Fingering instability on a sphere

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We investigate the stability of a gravity-driven thin film of viscous Newtonian fluid, coating the outer surface of a sphere, in the framework of lubrication theory. Near the advancing front, the emergence of a capillary ridge, prone to transverse instabilities, announces the breakup of axisymmetry. Ultimately, as thicker regions flow faster, the front splits into separate fingers (Figure 1).

The selection of the number of fingers is highly intricate due to the normal and tangential gravity components' variation along the substrate, combined with the increasing circumference that the front needs to cover. In fact, the width of the capillary ridge, which sets the characteristic length scale for the instability, is inversely proportional to the front's polar position (and hence, to its perimeter). The locally most unstable fingering number should increase as the front advances.

However, similar to the fingering instability on a horizontal cylinder[2], the draining flow's time-dependence makes the traditional modal stability analysis unsuitable. Thus, we perform a linear optimal transient growth analysis in order to determine the fastest growing initial perturbation of the film thickness profile for given fingering number and time horizon. This results in a prediction of the most amplified number of fingers, as a function of the time horizon, that we compare to experimental observations[1].

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[1] Takagi and Huppert, *J. Fluid Mech.* **647**, 221 (2010).

[2] Balestra et al., *J. Fluid Mech.* **868**, 726 (2019).

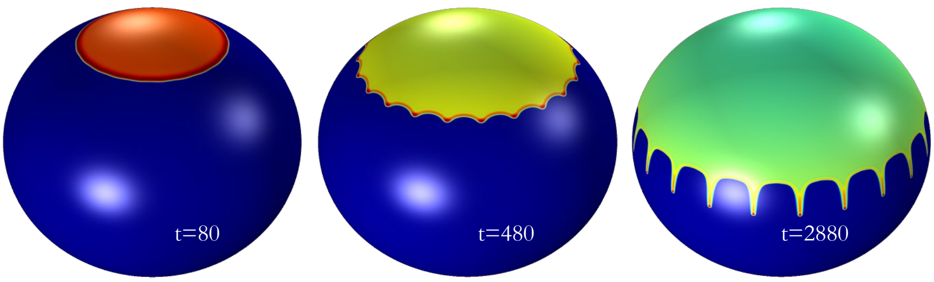

Figure 1: Simulation of the full non-linear 2-dimensional lubrication equation. The perturbation optimised for 21 fingers and for the time horizon T=120 is added to an axisymmetric initial condition.