  
if A[j] > A[j+1] then flag = 0; break;  
swap(A[j],A[j+1])  
if(flag == 1)
```



Selection Sort – Example 1

Sort the following elements in Ascending order

5	1	12	-5	16	2	12	14
---	---	----	----	----	---	----	----

Step 1 :

Unsorted Array

5	1	12	-5	16	2	12	14
---	---	----	----	----	---	----	----

1 2 3 4 5 6 7 8

5 1 12 -5 16 2 12 14 1 2 3 4 5 6 7 8

Step 2 :

Unsorted Array (elements 2 to 8)

- **Minj** denotes the current index and **Minx** is the value stored at current index.
- Assume that currently **Minx** is the smallest value.
- Now find the smallest value from the remaining entire Unsorted array.

-5 5

Swap

Index = 4, value = -5



Selection Sort – Example 1

Step 3 :

Unsorted Array (elements 3 to 8)

- Now **Minj** = 2, **Minx** = 1
- Find min value from remaining

-5 1 12 5 16 2 12 14

1

unsorted array

1 2 3 4 5 6 7 8

Index = 2, value = 1

No Swapping as min value is already at right place

Step 4 :

**Unsorted Array
(elements 4 to 8)**

-5 1 12 5 16 2 12 14
2 12

1 2 3 4 5 6 7 8

- Minj = 3, Minx = 12
- Find min value from remaining unsorted array

Index = 6, value = 2

Swap



Selection Sort – Example 1

8)

min value from remaining

Step 5 :

Unsorted Array (elements 5 to

-5 1 2 5 16 12 12 14

5

1 2 3 4 5 6 7 8

- Now Minj = 4, Minx = 5 ▪ Find

unsorted array

Index = 4, value = 5

(elements 6 to 8)

Step 6 :

No Swapping as min value is already

at right place ▪ Minj = 5, Minx = 16

-5 1 2 5 16 12 12 14
12 16

1 2 3 4 5 6 7 8 Swap

- Find min value from remaining unsorted array

Unsorted Array

Index = 6, value = 12



Selection Sort – Example 1

8)

Step 7 :

Unsorted Array (elements 7 to

-5 1 2 5 12 16 12 14

12 16

1 2 3 4 5 6 7 8 Swap

Index = 7, value = 12

Step 8 :

Unsorted Array
(element 8)

-5 1 2 5 12 12 16 14

14 16

1 2 3 4 5 6 7 8 Swap

- Minj = 7, Minx = 16
- Find min value from remaining unsorted array

Index = 8, value = 14

unsorted array

The entire array is sorted now.



Selection Sort

- Selection sort divides the array or list into two parts,
 1. The sorted part at the left end
 2. and the unsorted part at the right end.
- Initially, the sorted part is empty and the unsorted part is the entire list.
- The smallest element is selected from the unsorted array and swapped with the leftmost element, and that element becomes a part of the sorted array.
- Then it finds the second smallest element and exchanges it with the element in the second leftmost position.
- This process continues until the entire array is sorted.
- The time complexity of selection sort is $\Theta(n^2)$

Selection Sort - Algorithm

Input:

Array A

Output: Sorted array A

Algorithm: Selection_Sort(A) for i

$i \leftarrow 1$ to $n-1$ do

$minj \leftarrow i;$

$minx \leftarrow A[i];$

```
for j ← i + 1 to n do
if A[j] < minx then
minj ← j;
minx ← A[j];
A[minj] ← A[i];
A[i] ← minx;
```



Selection Sort – Example 2

```

Algorithm: Selection_Sort(A) for i ← 1 to n-1 do
    minj ← i; minx ← A[i];
    for j ← i + 1 to n do
        if A[j] < minx then A[j];
        minj ← j ; minx ← minj ← 1 34
        A[minj] ← A[i];
        A[i] ← minx;
        A[j] = 3456

```

Pass 1 :

<code>if A[j] < minx then A[j];</code> <code>minj ← j ; minx ← minj ← 1 34</code>	<code>minx ← 45</code> <code>No Change</code>
<code>A[minj] ← A[i];</code> <code>A[i] ← minx;</code> <code>A[j] = 3456</code>	<code>j = 2</code> <code>3</code>

Sort in Ascending order

45	34	56	23	12
----	----	----	----	----

1 2 3 4 5

Selection Sort – Example 2

Pass 1:

