Topology Inference for Radial Distribution Feeder based on Power Flow

Jie Xu (s181238)

2020-12-13

Contents

1 Introduction **5**

2 Radial Distribution Feeder **7**
2.1 Bus and Edge 7
2.2 Two Special Concepts 7
2.3 Case with 70 Buses 8

3 Problem Formulation **9**
3.1 Directed Graph 9
3.2 Remove Overlapping Edge 9
3.3 IP Formulation 10
3.4 Combinatorial Optimisation 11

4 AC Power Flow **13**
4.1 Two Matrices 13
4.2 Bus Impedance Matrix 14
4.3 Direct Impedance Method 14

5 Linearised Power Flow **15**
5.1 Linearised Voltage Drop 15
5.2 Linearised Voltage 16
5.3 Bus Resistance Matrix 16
5.4 Pseudo Linearised Power Flow 18
5.5 Assessment of Candidate 18
5.6 Error from Linearisation 19

6 Result and Discussion **21**
6.1 Result for Case-70 21
6.2 Summary 22

Chapter 1

Introduction

This website hosts slides for defence of my master graduation project in the Department of Electrical Engineering at Technical University of Denmark. How households are connected to distribution network is always unknown. A framework to infer such connections by utilising all kinds of information is proposed in this project.

Problem Setting

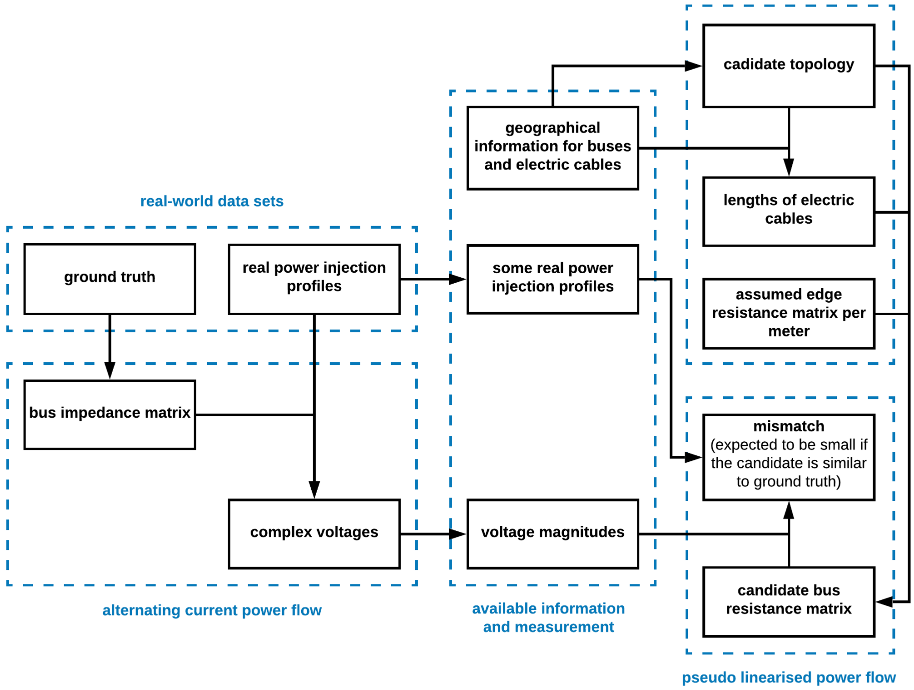
Available information and measurement:

- geographical information about buses
- voltage magnitudes of all the phases of all the buses
- some real power injection profiles

Association network inference:

- Correlation between buses.
- $|\mathcal{E}| = |\mathcal{N}| - 1$

Flowchart



Ground truth is not known, but used to simulate available measurement.

Two batches of computer programs:

- power flow
- three algorithms to handle directed graphs

Chapter 2

Radial Distribution Feeder

- bus and edge
- two special concepts for power flow
- case with 70 buses

2.1 Bus and Edge

There are roughly two types of electrical devices in power grids.

type	definition	examples
edge	transport power from one place to another	cable, transformer, capacitor
conversion element	convert power from or to another form	solar panel, battery
bus	where two edges joint or end of an edge	slack bus, PQ bus, PV bus

- Ignore conversion elements. Not necessary in power flow calculation.
- Cable.
- One slack bus -> **root**.

Impedance of cable is proportional to its length.

