



UIT

Unitedworld Institute Of Technology

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B.Tech. Computer Science & Engineering
Semester-3

Data Structures and Algorithms

Course Code: 71203002002



Analyzing Control Statements



For Loop # Input : `int A[n]`, array of `n` integers # Output : Sum of all numbers in array `A`

Algorithm: int Sum(int A[], int n)

{

for (int

i=0; i<n;

i++) s = s

+ A[i];

return s;

n

n+1

}

int s=0;

1

Running Time of Algorithm □ The time complexity of the algorithm is : $n^2 + n + 3$

$$n^2 + n + 3$$

□ Estimated running time for different values of n :

$$n = 2 \Rightarrow 2^2 + 2 + 3 = 23 \text{ steps}$$

$$n = 10 \Rightarrow 10^2 + 10 + 3 = 203 \text{ steps}$$

$$n = 100 \Rightarrow 100^2 + 100 + 3 = 2,003 \text{ steps}$$

$$n = 1000 \Rightarrow 1000^2 + 1000 + 3 = 20,003 \text{ steps}$$

- As n grows, the number of steps grow in linear proportion to n^2 for the given algorithm Sum.
- The dominating term in the function of time complexity is n^2 : As n gets large, the $+3$ becomes insignificant.

□ The time is linear in proportion to $\diamond\diamond$.



Analyzing Control Statements

Example 1:

$$\begin{aligned} &\diamond\diamond\diamond\diamond\diamond\diamond = \\ &\diamond\diamond + \diamond\diamond; \end{aligned} \quad \begin{aligned} &\begin{bmatrix} \diamond\diamond \end{bmatrix} = 1 \begin{bmatrix} \diamond\diamond\diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond \end{bmatrix} \\ &\begin{bmatrix} \diamond\diamond\diamond\diamond\diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond\diamond\diamond \end{bmatrix} \end{aligned} \quad \begin{aligned} &\begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond \end{bmatrix} + \begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond \end{bmatrix} \\ &\begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond \end{bmatrix} * \begin{bmatrix} \diamond\diamond \end{bmatrix} \end{aligned}$$

- Statement is executed once only
- So, The execution time $\diamond\diamond(\diamond\diamond)$ is some constant
- $\diamond\diamond \approx \diamond\diamond(\diamond\diamond)$

$$\begin{aligned} &\diamond\diamond\diamond\diamond\diamond\diamond \diamond\diamond = 1 \begin{bmatrix} \diamond\diamond\diamond\diamond \end{bmatrix} \\ &\begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond\diamond\diamond \end{bmatrix} \\ &\begin{bmatrix} \diamond\diamond\diamond\diamond\diamond\diamond \end{bmatrix} = \begin{bmatrix} \diamond\diamond \end{bmatrix} + \begin{bmatrix} \diamond\diamond \end{bmatrix}; \end{aligned}$$

- Analysis

Example 3:

$$\begin{aligned} &\begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond\diamond\diamond\diamond\diamond \end{bmatrix} + \begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond \end{bmatrix} \\ &\begin{bmatrix} \diamond\diamond \end{bmatrix} * \begin{bmatrix} \diamond\diamond \end{bmatrix} \end{aligned}$$

Example 2:

$$\begin{aligned} &\diamond\diamond\diamond\diamond\diamond\diamond \diamond\diamond = 1 \begin{bmatrix} \diamond\diamond\diamond\diamond \end{bmatrix} \\ &\begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond\diamond\diamond \end{bmatrix} \\ &\begin{bmatrix} \diamond\diamond\diamond\diamond\diamond\diamond \end{bmatrix} = \begin{bmatrix} \diamond\diamond \end{bmatrix} + \begin{bmatrix} \diamond\diamond \end{bmatrix}; \end{aligned}$$

$$\begin{aligned} &\begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond \end{bmatrix} * \begin{bmatrix} \diamond\diamond \end{bmatrix} + \begin{bmatrix} \diamond\diamond \end{bmatrix} \\ &\begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond \end{bmatrix} = \begin{bmatrix} \diamond\diamond \end{bmatrix}_1 \begin{bmatrix} \diamond\diamond \end{bmatrix} + 1 + \\ &\begin{bmatrix} \diamond\diamond \end{bmatrix}_2 \begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond \end{bmatrix} + 1 + \begin{bmatrix} \diamond\diamond \end{bmatrix}_3 \begin{bmatrix} \diamond\diamond \end{bmatrix} \\ &\begin{bmatrix} \diamond\diamond \end{bmatrix} \begin{bmatrix} \diamond\diamond \end{bmatrix} (\begin{bmatrix} \diamond\diamond \end{bmatrix}) = \begin{bmatrix} \diamond\diamond \end{bmatrix}_1 \begin{bmatrix} \diamond\diamond \end{bmatrix} + \\ &\begin{bmatrix} \diamond\diamond \end{bmatrix}_1 + \begin{bmatrix} \diamond\diamond \end{bmatrix}_2 \begin{bmatrix} \diamond\diamond \end{bmatrix}^2 + \begin{bmatrix} \diamond\diamond \end{bmatrix}_2 \begin{bmatrix} \diamond\diamond \end{bmatrix} + \\ &\begin{bmatrix} \diamond\diamond \end{bmatrix}_3 \begin{bmatrix} \diamond\diamond \end{bmatrix}^2 \begin{bmatrix} \diamond\diamond \end{bmatrix} (\begin{bmatrix} \diamond\diamond \end{bmatrix}) = \begin{bmatrix} \diamond\diamond \end{bmatrix}^2 (\begin{bmatrix} \diamond\diamond \end{bmatrix}_2 \end{aligned}$$

$$+ \diamond\diamond_3) + \diamond\diamond(\diamond\diamond_1 + \diamond\diamond_2) + \diamond\diamond_1$$

