Algorytmy Grafowe - najktrotsza sciezka w grafie

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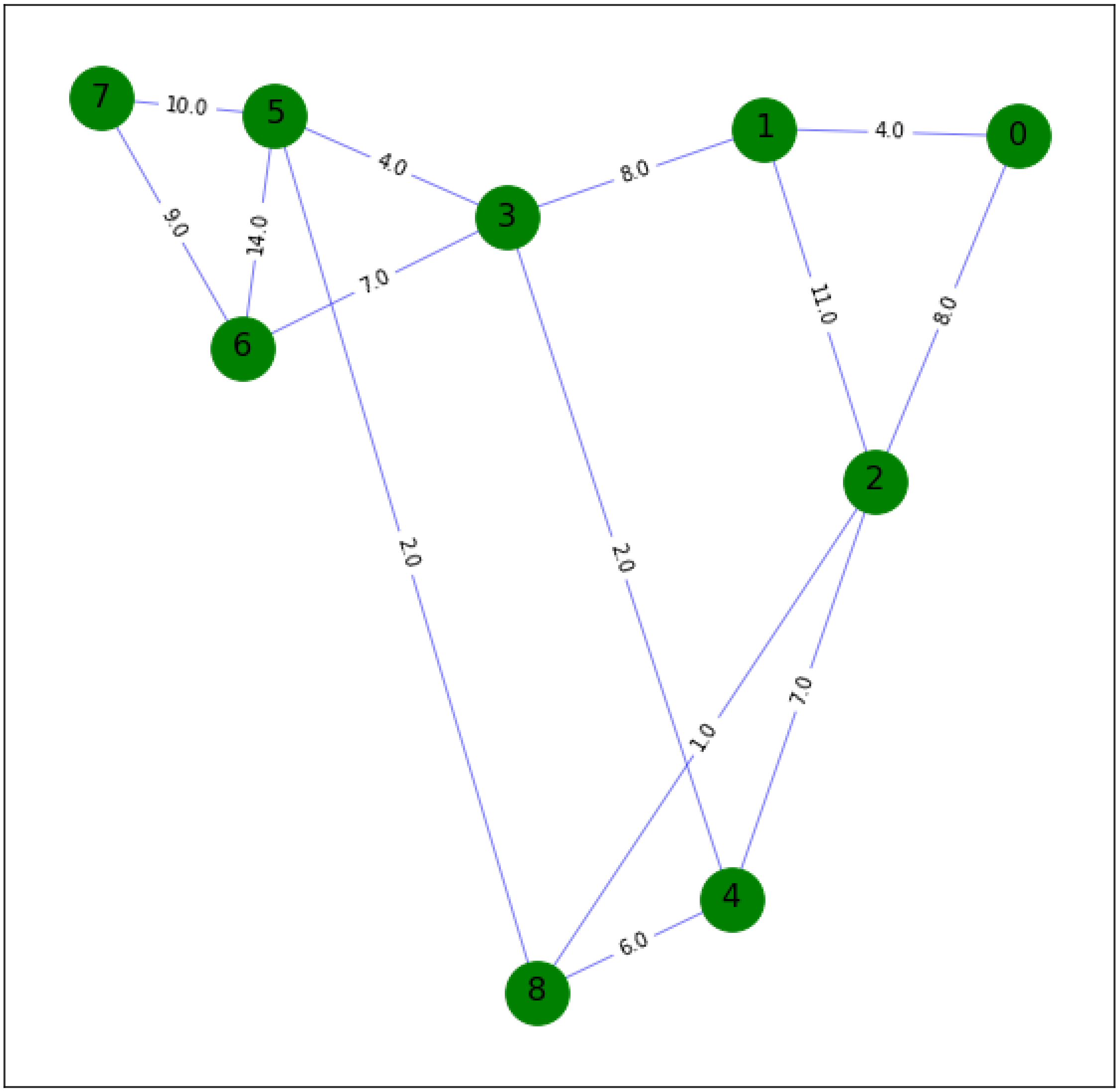
In [27]:
```
import networkx as nx
import numpy as np
import matplotlib.pyplot as plt
```