```
Algorithm: Selection_Sort(A) for i ← 1 to n-1 do
    i = 1
    minj ← i; minx ← A[i];
    4
    for j ← i + 1 to n do
        if A[j] < minx then minx ← 34 12
        minj ← j ; minx ← A[j]; 23
    minj ← 2 5
```

$A[minj] \leftarrow A[i];$ 3 4 5
 $j = 2$

$A[i] \leftarrow \min x;$

$$A[j] = 2312$$

Unsorted Array

Sort in Ascending order

45	34	56	23	12
----	----	----	----	----

45 12	34	56	23	1245
----------	----	----	----	------

1 2 3 4 5

Swap

45	
----	--

			34	
			56	



Insertion Sort – Example

Sort the following elements in Ascending order

5	1	12	-5	16	2	12	14
---	---	----	----	----	---	----	----

Step 1 : **Unsorted Array**

5	1	12	-5	16	2	12	14
---	---	----	----	----	---	----	----

1 2 3 4 5 6 7 8

Step 2 :

?? **1**

$$?? = ??, ?? = ??$$

$$?? > ??$$

$$?? = ?? - ?? \quad ?? - ??$$

5 1 12 -5 16 2 12 14 1 2 3 4 5 6 7 while ?? < ?? ?? do
8

$$?? ?? + 1 \leftarrow ?? ??$$

?? — — Shift down



Insertion Sort – Example

??

?? ??

?? ?? ?? ?? ?? ?? > ??

Step 3 :

$$?? = ??, ?? =$$

$$?? = ?? - ??$$

1 5 12 -5 16 2 12 14 1 2 3 4 5 6 7 8

while $?$ $?$ $<$ $?$ $?$ $?$ do $?$ $?$ $?$ +
 $1 \leftarrow ?$ $?$ $?$
 $?$ $?$ — —

No Shift will take place

Step 4 :

??

-5

while $?$ $?$ $<$ $?$ $?$ $?$ do

1 5 12 -5 16 2 12 14 1 2 3 4 5 6 7 8

— ??
?? = ?? - ??
?? ?? ?? ?? ?? >
??

?? = ??, ?? =

??

?? ?? + 1 \leftarrow ?? ??

?? — —

Shift down Shift down Shift down



Insertion Sort – Example

Step 5 :

-5 1 5 12 16 2 12 14 **1 2 3 4 5 6 7 8**

No Shift will take place

Step 6 :

? ? ?

-5 1 5 12 16 2 12 14
2

? ?

? ? = ? ?, ? ? =

? ? ? ?

? ? = ? ? - ? ?

? ? ? ? ? ? ? ? > ? ?

while ? ? < ? ? ? ? do ? ? ? ? +
1 ← ? ? ? ?
? ? — —

? ?

? ? = ? ? - ? ?

? ? ? ? ? ? ? ?

> ? ?

? ? = ? ?, ? ? =

1 2 3 4 5 6 7 8

while ? ? < ? ? ? ? do ? ? ? ?
+ 1 ← ? ? ? ?
? ? — —

down n
Shif Shift
Shift t down
dow



Insertion Sort – Example

Step 7 :

-5 1 2 5 12 16 12 14

12

1 2 3 4 5 6 7 8 Shift down

??

?? = ??, ?? =
?? ?? ?? ??
?? ?? ?? ?? ?? ?? > ??
?? ?? ?? + 1 ← ?? ?? ??
?? ?? — —

Step 8 :

while ?? < ?? ?? do

?? = ??, ?? = ?? ?? ?? ?? ?? = ?? - ??
?? ?? ?? ?? ?? ?? > ??

?? while ?? < ?? ?? ?? do

14

-5 1 2 5 12 12 16 14 1 2 3 4 5 6 7 8

Shift down

?? ?? + 1 ← ?? ??

The entire array is sorted now.

j ← i - 1;
while x < T[j] and j > 0 do

Insertion Sort - Algorithm # Input:

Array T

Output: Sorted array T

Algorithm:

Insertion_Sort(T[1,...,n]) for i ← 2

to n do

x ← T[i];



