# Topology Inference for RDF

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# Introduction

This website hosts slides for defence of my master thesis.

# Bus and Edge

There are roughly two types of electrical devices in power grids.

type	definition	examples
delivery element	transport power from one place to another	cable, transformer, capacitor
conversion element	convert power from or to another form	solar panel, battery

- Ignore conversion elements. Not necessary in power flow calculation.
- Delivery element will be called **edge**.

Another concept, **bus**, represent the place where two different delivery elements joint or end of a delivery element, but there is no physical entity corresponding to a bus. There are three common types of buses:

type	know quantities
slack bus	voltage magnitude and phase angle
PQ bus	real power injection and reactive power injection
PV bus	real power injection and voltage magnitude

It is sufficient to model most of RDFs with PQ buses and one kind of edges, cables:

- One slack bus in RDF, corresponding to the **root**.
- Root not in any matrix.
- Ignore other delivery elements.

# Two Special Concepts

Essential for power flow calculation.

### 3.1 Channel

### 3.2 Snapshot

 ${\bf Snapshot}$  is a concept to include power injections and voltages at one time index

- input: real power injections at all channels of PQ buses
- output: voltages, current flow, power flow

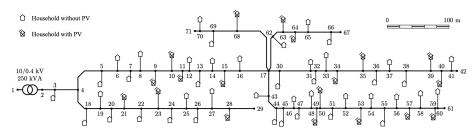
**Zero-load snapshot** is the snapshot where power injections at all the channels are zero and voltages equalt o rated voltages in corresponding phases.

### Radial Distribution Feeder

Assumptions about feeders:

- arborescence
- step-down transformer is not considered
- three-phase four-wire cable
- one phase star connection

A case with 70 buses is primarily used here:



- located in Belgium
- bus 1 is omitted
- Houses associated with buses 3, 7, 10, 13, 16, 20, 23, 26, 30, 33, 36, 39, 43, 46, 49, 52, 55, 58, 62, 65, 69 are connected through phase A.
- Houses associated with buses 5, 8, 11, 14, 18, 21, 24, 27, 31, 34, 37, 40, 44, 47, 50, 53, 56, 59, 63, 66, 70 are connected through phase B.

Power Flow

## Directed Graph

### weighted directed graph $G = (\mathcal{N}, \mathcal{E}, \sigma, \tau, \omega)$

- set of nodes:  $\mathcal{N}$
- set of edges:  $\mathcal{E}$
- incidence functions: source  $\sigma$ , target  $\tau$
- (edge) weighting function,  $\omega : E \to \mathbb{R}$ .
- 2-D Euclidean Distance as Weight

#### complete graph for a set of nodes

- ullet all edges are potential edges
- some are impossible to exist

#### arborescence

- subgraph of a directed graph
- root

### feasible region

- all the arborescences of a directed graph
- to count number of arborescences

## **IP** Formulation

Symbols and definitions of sets:

symbol	definition
$\mathcal{E}$	all the potential edges (edges in the complete graph)
${\mathcal C}$	available measurements of voltages and power injections
$\mathcal{E}_{\mathrm{impossible}}$	potential edges that are impossible to exist

Symbols, definitions, types and sets of variables:

symbol	definition	$\mathbf{type}$	$\mathbf{set}$
$x_{ij}$	if edge from i to j is in the solution	$\{0, 1\}$	$\mathcal{E}$

Symbols, definitions, sets of constants:

symbol	definition	$\overline{\mathbf{set}}$
$d_{i,j}$	weight of directed edge from i to j based on distance	$\mathcal{E}$

The formulation is:

$$\begin{split} \min_{x_{ij} \forall (i,j) \in \mathcal{E}} \quad & (1-\alpha) \sum_{(i,j) \in \mathcal{E}} d_{ij} x_{ij} + \alpha \mathcal{H} \left( \{ x_{ij} \forall (i,j) \in \mathcal{E} \}, \mathcal{C} \right) \\ \text{s.t.} \quad & \sum_{(i,j) \in \delta^-(j)} x_{ij} = 1 \quad \forall j \in V' \quad \text{(a directed forest)} \\ & \sum_{(i,j) \in \delta^-(S)} x_{ij} \geq 1 \quad \forall S \subseteq V', |S| \geq 2 \quad \text{(a connected graph)} \\ & x_{ij} = 0 \quad \forall (i,j) \in \mathcal{E}_{\text{impossible}} \quad \text{(remove impossible potential edges)} \end{split}$$

Two terms in the objective function:

term	definition	coefficient
$(1-\alpha)\sum_{(i,j)\in\mathcal{E}}d_{ij}x_{ij}$	weight of candidate arborescence	$1-\alpha$
	assessment of candidate arborescence	$\alpha$

### Three sets of constraints:

- $\bullet~$  First two sets ensure arborescence.
- $\bullet\,$  Last set removes impossible potential edges.