Topology Inference for Radial Distribution Feeder based on Power Flow

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2020-12-14

Contents

Chapter 1

Introduction

This website hosts slides for defence of my master graduation project in the Department of Electrical Engineering at Technical University of Denmark. How households are connected to distribution network is always unknown. A framework to infer such connections by utilising all kinds of information is proposed in this project.

Problem Setting

> How households are connected to distribution network is usually unknown.

Available information for topology inference:

> There are three kinds of available information in this project.

- geographical information about buses
- voltage magnitudes of all the phases of all the buses
- some real power injection profiles

> For example, where they are located and barriers between them. There is no literature mentioning how to handle geographical information.

Association network inference:

> In the literature, a technique called association network inference is usually used. Correlation between entities are derived based on some entity attributes. The face that number of edges in a mathematical tree is always 1 less than number of entities makes the problem simpler.
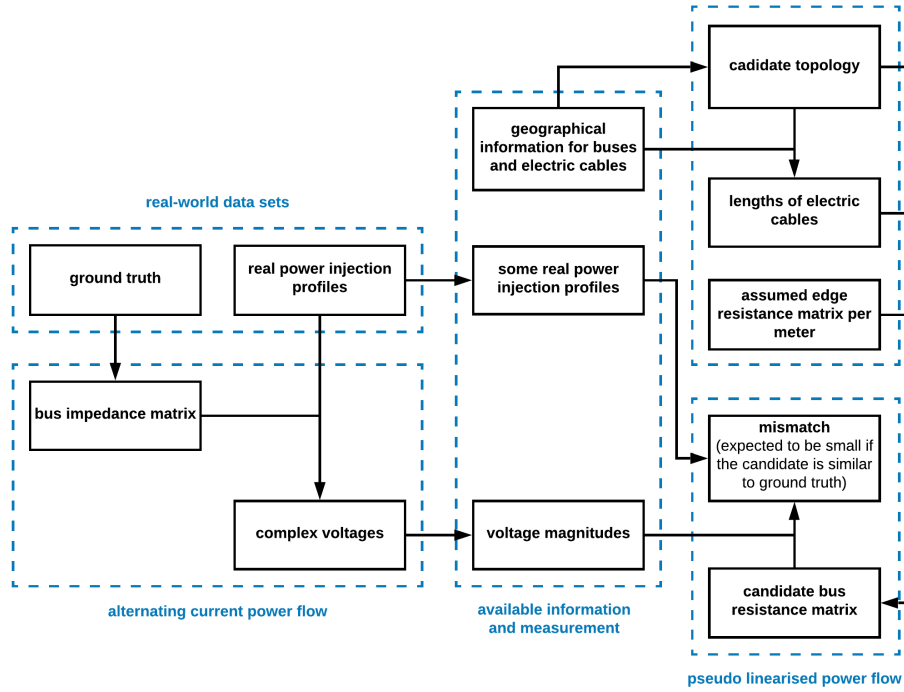
- Correlation between entities.
- $|\mathcal{E}| = |\mathcal{N}| - 1$

However, there are two reasons why such technique cannot be used
here.

- Spurious correlation resulted from correlated profiles.
- Missing entity attribute.

The second reason will be discussed in detail later.

Flowchart

There are three columns. In the left column, ground truth, the
true topology, is known, and used to simulate available measurement
using power flow program. Three boxes in the middle represents
three kinds of information disucssed before. Note that ground truth
is not known and we want to find it. Programs in the right column
are used to make inference about the topology.

Ground truth is not known, but used to simulate available measurement.

Two batches of computer programs:

- power flow
- three algorithms to handle directed graphs

The second batch will not be discussed in detail.

Chapter 2

Radial Distribution Feeder

Essential concepts are discussed in following three parts.

- bus and edge, -> 2.1
- two special concepts for power flow, -> 2.2
- case with 70 buses, -> 2.3

2.1   Bus and Edge

Power grids can be roughly described here by following three concepts.

type	definition	examples
edge	transport power from one place to another	cable, transformer, capacitor
conversion element	convert power from or to another form	solar panel, battery
bus	where two edges joint or end of an edge	slack bus, PQ bus, PV bus

Only one type of edge, cable, is considered. The method applies when there are transformers, capacitors, and other devices. Usually, there is one slack bus in distribution network and is referred to as root here.