Unit impedance matrix for case-70:

$$\bar{Z}_{abc} = \begin{bmatrix} 0.000412 + 1.558e^{-4}j & 0.000206 + 7.791e^{-5}j & 0.000206 + 7.791e^{-5}j \\ 0.000206 + 7.791e^{-5}j & 0.000412 + 1.558e^{-4}j & 0.000206 + 7.791e^{-5}j \\ 0.000206 + 7.791e^{-5}j & 0.000206 + 7.791e^{-5}j & 0.000412 + 1.558e^{-4}j \end{bmatrix}$$

2.2 Two Special Concepts

Essential for power flow calculation.

Channel

- **channel**: refer to one phase in some bus
- **active channel**: there are non-zero injections (connected to some conversion element)
- **observed active channel**: such non-zero injections are known

It is assumed that all active channels are known.

Snapshot

Snapshot is a concept to include power injections and voltages at one time index

- input: real power injections
- output: voltages, current flow, power flow
- Duration is 1 s in this project.

Zero-load snapshot is the snapshot where power injections at all the channels are zero and voltages equalt o rated voltages in corresponding phases.

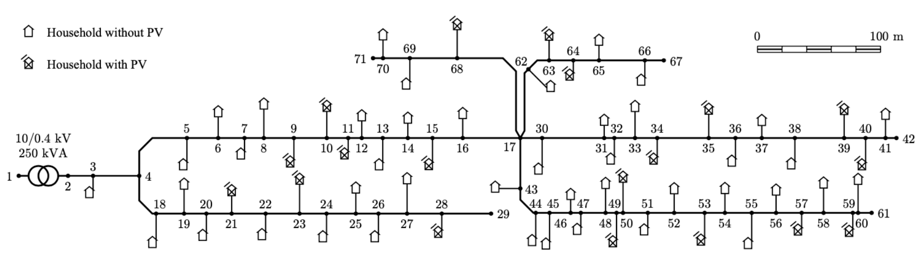
- $\bar{V}_{\text{zero}}$: voltages in zero-load snapshot
- V_{rate}: rated voltage magnitude, 230 V

2.3 Case with 70 Buses

Assumptions about feeders:

- one path to any bus
- step-down transformer is not considered
- three-phase four-wire cable
- one phase star connection

A case with 70 buses is primarily used here:



- located in Belgium
- bus 1 is omitted
- 70 buses
- 207 channels

Chapter 3

Problem Formulation

3.1 Directed Graph

weighted directed graph $G = (\mathcal{N}, \mathcal{E}, \sigma, \tau, \omega)$

- set of nodes: $\mathcal{N}$
- set of edges: $\mathcal{E}$
- incidence functions: source σ, target τ
- (edge) weighting function, $\omega : E \to \mathbb{R}$.

complete graph for a set of nodes

- all edges are **potential edges**
- some are impossible to exist
- possible to derive their lengths
- 2-D Euclidean distance as weight
- *association network inference cannot be used*

spanning arborescence (SA)

- subgraph of a directed graph
- root
- include every bus

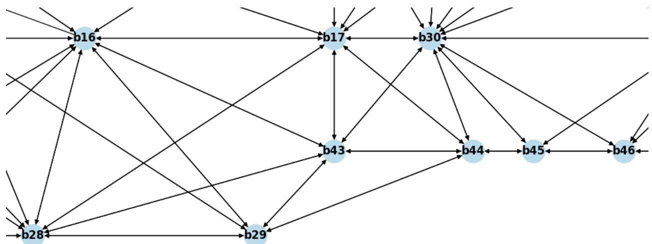
feasible region

- All the SAs.
- Number of SA is finite, making it a combinatorial optimisation problem.
- Count number of SA.

