Topology Inference for Radial Distribution Feeder based on Power Flow

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Contents

Chapter 1

Introduction

This website hosts slides for defence of my master graduation project in the Department of Electrical Engineering at Technical University of Denmark. How households are connected to distribution network is always unknown. A framework to infer such connections by utilising all kinds of information is proposed in this project.

Problem Setting

> How households are connected to distribution network is usually unknown.

Available information for topology inference:

> There are three kinds of available information in this project.

- geographical information about buses
- voltage magnitudes of all the phases of all the buses
- some real power injection profiles

> For example, where they are located and barriers between them. There is no literature mentioning how to handle geographical information.

Association network inference:

> In the literature, a technique called association network inference is usually used. Correlation between entities are derived based on some entity attributes. The face that number of edges in a mathematical tree is always 1 less than number of entities makes the problem simpler.

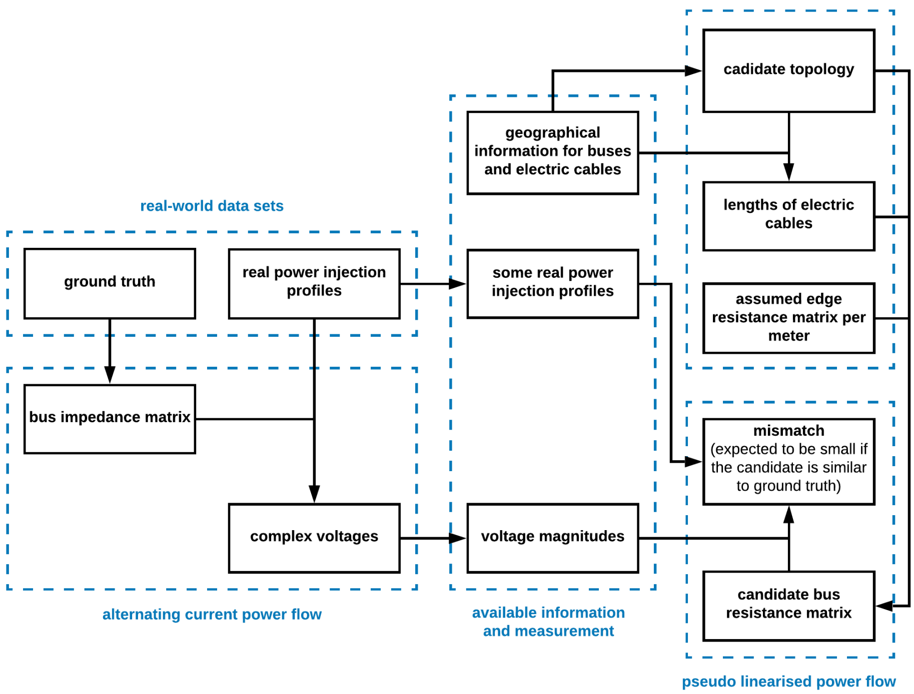
- Correlation between entities.
- $|\mathcal{E}| = |\mathcal{N}| - 1$

However, there are two reasons why such technique cannot be used here.

- Spurious correlation resulted from correlated profiles.
- Missing entity attribute.

The second reason will be discussed in detail later.

Flowchart



There are three columns. In the left column, ground truth, the true topology, is known, and used to simulate available measurement using power flow program. Three boxes in the middle represents three kinds of information disucssed before. Note that ground truth is not known and we want to find it. Programs in the right column are used to make inference about the topology.

Ground truth is not known, but used to simulate available measurement.

Two batches of computer programs:

- power flow
- three algorithms to handle directed graphs

The second batch will not be discussed in detail.

Chapter 2

Radial Distribution Feeder

Essential concepts are discussed in following three parts.

- bus and edge, -> 2.1
- two special concepts for power flow, -> 2.2
- case with 70 buses, -> 2.3

2.1 Bus and Edge

Power grids can be roughly described here by following three concepts.

type	definition	examples
edge	transport power from one place to another	cable, transformer, capacitor
conversion element	convert power from or to another form	solar panel, battery
bus	where two edges joint or end of an edge	slack bus, PQ bus, PV bus

Only one type of edge, cable, is considered. The method applies when there are transformers, capacitors, and other devices. Usually, there is one slack bus in distribution network and is referred to as root here.

