Problem Set - Logic

- Problems $1-13$ are based on the topic "Propositional Logic" (Sec 1.1).
- Problems $14-20$ are from the topic "Propositional Equivalence" (Sec 1.2).

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1. Which of these sentences are propositions? What are the truth values of those that are propositions?

   a) Read the paragraph carefully.
   b) Mumbai is the capital of India.
   c) $2+3=5$.
   d) Answer this question.
   e) $x+2=11$.
   f) $5+7=10$.

2. What is the negation of each of these propositions?

   a) Steve has more than 100 GB of free disk space on his laptop.
   b) Zach blocks e-mails and texts from David.
   c) $7.11.13=999$.
   d) Diane drove her bicycle 30 miles on Sunday.
   e) John walks or takes the bus to class.

3. Let p and q be the propositions "Swimming at the New Jersey shore is allowed" and "Sharks have been spotted near the shore", respectively. Express each of these compound propositions as an English sentence.

   a) $\neg q$
   b) $p \wedge q$
   c) $\neg p \vee q$
   d) $p \rightarrow \neg q$
   e) $\neg q \rightarrow p$
   f) $\neg p \rightarrow \neg q$
   g) $p \leftrightarrow \neg q$
   h) $\neg p \wedge (p \vee \neg q)$

4. Let p and q be the propositions

   p : It is below freezing.

   q : It is snowing.

   Write these propositions using p and q and logical connectives (including negations).

   a) It is below freezing and snowing.
   b) It is below freezing but not snowing.
   c) It is not below freezing and it is not snowing.
   d) It is either snowing or the temperature is below freezing.
   e) If it is below freezing, it is also snowing.
   f) Either it is below freezing or it is snowing, but it cannot be snowing if it is below freezing.
   g) That it is below freezing is necessary and sufficient for it to be snowing.

5. Let p, q and r be the propositions

   p : Grizzly bears have been seen in the area.

   q : Hiking is safe on the trail.

   r : Berries are ripe along the trail.

   Write these propositions using p and q and logical connectives (including negations).

   a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
   b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.
   c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
   d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.

f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

6. Write each of these statements in the form "if p, then q" in English.

a) It snows whenever the wind blows from the northeast.

b) The apple trees will bloom if it stays warm for a week.

c) That the Pistons win the championship implies that they beat the Lakers.

d) It is necessary to walk 8 miles to get to the top of Long's Peak.

e) To get tenure as a professor, it is sufficient to be world-famous.

f) If you drive more than 400 miles, you will need to buy gasoline.

g) Your guarantee is good only if you bought your CD player less than 90 days ago.

h) Jan will go swimming unless the water is too cold.

7. Write each of these sentences in the form "p if and only if q" in English.

a) If it is hot outside, you buy an ice cream cone, and if you buy an ice cream cone, it is hot outside.

b) For you to win the contest, it is necessary and sufficient that you have the only winning ticket.

c) You get promoted only if you have connections, and you have connections only if you get promoted.

d) If you watch television, your mind will decay, and conversely.

e) The trains run late on exactly those days when I take it.

8. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 2$, then $2 + 2 = 5$.

b) If $1 + 1 = 3$, then $2 + 2 = 4$.

c) If $1 + 1 = 3$, then $2 + 2 = 5$.

d) If monkeys can fly, then $1 + 1 = 3$.

9. Determine whether these biconditionals are true or false.

a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.

b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.

c) $1 + 1 = 3$ if and only if monkeys can fly.

d) $0 > 1$ if and only if $2 > 1$.

10. State the converse, contrapositive, and inverse of each conditional statement.

a) If it snows today, I will ski tomorrow.

b) I come to class whenever there is going to be a quiz.

c) A positive integer is a prime only if it has no divisors other than 1 and itself.

11. Construct a truth table for each of these compound propositions.

a) $p \to (\neg q \lor r)$

b) $\neg p \to (q \to r)$

c) $(p \to q) \lor (\neg p \to r)$

d) $(p \to q) \land (\neg p \to r)$

e) $(p \leftrightarrow q) \lor (\neg q \leftrightarrow r)$

12. Evaluate each of these expressions.

a) $11000 \land (01011 \lor 11011)$.

b) $(01111 \land 10101) \lor 01000$.

c) $(01010 \oplus 11011) \oplus 01000$.

d) $(11011 \lor 01010) \land (10001 \lor 11011)$.

13. How many different logical operators are possible, operating on two propositions?

14. Write the negation of each statement in English.

    a) If she works, she will earn money.
    b) He swims if and only if the water is warm.
    c) If it snows, then they do not drive the car.

15. Determine whether $\neg(p \to q) \equiv (p \wedge \neg q)$.

16. Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology.

17. Verify that the proposition $(p \wedge q) \wedge \neg(p \vee q)$ is a contradiction.

18. Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.

19. How many of the disjunctions $p \vee \neg q \vee s$, $\neg p \vee \neg r \vee s$, $\neg p \vee \neg r \vee \neg s$, $\neg p \vee q \vee \neg s$, $q \vee r \vee \neg s$, $q \vee \neg r \vee \neg s$, $\neg p \vee \neg q \vee \neg s$, $p \vee r \vee s$, and $p \vee r \vee \neg s$ can be made simultaneously true by an assignment of truth values to p, q, r, and s?

20. A compound proposition is **satisfiable** if there is at least one assignment of truth values to the variables in the compound proposition that makes the compound proposition true. Determine whether each of these compound propositions is satisfiable.

    a) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$
    b) $(p \to q) \wedge (p \to \neg q) \wedge (\neg p \to q) \wedge (\neg p \to \neg q)$
    c) $(p \leftrightarrow q) \wedge (\neg p \leftrightarrow q)$