Topology Inference for Radial Distribution Feeder based on Power Flow

Jie Xu (s181238)

2020-12-15

Contents

1 Introduction **5**

2 Radial Distribution Feeder **7**
2.1 Bus and Edge 7
2.2 Two Special Concepts 7
2.3 Case with 70 Buses 8

3 Problem Formulation **9**
3.1 Directed Graph 9
3.2 Remove Overlapping Edge 9
3.3 Integer Programming 10
3.4 Local Search 11

4 AC Power Flow **13**
4.1 Two Matrices 13
4.2 Bus Impedance Matrix 14
4.3 Direct Impedance Method 14

5 Linearised Power Flow **17**
5.1 Bus Resistance Matrix 17
5.2 Pseudo Linearised Power Flow 19
5.3 Assessment of Candidate 20
5.4 Error from Linearisation 20

6 Result and Summary **23**
6.1 Result for Case-70 23
6.2 Summary 24

Chapter 1

Introduction

Problem Setting

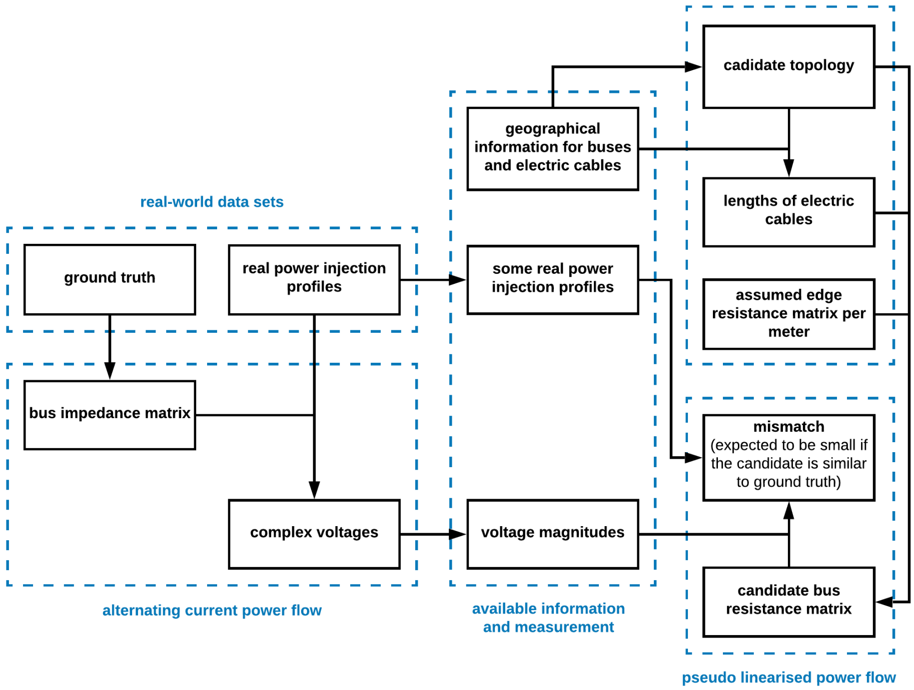
Available information for topology inference:

- geographical information about buses
- voltage magnitudes of all the phases of all the buses
- some real power injection profiles

Association network inference: correlation between buses

- Spurious correlation resulted from correlated profiles.
- Missing measurement.

Flowchart



Ground truth is to be found in topolgoy inference, but used to simulate available measurement first.

Two batches of computer programs:

- power flow
- three algorithms to handle directed graphs

Chapter 2

Radial Distribution Feeder

- bus and edge, -> 2.1
- two special concepts for power flow, -> 2.2
- case with 70 buses, -> 2.3

2.1 Bus and Edge

type	definition	examples
edge	transport power from one place to another	cable, transformer, capacitor
conversion element	convert power from or to another form	solar panel, battery
bus	where two edges joint or end of an edge	slack bus, PQ bus, PV bus

- Cable.
- One slack bus -> **root**.
- Ignore conversion elements. Not necessary in power flow calculation.

2.2 Two Special Concepts

Essential for power flow calculation.

Channel

- **channel**: refer to one phase in some bus
- **active channel**: connect to household
- **observed active channel**: power is measured

It is assumed that all inactive channels are observed.