- Ignore conversion elements. Not necessary in power flow calculation.
- Cable.
- One slack bus -> **root**.

Impedance of cable is proportional to its length.

Unit impedance matrix for case-70:

$$\bar{Z}_{abc} = \begin{bmatrix} 0.000412 + 1.558e^{-4}j & 0.000206 + 7.791e^{-5}j & 0.000206 + 7.791e^{-5}j \\ 0.000206 + 7.791e^{-5}j & 0.000412 + 1.558e^{-4}j & 0.000206 + 7.791e^{-5}j \\ 0.000206 + 7.791e^{-5}j & 0.000206 + 7.791e^{-5}j & 0.000412 + 1.558e^{-4}j \end{bmatrix}$$

It is assumed that there is only one type of cable in case-70, and its impedance matrix for one-meter-long cable is known. Diagonal values are the same, and two times of off-diagonal values. So it is sufficient to store this matrix with one number for resistance and one number for reactance.

2.2 Two Special Concepts

Essential for power flow calculation.

Such two concepts cannot be found in the literature, but are essential for power flow when taking multiple phases into account.

Channel

- **channel**: refer to one phase in some bus
- **active channel**: there are non-zero injections (connected to some conversion element)
- **observed active channel**: such non-zero injections are known

It is assumed that all inactive channels are observed.

That is, it is known that there is no power injection at those channels.

Snapshot

Snapshot: include power injections and voltages at one time index

- Duration is 1 s in this project.

Zero-load snapshot : when power injections at all the channels are zero

Such two symbols will be used later.

- $\bar{V}_{\text{zero}}$: voltages in zero-load snapshot
- V_{rate}: rated voltage magnitude, 230 V

2.3 Case with 70 Buses

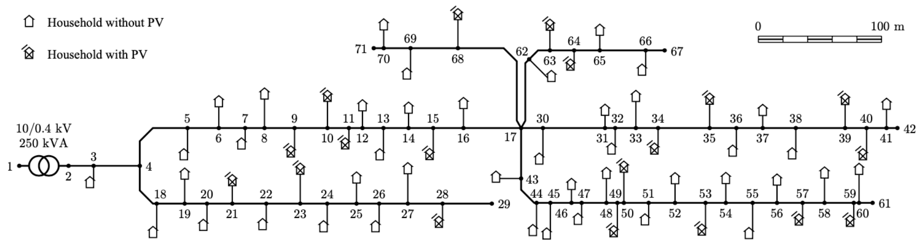
To make the project manageable:

Assumptions about feeders:

- one path to any bus (tree)

- one step-down transformer
- rated voltage, 230 V
- three-phase four-wire cable
- one phase star connection

A case with 70 buses is primarily used here:



- located in Belgium
- bus 1 is omitted
- 70 buses
- 207 channels

Chapter 3

Problem Formulation

- information in a directed graph -> 3.1
- integer programming formulation -> 3.3
- local search heuristic algorithm -> 3.4

There is an issue here that I will discuss after a summary in the end.

- remove overlapping edge -> 3.2

3.1 Directed Graph

All the information can be stored in a weighted directed graph.

weighted directed graph $G = (\mathcal{N}, \mathcal{E}, \sigma, \tau, \omega)$

- set of nodes: $\mathcal{N}$
- set of edges: $\mathcal{E}$
- incidence functions: source σ, target τ
- (edge) weighting function, $\omega : \mathcal{E} \to \mathbb{R}$.

complete graph for a set of nodes

- all edges are **potential edges**
- some are impossible to exist
- possible to derive their lengths
- 2-D Euclidean distance as weight
- *association network inference cannot be used*

spanning arborescence (SA)

- subgraph of a directed graph
- root

- include every bus

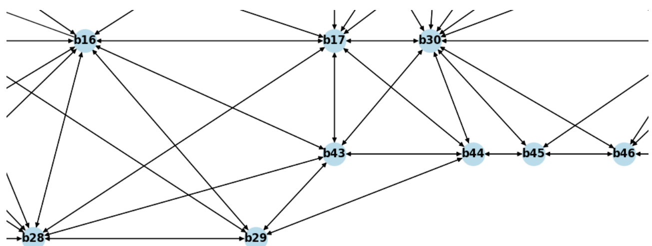
feasible region

- All the SAs.
- Number of SA is finite, making it a combinatorial optimisation problem.
- Count number of SA.