$$\diamond\diamond\diamond\diamond$$

▪ Total time is denoted as, $\diamond\diamond$

$$\diamond\diamond = \diamond\diamond_{\diamond\diamond}\diamond\diamond + \diamond\diamond_{\diamond\diamond} + \diamond\diamond_{\diamond\diamond}\diamond\diamond$$

$$* \diamond\diamond$$

$$\diamond\diamond(\diamond\diamond) = \diamond\diamond\diamond\diamond^2 + \diamond\diamond\diamond\diamond + \diamond\diamond\diamond\diamond(\diamond\diamond) = \diamond\diamond \blacksquare \diamond\diamond$$

$$\diamond\diamond(\diamond\diamond) = \diamond\diamond(\diamond\diamond_{\diamond\diamond} + \diamond\diamond_{\diamond\diamond}) + \diamond\diamond_{\diamond\diamond} \approx \diamond\diamond(\diamond\diamond)$$



$$\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond = \diamond\diamond\diamond\diamond\diamond\diamond$$

$$\diamond\diamond\diamond\diamond\diamond$$

$$\diamond\diamond = \diamond\diamond + 1$$

$$\diamond\diamond\diamond\diamond = \diamond\diamond \blacksquare \diamond\diamond$$

$$\diamond\diamond\diamond\diamond$$

$$\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond = 1 \diamond\diamond\diamond\diamond\diamond\diamond^3$$

$$\diamond\diamond\diamond\diamond$$

$$\diamond\diamond = \diamond\diamond + 1$$

$$\diamond\diamond\diamond\diamond = \diamond\diamond \blacksquare \diamond\diamond$$

Analyzing Control Statements

Example 4:

$$\diamond\diamond = 0$$

$$\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond = 1 \diamond\diamond\diamond\diamond\diamond\diamond$$

$$\diamond\diamond\diamond\diamond$$

$$\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond = 1 \diamond\diamond\diamond\diamond\diamond\diamond$$

$$\diamond\diamond\diamond\diamond$$

Example 5:

$$\diamond\diamond = 0$$

$$\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond = 1 \diamond\diamond\diamond\diamond\diamond\diamond$$

$$\diamond\diamond\diamond\diamond$$

$$\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond = 1 \diamond\diamond\diamond\diamond\diamond\diamond^2$$

Example 6:

$$\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond = 1 \diamond\diamond\diamond\diamond\diamond\diamond$$

???

+ 1

????? = 1 ? ? ? ? ?



printf("sum is now
%d", ? ? ? ? ? ? ?)

????

????? = ? ? ? ? ? + ? ? *

??

????? = 1 ? ? ? ? ?

?? ?? = ??  ?? + ?? ??
+ ?? ?? ?? ?? ?? ?? = ??  ??



????

????? = ? ? ? ? ? - ? ?



Sorting Algorithms

Bubble Sort, Selection Sort, Insertion Sort

Introduction

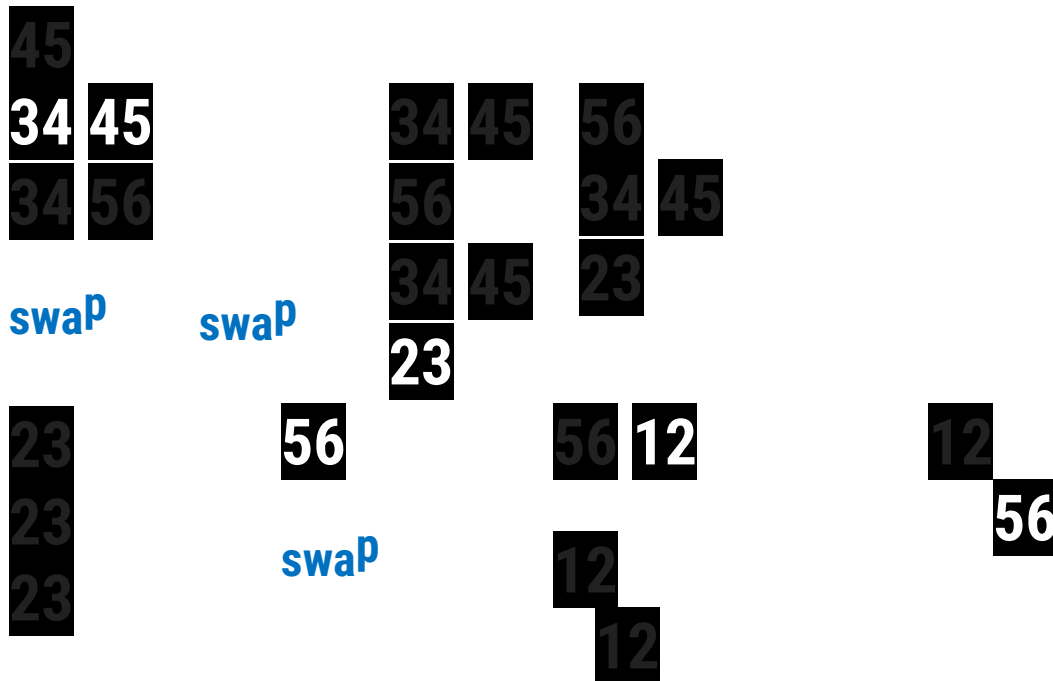
- Sorting is any process of arranging items systematically or arranging items in a sequence ordered by some criterion.
- Applications of Sorting
 1. Phone Bill: the calls made are date wise sorted.
 2. Bank statement or Credit card Bill: transactions made are date wise sorted.
 3. Filling forms online: “select country” drop down box will have the name of countries sorted in Alphabetical order.
 4. Online shopping: the items can be sorted price wise, date wise or relevance wise.
 5. Files or folders on your desktop are sorted date wise.