3.2 Remove Overlapping Edge

For example, in case-70:

shortest path	<	direct edge	×threshold	-> remove direct edge
”b17-b43-b29”	<	”b17-b29”	×1.1	-> remove ”b17-b29”
”b44-b43-b29”	>	”b44-b29”	×1.1	-> keep ”b44-b29”



However:

- 446 possible potential edges
- over 10^{45} spanning arborescences

-> summary

3.3 IP Formulation

Sets:

symbol	definition
$\mathcal{E}$	all the potential edges (edges in the complete graph)
$\mathcal{C}$	available measurements of voltages and power injections
$\mathcal{E}_{\text{impossible}}$	potential edges that are impossible to exist

Variables:

symbol	definition	type	set
x_{ij}	if edge from i to j is in the solution	$\{0,1\}$	$\mathcal{E}$

Constants:

symbol	definition	set
$d_{i,j}$	Euclidean distance from i to j	$\mathcal{E}$

Integer programming formulation:

$$
\begin{aligned}
\min_{x_{ij}\forall(i,j)\in\mathcal{E}} \quad & (1-\alpha)\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij} + \alpha\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right) \\
\text{s.t.} \quad & \sum_{(i,j)\in\delta^-(j)} x_{ij} = 1 \quad \forall j \in V' \quad \text{(a directed forest)} \\
& \sum_{(i,j)\in\delta^-(S)} x_{ij} \geq 1 \quad \forall S \subseteq V', |S| \geq 2 \quad \text{(a connected graph)} \\
& x_{ij} = 0 \quad \forall (i,j) \in \mathcal{E}_{\text{impossible}} \quad \text{(remove impossible potential edges)}
\end{aligned}
$$

Two terms in the objective function:

term	**definition**	**coefficient**
$(1-\alpha)\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij}$	weight of candidate arborescence	$1-\alpha$
$\alpha\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right)$	assessment of candidate arborescence	α

Three sets of constraints:

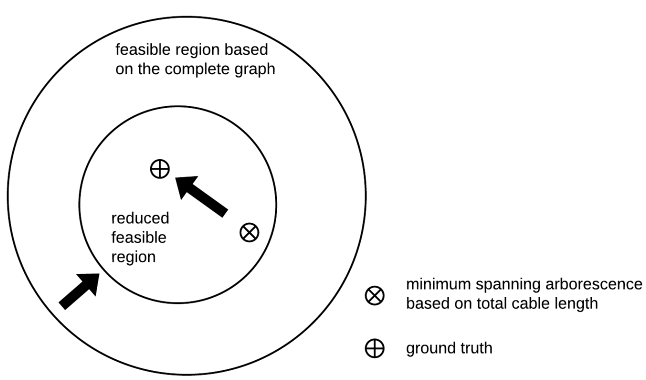
- First two sets ensure arborescence.
- Last set removes impossible potential edges.

3.4 Combinatorial Optimisation

At least two possible values for α:

value	**term lefted**	**to find**	**disadvantage**
1	$\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right)$	ground truth	NP-hard and non-linear
0	$\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij}$	topology with min total cable length	cannot find ground truth

Such two situations can be visualised:



A **local search heuristic algorithm** is proposed to to move from $\bigotimes$ to $\bigoplus$:

function	what it does	in this project
objective	assess candidate	pseudo linearised power flow
neighbourhood	generate candidate	rank spanning arborescence

- The starting point is found by minimum spanning arborescence
- Every candidate is reachable from the starting point.
- Ground truth should be found before long.
- Not in parallel.

Chapter 4

AC Power Flow

Discussion is based on one-line model. Can be generalised for multi-phase model.

- Model RDF with one bus impedance matrix.
- Calculate power flow using fixed point method.

4.1 Two Matrices

Current injection to flow:

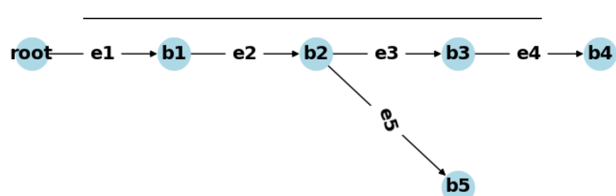
$$\bar{I}_{\text{edge}} = -K\bar{I}$$

where **edge path incidence matrix (EPI)**, K.