3.2 Remove Overlapping Edge

For example, in case-70:

shortest path	$<$	**direct edge**	$\times$**threshold**	**-> remove direct edge**
”b17-b43-b29”	$<$	”b17-b29”	$\times$1.1	-> remove ”b17-b29”
”b44-b43-b29”	$>$	”b44-b29”	$\times$1.1	-> keep ”b44-b29”



However:

- 446 possible potential edges
- over 10^{45} spanning arborescences

-> summary

3.3 Integer Programming

Sets:

symbol	**definition**
$\mathcal{E}$	all the potential edges (edges in the complete graph)
$\mathcal{C}$	available measurements of voltages and power injections
$\mathcal{E}_{\text{impossible}}$	potential edges that are impossible to exist

Variables:

symbol	**definition**	**type**	**set**
x_{ij}	if edge from i to j is in the solution	$\{0,1\}$	$\mathcal{E}$

Constants:

symbol	definition	set
$d_{i,j}$	Euclidean distance from i to j	$\mathcal{E}$

$$
\begin{aligned}
\min_{x_{ij}\forall(i,j)\in\mathcal{E}} \quad & (1-\alpha)\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij} + \alpha\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right) \\
\text{s.t.} \quad & \sum_{(i,j)\in\delta^-(j)} x_{ij} = 1 \quad \forall j \in V' \quad \text{(a directed forest)} \\
& \sum_{(i,j)\in\delta^-(S)} x_{ij} \geq 1 \quad \forall S \subseteq V', |S| \geq 2 \quad \text{(a connected graph)} \\
& x_{ij} = 0 \quad \forall (i,j) \in \mathcal{E}_{\text{impossible}} \quad \text{(remove impossible potential edges)}
\end{aligned}
$$

Two terms in the objective function:

term	definition	coefficient
$(1-\alpha)\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij}$	weight of candidate arborescence	$1-\alpha$
$\alpha\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right)$	assessment of candidate arborescence	α

Three sets of constraints:

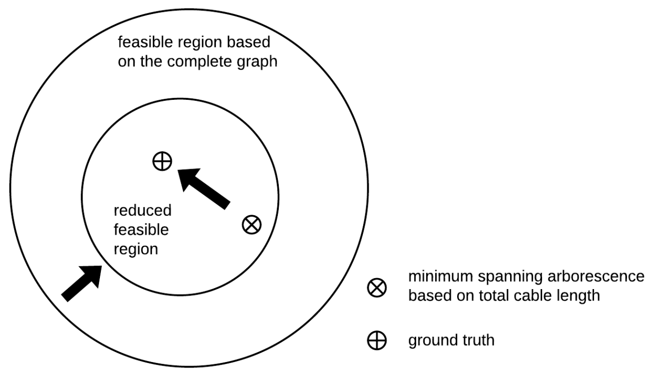
- First two sets ensure arborescence. (Fischetti and Vigo, 1997)
- Last set removes impossible potential edges.

3.4 Local Search

At least two possible values for α:

value	term lefted	to find	disadvantage
1	$\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right)$	ground truth	NP-hard and non-linear
0	$\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij}$	topology with min total cable length	cannot find ground truth

Such two situations can be visualised:



Combinatorial optimisation problem.

A **local search heuristic algorithm** is proposed to to move from $\otimes$ to $\oplus$. (Michiels et al., 2007)

> All the algorithms in this category have two parts, an objective funciton to assess candidates and an neighbourhood function to generate candidates systematically. Here, pseudo linearised power flow will be used as the objective function and resuted mean squared error is to be minimised. It will be discussed later. An algorithm to rank spanning arborescences according to their total cable lengths is implemented.

function	what it does	in this project
objective	assess candidate	pseudo linearised power flow
neighbourhood	generate candidate	rank spanning arborescence

- Starting point -> minimum spanning arborescence
- Every candidate is reachable.
- Ground truth should be found before long.
- Not in parallel.

