4) $Y = C_1 e^{(-5+2i)x} + C_2 e^{(-5-2i)x}$, $y'' + 10y' + 29y = 0$

$r^2 e^{rx} + 10re^{rx} + 29e^{rx} = 0$

$e^{rx}(r^2 + 10r + 29) = 0$

$r^2 + 10r + 29 = 0$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{-16}}{2}$

$\alpha = -5 \quad \pm \quad \beta = 2i \rightarrow r_1 = -5 + 2i \; ; \; r_2 = -5 - 2i$

$$\boxed{Y = C_1 e^{(-5+2i)x} + C_2 e^{(-5-2i)x}}$$ ✓

$Y_1 = e^{-5x} \cdot e^{2ix} = e^{-5x} \cos 2x$

$Y_2 = e^{-5x} \cdot e^{-2ix} = e^{-5x} \operatorname{sen} 2x$

$$\boxed{Y = C_1 e^{-5x} \cos 2x + C_2 e^{-5x} \operatorname{sen} 2x}$$

12) $8y'' - 2y' - y = 0$

$8r^2 e^{rt} - 2re^{rt} - e^{rt} = 0$

$e^{rt}(8r^2 - 2r - 1) = 0$

$8r^2 - 2r - 1 = 0$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{36}}{16}$

$r_1 = \frac{2+6}{16} = \frac{8}{16} = \frac{1}{2}$

$r_2 = \frac{2-6}{16} = \frac{-4}{16} = -\frac{1}{4}$

$y_1 = e^{1/2\,t}$

$y_2 = e^{-1/4\,t}$

$y_3 = C_1 e^{1/2\,t} + C_2 e^{-1/4\,t}$

20) $y'' - 6y' + 18y = 0 \qquad y(0) = 0 \qquad y'(0) = 6$

$r^2 e^{rt} - 6re^{rt} + 18e^{rt} = 0$

$e^{rt}(r^2 - 6r + 18) = 0$

$r^2 - 6r + 18 = 0$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{-36}}{2}$

$\alpha = 3 \qquad \pm \qquad \beta = \frac{6i}{2} = 3i$

$y_1 = e^{3t} \cdot e^{3it} = e^{3t}\cos 3t$

$y_2 = e^{3t} \cdot e^{-3it} = e^{3t}\operatorname{sen} 3t$

$y_3 = C_1 e^{3t}\cos 3t + C_2 e^{3t}\operatorname{sen} 3t$

$0 = C_1 e^0 (1) + C_2 e^0 (0)$

$\boxed{0 = C_1}$

$6 = C_1(3e^{3t}\cos 3t - 3e^{3t}\operatorname{sen} 3t) + C_2(3e^{3t}\operatorname{sen} 3t + 3e^{3t}\cos 3t)$

$6 = 3C_1 + 3C_2 \rightarrow 6 = 3(0) + 3C_2 \Rightarrow 6 = 3C_2 \rightarrow \boxed{C_2 = \frac{6}{3} = 2}$

$\star\, y = (0)e^{3t}\cos 3t + 2e^{3t}\operatorname{sen} 3t$

$y = 2e^{3t}\operatorname{sen} 3t$

25) $y'' - 16y = 0 \quad y(0) = 5 \quad y(1/4) = 5e$

$r^2 e^{rt} - 16 e^{rt} = 0$

$e^{rt}(r^2 - 16) = 0$

$r^2 - 16 = 0$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{0 \pm \sqrt{64}}{2}$

$r_1 = \frac{0+8}{2} = 4$

$r_2 = \frac{0-8}{2} = -4$

$y_1 = e^{4t}$

$y_2 = e^{-4t}$

$y_3 = C_1 e^{4t} + C_2 e^{-4t}$

★ $5 = C_1 e^0 + C_2 e^0$

$5 = C_1 + C_2$

$5e = C_1 e^{4(1/4)} + C_2 e^{-4(1/4)}$

$5e = C_1 e + C_2 e^{-1}$

$C_1 = 5 - C_2$

$5e = (5 - C_2)e + C_2 e^{-1}$

$5e = 5e - C_2 e + C_2 e^{-1}$

$5e - 5e = -C_2 e + C_2 e^{-1}$

$0 = -C_2 e + C_2 e^{-1}$

$0 = C_2(-e + e^{-1})$

$0 = C_2(1)$

$0 = C_2 \rightarrow C_1 = 5 - C_2$

$C_1 = 5 - 0$

$C_1 = 5$

$y = 5e^{4t} + (0)e^{-4t}$

$\boxed{y = 5e^{4t}}$