

# Topology Inference for RDF

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# Contents

<b>1</b>	<b>Introduction</b>	<b>5</b>
<b>2</b>	<b>Bus and Edge</b>	<b>7</b>
<b>3</b>	<b>Two Special Concepts for Power Flow</b>	<b>9</b>
3.1	Channel . . . . .	9
3.2	Snapshot . . . . .	9
<b>4</b>	<b>Radial Distribution Feeder</b>	<b>11</b>
<b>5</b>	<b>Power Flow</b>	<b>13</b>
<b>6</b>	<b>Directed Graph</b>	<b>15</b>
<b>7</b>	<b>Formulation using Integer Programming</b>	<b>17</b>



# Chapter 1

## Introduction

This website hosts

- ☐ Formulation using Integer Programming
- ☐ Direct Impedance Method
- ☐ Fixed Point Method for Power Flow



## Chapter 2

# Bus and Edge

There are roughly two types of electrical devices in power grids.

type	definition	examples
delivery element	transport power from one place to another	cable, transformer, capacitor
conversion element	convert power from or to another form	solar panel, battery

- Ignore conversion elements. Not necessary in power flow calculation.
- Delivery element will be called **edge**.

Another concept, **bus**, represent the place where two different delivery elements joint or end of a delivery element, but there is no physical entity corresponding to a bus. There are three common types of buses:

type	know quantities
slack bus	voltage magnitude and phase angle
PQ bus	real power injection and reactive power injection
PV bus	real power injection and voltage magnitude

It is sufficient to model most of RDFs with PQ buses and one kind of edges, cables:

- One slack bus in RDF, corresponding to the **root**.
- Root not in any matrix.
- Ignore other delivery elements.





## Chapter 3

# Two Special Concepts for Power Flow

### 3.1 Channel

### 3.2 Snapshot

**Snapshot** is a concept to include power injections and voltages at one time index

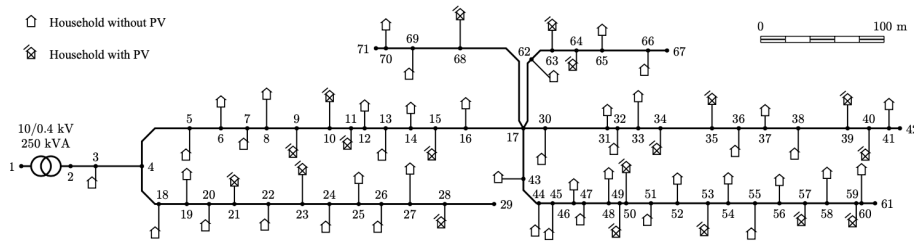
- input: real power injections at all channels of PQ buses
- output: voltages, current flow, power flow

**Zero-load snapshot** is the snapshot where power injections at all the channels are zero and voltages equal to rated voltages in corresponding phases.



## Chapter 4

# Radial Distribution Feeder



- located in Belgium
- one step-down transformer between bus 1 and bus 2 (not considered)
- bus 1 is omitted
- three-phase four-wire cables
- one phase star connection
- Houses associated with buses 3, 7, 10, 13, 16, 20, 23, 26, 30, 33, 36, 39, 43, 46, 49, 52, 55, 58, 62, 65, 69 are connected through phase A.
- Houses associated with buses 5, 8, 11, 14, 18, 21, 24, 27, 31, 34, 37, 40, 44, 47, 50, 53, 56, 59, 63, 66, 70 are connected through phase B.



## Chapter 5

# Power Flow



## Chapter 6

# Directed Graph

**weighted directed graph**  $G = (\mathcal{N}, \mathcal{E}, \sigma, \tau, \omega)$

- set of nodes:  $\mathcal{N}$
- set of edges:  $\mathcal{E}$
- incidence functions: source  $\sigma$ , target  $\tau$
- (edge) weighting function,  $\omega : E \rightarrow \mathbb{R}$ .
- 2-D Euclidean Distance as Weight

**arborescence**

- subgraph of a directed graph
- root

**feasible region**

- all the arborescences of a directed graph
- to count number of arborescences





## Chapter 7

# Formulation using Integer Programming

Symbols and definitions of sets:

symbol	definition
$\mathcal{E}$	all the potential edges (edges in the complete graph)
$\mathcal{C}$	available measurements of voltages and power injections
$\mathcal{E}_{\text{impossible}}$	potential edges that are impossible to exist

Symbols, definitions, types and sets of variables:

symbol	definition	type	set
$x_{ij}$	if edge from i to j is in the solution	$\{0, 1\}$	$\mathcal{E}$

Symbols, definitions, sets of constants:

symbol	definition	set
$d_{i,j}$	weight of directed edge from i to j based on distance	$\mathcal{E}$

Two terms in the objective function and three sets of constraints:

$$\begin{aligned}
& \min_{x_{ij} \forall (i,j) \in \mathcal{E}} (1 - \alpha) \sum_{(i,j) \in \mathcal{E}} d_{ij} x_{ij} + \alpha \mathcal{H}(\{x_{ij} \forall (i,j) \in \mathcal{E}\}, \mathcal{C}) \\
& \text{s.t.} \quad \sum_{(i,j) \in \delta^-(j)} x_{ij} = 1 \quad \forall j \in V' \quad (\text{a directed forest}) \\
& \quad \sum_{(i,j) \in \delta^-(S)} x_{ij} \geq 1 \quad \forall S \subseteq V', |S| \geq 2 \quad (\text{a connected graph}) \\
& \quad x_{ij} = 0 \quad \forall (i,j) \in \mathcal{E}_{\text{impossible}} \quad (\text{remove impossible potential edges})
\end{aligned}$$

- A nonlinear function  $\mathcal{H}$  to assess candidate topologies.
- A parameter  $\alpha$ .
- First two sets of constraints ensure arborescenc.
- Last set of constraints ensure impossible potential edges are removed.