> Because power grids are to be built with less cost, the total cable length of ground truth should not be too long, so we can find it before long,

Chapter 4

AC Power Flow

- two essential matrices -> 4.1
- bus impedance matrix -> 4.2
- direct impedance method for power flow calculation -> 4.2

Can be generalised for multi-phase model. (Hsieh et al., 2017)

4.1 Two Matrices

Current injection to flow:

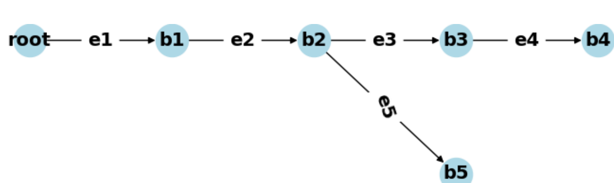
$$\bar{I}_{\text{edge}} = -K\bar{I}$$

where **edge path incidence matrix (EPI)**, K.

Voltage drop to nodal voltage:

$$\bar{V} = \bar{V}_{\text{zero}} - K^{\top}\bar{Z}_{\text{edge}}\bar{I}_{\text{edge}}$$

where **edge impedance diagonal block matrix (EIDB)**, $\bar{Z}_{\text{edge}}$.



$$
\begin{bmatrix} \bar{I}_{\mathrm{edge},1} \\ \bar{I}_{\mathrm{edge},2} \\ \bar{I}_{\mathrm{edge},3} \\ \bar{I}_{\mathrm{edge},4} \\ \bar{I}_{\mathrm{edge},5} \end{bmatrix} = - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \bar{I}_5 \end{bmatrix}
$$

$$
\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \\ \bar{V}_5 \end{bmatrix} - \begin{bmatrix} \bar{V}_{\mathrm{rate}} \\ \bar{V}_{\mathrm{rate}} \\ \bar{V}_{\mathrm{rate}} \\ \bar{V}_{\mathrm{rate}} \\ \bar{V}_{\mathrm{rate}} \end{bmatrix} = - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{\top} \begin{bmatrix} Z_{\mathrm{edge},1} & 0 & 0 & 0 & 0 \\ 0 & Z_{\mathrm{edge},2} & 0 & 0 & 0 \\ 0 & 0 & Z_{\mathrm{edge},3} & 0 & 0 \\ 0 & 0 & 0 & Z_{\mathrm{edge},4} & 0 \\ 0 & 0 & 0 & 0 & Z_{\mathrm{edge},5} \end{bmatrix} \begin{bmatrix} \bar{I}_{\mathrm{edge}} \\ \bar{I}_{\mathrm{edge}} \\ \bar{I}_{\mathrm{edge}} \\ \bar{I}_{\mathrm{edge}} \\ \bar{I}_{\mathrm{edge}} \end{bmatrix}
$$

Alternating current power flow: (Conti et al., 2006)

$$
\bar{V} = \bar{V}_{\mathrm{zero}} + \left(K^{\top} \bar{Z}_{\mathrm{edge}} K \right) \bar{I}
$$

4.2 Bus Impedance Matrix

Alternating current power flow:

$$
\bar{V} = \bar{V}_{\mathrm{zero}} + \left(K^{\top} \bar{Z}_{\mathrm{edge}} K \right) \bar{I}
$$

Bus impedance matrix (BIM), $\bar{Z}$, is defined as:

$$
\begin{aligned} \bar{Z} &= K^{\top} \bar{Z}_{\mathrm{edge}} K \\ &= R + jX \end{aligned}
$$

where **bus resistance matrix (BRM)**, R: real part of entries in BIM.

4.3 Direct Impedance Method

Build BIM directly and calculate power flow. (Schneider et al., 2017)

Five steps to build BIM:

1. Define a unit impedance matrix.
2. Calculate edge impedance matrices for cables.
3. Build EIDB.
4. Obtain EPI based on topology.
5. Calculate BIM using EIDB and EPI.

Fixed Point Method

To calculate power flow in one snapshot, given power injections, the following procedure is repeated:

$$
\begin{aligned}
\bar{I} &= \underline{P} \otimes \underline{V}_{\text{previous}} \\
\bar{V} &= \bar{Z}\bar{I} + \bar{V}_{\text{zero}} \\
\epsilon &= \left(\bar{V} - \bar{V}\right)^{\top} \left(\bar{V} - \bar{V}\right)
\end{aligned}
$$

until ϵ is smaller than a pre-defined threshold.