Voltage drop to nodal voltage:

$$\bar{V} = \bar{V}_{\text{zero}} - K^{\top}\bar{Z}_{\text{edge}}\bar{I}_{\text{edge}}$$

where **edge impedance diagonal block matrix (EIDB)**, $\bar{Z}_{\text{edge}}$.



$$\begin{bmatrix} \bar{I}_{\text{edge},1} \\ \bar{I}_{\text{edge},2} \\ \bar{I}_{\text{edge},3} \\ \bar{I}_{\text{edge},4} \\ \bar{I}_{\text{edge},5} \end{bmatrix} = - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \bar{I}_5 \end{bmatrix}$$

$$
\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \\ \bar{V}_5 \end{bmatrix} - \begin{bmatrix} \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \end{bmatrix} = - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^\top \begin{bmatrix} Z_{\text{edge},1} & 0 & 0 & 0 & 0 \\ 0 & Z_{\text{edge},2} & 0 & 0 & 0 \\ 0 & 0 & Z_{\text{edge},3} & 0 & 0 \\ 0 & 0 & 0 & Z_{\text{edge},4} & 0 \\ 0 & 0 & 0 & 0 & Z_{\text{edge},5} \end{bmatrix} \begin{bmatrix} \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \end{bmatrix}
$$

Alternating current power flow:

$$
\bar{V} = \bar{V}_{\text{zero}} + \left(K^\top \bar{Z}_{\text{edge}} K \right) \bar{I}
$$

4.2 Bus Impedance Matrix

Alternating current power flow:

$$
\bar{V} = \bar{V}_{\text{zero}} + \left(K^\top \bar{Z}_{\text{edge}} K \right) \bar{I}
$$

Bus impedance matrix (BIM), $\bar{Z}$, is defined as:

$$
\begin{aligned}
\bar{Z} &= K^\top \bar{Z}_{\text{edge}} K \\
&= R + jX
\end{aligned}
$$

where **bus resistance matrix (BRM)**, R: real part of entries in BIM.

4.3 Direct Impedance Method

Five steps to build BIM:

1. Define a unit impedance matrix.
2. Calculate edge impedance matrices for cables.
3. Build EIDB.
4. Obtain EPI based on topology.
5. Calculate BIM using EIDB and EPI.

Fixed Point Method

To calculate power flow in one snapshot, given power injections, the following procedure is repeated:

$$
\begin{aligned}
\bar{I} &= \underline{P} \otimes \underline{V}_{\text{previous}} \\
\bar{V} &= \bar{Z}\bar{I} + \bar{V}_{\text{zero}} \\
\epsilon &= \left(\bar{V} - \bar{V} \right)^\top \left(\bar{V} - \bar{V} \right)
\end{aligned}
$$

until ϵ is smaller than a pre-defined threshold.

Chapter 5

Linearised Power Flow

Three ways to calculated BRM:

- Real part of entries in BIM.
- Using EPI and ERDB.
- Lowest common ancestor problem.

5.1 Linearised Voltage Drop

With power flow at source of edge k, $\bar{S}_{\text{source},k}$:

$$\bar{I}_{\text{edge},k} = \frac{\underline{S}_{\text{source},k}}{\underline{V}_i}$$

Voltage drop:

$$\begin{aligned}
\bar{V}_{\text{edge},k} &= \bar{I}_{\text{edge},k}\bar{Z}_{\text{edge},k} \\
&= \frac{\underline{S}_{\text{source},k}\bar{Z}_{\text{edge},k}}{\underline{V}_i} \\
&= \frac{\left(P_{\text{source},k} - jQ_{\text{source},k}\right)\left(R_{\text{edge},k} + jX_{\text{edge},k}\right)}{\underline{V}_i} \\
&= \frac{R_{\text{edge},k}P_{\text{source},k} + X_{\text{edge},k}Q_{\text{source},k}}{\underline{V}_i} + j\frac{X_{\text{edge},k}P_{\text{source},k} - R_{\text{edge},k}Q_{\text{source},k}}{\underline{V}_i}
\end{aligned}$$

Then:

$$V_{\text{edge},k} = \frac{R_{\text{edge},k}}{V_{\text{rate}}}P_{\text{source},k}$$

- Ignore imaginary part.
- Replace $\underline{V}_i$ with V_{rate}.

5.2 Linearised Voltage

Voltage drop to nodal voltage:

$$V = V_{\text{zero}} - \frac{1}{V_{\text{rate}}} K^\top R_{\text{edge}} P_{\text{source}}$$