Bubble Sort – Example

Sort the following array in Ascending order

45	34	56	23	12
----	----	----	----	----

Pass 1 :



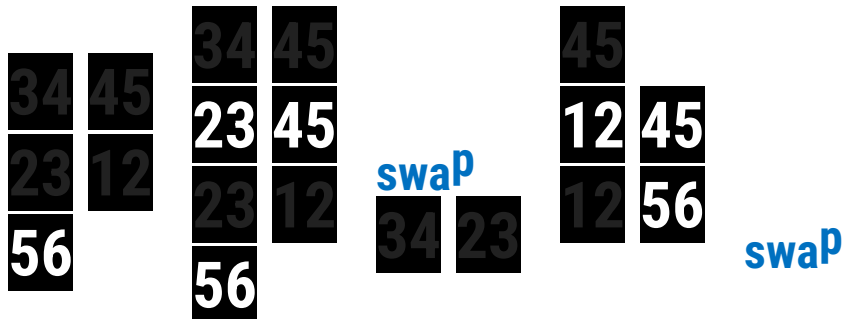
12

???(??[??] >
??[?? + 1])
?????(?
[??], ??[?? +
1])

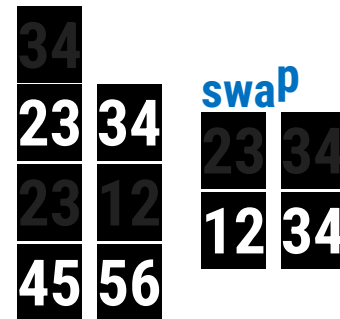


Bubble Sort – Example

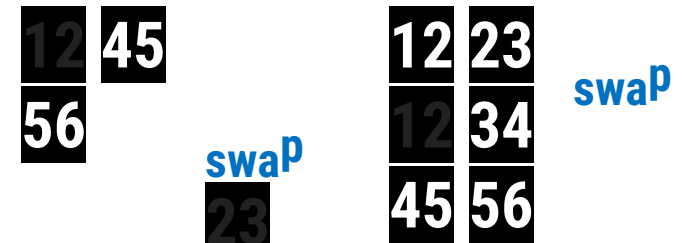
Pass 2 :



Pass 3 :



Pass 4 :



???(??[??] >
??[?? + 1])
????????(??
[??], ??[?? +
1])

Bubble Sort - Algorithm # Input:
Array A

Output: Sorted array A

for j ← 1 to n-i do if A[j] >
A[j+1] then

Algorithm: Bubble_Sort(A) for i ←
1 to n-1 do

temp
A[j] ←
A[j+1]
A[j+1] ←
temp
←
swap(A[j],
A[j]
A[j+1])

Bubble Sort

□ It is a simple sorting algorithm that works by comparing each pair of adjacent items and

swapping them if they are in the wrong order.

- The pass through the list is repeated **until no swaps are needed**, which indicates that the list is sorted.
- As it only uses comparisons to operate on elements, it is a **comparison sort**.
- Although the algorithm is simple, it is **too slow** for practical use.
- The time complexity of bubble sort is $O(n^2)$

Algorithm:
Bubble_Sort(A)

Bubble Sort Algorithm

– Best Case Analysis

Input: Array A

Output: Sorted array

A

for $i \leftarrow 1$ to $n-1$ do for $j \leftarrow 1$ to $n-i$ do

Condition never becomes

12
23
34

$i = 1$

$j = 1 \quad j = 2 \quad j = 3$

true

45 59

j = 4

```
cout<<"already sorted"<<endl  
if A[j] > A[j+1] then flag = 0; break;  
swap(A[j],A[j+1])  
if(flag == 1)
```



Selection Sort – Example 1

Sort the following elements in Ascending order

5	1	12	-5	16	2	12	14
---	---	----	----	----	---	----	----

Step 1 :

Unsorted Array

5	1	12	-5	16	2	12	14
---	---	----	----	----	---	----	----

1 2 3 4 5 6 7 8

Step 2 :

Unsorted Array (elements 2 to 8)

- **Minj** denotes the current index and **Minx** is the value stored at current index.

-5 5

Swap

Index = 4, value = -5



Selection Sort – Example 1

Step 3 :

Unsorted Array (elements 3 to 8)

- Now **Minj** = 2, **Minx** = 1 ▪ Find min value from remaining

-5 1 12 5 16 2 12 14
1

5 1 12 -5 16 2 12 14 1 2 3 4 5 6 7 8

unsorted array

1 2 3 4 5 6 7 8

Index = 2, value = 1

No Swapping as min value is already at right place

Step 4 :

Unsorted Array
(elements 4 to 8)

-5 1 12 5 16 2 12 14
2 12

1 2 3 4 5 6 7 8

- Minj = 3, Minx = 12
- Find min value from remaining unsorted array

Index = 6, value = 2

Swap



Selection Sort – Example 1

8)

min value from remaining

Step 5 :

Unsorted Array (elements 5 to

-5 1 2 5 16 12 12 14

5

1 2 3 4 5 6 7 8

▪ Now Minj = 4, Minx = 5 ▪ Find

unsorted array

Index = 4, value = 5

(elements 6 to 8)

-5 1 2 5 16 12 12 14

12 16

1 2 3 4 5 6 7 8 Swap

Step 6 :

No Swapping as min value is already

at right place ▪ Minj = 5, Minx = 16

▪ Find min value from remaining unsorted array

Unsorted Array

Index = 6, value = 12



Selection Sort – Example 1

8)

Step 7 :

Unsorted Array (elements 7 to

-5 1 2 5 12 16 12 14

1 2 3 4 5 6 7 8 Swap

- Now Minj = 6, Minx = 16 ▪ Find min value from remaining

12 16

Index = 7, value = 12

Step 8 :

Unsorted Array
(element 8)