Snapshot

Snapshot: include power injections and voltages at one time index

- duration: 1 s

Zero-load snapshot : when power injections at all the channels are zero

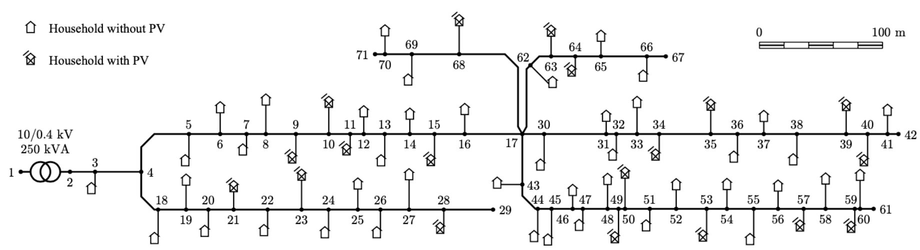
- $\bar{V}_{\text{zero}}$: voltages in zero-load snapshot
- V_{rate}: rated voltage magnitude, 230 V

2.3 Case with 70 Buses

Assumptions about feeders:

- spanning arborescence (SA)
- one step-down transformer
- rated voltage, 230 V
- three-phase four-wire cable
- one phase star connection

A case with 70 buses is primarily used here:



- located in Belgium
- bus 1 is omitted
- 62 households -> 62 active channels

Chapter 3

Problem Formulation

- information in a directed graph -> 3.1
- integer programming formulation -> 3.3
- local search heuristic algorithm -> 3.4

- remove overlapping edge -> 3.2

3.1 Directed Graph

weighted complete (directed) graph for a set of buses

- any pair of buses -> edge -> **potential edges** to be selected
- select a set of edges -> SA -> candidate
- **impossible potential edge**
- 2-D Euclidean distance -> cable length -> weight

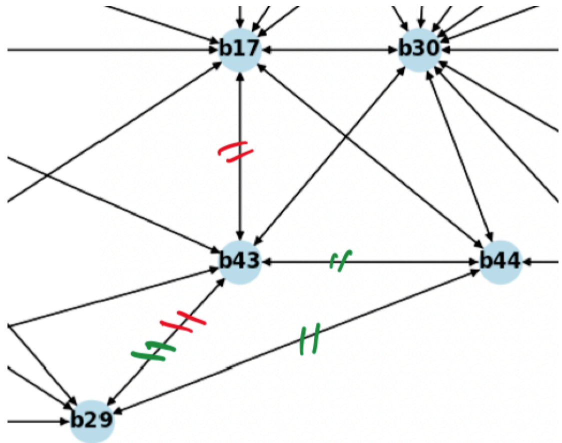
feasible region

- All the candidates (SAs).
- Number of SAs is finite, making it a combinatorial optimisation problem.
- Count number of SAs.

3.2 Remove Overlapping Edge

For example, in case-70:

shortest path	$<$	**direct edge**	$\times$**threshold**	-> **remove direct edge**
”b17-b43-b29”	$<$	”b17-b29”	$\times$1.1	-> remove ”b17-b29”
”b44-b43-b29”	$>$	”b44-b29”	$\times$1.1	-> keep ”b44-b29”



However:

- 446 possible potential edges
- over 10^{45} SAs

-> summary

3.3 Integer Programming

Sets:

symbol	definition
$\mathcal{E}$	all the potential edges
$\mathcal{C}$	available snapshots
$\mathcal{E}_{\text{impossible}}$	impossible potential edges

Variables:

symbol	definition	type	set
x_{ij}	whether to choose edge from i to j	binary	$\mathcal{E}$

Constants:

symbol	definition	set
$d_{i,j}$	Euclidean distance	$\mathcal{E}$

$$
\begin{aligned}
\min_{x_{ij}\forall(i,j)\in\mathcal{E}} \quad & (1-\alpha)\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij} + \alpha\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right) \\
\text{s.t.} \quad & \sum_{(i,j)\in\delta^-(j)} x_{ij} = 1 \quad \forall j\in V' \quad \text{(a directed forest)} \\
& \sum_{(i,j)\in\delta^-(S)} x_{ij} \geq 1 \quad \forall S\subseteq V', |S|\geq 2 \quad \text{(a connected graph)} \\
& x_{ij} = 0 \quad \forall(i,j)\in\mathcal{E}_{\text{impossible}} \quad \text{(remove impossible potential edges)}
\end{aligned}
$$