Power injection to flow:

$$P_{\text{source}} = -K \left(P - P_{\text{loss}} \right)$$

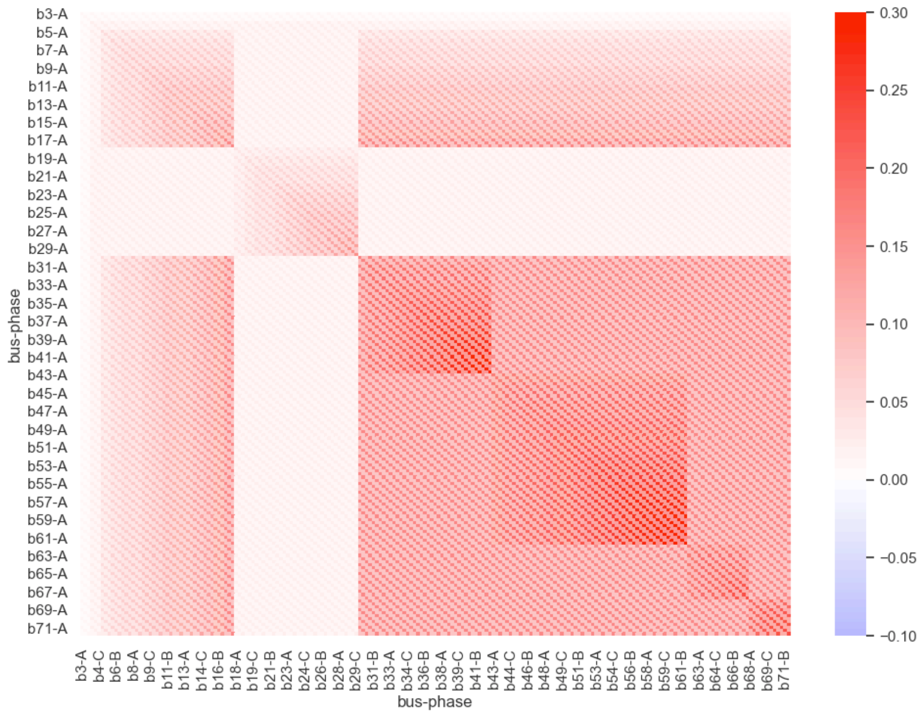
Voltage magnitude can be calculated using BRM and real power injections:

$$\begin{aligned} V &= V_{\text{zero}} + \frac{1}{V_{\text{rate}}} \left(K^\top R_{\text{edge}} K \right) P \\ &= V_{\text{zero}} + \frac{1}{V_{\text{rate}}} R P \end{aligned}$$

- To assess candidate by calculating power injection using voltage magnitudes.

5.3 Bus Resistance Matrix

- Step-down transformer is ignored, so bus 1 is not included.
- Bus 2 is the root.
- There are 69 PQ buses, and there are 207 channels.
- 207 rows and 207 columns.



Lowest Common Ancestor Problem

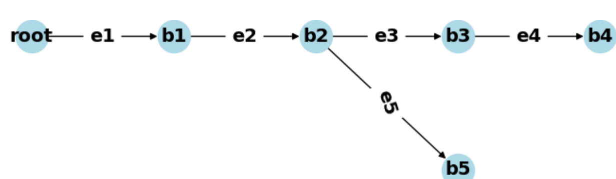
Entry (i, j) is the sum of edge resistances in the common path to the root of bus i and j. That is sum of edge resistances in the path from the root to the lowest common ancestor (LCA) of bus i and j:

$$R_{i,j} = \sum_{k \in U_i \cap U_j} R_{\text{edge },k}$$

where U_i is set of edges on path from the root to bus i.

- BRM can be calcualted efficiently using LCA for all pairs of buses.
- The pattern can be used in future work.

For example,



- LCA of b3 and b5 is b2. Entry for b3, b5 is $R_{\text{e1}} + R_{\text{e2}}$
- LCA of b4 and b5 is still b2. Entry for b4, b5 is still $R_{\text{e1}} + R_{\text{e2}}$.

