LOGIC

PROPOSITIONAL LOGIC (Deals with propositions)

1

Proposition! A declarative statement that is either true or false but not both.

Example J Ahmedabad is the capital of Gujarat. F | Proposition

2

5+2

=

4

F

=2 T

© 1+1

pass the paper. (not a proposition as not declaring anything)

4 Please

6

x+5=7 (not a proposition but a predicate logic) not propositions and expressions

- Requests, orders

that

are

cannot be evaluated as

Tor F are

not propositions

propositions.

Propositional variable

or statement variables

are denoted by

are denoted by

Tor F.

$,q, r, s,

Truth values

propositions, compound propositions

can be formed From simple

or

operators asing logical connectives

© Let & be a

proposition.

negation of denoted

The p

a

by Tp", is the

statement that,

The

a It is not the case that

p".. truth value of +p is the

opposite

F

the

@ P^q (conjunction/

and) is the statement "p

and q". P: Today is Friday

9! It is raining today

Example

$19: Today is friday and truth

value of p. it is raining

raining today.

The statement is T on

rainy Example : Today is friday.idays and is F on

any day

тp

I that

not the

case To: "It is not

Friday" or "Today is is

Friday

today is

not friday"

Truth Table

friday & on that is not a fridays when it does not

rain. Table (T.T.)

Truth Table

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3 Disjunction p vq : "Þ or q

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Example

(Inclusive

OR)

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Same as the previous

it is raining

today.

Today is Friday or

True

on any day that is either friday or a rainy

day (including the rainy fridays),

false on

on days

days that are not fridays when it also

does not rain.

$@q: " either & or q

but not both"

True when exactly one of them is true and is

false otherwise

G

ff

Conditional

Statements

pq is a false when

Conclusio

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consequence

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$ → q!" If & then q"

$→ 9:

pis T & q is F & true

otherwise.

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If þ then q

"If p, q

q

q

a

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is sufficient for q

if

when p

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necessary condition for p is q" unless + p

imp

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only if

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a sufficient condition for q is p"

P

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a

whenever p"

p"

is necessary for p.

q follows from p"

T. T.
lon verse

2

Inverse & Contra positive

9

Þ

is the

79

is the

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+ q → +p is the

7q

Biconditionals

bqpq

Converse of p→ q

inverse

7 4→q Contra positive of p→q.

449

и

I if and only if q"

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T. T.

Example

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q

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True only when both are ture or both are false & false otherwise.

Compound propositions

$(pv+q) \to (p19)$

s

+9

PV + q

$19

(pv+q)

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Precedence of logical

operators

operator

Precedence

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Propositional Equivalence

A compound proposition that is always true no matter what the truth values of the propositions that

Called

Tautolog

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that occur in it is

A compound proposition that is always false is

call ed

contradiction.

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Contingency : Compound prop. that is

neither tautology

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Contradiction.

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$ VTP is

a tautology & 61+þ

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Contradiction.

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Logical Equivalences

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always true

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compound propositions &

& q

pe p→ q a is a tautology

and

if

p

= q

Example

T. T.

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(рла) = трина

(pvq) = p λ + q

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pvq (pvq)

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Logically

equivalent

is denoted by

De

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$B \rightarrow q = +p \vee q$

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Example

Þv (q^r) = (pva) 1 (pur) Distributive

aar bv (918) pvq

Law

pur (puq) (pvr)

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+ (P1 VP2 V... VP2) =

(+P+1+P2 ^.

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V¬$n)

TCP1 ^ P2^... Apn) = (+ P1 V + Þ 2

V

Some

Logical Equivalences

PAT=P

Identity Laws

$ UT = T

Domination Laws.

b V F = p

p

PAFEF

Double

pvp = p Idempotent Laws

PAP = P /

pap

Þv q = q v p

рад= аль

}

Commutative

Laws

pu(918) =

(pvq)^(pvr). рисала)

Þλ(qvr) = (p^q) v (par)

Distributive Laws

pv (p ^

q) = P P ^

(pvq) = Þ

Absorption

Laws

+(+1) = Þ | Negation

Law

(pvq) vr = pv (qvr) & Associative ($19)^8 = P1

(910) Laws

$+(p1q) = +PV + q$

$+(pvq) = + Þ 1 + q$

De Morgan's

Laws

Витрет PAP

F Negation

Laws

Conditional logical Equi.

$Þ \rightarrow q = \neg þ \vee q$

ד

$Þ \; q = 7 q \rightarrow\rightarrow \; \rightarrow þ$

=

Þ vq Þ 19

$7(6\longrightarrow q) =$

ד

$7(9\longrightarrow\neg p)$

$p\wedge\neg+q$

$(p+c)\lambda(p+r) = b + (918)$

$p.\rightarrow r) 1 (q\longrightarrow r) = (pvq)$

$\rightarrow\alpha (b\longrightarrow q) v (b\rightarrow r) = p\rightarrow (qv)$

$(p\rightarrow r) \; v (q\longrightarrow r) = (\$19) \rightarrow r$

Biconditional

Equi

p← q = (p → q)^(q→

b) p↔ q = →→→ + q Þ

→→→ E = (p19)

√(76119) + (P←q) = Þ

79

+(p −q) = p 1 ¬ q

$ → q = +p v q

4→

+(+pva)

+ (pq) = +(→ vq)

+pvq

= Þλ+q (using De

Morgan's Laws).

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(Examp

le)

Show that

We

know that,

Then,

Bitwise operations

Bitwise OR

Bitwise AND, Bitwise

Bitwise XOR

Consider tw

fwo bit

x

y

bit string x

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and y=11011.

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Consider 0 as F

as T.

and computation follows bitwise

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