-5 1 2 5 12 12 16 14

1 2 3 4 5 6 7 8 Swap

- Minj = 7, Minx = 16
- Find min value from remaining unsorted array

14 16

Index = 8, value = 14

The entire array is sorted now.

unsorted array



Selection Sort

□ Selection sort divides the array or list into two parts,

1. The sorted part at the left end
2. and the unsorted part at the right end.

- Initially, the sorted part is empty and the unsorted part is the entire list.
- The smallest element is selected from the unsorted array and swapped with the leftmost element, and that element becomes a part of the sorted array.
- Then it finds the second smallest element and exchanges it with the element in the second leftmost position.
- This process continues until the entire array is sorted.
- The time complexity of selection sort is $O(n^2)$

Selection Sort - Algorithm

Array A

Output: Sorted array A

Algorithm: Selection_Sort(A) for i

← 1 to n-1 do

minj ← i;

minx ← A[i];

```
for j ← i + 1 to n do  
  if A[j] < minx then  
    minj ← j;  
    minx ← A[j];  
A[minj] ← A[i];  
A[i] ← minx;  
◆◆ ◆◆
```



Selection Sort – Example 2

```

Algorithm: Selection_Sort(A) for  $i \leftarrow 1$  to  $n-1$  do
    minj  $\leftarrow i$ ; minx  $\leftarrow A[i]$ ;
    for  $j \leftarrow i + 1$  to  $n$  do

```

Pass 1 :

```
if A[j] < minx then A[j];           minx ← 45
minj ← j ; minx ← minj ← 1 34      No Change
```

```
A[minj] ← A[i];
```

```
A[i] ← minx;
```

A[j] = 3456

Sort in Ascending order

45	34	56	23	12
----	----	----	----	----

1 2 3 4 5

Selection Sort – Example 2

Algorithm: Selection_Sort(A) for $i \leftarrow 1$ to $n-1$ do

$\text{minj} \leftarrow i$; $\text{minx} \leftarrow A[i]$;

 for $j \leftarrow i + 1$ to n do

 if $A[j] < \text{minx}$ then $\text{minx} \leftarrow A[j]$

$\text{minj} \leftarrow j$; $\text{minx} \leftarrow A[j]$;

$\text{minj} \leftarrow 2$ 5

$A[\text{minj}] \leftarrow A[i]$; $j = 2$ 3 4 5

$A[i] \leftarrow \text{minx}$;

$A[j] = 2312$

Unsorted Array

Sort in Ascending order

Pass 1 :

$i = 1$

4

$\text{minx} \leftarrow 34$ 12

23

3

4 5

45	34	56	23	12
----	----	----	----	----

1 2 3 4 5

Swap

45	
----	--

45 12	34	56	23	1245
----------	----	----	----	------

1 2 3 4 5

			34 56	
--	--	--	----------	--



Insertion Sort – Example

Sort the following elements in Ascending order

5	1	12	-5	16	2	12	14
---	---	----	----	----	---	----	----

Step 1 : **Unsorted Array**

5	1	12	-5	16	2	12	14
---	---	----	----	----	---	----	----

1 2 3 4 5 6 7 8

Step 2 :

?? 1

?? = ??, ?? = ?? ?? > ??

?? = ?? - ?? ?? ?? ?? ??

5 1 12 -5 16 2 12 14 1 2 3 4 5 6 7 while ?? < ?? ?? do
8 ?? ?? + 1 ← ?? ??

?? — — Shift down



Insertion Sort – Example

??

????

?????? > ??

Step 3 :

?? = ??, ?? =

?? = ?? - ??

1 5 12 -5 16 2 12 14 1 2 3 4 5 6 7 8

while $?? < ??$ do $?? +$
 $1 \leftarrow ??$
 $?? --$

No Shift will take place

Step 4 :

$--??$
 $?? = ?? - ??$
 $?? > ??$
 $??$

$?? = ??, ?? =$

$??$

$??$

-5

$?? + 1 \leftarrow ??$
 $?? --$

while $?? < ??$ do

1 5 12 -5 16 2 12 14 1 2 3 4 5 6 7 8

Shift down Shift down Shift down



Insertion Sort – Example

Step 5 :

??

?? = ??, ?? =
????

?? = ?? - ??
?????? > ??

-5 1 5 12 16 2 12 14 1 2 3 4 5 6 7 8

while ?? < ?? do
1 ← ??
?? --

No Shift will take place

Step 6 :

??
?? = ?? - ??
?????? > ??

?? = ??, ?? =

??
??

1 2 3 4 5 6 7 8

while ?? < ?? do
+ 1 ← ??
?? --

-5 1 5 12 16 2 12 14

2

Shift down
Shift down
Shift down
Shift down



Insertion Sort – Example

Step 7 :

-5 1 2 5 12 16 12 14

12

1 2 3 4 5 6 7 8 Shift down

Step 8 :

while $?? < ????$ do

$?? = ??, ?? =$
 $????$
 $?? + 1 \leftarrow ??$
 $?? - -$

$?? = ??, ?? =$
 $???? > ??$

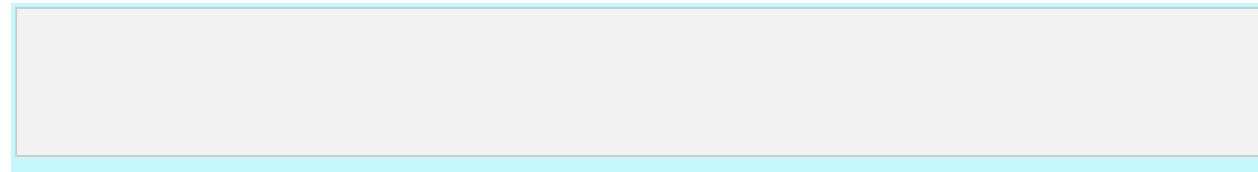
$??$ while $?? < ????$ do

14
-5 1 2 5 12 12 16 14 1 2 3 4 5 6 7 8

The entire array is sorted now.