```
j ← j - 1;  
T[j+1] ← x;  
  
T[j+1] ← ? ? ? ? ?  
T[j];
```

Insertion Sort Algorithm – Best Case Analysis

Input: Array T
Output: Sorted array T

Insertion_Sort(T[1,...,n])

Algorithm:

```
for i ← 2      ← T[i];  
to n do x    j ← i - 1; ? ? ? ?  
                34 45 59
```

12
Pass 1:

x=23	i=2	T[j]=12
x=34	i=3	T[j]=23
x=45	i=4	T[j]=34
x=59	i=5	T[j]=45

```
while x < T[j] and j > 0 do  
    T[j+1] ← T[j];  
    j ← j - 1;  
T[j+1] ← x;
```

The best case time complexity

of Insertion sort is ♦♦ ♦♦
The average and worst case
time complexity of Insertion
sort is ♦♦  ♦♦



Heap & Heap Sort Algorithm



Introduction

- A heap data structure is a binary tree with the following two properties.

1. It is a complete binary tree: Each level of the tree is completely filled, except possibly the bottom level. At this level it is filled from left to right.
2. It satisfies the **heap order** property: the data item stored in each node is **greater than or equal to** the data item stored in its children node.





Array Representation of Heap

□ Heap can be implemented using an Array.

- An array $\diamond\diamond$ that represents a heap is an object with two attributes: 1. $\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond h[\diamond\diamond]$, which is the number of elements in the array, and 2. $h\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond - \diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond$, the number of elements in the heap stored within array $\diamond\diamond$

16

14 10

Array representation of heap

8793

Heap

2 4 1

16	14	10

Array Representation of Heap

□ In

the array arr , that represents a heap

1. $\text{length}[\text{arr}] = \text{heap-size}[\text{arr}]$
2. For any node i the parent node is $\lceil i/2 \rceil$
3. For any node i , its left child is $2i+1$ and right child is $2i+2$

For node $i = 4$, parent node is $4/2 = 2$

??
16

?? ??

14 10

?? ?? ?? ?? ?? 8 7 9 3

The image shows a pattern of ten red diamond shapes, each containing a white question mark. The diamonds are arranged in two rows: a top row of five and a bottom row of five. The entire pattern is centered horizontally.

2⁴1 Heap

For node $\diamond\diamond = 4$,

Left child node is $2 * 4 = \text{node } 8$

Right child is $2 * 4 + 1 = \text{node } 9$



1. Max-Heap – Where the value of the root node is greater than or equal to either of its children.

9

67241

1

—24679

1

Types of Heap

2. Min-Heap – Where the value of the root node is less than or equal to either of its children.



Introduction to Heap Sort

1. Build the **complete binary tree** using given elements.
2. Create **Max-heap** to sort in ascending order.
3. Once the heap is created, **swap** the last node with the root node and **delete** the last node from the heap.
4. Repeat **step 2 and 3** until the heap is empty.



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3	5	1
---	----	---	---	---

Step 1 : Create Complete Binary Tree

1 2 3 4 5

4	10	3	5	1
---	----	---	---	---

10 3 5 1

Now, a binary tree is created
and we have to convert it into
a Heap.



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3
---	----	---

Step 2 : Create Max Heap

4

1 2 3 4 5 10

10 is greater than 4 So, swap 10 & 4

4	10	3	5	1
10	4			

to the child nodes.

10 3 4

Swap

5 1