In [28]:
```
weights = [
    (0, 1, 4),
    (0, 7, 8),
    (1, 7, 11),
    (2, 1, 8),
    (2, 8, 2),
    (2, 5, 4),
    (2, 3, 7),
    (3, 4, 9),
    (3, 5, 14),
    (4, 5, 10),
    (5, 6, 2),
    (6, 8, 6),
    (6, 7, 1),
    (7, 8, 7),
]

G = nx.Graph()
G.add_weighted_edges_from(weights)
M = nx.to_numpy_array(G)
G = nx.Graph(M)

fig = plt.figure(figsize=(10, 10))

pos = nx.spring_layout(G)

nx.draw_networkx_nodes(G, pos, nodelist=[i for i in range(9)], node_color='g', node_size
nx.draw_networkx_edges(G, pos, width=1,alpha=0.5,edge_color='b')

nx.draw_networkx_edge_labels(G, pos, font_size=10, edge_labels = nx.get_edge_attributes(
nx.draw_networkx_labels(G, pos, font_size=16)

plt.title("M")
plt.show()
```

M

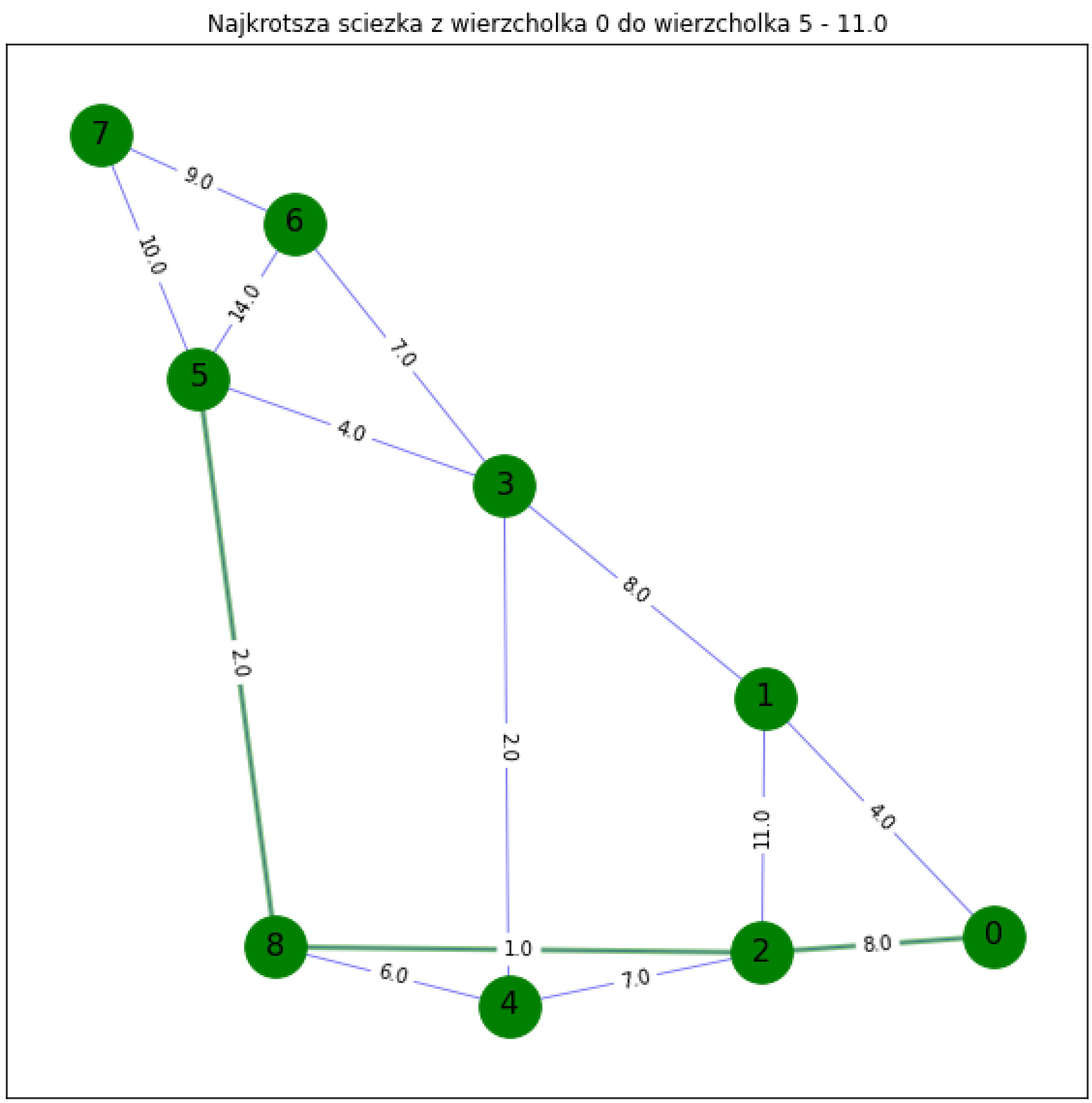


In [29]:
```
def bellmanFordAlgorithm(M, start):
    n = len(M)
    prev = [None for _ in range(n)]
    d = [np.inf for _ in range(n)]
    d[start] = 0

    for i in range(n):
        for j in range(n):
            if d[j] > d[i] + M[i][j] and M[i][j] > 0:
                d[j] = d[i] + M[i][j]
                prev[j] = i

    return d, prev
```

In [30]:
```
def getPath(prev, start, end):
    i = start
    j = end
    res = []
    while i != j:
        res.append((prev[j], j))
        j = prev[j]

    return res
```

In [31]:
```
M = nx.to_numpy_array(G)

start = 0
end = 5
```

```
distances, prev = bellmanFordAlgorithm(M, start)

edgeList = getPath(prev, start, end)

G = nx.Graph(M)

fig = plt.figure(figsize=(10, 10))

pos = nx.spring_layout(G)


nx.draw_networkx_nodes(G, pos, nodelist=[i for i in range(9)], node_color='g', node_size
nx.draw_networkx_edges(G, pos, width=1,alpha=0.5,edge_color='b')
nx.draw_networkx_edges(G, pos, edgelist=edgeList, width=3, alpha=0.5,edge_color='g')