- Ignore conversion elements. Not necessary in power flow calculation.
- Cable.
- One slack bus -> **root**.

2.2   Two Special Concepts

Essential for power flow calculation.

Such two concepts cannot be found in the literature, but are essential for power flow when taking multiple phases into account.

Channel

- **channel**: refer to one phase in some bus
- **active channel**: there are non-zero injections (connected to some conversion element)
- **observed active channel**: such non-zero injections are known

It is assumed that all inactive channels are observed.

That is, it is known that there is no power injection at those channels.

Snapshot

Snapshot: include power injections and voltages at one time index

- duration: 1 s

Zero-load snapshot : when power injections at all the channels are zero

Such two symbols will be used later.

- $\bar{V}_{\text{zero}}$: voltages in zero-load snapshot
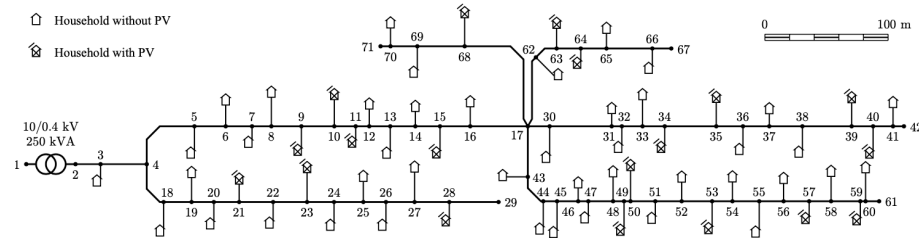- V_{rate}: rated voltage magnitude, 230 V

2.3   Case with 70 Buses

To make the project manageable:

Assumptions about feeders:

- spanning arborescence (SA)
- one step-down transformer
- rated voltage, 230 V
- three-phase four-wire cable
- one phase star connection

———————————

A case with 70 buses is primarily used here:

- located in Belgium
- bus 1 is omitted
- 70 buses
- 207 channels

Chapter 3

Problem Formulation

- information in a directed graph -> 3.1
- integer programming formulation -> 3.3
- local search heuristic algorithm -> 3.4

There is an issue here that I will discuss after a summary in the end.

- remove overlapping edge -> 3.2

3.1 Directed Graph

All the information can be stored in a complete directed graph.

complete (directed) graph for a set of buses

All edges are **potential edges**, and they must have weights. In association network inference, it is impossible when some entity attributes are missing.

- all edges are **potential edges**
- select a set of edges -> SA -> candidate
- **impossible potential edge**
- 2-D Euclidean distance as weight

feasible region

- All the candidates (SAs).
- Number of SA is finite, making it a combinatorial optimisation problem.
- Count number of SA.

3.2   Remove Overlapping Edge

A potential edge can overlap some bus on its way from source to
target.

For example, in case-70:

shortest path	<	direct edge	×threshold	-> remove direct edge
"b17-b43-b29"	<	"b17-b29"	×1.1	-> remove "b17-b29"
"b44-b43-b29"	>	"b44-b29"	×1.1	-> keep "b44-b29"
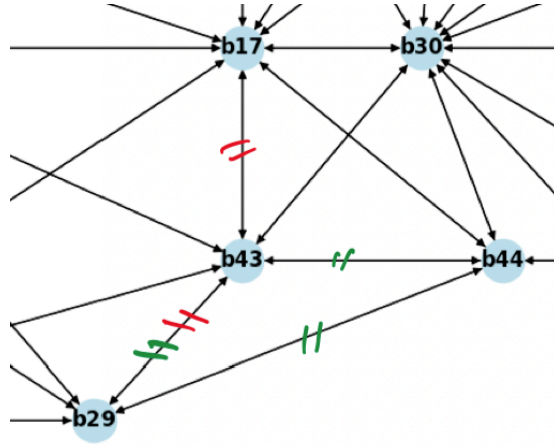
However:

After all overlapping edges are removed (or saying detected as im-
poosible to exist), there are still 446 possible potential edges and over
10^{45} SAs. This is the only technique used in this project. When more
information is available, more advanced techniques can be applied.

