Definition ! A graph G = (V,E) or <V, E> consists

= <V, *E*> of

non-empty set

v, a

and

E,

a

has either

with it

called

201

one

of vertices (or nodes) set of

edges, Each edge or

two edges associated

end

points.

the

24

$V = \{21, 22, 23, 24\}$

2

23

$E = \{(01, 22), (22, 23), (03, 04), (04, 04)\}$

graph $G = (V, E)$, where

non-empty set.

Consider a

of

$|V| <$

ar

✓ is

a

∞ =

or

V

then the graph G

is

& IVI

.

1

order, then

the graph G

A simple Graph ! Consider

V is a

of finite order, finite graph. is of infinite

is an infinite

graph

a graph in which

each edge connected two different vertices and

Connect the same pair of vertices. edge

Connect

по

two

Then

the graph

V 2

201

$V = \{21, 22, 23$

$E =$

$\}$

гез

$\{(201322),$

$(22, 23)\}$

is

a

201

simple

graph.

24

05

чег

203

V= { 21, 22, 23, 24, 25) == {{ (11,

12), (02, US),

(1202), (14, 21),

(22, 25

مرد

Multigraph: Multigraph is a

(2)

has multiple edges connecting the

same

that

pair of

vertices, *but*

two different vertices

го

202

graph

be edge

connects

гел

each

201

гез

202

20

чез

3

V = {21, 22,

23} E = { (11, 12), (12, 13) }

The edge (22, 23) has multiplicity 2.

E =

V = {21, 22, 237

2)

ろ

1,22), (22, 23), (23,24)

The edges (21, 22), (22,03) and (103, 24) have multiplicity 2.

An edge that connects

called

Loops !

itself

is

Coops and

between

the

a

Pseudograph! A

graph

possibly

a

<

201

Coop.

a vertex to

that may

include

contains multiple edges

same fair of vertices is

calle

pseudograph :

○ v =

{27}

{ (0, 0)

)

E

حيا
N

ro

مرد

E

201

13

re

V = { 21, 22, 23,

04}

= {201,

(= {(21, 22), (22, 22). (V2, 24), (4,

V

(23, 201), (201,

284)}

The edge (293, 19) has multiplicity

2.

Undirected Graph: An undricted graph is a

graph

with

fo

that

a re

end

associated

Directed

@ undirected

edge

s

that

are

an a unordered pair of vertices

points

Graph A

128

directed

(V,E) consists of

a

non-

a

of

is

set

directed

edge pair of

graph (or
digap):
empty set V and edges (or arcs) F.
fach
directed
edges
associated
F,
with an ordered
is
at 29
vertices (u, v) that
start from
и
and
end
read as
?

simple directed

directed

graph!

graph : Direded

Directed

graphs

WOOD
S

with

по

то

and between the

same

the same direction

becimple

人

are

127

of vertices hs.

cted graph dire for

gtiple edges in

Directed graphs

multiple edges

Directed

Multigraph :

with

in

no the

wops

Same

pair of vertices

Multigraph

ro

1

V2

direction between the same

and have

are

called

Directed

201

20

J

ro

4

15

ro

3

Simple directed

graph.

No two vertices are connected

by

& multiple edges

in the same direction.

re
rer

Not a simple

directed graph. The edge

starts

at 13 and endsatuz (13, 192) has multiplicity 2.

So it is a directed

multigrap

GRAPH TERMINOLOGY & SPECIAL TYPES OF GRAPHS

undirected graph G = (V,E), two vertices

adjacent in G if an edge of

G!

In

an

a, v E V

are

end points

J

The edge

the edge

e

ce

and re.

u and we are

is associated with Eu,v. Then is called

incident

Degree of a vertex

in

Consider

an

with vertices

an undirected graph!

undirected graph G =

(V,E) and

re

LEV. Then the degree of ve is

denoted by deg (20) and is the

are

a

Ex:

number of edges

that

except that a loop at

to the degree.

incident with it

contributes twice

vert ex

7

rer

203

24

deg(201) = 2 deg(105) = 0 deg (192) = 4 deg (286) = 3 deg(23) = 4 deg (207) = 4 deg (294) = 1

25

of deg o

is

called

an

isolated

In

- A vertex

vertex.

20.

197

A vertex is

the

bendant

o

pendant iff it has deg 1.

above example, the

The orem?