In a Max Heap, parent node is
always greater than or equal



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3	5	1
---	----	---	---	---

Step 2 : Create Max Heap

10

1 2 3 4 5

10	4	5	3	5	4	1
----	---	---	---	---	---	---

5

5 is greater than 4 So, swap 5 & 4

Swap

In a Max Heap, parent node

4

is always greater than or equal
to the child nodes.

Max Heap is created

5 1



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3	5	1
---	----	---	---	---

Step 3 : Apply Heap Sort

1 2 3 4 5

10	5	3	4	110
1				

1

1. Swap the first and the last nodes and
2. Delete the last node.

10

Swap



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3	5	1
---	----	---	---	---

Step 3 : Apply Heap Sort

5

1 2 3 4 5 Max Heap Property is

violated so, create a

1	5 1	3	4 1	10
5	4			

4
1

1
4

Swap

Max Heap again.

5 3



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3	5	1
---	----	---	---	---

Step 3 : Apply Heap Sort

1

1 2 3 4 5 Max Heap is created 5

5	4	3	1	5	10
1					

Swap

1. Swap the first and the

5

1

last nodes and
2. Delete the last node.

4 3

Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3
Create Max Heap		

Step 3 : Apply Heap Sort

3

1	2	3	4	5
1	4	4	1	3
3				10

Swap

1. Swap the first and the last nodes and

2. Delete the last node.

4

4
3
1
4

again

Max Heap is created

Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3
---	----	---

Step 3 : Apply Heap Sort

1 2 3 4 5

3	1	1	3	4	5	10
---	---	---	---	---	---	----

3

1

Swap

1. Swap the first and the last nodes and
2. Delete the last node.

Step 3 : Apply Heap Sort

Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3
---	----	---

Remove the last element from heap and the sorting is over.

1
1 2 3 4 5

1	1	3	4	5	10
---	---	---	---	---	----

Already a Max Heap



Heap Sort – Example 2 □ Sort given element in ascending order using heap sort. 19, 7, 16, 1, 14, 17

19	7	16	1	14	17
----	---	----	---	----	----

	14	17		7	6
--	----	----	--	---	---

Step 1: Create binary tree Step 2: Create Max-heap

19

1 14 17

7 16 17

7 16

19

1 14 16 7 7

14

Heap Sort – Example 2 Step 3 Step 4

19	14	17	1	7	16 1 9
----	----	----	---	---	-----------

16	14	17	1	7	
17		16			

19

Swap & remove the
last element

14 17

17

16

17 16 19

14 17 16

17

Heap Sort – Example 2 Step 5 Step 6

17	14	16	1	7	19
7				17	

17

7	14	16	1	17	
16		7		17	

the last
element

14 16

16

7

Swap &
remove

17

17

14 16 1

7

16

Heap Sort – Example 2 Step 7 Step 8

16	14	7	1	17	19
1			16		

1 14

1	14	7	16	17	
14	1				

Create Max-heap

1

Swap &
element

14 7

16
1

remove the last

14 7 1

Swap &
remove the

7 1	1 7	14	16	17	
-----	-----	----	----	----	--

Heap Sort – Example 2 Step 9 Step 10

14	1	7	16	17	19
7		14			

Already a Max-heap

7
1

Swap & remove the last

1

last element

17 14

7

Step 11

element

1	1	7	1
			Remove the last element



The entire array is sorted now.

Exercises

- Sort the following elements using Heap Sort Method.

1. 34, 18, 65, 32, 51, 21
2. 20, 50, 30, 75, 90, 65, 25, 10, 40

- Sort the following elements in Descending order using Hear Sort Algorithm.
- Algorithm.
1. 65, 77, 5, 23, 32, 45, 99, 83, 69, 81





Binary Tree Analysis



Heap Sort – Algorithm # Input: Array A
Output: Sorted array A

Algorithm: `Heap_Sort(A[1,...,n])`

`BUILD-MAX-HEAP(A)`