Chapter 5

Linearised Power Flow

Two steps:

- linearise voltage drop -> 5.1
- ignore power loss -> 5.2

Then:

- assessment of candidate -> 5.5

- bus resistance matrix -> 5.3
- inversed bus resistance matrix -> 5.4
- error from linearisation -> 5.6

5.1 Linearised Voltage Drop

With power flow at source of edge k, $\bar{S}_{\text{source},k}$:

$$\bar{I}_{\text{edge},k} = \frac{\underline{S}_{\text{source},k}}{\underline{V}_i}$$

Voltage drop:

$$\begin{aligned}
\bar{V}_{\text{edge},k} &= \bar{I}_{\text{edge},k}\bar{Z}_{\text{edge},k} \\
&= \frac{\underline{S}_{\text{source},k}\bar{Z}_{\text{edge},k}}{\underline{V}_i} \\
&= \frac{\left(P_{\text{source},k} - jQ_{\text{source},k}\right)\left(R_{\text{edge},k} + jX_{\text{edge},k}\right)}{\underline{V}_i} \\
&= \frac{R_{\text{edge},k}P_{\text{source},k} + X_{\text{edge},k}Q_{\text{source},k}}{\underline{V}_i} + j\frac{X_{\text{edge},k}P_{\text{source},k} - R_{\text{edge},k}Q_{\text{source},k}}{\underline{V}_i}
\end{aligned}$$

Then: (Conti et al., 2006)

$$V_{\text{edge},k} = \frac{R_{\text{edge},k}}{V_{\text{rate}}} P_{\text{source},k}$$

- Ignore imaginary part.
- Replace $\underline{V}_i$ with V_{rate}.

5.2 Linearised Voltage

Voltage drop to nodal voltage:

$$V = V_{\text{zero}} - \frac{1}{V_{\text{rate}}} K^{\top} R_{\text{edge}} P_{\text{source}}$$

Power injection to flow:

$$P_{\text{source}} = -K\left(P - P_{\text{loss}}\right)$$

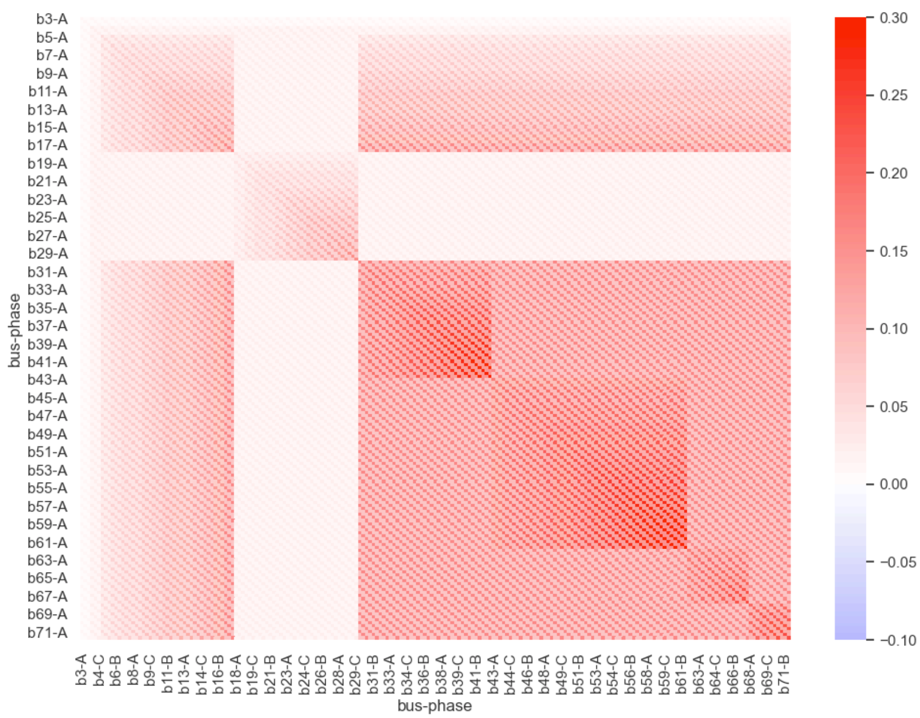
Voltage magnitude can be calculated using BRM and real power injections:

$$\begin{aligned} V &= V_{\text{zero}} + \frac{1}{V_{\text{rate}}} \left(K^{\top} R_{\text{edge}} K\right) P \\ &= V_{\text{zero}} + \frac{1}{V_{\text{rate}}} R P \end{aligned}$$

- To assess candidate by calculating power injection using voltage magnitudes.