The

Consider an

e

and

vartex

24

is

a

isolated.

295 is

apple Lemma

Handshaking

undirected graph $G = (V, E)$

with

number of edges

Then,

[True even

2e

Σ

deg

(20)

29€ V

even if multiple edges and loops are

present J

Each edge contributes

2

to the

sum of

degrees.

1

Ex !

with

How

many

10

vertices

не

клого

that,

i.e., the

graph

graph has

I

(..

there

in

a

ed gas are

each having deg

Σ deg

(20)

WEV

10X6

01

13

graph

6.

113

€60

-> 2e = 60 [From Hand

shakin

[emma]

e 30.

30 edges in total.

Theorem 2

An

4

of vertices

Consider

of

undirected graph

odd degrees.

has

even numbe

a graph

$G = (V, E)$

Then,

V_1

is the set

of

vertices of even

12

even

degree odd

degree

V_2 is

$$2e = \sum_{v \in V} \deg(v)$$

So,

$2e$

H

even

Σ deg (2) + Σ

deg(w)

20€V1

even

10 EV2

must be

even

Degree of a vertex in a directed graph !

Consider

a directed graph

(u, v) €£. Then

initial vertex

as

an edge

defined

respectively.

For a loop

G = (V,E) and

a

and

and re

are

terminal vertex

initial vertex

=P

terminal vertex.

– In-degree (deg (1)) : The in-degree of a vertex in a directed graph is the no. of edges with

20

as their terminal

vertex.

Out - degree (deg+ (20)): The out-degree of a vertex in a directed graph is the no. of edges with is as their

initial vertex

Note!. A loop at a vertex

contributes

in-deg and out-deg of the vertex.

1 to both

re

255

204

Theorem

20

3

$$\deg(201) = 2\deg + (101) = 4$$

değ

2

$$\deg(194) = 2 \quad \deg + (194) = 2$$

deg-(105)

=3 dag +

(295) = 3

Consider

a

$$\Sigma \deg(20)$$

VE v

deg (192) deg + (92) = 1

deg (203)=3 deg + (203) =

2

deg (106) =

0 deg +

(16)=0

directed graph Σ degt (20) =

(E) .

WEV

G =

(V, E),

Then

Each edge has initial vertex and fermal vertex

va

and

hence

of the

Contributes

in-degrees of all

1

to

the summation the vertices and 1 to

"out-degree

the summation of the out- degrees of all the vertices.

SPECIAL GRAPHS

a simple undirected

edge between

A. Complete Graph! Consider

graph that contains exactly one

distinct vertices and each

pair of

is

no vertex it is called

connected to itself. Then Complete Graph and

is denoted by Kn, where

vertices.

a

is

the

no. of

n

K3

кл

K2

K4

KБ

B. Cycles: Consider a graph G =

(V,E) with

the vertex set V = {0, 02, on? and the edge set

го

n}

(20 n-1,

On) (20m, 2017) Then the graph is a cycle and is denoted by en.

E = {(v1, v2), (12, 13), (13, 14)

CT

CA

C5

ہاں

c. Wheels:

and

add

that the

n vertices

graph

W3

Consider

Ain

new

the

a cycle cn with n vertices additional

vertex to the cycle such

to each of connected

vertex

in the

is

cycle by new edges. The

new

wheel and is denoted by Wn.

is

a

W4

veet

Some other graphs

W5

W6

Mixed graph! A graph with both directed and

h

is called

undirected grap

weighted graph:

Consider a

a

mixed

graph.

graph in which each

weight

the edges is assigned with some value. The graph is called a

of the

or

Ɵr

10

7

20

01

6

3

3

weighted graph.

Wt. is nothing but the cost of reaching from source to the destination.

Bipartite Graph: A simple graph G = (V,E) is called bipartite if the vertex set V can be partitioned into two disjoint sets V1 & V2 (V =

V1 BUV2)

(V

=V1 Such that every edge in the

act graph connects vertex in *V2.* furthermore

a vertex

no

ɤ

in V1 and

a

edge in G connects either two

vertices in V1

vertices in

two *(*V1,

V2) *is* the

Ex :

C6 гел

is

همراه

15

24

V2.

bipartition of the *vertex* set

V.

bipartite

2무

23

25

but Ks is not.

