2) $3t^2 - e^{2t}$

$$\frac{3 \cdot 2!}{s} - \frac{1}{s-2} = \frac{6}{s} - \frac{1}{-(2-s)}$$

$$\frac{6}{s^3} + \frac{1}{2-s}$$

4) $3t^4 - 2t^2 + 1$

$$\frac{3 \cdot 4!}{s^3} - \frac{2 \cdot 2!}{s^3} + \frac{1}{s}$$

$$\frac{72}{s^5} - \frac{4}{s^3} + \frac{1}{s}$$

Tema

Día: / Mes: / Año:

6) $e^{-2t}\,\text{sen}\,2t + e^{3t}t^2$

$\mathcal{L}\{e^{-2t}\,\text{sen}\,2t\} + \mathcal{L}\{e^{3t}t^2\}$

$$\frac{2}{(s+2)^2+4} + \frac{2}{(s-3)^3}$$

8) $(1+e^{-t})^2$

$1 + 2e^{-t} + e^{-2t}$

$$\frac{1}{s} + \frac{2}{s+1} + \frac{1}{s+2}$$

10) $t\,e^{2t}\cos 5t \rightarrow$

$$\mathcal{L}\{e^{2t}\cos 5t\} = \frac{(s-2)}{(s-2)^2+25}$$

$$= (-1)^1 \frac{d}{ds}\left[\frac{(s-2)}{(s-2)^2+25}\right] = \frac{(s-2)^2+25-(s+2)2(s-2)}{((s-2)^2+25)^2}$$

$$\frac{(s-2)^2-2(s-2)^2}{((s-2)^2+25)^2} = \frac{-(s-2)^2}{((s-2)^2+25)^2} = \frac{(s-2)}{((s-2)^2+2}$$

1) t

$$\mathcal{L}\{t\}(s) = \lim_{N\to\infty} \int_0^N e^{-st}\, t\, dt$$

$$u = t \qquad du = dt$$
$$dv = e^{-st}dt \qquad v = \frac{e^{-st}}{-s}$$

$$= \lim_{N\to\infty} \left[\frac{t \cdot e^{-st}}{-s} - \frac{1}{-s} \int_0^N e^{-st}\, dt \right]$$

$$= \lim_{N\to\infty} \left[\frac{t \cdot e^{-st}}{-s} + \frac{1}{s} \cdot \frac{e^{-st}}{-s} \right] \Bigg|_0^N$$

$$= \lim_{N\to\infty} \left[\frac{t \cdot e^{-st}}{-s} - \frac{e^{-st}}{s^2} \right] \Bigg|_0^N$$

$$= \lim_{N\to\infty} \left[\frac{N e^{-Ns}}{-s} - \frac{e^{-Ns}}{s^2} - \left(\frac{0 e^{-0s}}{-s} - \frac{e^{-0s}}{s^2} \right) \right]$$

$$\mathcal{L}\{t\}(s) = \frac{1}{s^2}$$

2) t^2

$$\mathcal{L}\{t^2\}(s) = \lim_{N\to\infty} \int_0^N e^{-st}\, t^2\, dt$$

$$u = t^2 \quad du = 2t\,dt \qquad dv = e^{-st}dt \qquad v = \frac{e^{-st}}{-s}$$

$$= \lim_{N\to\infty} \left[\frac{t^2 \cdot e^{-st}}{-s} + \frac{2}{s} \int e^{-st}\, t\, dt \right]$$

$$= \lim_{N\to\infty} \left[\frac{t^2 e^{-st}}{-s} + \frac{2}{s} \left(\frac{t e^{-st}}{-s} - \frac{e^{-st}}{s^2} \right) \right] \Bigg|_0^N$$

$$= \lim_{N\to\infty} \left[\frac{t^2 e^{-st}}{-s} - \frac{2t e^{-st}}{s^2} - \frac{2 e^{-st}}{s^3} \right] \Bigg|_0^N$$

$$= \lim_{N\to\infty} \left[\frac{N^2 e^{-sN}}{-s} - \frac{2N e^{-sN}}{s^2} - \frac{2 e^{-sN}}{s^3} - \left(\frac{0^2 e^{-s0}}{-s} - \frac{2 \cdot 0 e^{-s0}}{s^2} - \frac{2 e^{-s0}}{s^3} \right) \right]$$