- 446 possible potential edges
- over 10^{45} SAs

-> summary

3.3   Integer Programming

Sets:

symbol	definition
$\mathcal{E}$	all the potential edges
$\mathcal{C}$	available measurements
$\mathcal{E}_{\text{impossible}}$	impossible potential edges

Variables:

symbol	definition	type	set
x_{ij}	whether to choose edge from i to j	binary	$\mathcal{E}$

Constants:

symbol	definition	set
$d_{i,j}$	Euclidean distance	$\mathcal{E}$

$$\min_{x_{ij}\forall(i,j)\in\mathcal{E}} \quad (1-\alpha)\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij} + \alpha\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right)$$

$$\text{s.t.} \quad \sum_{(i,j)\in\delta^-(j)} x_{ij} = 1 \quad \forall j \in V' \quad \text{(a directed forest)}$$

$$\sum_{(i,j)\in\delta^-(S)} x_{ij} \geq 1 \quad \forall S \subseteq V', |S| \geq 2 \quad \text{(a connected graph)}$$

$$x_{ij} = 0 \quad \forall(i,j)\in\mathcal{E}_{\text{impossible}} \quad \text{(remove impossible potential edges)}$$

Two terms in the objective function:

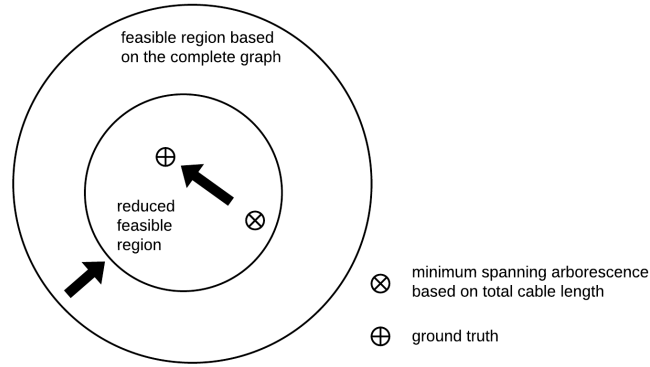| term	definition	coefficient
$(1-\alpha)\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij}$	total weight of candidate	$1-\alpha$
$\alpha\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right)$	assessment of candidate	α

Three sets of constraints:

- First two sets ensure arborescence. (Fischetti and Vigo, 1997)
- Last set removes impossible potential edges.

3.4 Local Search

At least two possible values for α:

value	term lefted	to find	disadvantage
1	$\mathcal{H}\left(\{x_{ij}\forall(i,j)\in\mathcal{E}\},\mathcal{C}\right)$	ground truth	NP-hard and non-linear
0	$\sum_{(i,j)\in\mathcal{E}} d_{ij}x_{ij}$	topology with min total cable length	cannot find ground truth

Such two situations can be visualised:

feasible region based
on the complete graph

$\oplus$

reduced
feasible
region

$\otimes$

$\otimes$ minimum spanning arborescence
based on total cable length

$\oplus$ ground truth

For this combinatorial optimisation problem, a **local search heuristic algorithm** is proposed to to move from $\otimes$ to $\oplus$. (Michiels et al., 2007)

> All the algorithms in this category have two parts, an objective funciton to assess candidates and an neighbourhood function to generate candidates systematically. Here, pseudo linearised power flow will be used as the objective function and resulted mean squared error is to be minimised. It will be discussed later. An algorithm to rank spanning arborescences according to their total cable lengths is implemented.

function	what it does	in this project
objective	assess candidate	pseudo linearised power flow
neighbourhood	generate candidate	rank spanning arborescence

> Because power grids are to be built with less cost, the total cable length of ground truth should not be too long, so we can find it before long,

- Ground truth should be found before long.
- Not in parallel.

