LOGIC

PROPOSITIONAL LOGIC (Deals with propositions)

Proposition: A declarative statement that is either true or false but not both.

Example
① Ahmedabad is the capital of Gujarat. F
② $5+2=4$ F
③ $1+1=2$ T
} Proposition

④ Please pass the paper. (not a proposition as not declaring anything)
⑤ $x+5=7$ (not a proposition but a predicate logic)

- Requests, orders are not propositions and expressions that cannot be evaluated as T or F are not ~~proposition~~ propositions.

– Propositional variable or statement variables are denoted by $p, q, r, s, \ldots$

– Truth values are denoted by T or F.

From simple propositions, compound propositions can be formed using logical connectives or operators.

① Let p be a proposition. The negation of p, denoted by "$\neg p$", is the statement that, "It is not the case that p".
The truth value of $\neg p$ is the opposite of the truth value of p.

Example p: Today is friday.
$\neg p$: "It is not the case that today is Friday" or "Today is not friday".

Truth Table

$\neg p$:

p	$\neg p$
T	F
F	T

② $p \wedge q$ (conjunction/and) is the statement "p and q".

Example p: Today is Friday
q: It is raining today

$p \wedge q$: Today is friday and it is raining today.
The statement is T on rainy fridays and is F on any day that is not a friday & on fridays when it does not rain.

Truth Table (T.T.)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

③ Disjunction $p \vee q$: "p or q" (Inclusive OR)

T.T

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Example same as the previous

Today is Friday or it is raining today.

True on any day that is either friday or a rainy day (including the rainy fridays). false on days that are not fridays when it also does not rain.

④ Exclusive OR $p \oplus q$: "either p or q but not both"

True when exactly one of them is true and is false otherwise.

T.T

p	q	$p \oplus q$
T	T	F
T	f	T
f	T	T
F	F	F

⑤ Conditional Statements $p \rightarrow q$: "If p then q".

$p \rightarrow q$ is ~~pla~~ false when p is T & q is F & true otherwise.

p ↙ hypothesis, q ↘ conclusion/consequence

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \rightarrow q$,

"If p then q"
"If p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"
"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

Converse, Inverse & Contrapositive

- $q \to p$ is the converse of $p \to q$
- $\neg p \to \neg q$ is the inverse of $p \to q$
- $\neg q \to \neg p$ is the contrapositive of $p \to q$.

Biconditionals

$p \leftrightarrow q$: " p if and only if q "

True only when both are ture or, both are false & false otherwise.

T. T.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

T. T. of compound propositions

Example $(p \vee \neg q) \to (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \to (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of logical operators

operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\to$	4
$\leftrightarrow$	5

Propositional Equivalence

- A compound proposition that is always true no matter what the truth values of the propositions that occur in it is called Tautology.
- A compound proposition that is always false is called contradiction.
- Contingency : Compound prop. that is neither a tautology nor a contradiction.

Example $p \vee \neg p$ is a tautology & $p \wedge \neg p$ is a contradiction.

T. T.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F
		always true	always false.

Logical Equivalences

compound propositions p & q are logically equivalent if $p \leftrightarrow q$ is a tautology and is denoted by "$p \equiv q$" or "$p \Leftrightarrow q$".

Example

$$\left.\begin{array}{l} \neg(p \wedge q) \equiv \neg p \vee \neg q \\ \neg(p \vee q) \equiv \neg p \wedge \neg q \end{array}\right\} \text{De Morgan's Law}$$

T. T.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T ✓	T	T	T ✓

Example $p \rightarrow q \equiv \neg p \vee q$

T. T.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T ✓	T ✓

Example $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ Distributive Law

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

$$\neg (p_1 \vee p_2 \vee \dots \vee p_n) \equiv (\neg p_1 \wedge \neg p_2 \wedge \dots \wedge \neg p_n)$$
$$\neg (p_1 \wedge p_2 \wedge \dots \wedge p_n) \equiv (\neg p_1 \vee \neg p_2 \vee \dots \vee \neg p_n)$$

Some Logical Equivalences

$p \wedge T \equiv p$
$p \vee F \equiv p$
Identity Laws

$p \vee T \equiv T$
$p \wedge F \equiv F$
Domination Laws

$\neg(\neg p) \equiv p$ Double Negation Law

$p \vee p \equiv p$
$p \wedge p \equiv p$
Idempotent Laws

$(p \vee q) \vee r \equiv p \vee (q \vee r)$
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Associative Laws

$p \vee q \equiv q \vee p$
$p \wedge q \equiv q \wedge p$
Commutative Laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$
De Morgan's Laws

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
Distributive Laws

$p \vee \neg p \equiv T$
$p \wedge \neg p \equiv F$
Negation Laws

$p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$
Absorption Laws

Conditional logical Equi.

$p \to q \equiv \neg p \vee q$

$p \to q \equiv \neg q \to \neg p$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg (q \to \neg p)$

$\neg (p \to q) \equiv p \wedge \neg q$

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

Biconditional Equi

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg (p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

$\boxed{\text{Example}}$ Show that $\neg (p \to q) \equiv p \wedge \neg q$

→ We know that, $p \to q \equiv \neg p \vee q$

Then, $\neg (p \to q) \equiv \neg (\neg p \vee q)$

$\equiv p \wedge \neg q$ (using De Morgan's Laws)

Bitwise operations

Bitwise OR, Bitwise AND, Bitwise XOR

Consider two bit string $x = 10110$ and $y = 11011$.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
1	1	1	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0
0	1	1	0	1

Consider 0 as F and 1 as T. and computation follows bitwise.