V1 = {21, 23,

15 }

re

24

V6

√2 = {{ 292,

14, 16 }

V2

each vertex in

connect

vertex

hoo

V2

cь

connects a vertex in

in V2. Also, there vertices in V1

are

по

edga
s

or two vertice's

V1

that

Note that,

and

a

√2

K3

гез

co

чет

Since

there are

3 vertices

One

of

the set

V1 or V2 will

Contain two vertices that will

be connected by an

not a

2

So, K3 is

edge.

bipartite

graph.

Check whether the following graph are bipartite.

21

19-2

Ex :

201

20

gx ge

rer

rez

96

v.

15

e

5

Solution!

01

G

gr
да
29
до
25
26
07
гез
H

It is not a bipartite graph since it is not possible to find two disjoint set of no edge vertices in which will connect the vertices in the same set.

Bipartite graph

Theorem: A simple graph is bipartite iff it is

assign one of the

possible to

assign

colours

to

each

two different

no two adjacent colour.

vertex such that

vertices are assigned the same

A

ᴨ

13

graph

Complete Bipartite graph : The complete

bipartite

that has its vertex sel partitioned

is the graph

that

and n

Kmin into two subsets

of

m

vertices,

respectively.

There is an edge between two vertices if and only if one verfex is in the first subset and the other vertex

is in the second subset.

1

2

V1

V2

5

K293

3

$V1 = \{1,2\}$ $V2 =$

{{3,4,5)

V2

V1 UV 2 = V V1

2

V2 = *0*

V1 V1n V2=0

1

8

00

구

6

3

2

3

1

No1

งาน

5

6

K31

3

K2,6

5

NEW GRAPHS FROM OLD

Subgraph of a graph! A subgraph of a

graph G =

(V,E) graph H = (W, F), where WCV and FCE.

of G is

is a

A subgraph H if

HG. Example!

V5

291

a proper subgraph of G

25

го г

че

21

v

2

24

G = (V,E)

гез

CV

V2 = { 01, 02,

03, 05y =

re

3

is a subgraph H =

(V', E')

of G

E = { (11, 12), (22,03),
(13,0), (12,05) CE

Union of two
graph : graphs
graph

UE2
E 1 UE 2
E1

van

G1

=

1

The union
union of two simple

2

(10)

2) (V11, E1) and 92 =
(V2, E2) is the simple with vertex set V1 UV2
and the edge set

The union is

412

tz

denoted by G1 VG2.

U2

13

G1

Иб

Uz

G2

Now, G1 UG2 = (V1 U V2, E1 U E 2) = (V, E)

2

sewer { U1, U2, 43, 44, 45,4%

W =

из

U2

ил

G1092

иб

Иб

иа

06

12

чет

v2

20
1

rer
2

15

че

V4

3

24

H2

на

25

re

REPRESENTING GRAPHS

One way to represent a

283

graph without multiple edges is to list all the edges of this graph.

Another way

is to

represent

multiple edges is

to specify the

vertices

vertex of the graph.

Use

that

a

graph with no

adjacency lists,

that are adjacent to

each

Tsidence

T

яz

Matrix.

Lof

(13) undirected

he

Adjacency List

(11)

Vertex

ген

02

Adjacent vertices

V2, 293, 25

3,25

re

1

21, 24, 25

25

20

4

simple

graph

23

ro

29

5

23, 295

21. 13.2g

For directed graphs, adjacency list can be formed by listing all the vertices that are terminal vertices of edges starting at each vertex of the graph

21

Adjacency `List` Initial vertex

201

Terminal Verties

42,43,44,45

V4

25

22, 24

re

2

reз

11, 193, 195

295

192, 193, 194

Adjacency Matrices! Suppose that G = (V, E) is a simple

graph where

lvl=n, Suppose that the vertices of G are

listed arbitrarily

matrix A of G is

as 11, 12, un. The adjacency

a binary matrix with 1 as

vi v;

its (i, j)th entry when v; and ; are adjacent

and

o

its

as

are

زمه

(i, j)th entry when vi and

A = [ai;], then if.

(vi, v;) is an edge in

not adjacent. So if

1

a;; =

{ 0

aij

G

otherwise.

Ex.

201

02

21

ro

2

श्र o

V = { 21, V2, V3, 04

203

VA.