Chapter 4

AC Power Flow

- two essential matrices -> 4.1
- bus impedance matrix -> 4.2
- direct impedance method for power flow calculation -> 4.2

Can be generalised for multi-phase model. (Hsieh et al., 2017)

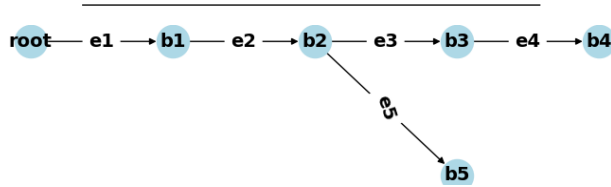## 4.1 Two Matrices

Current injection to flow:
$$\bar{I}_{\text{edge}} = -K\bar{I}$$

where **edge path incidence matrix (EPI)**, K.

Voltage drop to nodal voltage:
$$\bar{V} = \bar{V}_{\text{zero}} - K^{\top}\bar{Z}_{\text{edge}}\bar{I}_{\text{edge}}$$

where **edge impedance diagonal block matrix (EIDB)**, $\bar{Z}_{\text{edge}}$.

Each row indicates whether power injection from some bus will pass the edge.

$$\begin{bmatrix} \bar{I}_{\text{edge},1} \\ \bar{I}_{\text{edge},2} \\ \bar{I}_{\text{edge},3} \\ \bar{I}_{\text{edge},4} \\ \bar{I}_{\text{edge},5} \end{bmatrix} = - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \bar{I}_5 \end{bmatrix}$$

$$\begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \\ \bar{V}_5 \end{bmatrix} - \begin{bmatrix} \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \\ \bar{V}_{\text{rate}} \end{bmatrix} = - \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{\top} \begin{bmatrix} Z_{\text{edge},1} & 0 & 0 & 0 & 0 \\ 0 & Z_{\text{edge},2} & 0 & 0 & 0 \\ 0 & 0 & Z_{\text{edge},3} & 0 & 0 \\ 0 & 0 & 0 & Z_{\text{edge},4} & 0 \\ 0 & 0 & 0 & 0 & Z_{\text{edge},5} \end{bmatrix} \begin{bmatrix} \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \\ \bar{I}_{\text{edge}} \end{bmatrix}$$

Alternating current power flow: (Conti et al., 2006)

$$\bar{V} = \bar{V}_{\text{zero}} + \left(K^{\top} \bar{Z}_{\text{edge}} K \right) \bar{I}$$

4.2   Bus Impedance Matrix

Alternating current power flow:

$$\bar{V} = \bar{V}_{\text{zero}} + \left(K^{\top} \bar{Z}_{\text{edge}} K \right) \bar{I}$$

Bus impedance matrix (BIM), $\bar{Z}$, is defined as:

$$\bar{Z} = K^{\top} \bar{Z}_{\text{edge}} K$$
$$= R + jX$$

where **bus resistance matrix (BRM)**, R: real part of entries in BIM.

4.3   Direct Impedance Method

Build BIM directly and calculate power flow. (Schneider et al., 2017)

Five steps to build BIM:

1. Define a unit impedance matrix.
2. Calculate edge impedance matrices for cables.
3. Build EIDB.
4. Obtain EPI based on topology.
5. Calculate BIM using EIDB and EPI.

Fixed Point Method

The following procedure is repeated:

$$\bar{I} = \underline{P} \otimes \underline{V}_{\text{previous}}$$
$$\bar{V} = \bar{Z}\bar{I} + \bar{V}_{\text{zero}}$$
$$\epsilon = \left(\bar{V} - \bar{V}\right)^{\top} \left(\bar{V} - \bar{V}\right)$$

until ϵ is smaller than a pre-defined threshold.

Chapter 5

Linearised Power Flow

Two steps:

- linearise voltage drop -> 5.1
- ignore power loss -> 5.2

Then:

- assessment of candidate -> 5.5

Besides, there are three parts that I will discussed in detail after the summary.