-> summary

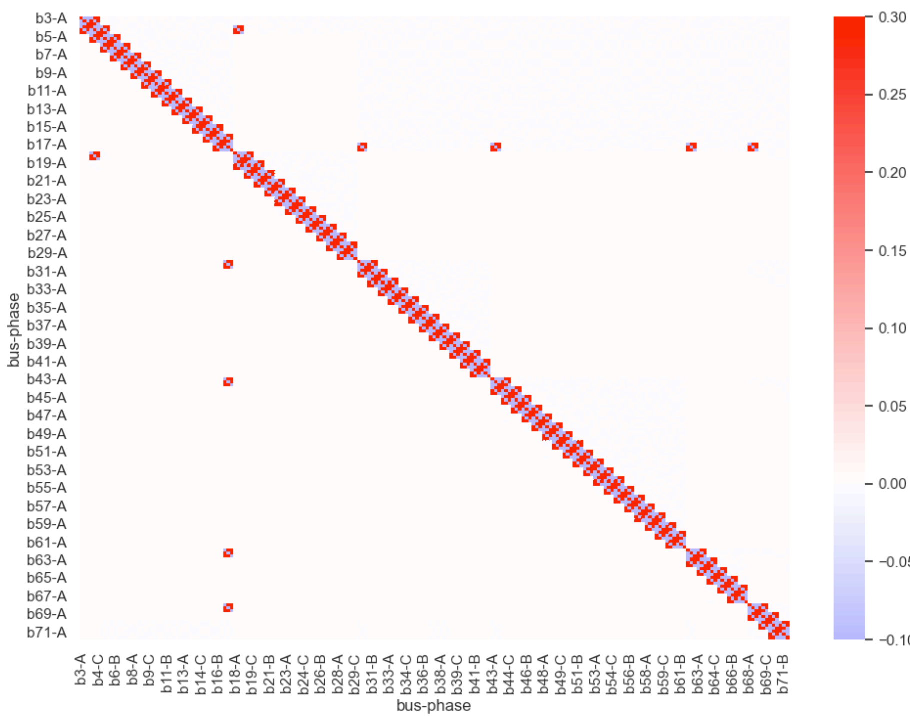
5.4 Pseudo Linearised Power Flow

Based on linearised power flow, $V = V_{\text{zero}} + \frac{1}{V_{\text{rate}}} RP$:

$$P_{\text{assess}} = V_{\text{rate}} R^{\top} (V - V_{\text{zero}})$$

which is referred to as **pseudo linearised power flow**.

The inversed BRM for case-70 looks like:



- Voltage magnitude at any channel can have a huge impact.
- The pattern can be used in future work.

5.5 Assessment of Candidate

Calculate P_{assess} using voltage magnitudes.

The second term in the objective function (mean squared error):

$$\mathcal{H}(R) = [(P_{\text{assess}} - P) \otimes O]^{\top} \cdot [(P_{\text{assess}} - P) \otimes O] / |\mathcal{O}|$$

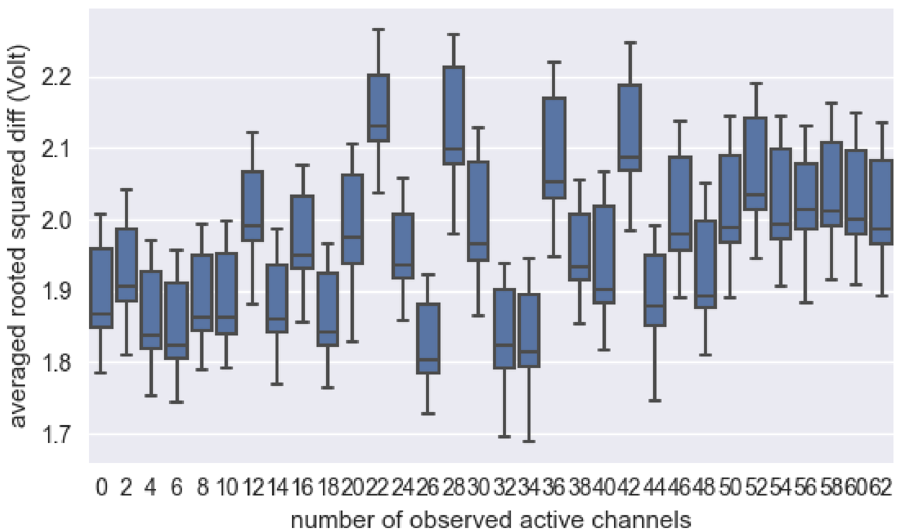
where:

- $\mathcal{O}$: set of observed active channels and inactive channels
- O: binary vector indicating observed active channels
- Entries for unobserved active channels are ignored.

5.6 Error from Linearisation

Box plot:

- with respect to different number of observed active channels
- based on ground truth and 50 snapshots[1]



- Error is aleady reduced to 1.7 ~ 2.2.
- Rated voltage magnitudes will increase the error dramatically.
- Full observability over voltage magnitudes for now.