Two terms in the objective function:

term	**definition**	**coefficient**
$(1-\alpha)\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij}$	total cable length of candidate	$1-\alpha$
$\alpha\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right)$	assessment of candidate	α

Three sets of constraints:

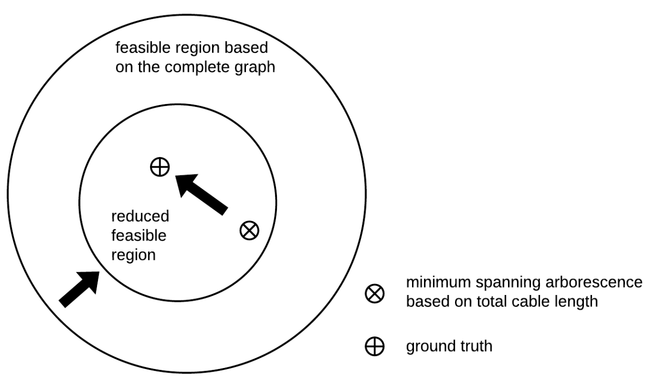
- First two sets ensure SA. (Fischetti and Vigo, 1997)
- Last set removes impossible potential edges.

3.4 Local Search

At least two possible values for α:

value	**term lefted**	**to find**	**disadvantage**
1	$\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right)$	ground truth	NP-hard and non-linear
0	$\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij}$	topology with min total cable length	cannot find ground truth

Such two situations can be visualised:



For this combinatorial optimisation problem, a **local search heuristic algorithm** is proposed to move from $\otimes$ to $\oplus$. (Michiels et al., 2007)

function	what it does	in this project
objective	assess candidate	pseudo linearised power flow
neighbourhood	generate candidate	rank spanning arborescence

- Ground truth should be found before long.
- Not in parallel.

Chapter 4

AC Power Flow

- two essential matrices -> 4.1
- bus impedance matrix -> 4.2
- direct impedance method for power flow calculation -> 4.2

Can be generalised for multi-phase model. (Hsieh et al., 2017)

4.1 Two Matrices

current injection -> current flow:

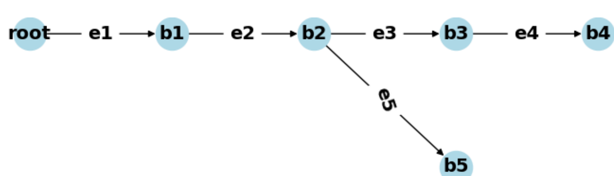
$$\bar{I}_{\text{edge}} = -K\bar{I}$$

where **edge path incidence matrix (EPI)**, K.

voltage drop -> voltage:

$$\bar{V} = \bar{V}_{\text{zero}} - K^{\top}\bar{Z}_{\text{edge}}\bar{I}_{\text{edge}}$$

where **edge impedance diagonal block matrix (EIDB)**, $\bar{Z}_{\text{edge}}$.



$$\begin{bmatrix} \bar{I}_{\text{edge},1} \\ \bar{I}_{\text{edge},2} \\ \bar{I}_{\text{edge},3} \\ \bar{I}_{\text{edge},4} \\ \bar{I}_{\text{edge},5} \end{bmatrix} = - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \bar{I}_5 \end{bmatrix}$$

$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \\ \bar{V}_5 \end{bmatrix} - \begin{bmatrix} \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \end{bmatrix} = - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{\top} \begin{bmatrix} Z_{\text{edge},1} & 0 & 0 & 0 & 0 \\ 0 & Z_{\text{edge},2} & 0 & 0 & 0 \\ 0 & 0 & Z_{\text{edge},3} & 0 & 0 \\ 0 & 0 & 0 & Z_{\text{edge},4} & 0 \\ 0 & 0 & 0 & 0 & Z_{\text{edge},5} \end{bmatrix} \begin{bmatrix} \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \end{bmatrix}$$

Alternating current power flow: (Conti et al., 2006)

$$\bar{V} = \bar{V}_{\text{zero}} + \left(K^{\top} \bar{Z}_{\text{edge}} K \right) \bar{I}$$

4.2 Bus Impedance Matrix

Alternating current power flow:

$$\bar{V} = \bar{V}_{\text{zero}} + \left(K^{\top} \bar{Z}_{\text{edge}} K \right) \bar{I}$$

Bus impedance matrix (BIM), $\bar{Z}$, is defined as:

$$\begin{aligned} \bar{Z} &= K^{\top} \bar{Z}_{\text{edge}} K \\ &= R + jX \end{aligned}$$

where **bus resistance matrix (BRM)**, R: real part of entries in BIM.