- bus resistance matrix -> 5.3
- inversed bus resistance matrix -> 5.4
- error from linearisation -> 5.6

5.1   Linearised Voltage Drop

With power flow at source of edge k, $\bar{S}_{\text{source},k}$, voltage drop:

$$
\begin{aligned}
\bar{V}_{\text{edge},k} &= \bar{I}_{\text{edge},k}\bar{Z}_{\text{edge},k} \\
&= \frac{\underline{S}_{\text{source},k}\bar{Z}_{\text{edge},k}}{\underline{V}_i} \\
&= \frac{\left(P_{\text{source},k} - jQ_{\text{source},k}\right)\left(R_{\text{edge},k} + jX_{\text{edge},k}\right)}{\underline{V}_i} \\
&= \frac{R_{\text{edge},k}P_{\text{source},k} + X_{\text{edge},k}Q_{\text{source},k}}{\underline{V}_i} + j\frac{X_{\text{edge},k}P_{\text{source},k} - R_{\text{edge},k}Q_{\text{source},k}}{\underline{V}_i}
\end{aligned}
$$

Then: (Conti et al., 2006)

$$V_{\text{edge},k} = \frac{R_{\text{edge},k}}{V_{\text{rate}}} P_{\text{source},k}$$

- Ignore imaginary part.
- Use V_{rate}.

5.2   Linearised Voltage

Similar to AC power flow:

Power injection to flow:

$$P_{\text{source}} = -K\left(P - P_{\text{loss}}\right)$$

Voltage drop to nodal voltage:

$$V = V_{\text{zero}} - \frac{1}{V_{\text{rate}}} K^{\top} R_{\text{edge}} P_{\text{source}}$$

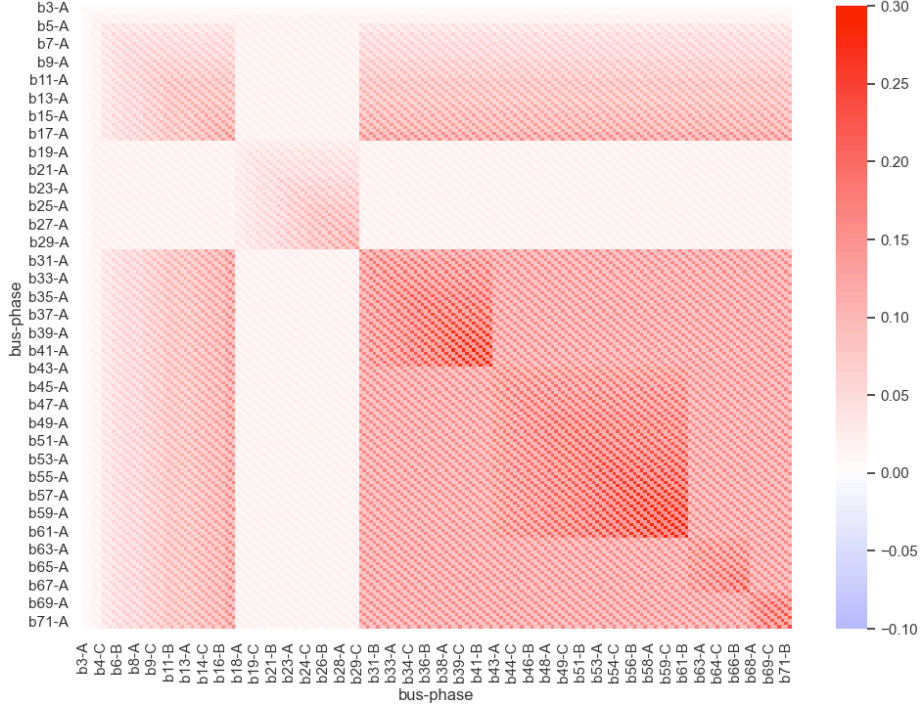Voltage magnitudes can be calculated using BRM and real power injections:

$$V = V_{\text{zero}} + \frac{1}{V_{\text{rate}}} \left(K^{\top} R_{\text{edge}} K\right) P$$
$$= V_{\text{zero}} + \frac{1}{V_{\text{rate}}} RP$$

- To assess candidate by calculating power injections using voltage magnitudes.

5.3   Bus Resistance Matrix

BRM of case-70:

- Bus 2 -> root
- 69 PQ buses
- 207 channels
- 207 rows and 207 columns

There is some pattern that can be explained by the analytical expression.