-> summary

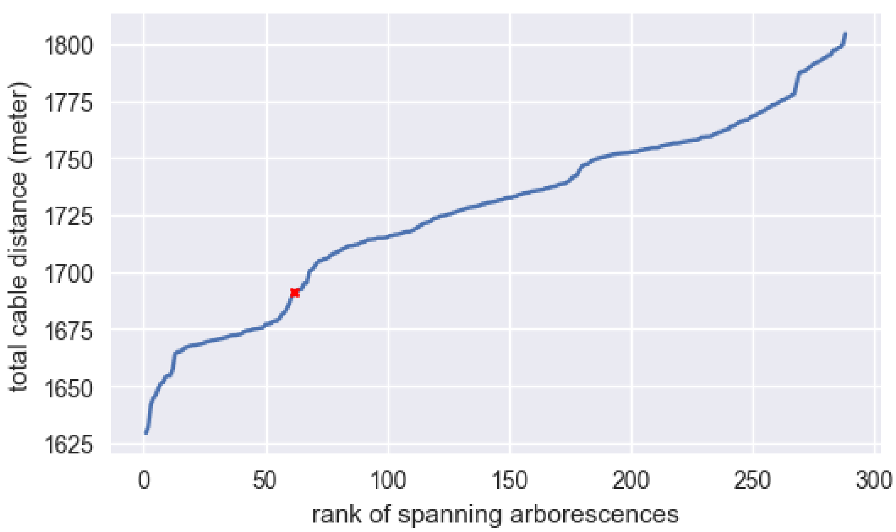
[1] during 00:00:00 and 00:00:50 on Dec 2, 2020 from Sonnen data set.

Chapter 6

Result and Discussion

6.1 Result for Case-70

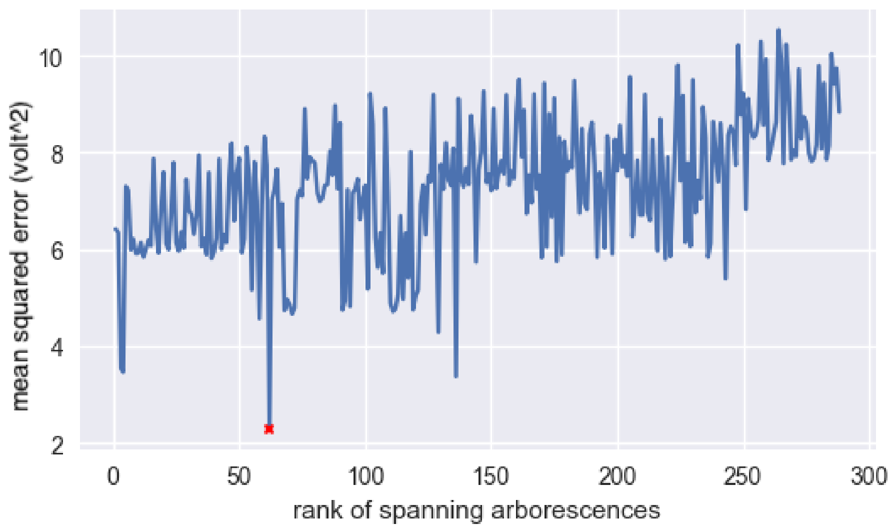
Rank SA:



Assessment (mean squared error):

- based on 50 snapshots[1]

[1] during 00:00:00 and 00:00:50 on Dec 2, 2020 from Sonnen data set.



6.2 Summary

- Topology inference -> combinatorial optimisation problem.
- A framework is proposed.
- Core: local search heuristic algorithm.

Advantages:

- Robust to partial observability.
- Integrate all kinds of information in weight and direction.

Four steps:

1. Shrink feasible region (reduce the number of SAs) by removing overlapping edges.
2. Measure the size of feasible region by counting number of SAs.
3. Get candidates sequentially by ranking SAs according to total cable lengths.
4. Assess candidates based on available measurements.

Issues

1. Too many spanning arborescences. (remove overlapping edges)
2. Full observability over voltage magnitudes. (matrices with full rank)
3. Error in linearised power flow calculation. (error from linearisation)

Future Work

- How to detect more impossible potential edges. (for issue 1)

- How to assess candidates based on a fraction. (for issue 2)
- How to use voltage sensitivity matrix in linearised power flow. (for issue 3)