(12)

1

A = 2

1

293

1

1

o

o

23

94

1

1

10

Ex:

Let

A =

イ

001

1

001

O

110

find the graph G

respect to the ordering of

with respect

V1, V2, V3

and 14

чет

vertices as

гег

V1

гег

гез

V4

29

1

1

A =

o

1

22

1

o

203

1

1

10

14

011

= 0

means there is no

edge between 2 and

Vegglie,

no loop at

roto

034

= 1

means there is

an

between 20 3

and V4.

ro3

edge

Ex: find the adjacency matrix to represent the following pseudo graph

V1

3

o сооб

g

t

N

F

N

O

Из

21

2

02

22

3

1

1

A:

23

o 1

1

2

V4

2

1

2

V4

re

3

2

an undirected

• Incidence matrix! Let G = (V, E) be graph. Suppose that 11, 192, . . ., on are the vertices

Q

and 21.02.

em are

incidence matrix with respect to this ordering of

the edges of G. Then the

V

and

E is

the

nxm matrix. M = [mij],

where,

mij

=

أ

1

when edge e; is incident withi

otherwise.

Ex!

ел

22

ез

24

25.

292

201

26

гез

гол

1

01

o

1

1

292

M=

20

3

o

o

O

O

204

05

1

O

1.4

25

1

1

O

イ

1

Ex

11

ez

e3

O

221

イ

ez eq 25

26

O

O

rer

es

13

1

1

1 10

M=

1

оба

O

O

16

25

011

204

o

1

10

V5

o

• Mixed Graph: A graph with both directed and undirected edges is called

Graph terminology!

a

Mixed Graph.

Type

1) simple graph

Edges

Undirected

raultiple

edges

Loops allowed?

allowed

No

No

OΓ

Yes

11) Multigraph

"

Yes

111)

Pseudograph

"J

IV) Simple directed graph

- Directed

No

No

Yes

Yes

1) Directed

multigraph

))

vi) Mixed

graph

both

Yes

Yes

directed and,

undirected

Isomorphism of Graphs

A4

We often need to know whether it is possible to draw two graphs in the

in the same way.

a

one-to-one

to-one and onto

Definition: The simple graphs $G1 = (V, En)$ and $G2 = (V2, F2)$ are isomorphic if there is function f from V1 to V2 with the property that

are adjacent in G1 if and only if

a

and

J

an isomorphism.

f(a) and f(b) are adjacent in G2 for all a, b.

function f is called

in V1. Such a

好

Ex! Show that the following graphs are isomorphie.

U2

02

201

23

H=(W,F)

Then f is

an

Now

we

из

u4

G =

(V,E)

Consider

the function f,

14

f(a1) = 21, f(u2) = 11,

l

one-to-one correspondence between

Vardhe

consider the adjacency in both the

f(93)=293, f(94)=12+

grap

hs.

In G

In H

(Uns

(U1,42) is an

edge

f(un) "

(21, 294) is

if (42)

an

edge

"

n *11*

11

")

И

3) (U2,

U4) (44,

43)

"l

12

the graphs

is denoted

Hence,

and is

VA

The isomorph

is

omorphism

relation.

By

11

"

(201203)

(194,

102)

is

202) is

(292 293)

)

an

edge

n

12

n

n

n

G and

H

are

is omorphic

211

G=H.

is

an

equivalence

Ex !

ал

Consider the following graphs

en G

61

a2

62

@2

d2

H

сл

02

order of

5

the

and 6

are

da

Note that the

sels of edges

graphs"

.

(15)

sets of vertices and

However, the graph I

contains

↑ (deg (ez) = 1),

whereas

in

both the

with

a vertex ez

no vertex G and

deg

ree

G

has

7

G F Hor

of deg 1. Therefore

not is omorphic.

7.

H

are

201

22

оз

Ex :

12

ил

296

го

both

ил

Iv1 = 6

иб

The deg of the

us

из

G

1E1 = 7

for vertices

G

and H

So,

f may

exist.

Consider

$f(91) = 26$

$f(14) = 195$

$f(15)=291$

H

both

G and H.

are also matching

in

the following function.

f(a2) = 293

3

f(uz)=14

f(46) = 292

the assignment f

is

one- one, caref

Now **bony** comparing the adjacency matrices of both the graphs w.r.t

can determine

and the adjan cency graphs

accordingly. Hence,

we

onto

that f

matches in both the

G=H.