Shift down

?? + 1 ← ??



Insertion Sort - Algorithm # Input:

Array T

Output: Sorted array T

Algorithm:

Insertion_Sort(T[1,...,n]) for i ← 2

to n do

x ← T[i];

??

j ← i - 1;
while x < T[j] and j > 0 do

??

```

j ← j - 1;
T[j+1] ← x;

```

```

T[j+1] ←
T[j];

```



Insertion Sort Algorithm – Best

Case Analysis # Input: Array T

Output: Sorted array T

12

Pass 1 :

```

Insertion_Sort(T[1,...,n
])

```

23

x=23 i=2

T[j]=12

Algorithm:

```

← T[i];
for i ← 2
to n do x
j ← i - 1;

```



i=3 i=4 i=5

x=34 x=45

x=59

T[j]=23

T[j]=34

T[j]=45

```
while x < T[j] and j > 0 do
    T[j+1] ← T[j];
    j ← j - 1;
T[j+1] ← x;
```

The best case time complexity

of Insertion sort is $O(n^2)$
The average and worst case
time complexity of Insertion
sort is $O(n^2)$



Heap & Heap Sort Algorithm



Introduction □ A heap data structure is a binary tree with the following two properties.

1. It is a complete binary tree: Each level of the tree is completely filled, except possibly the bottom level. At this level it is filled from left to right.
2. It satisfies the **heap order** property: the data item stored in each node is **greater than or equal to** the data item stored in its children node.



Binary Tree but not a Heap Complete
Binary Tree - Heap Not a Heap
Heap



Array Representation of Heap

□ Heap can be implemented using an Array.

- An array h that represents a heap is an object with two attributes:
1. $h[0]$, which is the number of elements in the array, and
 2. $h[1] - h[h[0]]$, the number of elements in the heap stored within array h

16

14 10

8 7 9 3

Array representation of heap

Heap

2 4 1

16	14	10
----	----	----

Array Representation of Heap

In the array arr , that represents a heap

1. $length[arr] = heap-size[arr]$
2. For any node i the parent node is $i/2$
3. For any node i , its left child is $2i$ and right child is $2i+1$

For node $i = 4$, parent node is $4/2 = 2$

$arr[1]$
16

$arr[2]$ $arr[3]$ $arr[4]$ $arr[5]$

14 10

$arr[6]$ $arr[7]$ $arr[8]$ $arr[9]$ $arr[10]$ 8 7 9 3



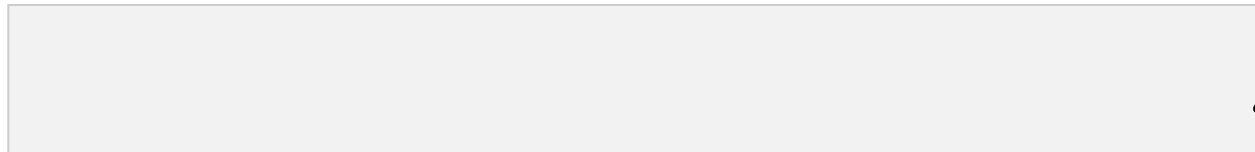
$2^4 1$ Heap

For node $\diamond\diamond = 4$,

Left child node is $2 * 4 = \text{node } 8$

Right child is $2 * 4 + 1 = \text{node } 9$

$\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond\diamond$									
16	14	10	8	7	9	3	2 2	4 4	1



Types of Heap

1. Max-Heap – Where the value of the root node is **greater than or equal to** either of its children.

9

6 7 2 4 1

1

2 4 6 7 9

2. Min-Heap – Where the value of the root node is **less than or equal to** either of its children.



- ## Introduction to Heap Sort
1. Build the **complete binary tree** using given elements.
 2. Create **Max-heap** to sort in ascending order.
 3. Once the heap is created, **swap** the last node with the root node and **delete** the last node from the heap.
 4. Repeat **step 2 and 3** until the heap is empty.



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3	5	1
---	----	---	---	---

Step 1 : Create Complete Binary Tree

1 2 3 4 5

4	10	3	5	1
---	----	---	---	---

4
10 3 5 1

Now, a binary tree is created
and we have to convert it into
a Heap.

Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3
---	----	---

Step 2 : Create Max Heap

4

1 2 3 4 5 10

10 is greater than 4 So, swap 10 & 4

4	10	3	5	1
10	4			

to the child nodes.

10 3 4

Swap

5 1

In a Max Heap, parent node is
always greater than or equal



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3	5	1
---	----	---	---	---

Step 2 : Create Max Heap

10

1 2 3 4 5

10	4 5	3	5 4	1
----	-----	---	-----	---

5

5 is greater than 4 So, swap 5 & 4

4 3

Swap

In a Max Heap, parent node

4

is always greater than or equal
to the child nodes.

Max Heap is created

5 1



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3	5	1
---	----	---	---	---

Step 3 : Apply Heap Sort

1

1 2 3 4 5

10	5	3	4	1	10
1					

1. Swap the first and the last nodes and
2. Delete the last node.

Swap

10

4 1

5 3

10



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3	5	1
---	----	---	---	---

Step 3 : Apply Heap Sort

5

1 2 3 4 5 **Max Heap Property is**

1

violated so, create a

1	5 1	3	4 1	10
5	4			

4
1

Swap
Max Heap again.

1
4

5 3



Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3	5	1
---	----	---	---	---

Step 3 : Apply Heap Sort

1

1 2 3 4 5 Max Heap is created 5

5	4	3	1 5	10
1				

- 1 last nodes and
2. Delete the last node.