Lowest Common Ancestor Problem

Entry (i, j) -> sum of edge resistances in their common path to root:

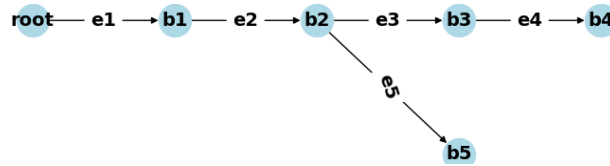$$R_{i,j} = \sum_{k \in U_i \cap U_j} R_{\text{edge},k}$$

where U_i is set of edges on the path from root to bus i.

Entry (i, j) is the sum of edge resistances in the path from root to their lowest common ancestor (LCA) of bus i and j.

- Calcualted efficiently using LCA for all pairs.
- Useful pattern.

BRM can be calcualted efficiently using LCA for all pairs of buses. and the pattern can be used in future work.

For example,

pair of buses	entry in BRM
b3-b5	$R_{e1} + R_{e2}$
b4-b5	$R_{e1} + R_{e2}$

-> summary

5.4   Pseudo Linearised Power Flow

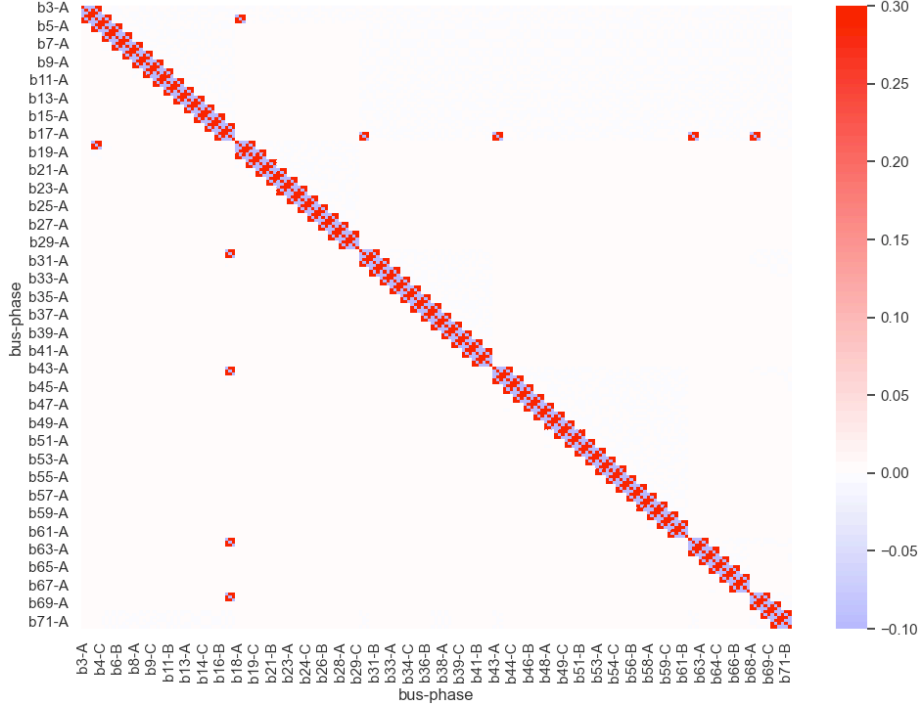Based on linearised power flow, $V = V_{\text{zero}} + \frac{1}{V_{\text{rate}}} RP$:

$$P_{\text{assess}} = V_{\text{rate}} R^\top (V - V_{\text{zero}})$$

-> **pseudo linearised power flow**.

which is referred to as pseudo linearised power flow.

Inversed BRM for case-70:

Inversed BRM for the case looks like:

- Sparse.
- Full rank.
- Voltage magnitude at any channel can have a huge impact.
- Useful pattern.

5.5 Assessment of Candidate

- Calculate P_{assess} using voltage magnitudes.
- Compare P_{assess} with available power measurements.