5.3 Bus Resistance Matrix

- Bus 2 -> root
- 69 PQ buses
- 207 channels
- 207 rows and 207 columns



There is some pattern that can be explained by the analytical expression for entries in BRM.

Lowest Common Ancestor Problem

Entry (i, j) -> sum of edge resistances in their common path to root:

$$R_{i,j} = \sum_{k \in U_i \cap U_j} R_{\text{edge },k}$$

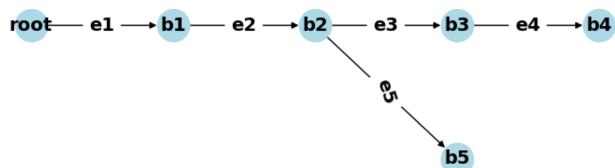
where U_i is set of edges on the path from root to bus i.

Entry (i, j) is the sum of edge resistances in the path from root to their lowest common ancestor (LCA) of bus i and j.

- Calcualted efficiently using LCA for all pairs.
- Useful pattern.

BRM can be calcualted efficiently using LCA for all pairs of buses. and the pattern can be used in future work.

For example,



pair of buses	entry in BRM
b3-b5	$R_{\mathrm{e1}} + R_{\mathrm{e2}}$
b4-b5	$R_{\mathrm{e1}} + R_{\mathrm{e2}}$

-> summary

5.4 Pseudo Linearised Power Flow

Based on linearised power flow, $V = V_{\text{zero}} + \frac{1}{V_{\text{rate}}} RP$:

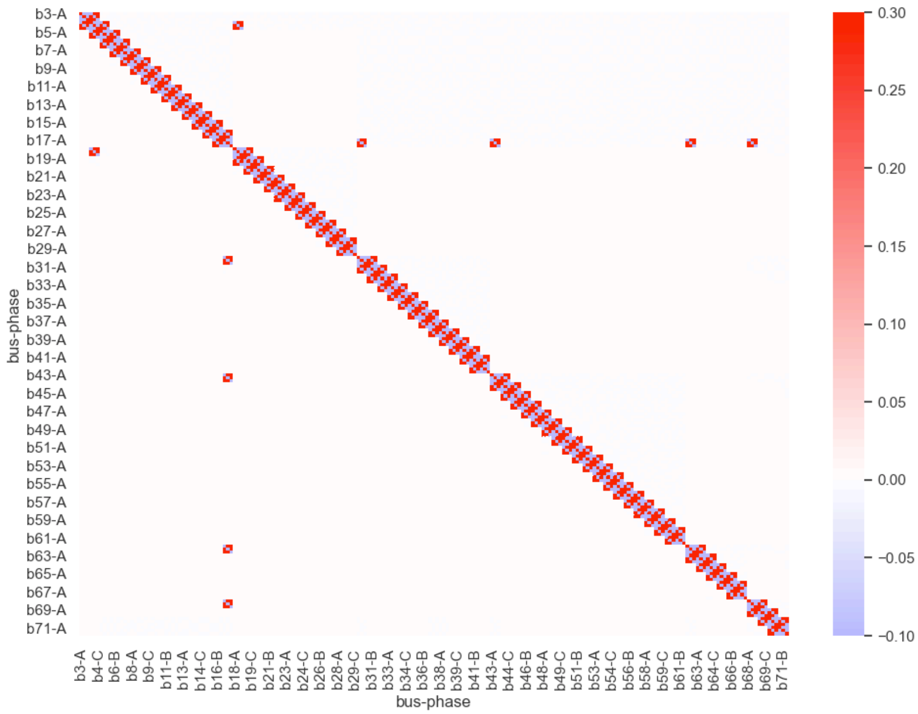
$$P_{\text{assess}} = V_{\text{rate}} R^{\top} \left(V - V_{\text{zero}} \right)$$

-> **pseudo linearised power flow**.

which is referred to as pseudo linearised power flow.

Inversed BRM for case-70:

Inversed BRM for the case looks like:



- Sparse.
- Full rank.
- Voltage magnitude at any channel can have a huge impact.
- Useful pattern.

5.5 Assessment of Candidate

- Calculate P_{assess} using voltage magnitudes.
- Compare P_{assess} with available power measurements.