4 3

Swap

1. Swap the first and the

5

Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3
---	----	---

Create Max Heap

Step 3 : Apply Heap Sort

3

1 2 3 4 5

1 4	4 1	3 4	5	10
3				

Swap

1. Swap the first and the last nodes and
2. Delete the last node.

4

4 3 1 4

again

Max Heap is created

Already a Max Heap

Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3
---	----	---

Step 3 : Apply Heap Sort

1

3

1	2	3	4	5
3	1	1	3	4
			5	10

3
1

Swap

1. Swap the first and the last nodes and
2. Delete the last node.

Step 3 : Apply Heap Sort

Heap Sort – Example 1

Sort the following elements in Ascending order

4	10	3
---	----	---

Remove the last element
from heap and the
sorting is over.

1
1 2 3 4 5

1	1	3	4	5	10
---	---	---	---	---	----

Already a Max Heap



Heap Sort – Example 2 □ Sort given element in ascending order using heap sort. 19, 7,
16, 1, 14, 17

19	7	16	1	14	17	1
----	---	----	---	----	----	---

	14	17		7	6
--	----	----	--	---	---

Step 1: Create binary tree Step 2: Create Max-heap

19

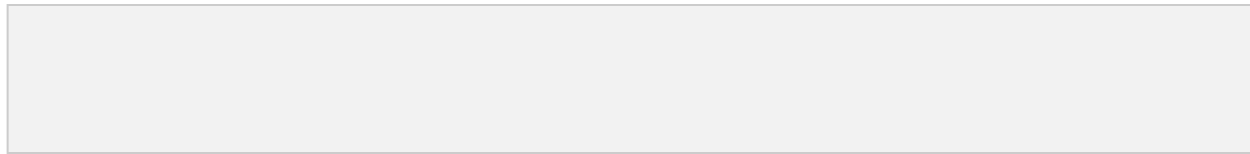
1 14 17

7 16 17

7 16

19

14



Heap Sort – Example 2

Step 3 Step 4

19	14	17	1	7	16	1
16					9	

16	14	17	1	7	
17		16			

16 Create Max-heap

19

Swap & remove the last element

14 17

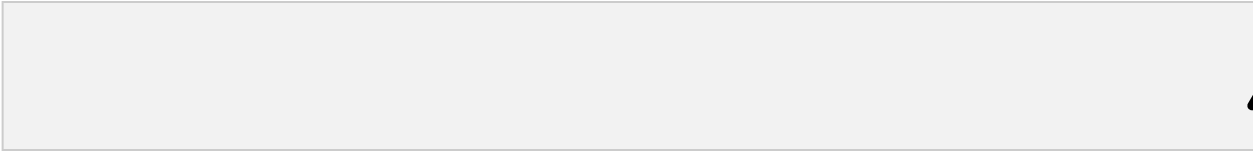
17

16

14 17 16

1 7 16 19

17



Heap Sort – Example 2 Step 5 Step 6

17	14	16	1	7	19
7				17	

17

7	14	16	1	17	
16		7			

7 Create Max-heap

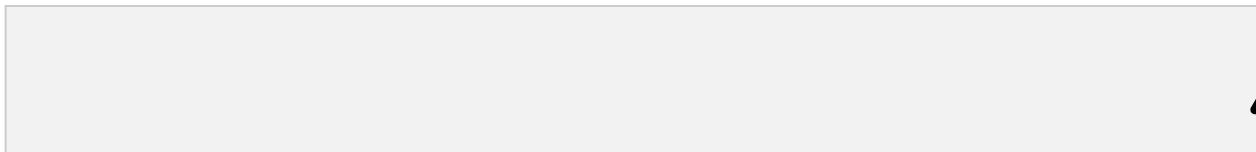
the last
element

14 16
16
7

Swap &
remove

1 7
17
14 16 1

7



16

Heap Sort – Example 2

Step 7 Step 8

16	14	7	1	17	19
1			16		

1 14

1	14	7	16	17	
14	1				

Create Max-heap

Heap Sort – Example 2

14	1	7	16	17	19
7		14			

7	1	1	7	14	16	17	
---	---	---	---	----	----	----	--

Swap &

element

16

remove the last

Swap &
remove the

Already a Max-heap

Swap & remove the last

last element

Step 11

element

1	1
	7

Remove the
last element

The entire array is sorted
now.



Exercises □ Sort the following elements using Heap Sort Method.

1. 34, 18, 65, 32, 51, 21

2. 20, 50, 30, 75, 90, 65, 25, 10, 40

□ Sort the following elements in Descending order using Hear Sort
Algorithm. 1. 65, 77, 5, 23, 32, 45, 99, 83, 69, 81





Binary Tree Analysis



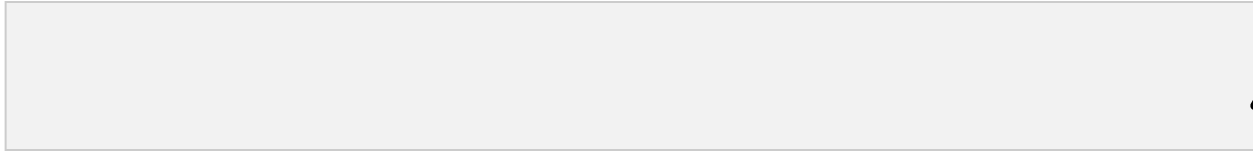
Heap Sort – Algorithm # Input: Array A
Output: Sorted array A

Algorithm: Heap_Sort($A[1, \dots, n]$)
 BUILD-MAX-HEAP(A)

```

for i ← length[A] downto 2
    do exchange A[1]   A[i]
    heap-size[A] ← heap-size[A] - 1
    MAX-HEAPIFY(A, 1, n)