```

for i ← length[A] downto 2
    do exchange A[1] □ A[i]
    heap-size[A] ← heap-size[A] - 1
    MAX-HEAPIFY(A, 1, n)

```

4

Heap Sort – Algorithm

BUILD-MAX-HEAP(A) heap-size[A] ←
length[A] for i ← [length[A]/2] downto
1 do MAX-HEAPIFY(A, i)

heap-size[A] = 6

4	1	7	2	9	3
---	---	---	---	---	---

i = 3

4	9	7	2	1	3
---	---	---	---	---	---

1

1
4
2 3
1 3
4 5 6
2 9 7

9	4	7	2	1	3
---	---	---	---	---	---

i = 2 i = 1



Output: Sorted array A

39	4	7	2	1	93
----	---	---	---	---	----

Heap Sort – Algorithm # Input:
Array A

Algorithm: `Heap_Sort(A[1,...,n])`

BUILD-MAX-HEAP(A)

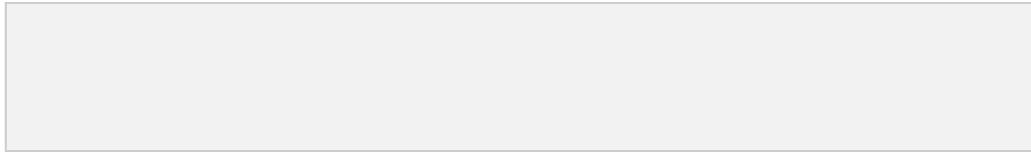
for i ← length[A] downto 2

```

do exchange A[1] A[i]
heap-size[A] ← heap-size[A] - 1
1 MAX-HEAPIFY(A, 1, n)

```

23
47
4 56213



$l \leftarrow \text{LEFT}(i)$ $r \leftarrow \text{RIGHT}(i)$

$l \leftarrow 21$

$r \leftarrow 3$

if $l \leq n$ and $A[l] > A[i]$

1

then $\text{largest} \leftarrow l$

else $\text{largest} \leftarrow i$

Yes

3

$\text{largest} \leftarrow 2$

23

Heap Sort – Algorithm

Algorithm: Max-heapify(A, i, n)

3	4	7	2	1	9
---	---	---	---	---	---

47

45

if $r \leq n$ and $A[r] >$

$A[\text{largest}]$

Yes

```

then           largest← 3
largest←r
if           Yes
largest≠i
  then exchange A[i]   A[largest]
MAX-HEAPIFY(A, largest, n)

```

Output: Sorted array A

3	4	7	2	1	9
---	---	---	---	---	---

Heap Sort – Algorithm # Input:

Array A

Algorithm: Heap_Sort(A[1,...,n])

BUILD-MAX-HEAP(A)

for i ← length[A] downto 2

do exchange A[1] A[i]

heap-size[A] ← heap-size[A] -

7

1

1 MAX-HEAPIFY(A, 1, n)

23

43

4
5

21

1

Heap Sort Algorithm – `Heap_Sort(A[1,...,n])`

Analysis # Input:

Array A

Output: Sorted array

A

```
heap-size[A] ← length[A]  
for i ← ⌊length[A]/2⌋ downto
```

BUILD-MAX-HEAP (A)

◇ ◇ T ◇ ◇

heap-size[A] \leftarrow
heap-size[A] - 1

$$? ? (? ? - ? ?) \\ (? ? ? ? ? ? ? ?)$$

?

MAX-HEAPIFY(A, 1, n)

Running time of heap sort algorithm is:

$$? ? ? ? ? ? ? ? ? ? ? + ? ? (? ? ? ? ? ? ? ?) ? ? - ? ? + ? ? (? ? - ? ?) = ? ? (? ? ? ? ? ? ? ?)$$



Sorting Algorithms

Radix Sort, Bucket Sort, Counting Sort



Radix Sort □ Radix Sort puts the elements in order by comparing the digits of the numbers.
□ Each element in the $\diamond\diamond$ -element array $\diamond\diamond$ has $\diamond\diamond$ digits, where digit 1 is the lowest-order

digit and digit $\diamond\diamond$ is the highest order digit.

Algorithm: $RADIX-SORT(A, d)$

for $i \leftarrow 1$ to d

do use a stable sort to sort array A on digit i

□ Sort following elements in Ascending order using radix sort.

363, 729, 329, 873, 691, 521, 435, 297



5	2	1	7	2	9	3	2	9	4	3	5	3	6	3	8	7	3	6	9	1	2	9	7
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Sort on column 3

Sort on column 2

Sort on column 1

The entire array is sorted now.



Bucket Sort – Introduction

□ Sort the following elements in Ascending order using bucket sort.

45	96	29	30	27	12	39	61	91
----	----	----	----	----	----	----	----	----

1. Create ♦♦ empty buckets.
2. Add each input element to appropriate bucket as,
 - a. Bucket ♦♦ holds values in the half-open interval,
$$\text{low} * \text{scale} \leq \text{value} < (\text{high} + \text{scale}) * \text{scale}$$
3. Sort each bucket queue with insertion sort.

4. Merge all bucket queues together in order.

- Expected running time is $\Theta(n^2(\log n + \sqrt{n}))$, with n = size of original sequence. If n is $\Theta(n^2)$ then sorting algorithm is $\Theta(n^2(\log n))$.



Bucket Sort – Example

45	96	29	30	27	12	39	61	91
----	----	----	----	----	----	----	----	----

45 96 29 30 27 12 39 61 91

0 1 2 3 4 5 6 7 8 9

Sort each bucket queue with insertion sort

Merge all bucket queues together in order

12	27	29	30	39	45	61	91	96
----	----	----	----	----	----	----	----	----



Bucket Sort - Algorithm # Input: Array A

Output: Sorted array A

Algorithm: Bucket-Sort(A[1,...,n])

n \leftarrow length[A]

for i \leftarrow 1 to n do

 insert A[i] into bucket B[|A[i] \div n|]