4.3 Direct Impedance Method

Five steps to build BIM:

1. Define a unit impedance matrix.
2. Calculate edge impedance matrices for cables.
3. Build EIDB.
4. Obtain EPI based on topology.
5. Calculate BIM using EIDB and EPI.

Fixed Point Method

The following procedure is repeated:

$$\begin{aligned}
\bar{I} &= \underline{P} \otimes \underline{V}_{\text{previous}} \\
\bar{V} &= \bar{Z}\bar{I} + \bar{V}_{\text{zero}} \\
\epsilon &= \left(\bar{V} - \bar{V}\right)^{\top} \left(\bar{V} - \bar{V}\right)
\end{aligned}$$

until ϵ is smaller than a pre-defined threshold.

Chapter 5

Linearised Power Flow

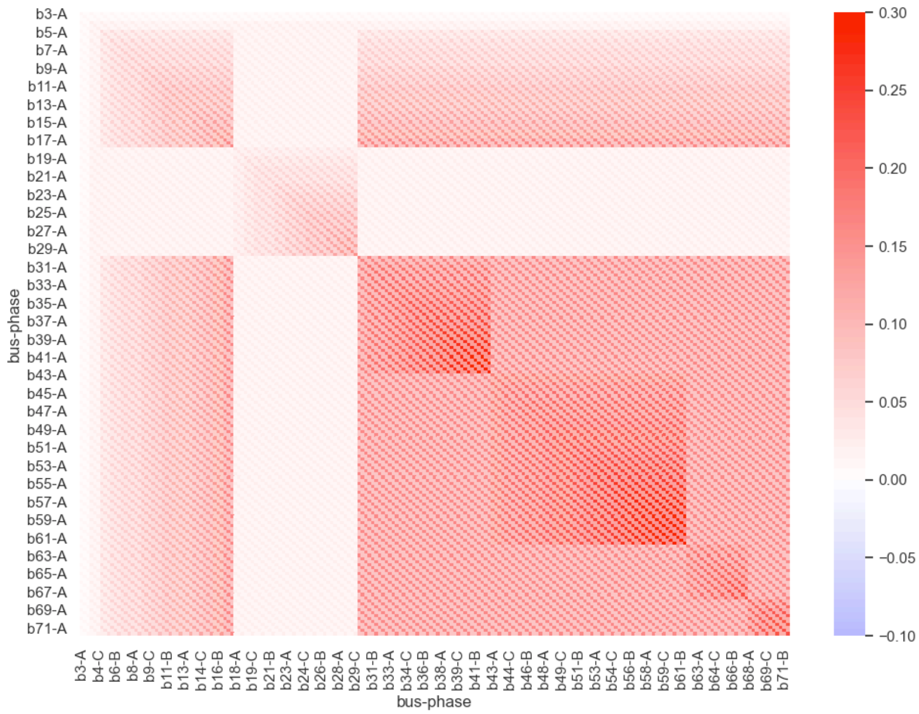
- assessment of candidate -> 5.3

- bus resistance matrix -> 5.1
- inversed bus resistance matrix -> 5.2
- error from linearisation -> 5.4

5.1 Bus Resistance Matrix

BRM of case-70:

- bus 2 -> root
- 69 PQ buses
- 207 channels -> 207 rows and 207 columns



Lowest Common Ancestor Problem

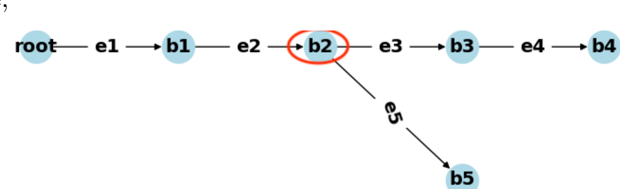
Entry (i, j) -> sum of edge resistances in the path from root to their lowest common ancestor (LCA):

$$R_{i,j} = \sum_{k \in U_i \cap U_j} R_{\text{edge },k}$$

where U_i is set of edges on the path from root to bus i.