$$\mathcal{L}\{t^2\}(s) = \frac{2}{s^3}$$

3) e^{6t}

$$\mathcal{L}\{e^{6t}\}(s) = \lim_{N\to\infty} \int_0^N e^{-st} e^{6t}\, dt$$

$$= \lim_{N\to\infty} \int_0^N e^{(-st+6t)}\, dt$$

$$= \lim_{N\to\infty} \int_0^N e^{-(s-6)t}\, dt$$

$u = -(s-6)t \qquad du = -(s-6)\,dt \qquad \frac{du}{-(s-6)} = dt$

$$= \lim_{N\to\infty} \int_0^N \frac{e^u\, du}{-(s-6)}$$

$$= \lim_{N\to\infty} \frac{1}{-(s-6)} e^u \Big|_0^N$$

$$= \lim_{N\to\infty} \frac{e^{-(s-6)t}}{-(s-6)} \Big|_0^N$$

$$= \frac{1}{-(s-6)} \cdot \lim_{N\to\infty} \left(e^{-(s-6)N} - e^{-(s-6)0}\right)$$

$$\mathcal{L}\{e^{6t}\}(s) = \frac{1}{s-6}$$

4) $t e^{3t}$

$$\mathcal{L}\{t e^{3t}\}(s) = \lim_{N\to\infty} \int_0^N e^{-st}\, t\, e^{3t}\, dt$$

$$= \lim_{N\to\infty} \int_0^N e^{(-s+3)t}\, t\, dt$$

$u = t \qquad du = dt \qquad dv = e^{(-s+3)t}\,dt \qquad v = \frac{e^{(-s+3)t}}{(-s+3)}$

$$= \lim_{N\to\infty} \left[\frac{t e^{(-s+3)t}}{(-s+3)} - \frac{1}{(-s+3)} \int e^{(-s+3)t}\, dt \right]$$

$$= \lim_{N\to\infty} \left[\frac{t e^{(-s+3)t}}{(-s+3)} - \frac{1}{(-s+3)} \left(\frac{e^{(-s+3)t}}{(-s+3)} \right) \right] \Big|_0^N$$

$$= \lim_{N\to\infty} \left[\frac{t e^{(-s+3)t}}{(-s+3)} - \frac{e^{(-s+3)t}}{(-s+3)^2} \right] \Big|_0^N$$

$$= \lim_{N\to\infty} \left[\frac{N e^{(-s+3)N}}{(-s+3)} - \frac{e^{(-s+3)N}}{(-s+3)^2} - \left(\frac{0 e^{(-s+3)0}}{(-s+3)} - \frac{e^{(-s+3)0}}{(-s+3)^2} \right) \right]$$

$$\mathcal{L}\{t e^{3t}\}(s) = -\frac{1}{(-s+3)^2}$$

13) $\mathcal{L}\{6e^{-3t}-t^2+2t-8\}$

$$\mathcal{L}\{6e^{-3t}-t^2+2t+8\} = \frac{6}{s+3} - \frac{2}{s^3} + \frac{2}{s^2} - \frac{8}{s}$$

14) $\mathcal{L}\{5-e^{2t}+6t^2\}$

$$\mathcal{L}\{5-e^{2t}+6t^2\} = \frac{5}{s} - \frac{1}{s-2} + \frac{12}{s^3}$$

15) $\mathcal{L}\{t^3-te^{t}+e^{4t}\cos t\}$

$$\mathcal{L}\{t^3-te^{t}+e^{4t}\cos t\} = \frac{6}{s^4} - \frac{1}{(s-1)^2} + \frac{s-4}{(s-4)^2+1}$$

16) $\mathcal{L}\{t^2-3t-2e^{-t}\operatorname{sen} 3t\}$

$$\mathcal{L}\{t^2-3t-2e^{-t}\operatorname{sen} 3t\} = \frac{2}{s^3} - \frac{3}{s^2} - \frac{6}{(s+1)^2+9}$$

17). $\mathcal{L}\{e^{3t}\operatorname{sen} 6t - t^3 + e^{t}\}$

$$\mathcal{L}\{e^{3t}\operatorname{sen} 6t - t^3 + e^{t}\} = \frac{6}{(s-3)^2+36} - \frac{6}{s^4} + \frac{1}{s-1}$$