```



4	
---	--

Heap Sort – Algorithm

Algorithm:

```

BUILD-MAX-HEAP(A) heap-size[A] ←
length[A] for i ← ⌊length[A]/2⌋ downto
1 do MAX-HEAPIFY(A, i)

```

heap-size[A] = 6

4	1	7	2	9	3
---	---	---	---	---	---

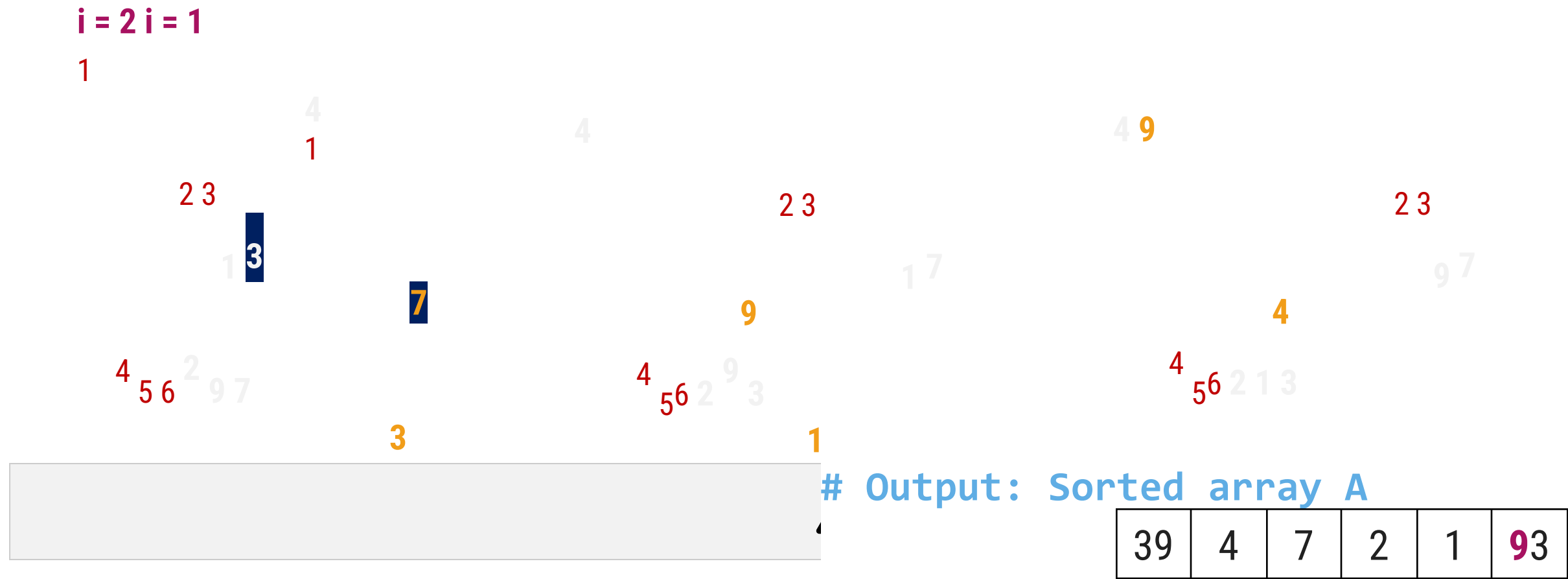
i = 3

4	9	7	2	1	3
---	---	---	---	---	---

1

9	4	7	2	1	3
---	---	---	---	---	---

1
4
2 3
1 3
4 5 6
2 9 7



Heap Sort – Algorithm

Input: Array A

Algorithm: Heap_Sort(A[1,...,n])

BUILD-MAX-HEAP(A)

for i ← length[A] downto 2


9

1

Heap Sort – Algorithm

Algorithm: Max-heapify(A, i, n)

3	4	7	2	1	9
---	---	---	---	---	---

```
do exchange A[1]  A[i]
heap-size[A] ← heap-size[A] - 1
MAX-HEAPIFY(A, 1, n)
```

2 3

4 7

4 5 6 2 1 3



$l \leftarrow \text{LEFT}(i)$ $r \leftarrow \text{RIGHT}(i)$

$l \leftarrow 2$

1

$r \leftarrow 3$

if $l \leq n$ and $A[l] > A[i]$

then $\text{largest} \leftarrow l$

else $\text{largest} \leftarrow i$

Yes

3

$\text{largest} \leftarrow 2$

2 3

4 7

4 5

if $r \leq n$ and $A[r] > A[\text{largest}]$

Yes

21

```
then largest ← 3
largest ← r
if Yes
largest ≠ i
then exchange A[i] A[largest]
MAX-HEAPIFY(A, largest, n)
```

Output: Sorted array A

3	4	7	2	1	9
---	---	---	---	---	---

Heap Sort – Algorithm

Input:
Array A

Algorithm: Heap_Sort(A[1,...,n])

BUILD-MAX-HEAP(A)

for i ← length[A] downto 2

do exchange A[1] A[i]

heap-size[A] ← heap-size[A] -

1

7

1 MAX-HEAPIFY(A, 1, n)

2 3

4 3

4 5

2 1

1

Heap Sort Algorithm –

Analysis # Input:

Array A

Output: Sorted array

A

Algorithm:

Heap_Sort(A[1,...,n])

??T??

heap-size[A] ← length[A]

for i ← ⌊length[A]/2⌋ downto

(A)

???)

i)

BUILD-MAX-HEAP

??(??

do MAX-HEAPIFY(A, ??(??))

for i ← length[A]
downto 2

do exchange A[1] 
A[i]

$\text{heap-size}[A] \leftarrow$
 $\text{heap-size}[A] - 1$

$?? - ??$

$??(?? - ??)$
 $(???????)$

MAX-HEAPIFY(A, 1, n)

Running time of heap sort algorithm is:

$?? \text{ } ?? \text{ } ?? \text{ } ?? \text{ } ?? \text{ } ?? \text{ } ?? \text{ } ?? \text{ } ?? \text{ } ?? + ??(?? \text{ } ?? \text{ } ?? \text{ } ?? \text{ } ?? \text{ } ??) ?? - ?? + ??(?? - ??) = ??(??$
 $?? \text{ } ?? \text{ } ?? \text{ } ?? \text{ } ?? \text{ } ??)$



Sorting Algorithms

Radix Sort, Bucket Sort, Counting Sort



- Radix Sort** □ Radix Sort puts the elements in order by **comparing the digits of the numbers**.
- Each element in the n -element array has d digits, where digit 1 is the lowest-order

digit and digit $??$ is the highest order digit.