Mean squared error (MSE):

$$\mathcal{H}(R) = \left[(P_{\text{assess}} - P) \otimes O\right]^{\top} \cdot \left[(P_{\text{assess}} - P) \otimes O\right] / |\mathcal{O}|$$

where:

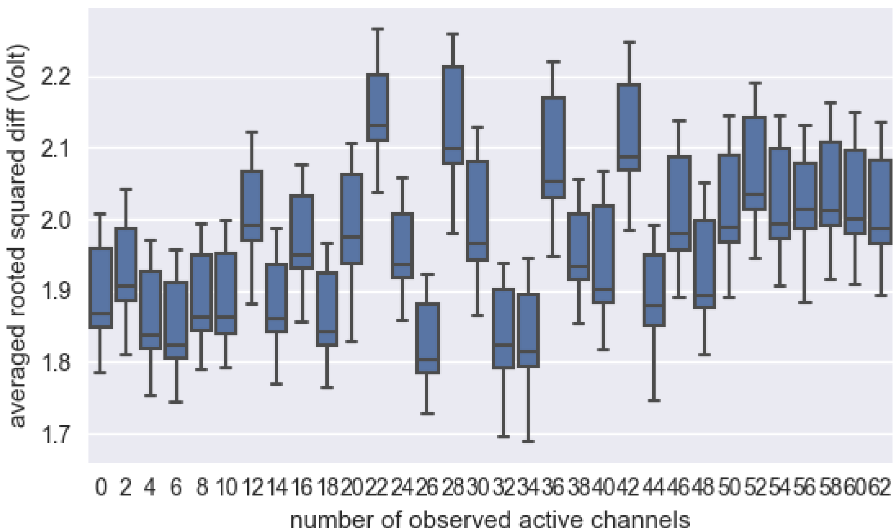
- $\mathcal{O}$: set of observed active channels and inactive channels.
- O: binary vector indicating observed active channels.

  It is the second term in the objective function. Entries for unobserved active channels are ignored.

5.6 Error from Linearisation

Box plot:

- with respect to different number of observed active channels
- based on ground truth and 50 snapshots[1]



- Error is aleady reduced to 1.7 ~ 2.2.
- Rated voltage magnitudes will increase the error dramatically.

  Rated voltage magnitudes will increase the error dramatically, so full observability of voltage magnitudes are still required for now.

-> summary

[1]during 00:00:00 and 00:00:50 on Dec 2, 2020 from Sonnen data set.

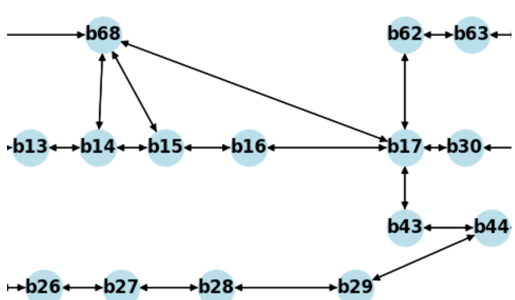
Chapter 6

Result and Discussion

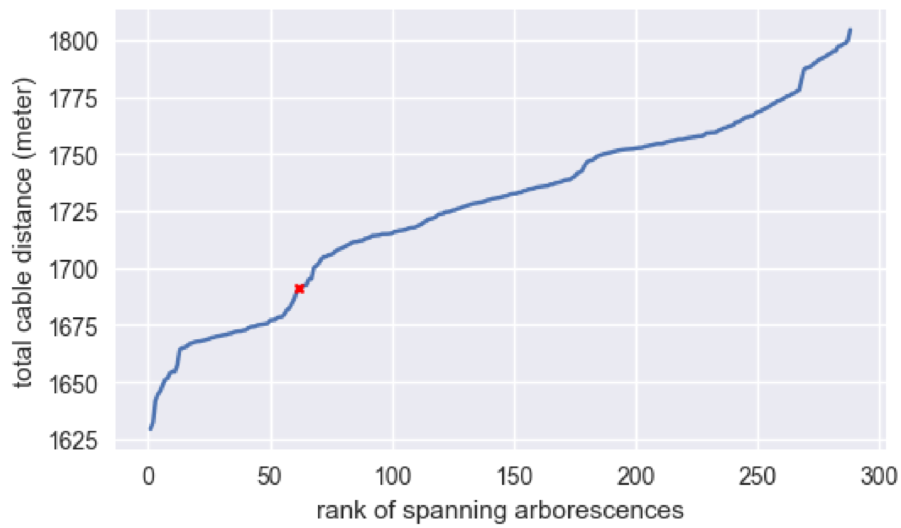
- result for case-70 -> 6.1
- summary -> 6.2