Mean squared error (MSE):

$$\mathcal{H}(R) = \left[(P_{\text{assess}} - P) \otimes O\right]^{\top} \cdot \left[(P_{\text{assess}} - P) \otimes O\right] / |\mathcal{O}|$$
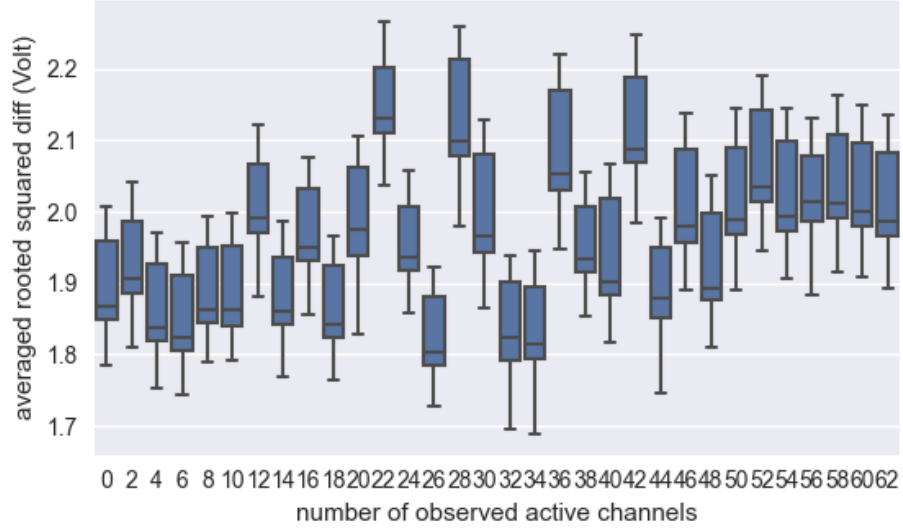
where:

- $\mathcal{O}$: set of observed active channels and inactive channels.
- O: binary vector indicating observed active channels.

  It is the second term in the objective function. Entries for unobserved active channels are ignored.

5.6   Error from Linearisation

Box plot:

- with respect to different number of observed active channels
- based on ground truth and 50 snapshots[1]

- Error is aleady reduced to 1.7 ~ 2.2.
- Rated voltage magnitudes will increase the error dramatically.

  Rated voltage magnitudes will increase the error dramatically, so full observability of voltage magnitudes are still required for now.
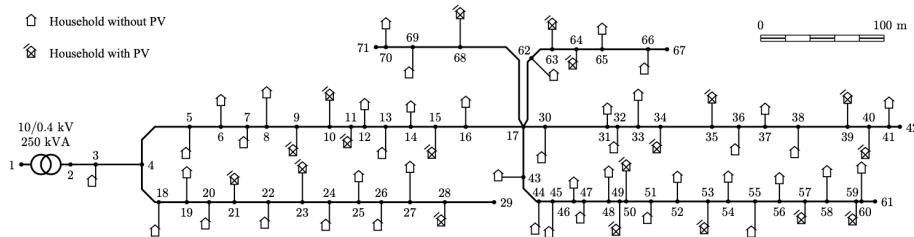
-> summary
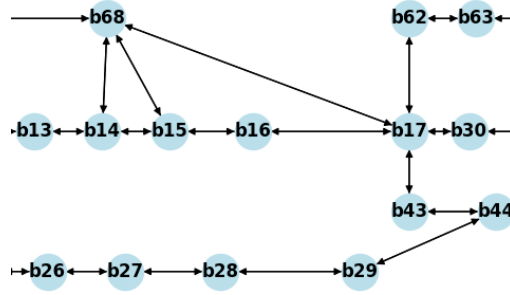
Chapter 6