```
for i ← 0 to n - 1 do
    sort bucket B[i] with insertion sort
concatenate the buckets B[0], B[1], . . . , B[n - 1] together in
order.
```



Counting Sort – Example

□ Sort the following elements in Ascending order using counting sort.

3	6	4	1	3	4	1	4	2
---	---	---	---	---	---	---	---	---

Step 1

Given elements are stored in an input array $\text{arr}[1, \dots, 9]$

1 2 3 4 5 6 7 8 9

3	6	4		1 3	4	1	4	2
---	---	---	--	-----	---	---	---	---

Index

Elements

Step 2

Define a temporary array $\text{temp}[]$.
The size of an array $\text{temp}[]$ is

equal to the **maximum element**, 6] to 0.
in array arr . Initialize $\text{count}[1, \dots]$

Index

Elements

1	2
---	---

0	0	0	0	0	0
---	---	---	---	---	---



Counting Sort – Example

Sort the following elements in Ascending order using counting sort.

Input array arr

3	6	4
---	---	---

Step 3

Update an array C with the occurrences of each value of array arr

Index

Elements

1	2	3	4	5	6
02	10	20	03	00	10

++

Step 4

In array arr , from index 2 to $\text{arr.length} - 1$

add the value with previous element

Index

Elements

1	2
---	---

2	3	5	8	8	9
---	---	---	---	---	---



Counting Sort – Example □ Create an output array $\text{output}[1 \dots 9]$. Start positioning elements of Array arr as shown below.

Input array arr

1 2 3 4 5 6 7 8 9

3	6	4	1	3	4	1	4	2
---	---	---	---	---	---	---	---	---

Temporary Array C

		Output Array B								
1	2	1	2	3	4	5	6	7	8	9
2	0	1	3							



Counting Sort - Procedure

- Counting sort assumes that each of the $\diamond\diamond$ input elements is an integer in the range 0 to $\diamond\diamond$, for some integer $\diamond\diamond$.
- When $\diamond\diamond = \diamond\diamond(\diamond\diamond)$, the counting sort runs in $\diamond\diamond(\diamond\diamond)$ time.
- The basic idea of counting sort is to determine, for each input element $\diamond\diamond$, the number of elements less than $\diamond\diamond$.
- This information can be used to place element $\diamond\diamond$ directly into its position in the output array.



Counting Sort - Algorithm # Input: Array A

Output: Sorted array A

Algorithm: Counting-Sort(A[1,...,n], B[1,...,n],

k) for i ← 1 to k do

 c[i] ← 0

for j ← 1 to n do

 c[A[j]] ← c[A[j]]

 + 1

for i ← 2 to k do

 c[i] ← c[i] +
 c[i-1]

for j ← n downto 1 do

 B[c[A[j]]] ← A[j]

 c[A[j]] ← c[A[j]]





Thank You!