- Calcualted efficiently using LCA for all pairs.
- Useful pattern.

For example,



pair of buses	entry in BRM
b3-b5	$R_{\text{e1}} + R_{\text{e2}}$
b4-b5	$R_{\text{e1}} + R_{\text{e2}}$

-> summary

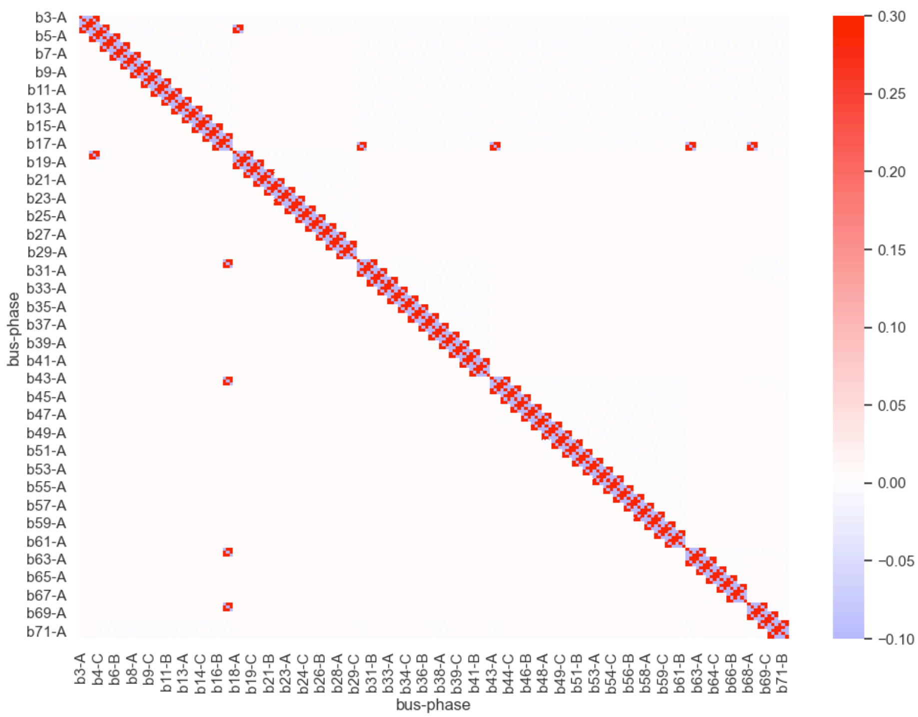
5.2 Pseudo Linearised Power Flow

Based on linearised power flow, $V = V_{\text{zero}} + \frac{1}{V_{\text{rate}}} RP$:

$$P_{\text{assess}} = V_{\text{rate}} R^{\top} \left(V - V_{\text{zero}} \right)$$

-> **pseudo linearised power flow**.

Inversed BRM for case-70:



- Sparse.
- Full rank.
- Voltage magnitude at any channel can have a huge impact.
- Useful pattern.

-> summary

5.3 Assessment of Candidate

Linearised power flow:

$$
\begin{aligned}
V &= V_{\text{zero}} + \frac{1}{V_{\text{rate}}} \left(K^{\top} R_{\text{edge}} K \right) P \\
&= V_{\text{zero}} + \frac{1}{V_{\text{rate}}} R P
\end{aligned}
$$

- Calculate P_{assess} using voltage magnitudes.
- Compare with available power measurements.

Mean squared error (MSE):

$$
\mathcal{H}(R) = \left[(P_{\text{assess}} - P_{\text{measure}}) \otimes O \right]^{\top} \cdot \left[(P_{\text{assess}} - P_{\text{measure}}) \otimes O \right] / |\mathcal{O}|
$$

where:

- $\mathcal{O}$: set of observed active channels and inactive channels.
- O: binary vector indicating observed active channels.