18). $\mathcal{L}\{t^4-t^2-t+\operatorname{sen}\sqrt{2}t\}$

$$\mathcal{L}\{t^4-t^2-t+\operatorname{sen}\sqrt{2}t\} = \frac{24}{s^5} - \frac{2}{s^3} - \frac{1}{s^2} + \frac{\sqrt{2}}{s^2+2}$$

19) $\mathcal{L}\{t^4e^{5t}-e^{t}\cos\sqrt{7}t\}$

$$\mathcal{L}\{t^4e^{5t}-e^{t}\cos\sqrt{7}t\} = \frac{24}{(s-5)^5} - \frac{s-1}{(s-1)^2+7}$$

20). $\mathcal{L}\{e^{-2t}\cos\sqrt{3}t - t^2e^{-2t}\}$

$$\mathcal{L}\{e^{-2t}\cos\sqrt{3}t - t^2e^{-2t}\} = \frac{s+2}{(s+2)^2+3} - \frac{2}{(s+2)^3}$$

1). $t^2 + e^t \operatorname{sen} 2t$

$\mathcal{L}\{t^2\} + \mathcal{L}\{e^t \operatorname{sen} 2t\}$

$\frac{2!}{s^3} + \frac{2}{(s-1)^2+4}$

2) $3t^2 - e^{2t}$

$\mathcal{L}\{3t^2\} - \mathcal{L}\{e^{2t}\}$

$3\mathcal{L}\{t^2\} - \mathcal{L}\{e^{2t}\}$

$3\,\frac{2!}{s^3} - \frac{1}{s-2}$

3) $e^{-t}\cos 3t + e^{6t} - 1$

$\mathcal{L}\{e^{-t}\cos 3t\} + \mathcal{L}\{e^{6t} - 1\}$

$\frac{s+1}{(s+1)^2+9} + \mathcal{L}\{e^{6t}\} - \mathcal{L}\{1\}$

$\frac{s+1}{(s+1)^2+9} + \frac{1}{s-6} - \frac{1}{s}$

5) $2t^2 e^{-t} - t + \cos 4t$

$\mathcal{L}\{2t^2 e^{-t}\} - \mathcal{L}\{t\} + \mathcal{L}\{\cos 4t\}$

$2 \cdot \frac{2!}{(s+1)^3} - \frac{1}{s^2} + \frac{s}{s^2+16}$

7) $(t-1)^4$

$t^4 - 4t^3 + 6t^2 - 4t + 1$

$\mathcal{L}\{t^4\} - 4\mathcal{L}\{t^3\} + 6\mathcal{L}\{t^2\} - 4\mathcal{L}\{t\} + \mathcal{L}\{1\}$

$\frac{4!}{s^5} - 4\frac{3!}{s^4} + 6\frac{2!}{s^3} - 4\frac{1}{s^2} + \frac{1}{s}$

$\frac{24}{s^5} - \frac{24}{s^4} + \frac{12}{s^3} - \frac{4}{s^2} + \frac{1}{s}$

9) $e^{-t} t \operatorname{sen} 2t$

$\mathcal{L}\{e^{-t} t \operatorname{sen} 2t\}$

11) $\cosh Bt$ $\qquad \cosh x = \frac{e^{x}+e^{-x}}{2}$

$$\mathcal{L}\left\{\frac{e^{at}+e^{-at}}{2}\right\}$$

$$\frac{1}{2}\mathcal{L}\left\{e^{at}+e^{-at}\right\}$$

$$\frac{1}{2}\left(\mathcal{L}\left\{e^{at}\right\}+\mathcal{L}\left\{e^{-at}\right\}\right)$$

$$\frac{1}{2}\left(\frac{1}{s-a}+\frac{1}{s+a}\right)=\frac{1}{2}\left(\frac{s+\cancel{a}+s-\cancel{a}}{s^2-a^2}\right)$$

$$\frac{1}{\cancel{2}}\left(\frac{\cancel{2}s}{s^2-a^2}\right)=\frac{s}{s^2-a^2}$$

13) $\operatorname{sen}^2 t$ $\qquad \operatorname{sen}^2\theta=\frac{1}{2}-\frac{1}{2}\cos 2\theta$

$$\mathcal{L}\left\{\frac{1}{2}-\frac{1}{2}\cos 2t\right\}$$

$$\mathcal{L}\left\{\frac{1}{2}(1-\cos 2t\right\}=\frac{1}{2}\mathcal{L}\left\{1-\cos 2t\right\}$$

$$\frac{1}{2}\left(\frac{1}{s}-\frac{s}{s^2+4}\right)=\frac{1}{2s}-\frac{s}{2s^2+8}$$