Algorithm: RADIX-SORT(A, d)

for $i \leftarrow 1$ to d

do use a stable sort to sort array A on digit i

□ Sort following elements in Ascending order using radix sort.

363, 729, 329, 873, 691, 521, 435, 297

Radix Sort - Example

3 6 3

7 2 9

3 2 9

8 7 3

6 9 1

5 2 1

4 3 5

2 9 7

6 9 1 5 2 1 3 6 3 8 7 3 4 3

5 2 9 7 7 2 9 3 2 9

5 2 1 7 2 9 3 2 9 4 3 5 3 6 3 8 7 3 6 9 1 2 9 7

Sort on column 1

The entire array is sorted now.

Sort on column 3

Sort on column 2



Bucket Sort – Introduction □ Sort the following elements in Ascending order using bucket sort.

45	96	29	30	27	12	39	61	91
----	----	----	----	----	----	----	----	----

1. Create $\diamond\diamond$ empty buckets.
2. Add each input element to appropriate bucket as,
 - a. Bucket $\diamond\diamond$ holds values in the half-open interval,

$$\diamond\diamond * \diamond\diamond\diamond\diamond \leq \diamond\diamond[\diamond\diamond] < (\diamond\diamond + \diamond\diamond) * \diamond\diamond\diamond\diamond$$

3. Sort each bucket queue with insertion sort.

4. Merge all bucket queues together in order.

□ Expected running time is $O(n \log n)$, with n = size of original sequence. If n is $O(n^2)$ then sorting algorithm is $O(n^2 \log n)$.



Bucket Sort – Example

45	96	29	30	27	12	39	61	91
----	----	----	----	----	----	----	----	----

45 96 29 30 27 12 39 61 91

0 1 2 3 4 5 6 7 8 9

Sort each bucket queue with insertion sort

Merge all bucket queues together in order

12	27	29	30	39	45	61	91	96
----	----	----	----	----	----	----	----	----



Bucket Sort - Algorithm

Input: Array A

Output: Sorted array A

Algorithm: Bucket-Sort($A[1, \dots, n]$)

$n \leftarrow \text{length}[A]$

for $i \leftarrow 1$ to n do

 insert $A[i]$ into bucket $B[\lfloor A[i] \div n \rfloor]$

```

for i ← 0 to n - 1 do
    sort bucket B[i] with insertion sort
concatenate the buckets B[0], B[1], . . ., B[n - 1] together in
order.

```



Counting Sort – Example □ Sort the following elements in Ascending order using counting sort.

3	6	4	1	3	4	1	4	2
---	---	---	---	---	---	---	---	---

Step 1

Given elements are stored in an input array $arr[1, \dots, 9]$

1 2 3 4 5 6 7 8 9

3	6	4	1 3	4	1	4	2
---	---	---	-----	---	---	---	---

Index

Elements

Step 2

Define a temporary array $count$.

The size of an array $count$ is

equal to the **maximum element** , 6] to 0.
in array **??**. Initialize **??**[1, ...

Index

Elements

0	0	0	0	0	0
---	---	---	---	---	---

1	2
---	---



Counting Sort – Example

Sort the following elements in Ascending order using counting sort.

Input array **??**

3	6	4
---	---	---

Step 3
Update an array C with the occurrences of each value of array **??**

Index

Elements

1	2	3	4	5	6
0 2	1 0	2 0	0 3	0 0	1 0

++

Step 4

In array $??$, from index 2 to $??$

add the value with previous element

2	3	5	8	8	9
---	---	---	---	---	---

Index

Elements

1	2
---	---



Counting Sort – Example

Create an output array $??[1...9]$. Start positioning elements of Array $?? ?? ?? ?? ?? ?? ??$ as shown below.

Input array $??$

1 2 3 4 5 6 7 8 9

3	6	4	1	3	4	1	4	2
---	---	---	---	---	---	---	---	---

Temporary Array C

1	2
2 0 1	3

Output Array B

1 2 3 4 5 6 7 8 9

1	1	2	3	3	4	4	4
---	---	---	---	---	---	---	---



Counting Sort - Procedure

- Counting sort assumes that each of the n input elements is an integer in the range 0 to k , for some integer k .
- When $k = O(n)$, the counting sort runs in $O(n)$ time.
- The basic idea of counting sort is to determine, for each input element x , the number of elements less than x .
- This information can be used to place element x directly into its position in the output array.



Counting Sort - Algorithm # Input: Array A

Output: Sorted array A

Algorithm: Counting-Sort($A[1, \dots, n]$, $B[1, \dots, n]$,

k) **for** $i \leftarrow 1$ **to** k **do**

$c[i] \leftarrow 0$

for $j \leftarrow 1$ **to** n **do**

$c[A[j]] \leftarrow c[A[j]]$
 $+ 1$

for $i \leftarrow 2$ **to** k **do**

$c[i] \leftarrow c[i] +$
 $c[i-1]$

for $j \leftarrow n$ **downto** 1 **do**

$B[c[A[j]]] \leftarrow A[j]$
 $c[A[j]] \leftarrow c[A[j]] - 1$



- 1



Thank You!