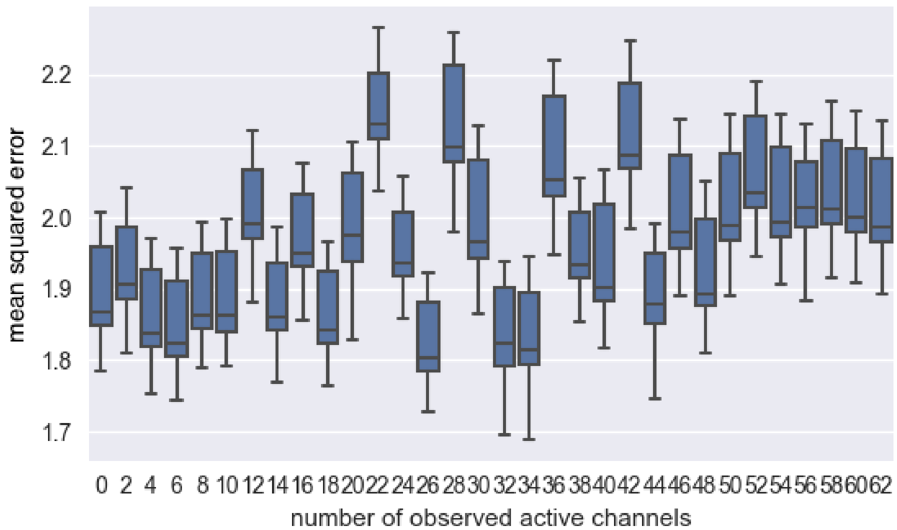
-> summary

5.4 Error from Linearisation

Box plot:

- with respect to different number of observed active channels
- based on ground truth and 50 snapshots[1]

[1] during 00:00:00 and 00:00:50 on Dec 2, 2020 from Sonnen data set.



- ~~Number of observed active channels.~~
- V_{rate} will increase the error dramatically.