"$\cos^2 t = \frac{1}{2} + \frac{1}{2}\cos 2t$"

15) $\cos^3 t = \mathcal{L}\left\{\cos t\,(\cos^2 t)\right\}$

$\mathcal{L}\left\{\cos t\left(\frac{1}{2} + \frac{1}{2}\cos 2t\right)\right\} = \frac{1}{2}\mathcal{L}\left\{\cos t\,(1 + \cos 2t)\right\}$

$\frac{1}{2}\mathcal{L}\left\{\cos t + \cos t \cos 2t\right\}$

"$\cos A \cos B = \frac{1}{2}\cos(A+B) + \cos(A-B)$"

$\frac{1}{2}\mathcal{L}\left\{\cos t + \frac{1}{2}(\cos 3t + \cos(-t))\right\} =$

$\frac{1}{2}\mathcal{L}\left\{\cos t + \frac{1}{2}\cos 3t + \frac{1}{2}\cos t\right\} =$

$\frac{1}{2}\mathcal{L}\left\{\frac{3}{2}\cos t + \frac{1}{2}\cos 3t\right\} = \frac{1}{2}\mathcal{L}\left\{\frac{1}{2}(3\cos t + \cos 3t)\right\} =$

$\frac{1}{4}\mathcal{L}\left\{3\cos t + \cos 3t\right\} = \frac{1}{4}\left\{\frac{3s}{s^2+1} + \frac{s}{s^2+9}\right\} =$

$\frac{1}{\cancel{4}}\left(\frac{\cancel{4}s^3 + \cancel{28}s}{(s^2+1)(s^2+9)}\right) = \frac{s^3 + 7s}{(s^2+1)(s^2+9)}$

21). $\mathcal{L}\{\cos bt\} = \frac{s}{(s^2+b^2)}$ Translación.

$$\mathcal{L}\{e^{at}\cos bt\} = \mathcal{L}\{\cos bt\}\Big|_{s \to s-a}$$

$$\frac{s-a}{(s-a)^2+b^2}$$

25)

a) $\mathcal{L}\{t\cos bt\}$ B) $\mathcal{L}\{t^2\cos bt\}$

a) $\mathcal{L}\{t\cos bt\}$ $(-1)^n \frac{d^n F}{ds^n}(s)$

$$\cos bt \xrightarrow{\mathcal{L}} \frac{s}{s^2+b^2} \xrightarrow{\frac{d}{ds}} \frac{1\cdot(s^2+b^2)-2s\cdot s}{(s^2+b^2)^2} =$$

$$\frac{\cancel{s^2}+b^2-\cancel{2}s^2}{(s^2+b^2)^2} = (-1)^1\frac{-s^2+b^2}{(s^2+b^2)^2} = \frac{s^2-b^2}{(s^2+b^2)^2}$$

$v'' + 3v = -9 \rightarrow 3A = -9$

b) $\mathcal{L}\{t^2 \cos bt\}$

$$\cos bt \xrightarrow{\mathcal{L}} \frac{s}{s^2+b^2} \xrightarrow{\frac{d}{ds}} \frac{\cancel{s^2}+b^2-\cancel{2}s^2}{(s^2+b^2)^2} =$$

$$\frac{-s^2+b^2}{(s^2+b^2)^2} \xrightarrow{\frac{d}{ds}} \frac{-2s^2(s^2+b^2)^2 - 4s(s^2+b^2)(-s^2+b^2)}{((s^2+b^2)^2)^2} =$$

$$\frac{-2s(s^2+b^2)(s^2+b^2) - 4s(s^2+b^2)(-s^2+b^2)}{(s^2+b^2)^4} =$$

$$\frac{-2s(s^2+b^2)(s^2+b^2+2(-s^2+b^2)}{(s^2+b^2)^4} =$$

$$\frac{-2s\cancel{(s^2+b^2)}(-s^2+3b^2)}{(s^2+b^2)^{\cancel{4}}} = \frac{-2s(-s^2+3b^2)}{(s^2+b^2)^3} =$$

$$(-1)^2 \frac{2s^3-6sb^2}{(s^2+b^2)^3} = \frac{2s^3-6sb^2}{(s^2+b^2)^3}$$