Question Bank - Graph Theory I

• Problems 1 − 5 are based on the topic "Basic Graph Theory Models". • Problems 6 − 11 are from the topic "Basic Terminologies & Properties of Graphs". • Problems 12 − 14 are from the topic "Mathematical Operations on Graphs". • Problems 15 − 20 are from the topic "Representation of Graphs".

• Problems 21 − 23 are from the topic "Special Graphs".

• Problems 24 − 26 are from the topic "Isomorphism of Graphs".

1. The Hollywood graph represents actors by vertices and connects two vertices when the actors represented by these vertices have worked together on a movie. Describe a graph model to represent the graph.

2. Describe a graph model that represents whether each person at a party knows the name of each other person at the party. Should the edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed?

3. The intersection graph of a collection of sets $A_1, A_2, \ldots, A_n$ is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

   (a) $A_1 = \{0, 2, 4, 6, 8\}$, $A_2 = \{0, 1, 2, 3, 4\}$, $A_3 = \{1, 3, 5, 7, 9\}$,
   $A_4 = \{5, 6, 7, 8, 9\}$, $A_5 = \{0, 1, 8, 9\}$

   (b) $A_1 = \{\ldots, -4, -3, -2, -1, 0\}$,
   $A_2 = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$,
   $A_3 = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$,
   $A_4 = \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$,
   $A_5 = \{\ldots, -6, -3, 0, 3, 6, \ldots\}$

   (c) $A_1 = \{x|x < 0\}$,
   $A_2 = \{x| - 1 < x < 0\}$,
   $A_3 = \{x|0 < x < 1\}$,
   $A_4 = \{x| - 1 < x < 1\}$,
   $A_5 = \{x|x > -1\}$,
   $A_6 =$ R (The set of real numbers)

4. For each course at a university, there may be one or more other courses that are prereq uisites for it. How can a graph be used to model these courses, and which courses are prerequisites for which courses? Should edges be directed or undirected? Looking at the graph model, how can we find courses that do not have any prerequisites, and how can we find courses that are not prerequisites for any other courses?

5. Describe a discrete structure based on a graph that can be used to model relationships between pairs of individuals in a group, where each individual may either like, dislike, or be neutral about another individual, and the reverse relationship may be different.

6. Find the number of vertices, the number of edges, and the degree of each vertex in the given graph. Identify all isolated and pendant vertices.

(a)

a b c

e
f d

(b)

a b c

d e

(c)

a b

d c

7. Can a simple graph exist with 15 vertices, each of degree five?

8. How many edges are there in a graph with 10 vertices, each of degree 6?

9. Establish the Handshaking Lemma for the Cycle graphs C_n.

10. Prove that any simple graph with n number of vertices has at most $\frac{n(n-1)}{2}$ edges. For each n, provide an example of such graphs.

11. A sequence $d_1, d_2, \ldots, d_n$ is called graphic if it is the degree sequence of a simple graph. Determine whether each of these sequences is graphic. For those sequences that are graphic, draw a graph having the given degree sequence.

(a) 3, 3, 3, 3, 2
(b) 5, 4, 3, 2, 1
(c) 4, 4, 3, 2, 1
(d) 4, 4, 3, 3, 3
(e) 3, 2, 2, 1, 0
(f) 1, 1, 1, 1, 1

12. How many subgraphs with at least one vertex does K_3 have? 2

13. Draw all subgraphs of this graph.

a b

c d

14. Find the union and intersection of the given pair of simple graphs. (Assume edges with the same endpoints are the same.)

(a)

f
a b

a b e

e

c

c d

g

(b)

d

a e h

a b
c d

f

g

15. Use an Adjacency List to represent the following graph. Also, determine the Adjacency Matrix and the Incident Matrix of the graph.

a b c

d e

16. Use an Adjacency List to represent the following graph. Also, determine the Adjacency Matrix of the graph.

a b c

d e

3

17. Draw a graph with the given adjacency matrix.

(i)
0 0 1 0
1 1
0 0 1 0 1
1 0 1 1 1

(ii)
0 0 1 0 1 1 1
1 0
1 1
0 0 1 0 1

(iii)
0 0 2 2
1 2 1 2 0

18. Represent the given graphs a b c d using an adjacency matrix. a b

c d

19. Draw an undirected graph represented by the given adjacency matrix.

0 0 2 4
0 1 2 3
0 1 3
0 4
1 2 1 3
0 3 1 1
0 1 0 3

20. Find the adjacency matrix of the given directed multigraph with respect to the vertices listed in alphabetic order.

a b

c d

21. Draw the following graphs and determine which of these graphs are bipartite: (a) K_7, (b) C_7, (c) W_7, (d) $K_{4,4}$.

22. Determine which of the following graphs are bipartite.

(a)

b c

a

d

e

(b)

b

a

c

f

d

e

23. A simple graph is called regular if every vertex of the graph has the same degree. Then, for what values of m and n, the graph $K_{m,n}$ will be regular?

24. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or

provide a rigorous argument that none exists.

(a)

u_1 u_2

v_5

[redacted]3

v_4

u_5 u_4 u_3

1

(b) v v_2

v_6

v_7

u_6 u_7

v_1 v_2 v_3 v_4 v_5

u_1 u_2 u_3 u_4 u_5

u_8 u_9 u_{10} 5

v_{10} v_8 v_9

(c) u_2

v_3

v_{10}

u_9

v_1 v_8

u_1 u_{10} u_3 u_8 v_2

u_5 u_4 v_4 v_9

6

u u_7

v_6 v_7 v_5

25. Determine whether the graphs without loops with these incidence matrices are isomorphic. (a)

□ □ 1 0 1 0 1 1 □ , 1 1 0 1 0 1

□ 1 1 0 □ □ 0 1 1

□

□ (b)

1 0 1 0 □□ 0 1 0 0

1 0 0 0 □, 1 0 1 0

□ 1 1 0 1 □ 1 0 1

□□ 1 1 0 □□ □□

□1 1 □ □0 1 □ □

0 0 0 0 0 1

0 1 1 1

26. Determine whether the given pair of directed graphs is

isomorphic. (a)

(b)

u_1

u_4

v_3

u_2 u_3

v_5 v_4

u_1 u_2 u_3 u_4 u_5 u_6

6

v_6

v_1

v_4

v_2 v_3

v_1 v_2