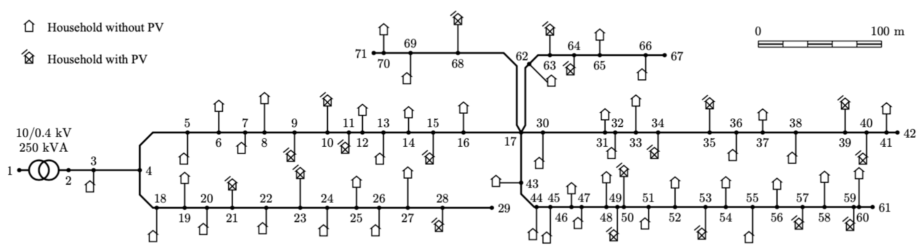
-> summary

Chapter 6

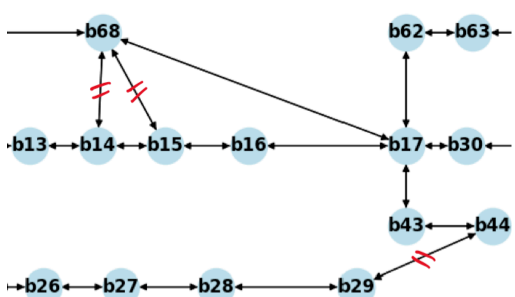
Result and Summary

- result for case-70 -> 6.1
- summary -> 6.2

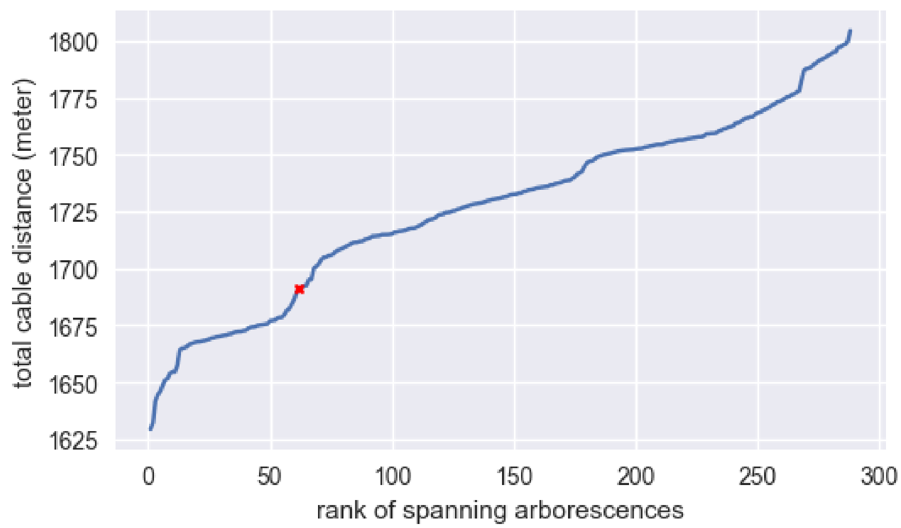
6.1 Result for Case-70



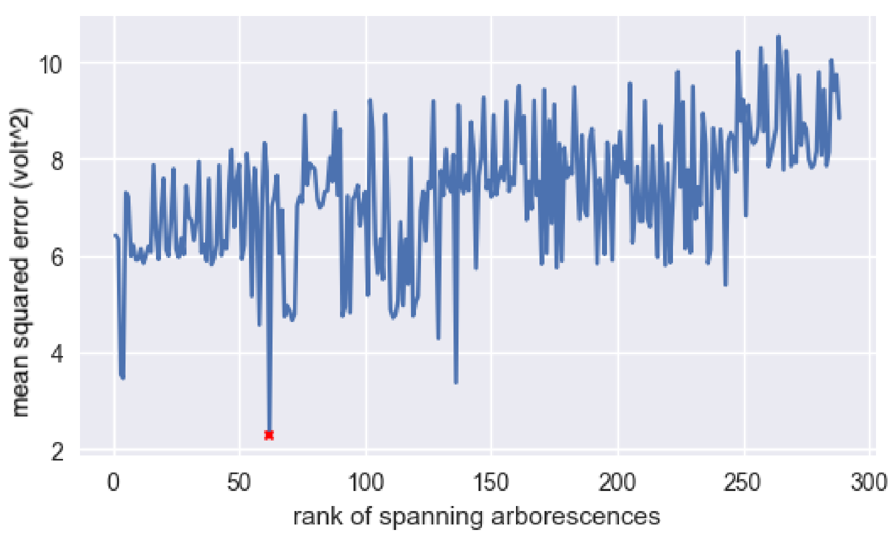
- three new pairs, “b29-b44”, “b68-b14”, and “b68-b15”
- 144 potential edges in total
- 288 SAs rooted at bus 2
- full observability



Rank candidates according to total cable lengths:



Assessment based on 50 snapshots[1]:



6.2 Summary

- Topology inference -> combinatorial optimisation problem.
- A new framework is proposed.

[1]during 00:00:00 and 00:00:50 on Dec 2, 2020 from Sonnen data set.

- Core: local search heuristic algorithm.

Four steps:

1. Shrink feasible region (reduce number of SAs).
2. Measure the size of feasible region.
3. Get candidates sequentially according to total cable lengths.
4. Assess candidates based on available measurements.

Advantages:

- Robust to partial observability.
- Integrate all kinds of information in weights and directions.

Issues

1. Too many candidates. (remove overlapping edges)
2. Full observability over voltage magnitudes. (matrices with full rank)
3. Error in linearised power flow calculation. (error from linearisation)

Future Work

- How to detect more impossible potential edges. (for issue 1)
- How to assess candidates based on a fraction. (for issue 2)
- How to use voltage sensitivity matrix in linearised power flow. (for issue 3)

Bibliography

Conti, S., Greco, A., and Raiti, S. (2006). Voltage sensitivity analysis in mv distribution networks. In *Proceedings of the 6th WSEAS/IASME International Conference on Electric Power Systems, High Voltages, Electric Machines, Tenerife, Spain*, pages 16–18.

Fischetti, M. and Vigo, D. (1997). A branch-and-cut algorithm for the resource-constrained minimum-weight arborescence problem. *Networks: An International Journal*, 29(1):55–67.

Hsieh, T.-Y., Chen, T.-H., and Yang, N.-C. (2017). Matrix decompositions-based approach to z-bus matrix building process for radial distribution systems. *International Journal of Electrical Power & Energy Systems*, 89:62–68.

Michiels, W., Aarts, E., and Korst, J. (2007). *Theoretical aspects of local search*. Springer Science & Business Media.