6.1 Result for Case-70

- three new pairs, “b29-b44”, “b68-b14”, and “b68-b15”
- 144 edges in total
- 288 SAs rooted at “b2”

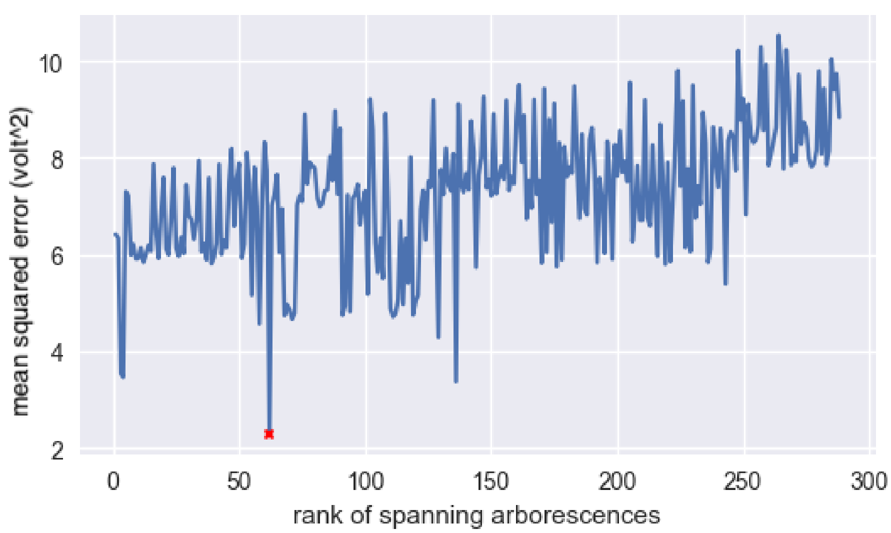


Rank SAs according to total cable lengths:



Total cable length increases all the time. That is, we rank spanning arborescences according to their total cable lengths.

Assessment based on 50 snapshots[1]:



6.2 Summary

- Topology inference -> combinatorial optimisation problem.

[1]during 00:00:00 and 00:00:50 on Dec 2, 2020 from Sonnen data set.

- A framework is proposed.
- Core: local search heuristic algorithm.

Advantages:

- Robust to partial observability.
- Integrate all kinds of information in weight and direction.

Four steps:

1. Shrink feasible region (reduce number of SAs).
2. Measure the size of feasible region by counting number of SAs.
3. Get candidates sequentially by ranking SAs according to total cable lengths.
4. Assess candidates based on available measurements.

Issues

1. Too many spanning arborescences. (remove overlapping edges)
2. Full observability over voltage magnitudes. (matrices with full rank)
3. Error in linearised power flow calculation. (error from linearisation)

Future Work

- How to detect more impossible potential edges. (for issue 1)
- How to assess candidates based on a fraction. (for issue 2)
- How to use voltage sensitivity matrix in linearised power flow. (for issue 3)

Bibliography

Conti, S., Greco, A., and Raiti, S. (2006). Voltage sensitivity analysis in mv distribution networks. In *Proceedings of the 6th WSEAS/IASME International Conference on Electric Power Systems, High Voltages, Electric Machines, Tenerife, Spain*, pages 16–18.

Fischetti, M. and Vigo, D. (1997). A branch-and-cut algorithm for the resource-constrained minimum-weight arborescence problem. *Networks: An International Journal*, 29(1):55–67.

Hsieh, T.-Y., Chen, T.-H., and Yang, N.-C. (2017). Matrix decompositions-based approach to z-bus matrix building process for radial distribution systems. *International Journal of Electrical Power & Energy Systems*, 89:62–68.

Michiels, W., Aarts, E., and Korst, J. (2007). *Theoretical aspects of local search*. Springer Science & Business Media.

Schneider, K. P., Mather, B., Pal, B., Ten, C.-W., Shirek, G. J., Zhu, H., Fuller, J. C., Pereira, J. L. R., Ochoa, L. F., de Araujo, L. R., et al. (2017). Analytic considerations and design basis for the ieee distribution test feeders. *IEEE Transactions on power systems*, 33(3):3181–3188.