Weakly non-parallel linear instability of a condensing film flow

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We investigate the linear response to harmonic forcing of Nusselt's condensation film[1]. A quiescent, saturated vapour condenses onto a uniformly cooled, inclined substrate and forms a film of condensate, assumed of constant material properties, falling under the effect of gravity (fig. 1a). This flow has a spatially-developing nature: the film thickness grows downstream as $x^{1/4}$.

As regards its local linear stability, a unique spatial branch, associated to downstream-propagating waves, becomes unstable. This is the Kapitza instability[2], the convective nature of which ensures the well-posedness of the signalling problem. Compared to the classical case of an isothermal, uniform film flow, where destabilising inertia competes against the stabilising effects of gravity and capillarity, the condensing film flow is additionally locally stabilised by the phase change: as thinner liquid layers promote heat transfer, vapour tends to condense more in the perturbation's troughs rather than on the crests, thus damping perturbations. Nevertheless, as the basic film thickness grows downstream, inertia experienced by perturbations increases and after a critical distance Kapitza waves can develop.

The basic flow's weak non-parallelism enables the use of the Wentzel–Kramers–Brillouin–Jeffreys (WKBJ) formalism[3], which consists in the "stitching" of successive local analyses, corrected by a slowly-varying amplitude. By setting some threshold for nonlinear saturation, our analysis results in a prediction of the linearly most prevalent angular frequency, expected to reach this threshold the earliest (fig. 1b).

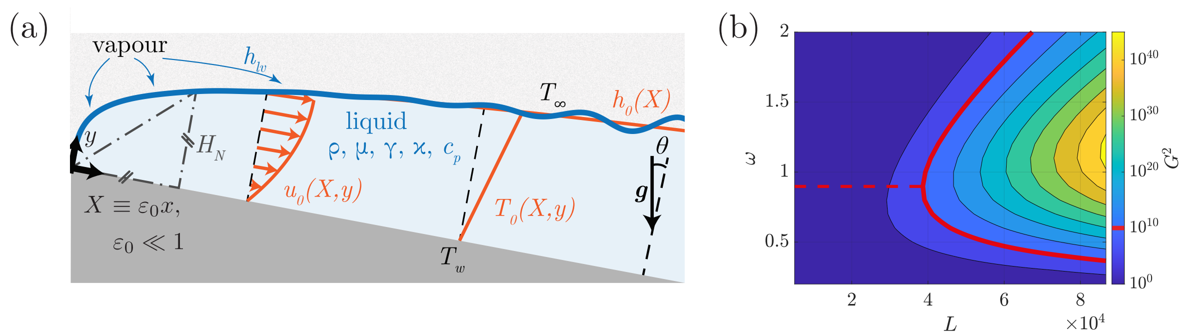

Figure 1: (a) Sketch of the problem at hand. (b) Heatmap of the largest surface deformation amplification G^2 as a function of dimensionless plate length L and angular frequency ω. The red solid line follows the isoline of $G^2 = 10^{10}$ and the dashed line indicates the frequency, predicted to reach this threshold the earliest. Here, $\theta = \pi/4$ and the Jakob number, which measures the ratio of sensible to latent heat, $Ja \approx 0.02$; for condensing water, this represents 10 K of cooling. Under these conditions, the largest considered plate length $L \approx 9 \times 10^4$ represents 1 m and the prevailing frequency $\omega \approx 0.9$ scales to around 11 Hz.

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[1] Nusselt, *Z. VDI*, **60** (1916).

[2] Kapitza, *Zh. Eksp. Teor. Fiz.*, **18** (1948).

[3] Huerre & Rossi, *Cambridge University Press* (1998).