Result and Summary

- result for case-70 -> 6.1
- summary -> 6.2

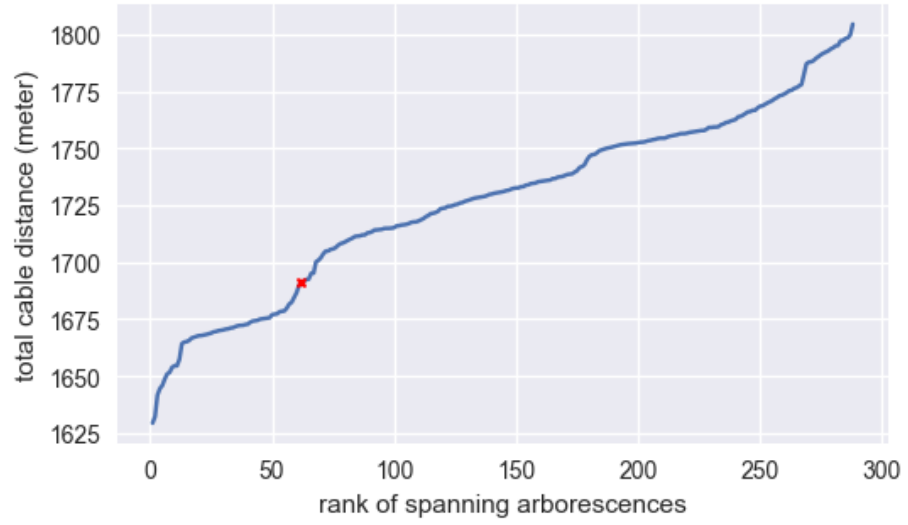## 6.1   Result for Case-70

Here we only show how result might look like. Because there are only 75 new possible potential edges here. There is an issue that will be discussed later.

- three new pairs, "b29-b44", "b68-b14", and "b68-b15"
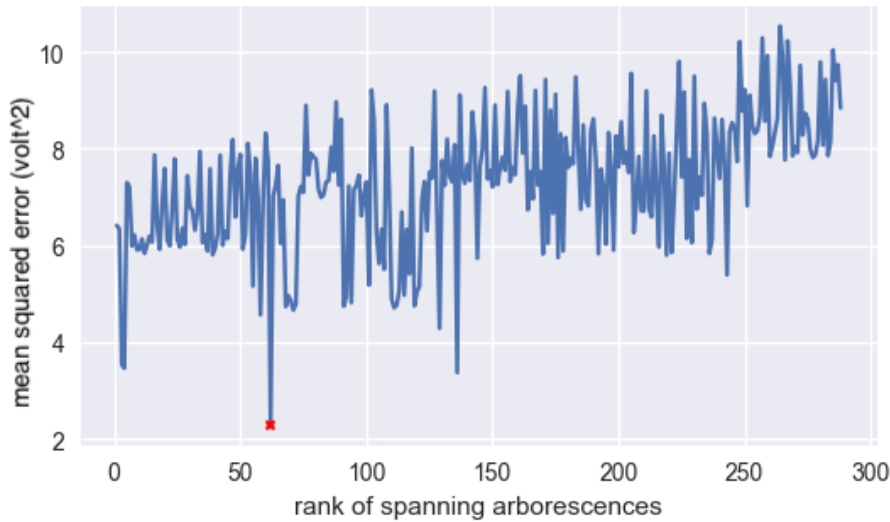- 144 edges in total
- 288 SAs rooted at "b2"

Rank candidates according to total cable lengths:

It increases all the time.

Assessment based on 50 snapshots[1]:

The red cross point, with minimum mean squared error, is ground truth.

6.2 Summary

The project is summarised here. We turn topology inference into a combinatorial optimisation problem. A new framework is proposed, and its core is a local search heuristic algorithm.

- Topology inference -> combinatorial optimisation problem.
- A new framework is proposed.
- Core: local search heuristic algorithm.

Four steps:

1. Shrink feasible region (reduce number of SAs).
2. Measure the size of feasible region by counting number of SAs.
3. Get candidates sequentially by ranking SAs according to total cable lengths.
4. Assess candidates based on available measurements.

Advantages:

There are two advantages. It is robust to partial observability over power injections, and is capable to integrate all kinds of information in weights and directions.

- Robust to partial observability.
- Integrate all kinds of information in weights and directions.

Issues

There are three issues that I want to discuss in detail here. Firstly, there are too many SAs, so we have to reduce the number, for example, by removing overlapping edges.

1. Too many SAs. (remove overlapping edges)
2. Full observability over voltage magnitudes. (matrices with full rank)
3. Error in linearised power flow calculation. (error from linearisation)

Future Work

- How to detect more impossible potential edges. (for issue 1)
- How to assess candidates based on a fraction. (for issue 2)
- How to use voltage sensitivity matrix in linearised power flow. (for issue 3)

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