nx.draw_networkx_edge_labels(G, pos, font_size=10, edge_labels = nx.get_edge_attributes(
nx.draw_networkx_labels(G, pos, font_size=16)

plt.title(f"Najkrotsza sciezka z wierzcholka {start} do wierzcholka {end} - {distances[e
plt.show()
```



Algorytm A*

In [32]:
```
# Stowrzylem kolejke priorytetowa ktorej priorytet to bedzie odleglosc wierzcholka od ce
class PriorityQueue:
    def __init__(self):
```

```python
        self.queue = []

    def enqueue(self, val, priority):
        self.queue.append((val, priority))
        self.queue.sort(key=lambda x: x[1])

    def dequeue(self):
        return self.queue.pop(0)[0]

    def is_empty(self):
        return len(self.queue) == 0
```

In [33]:

```python
def heuristic(P1, P2) -> float:
    return ((P1[0] - P2[0])**2 + (P1[1] - P2[1]))**(1/2)


def a_star_search(M, start, goal, points):
    n = len(M) # Ilosc wierzcholkow
    # Tworze kolejke priorytetowa, ktora jako priorytet bierze sume funkcje heurystyki,
    queue = PriorityQueue()
    queue.enqueue(start, 0)

    # Lista poprzednich wierzcholkow
    prev = [None for _ in range(n)]
    # Lista najkrotszych dystansow
    distances = [np.inf for _ in range(n)]

    distances[start] = 0
    prev[start] = start

    while not queue.is_empty():
        # Pobieram z kolejki kolejny wierzcholek
        current = queue.dequeue()

        # Jezeli wierzcholek jest rozwiazaniem to koncze petle
        if current == goal:
            break

        # Iteruje po mozliwych przejsciach bierzacego wierzcholka
        for i in range(n):
            if M[current][i] > 0:
                # Obliczam koszt przejscia do kolejnego wierzcholka
                new_cost = distances[current] + M[current][i]
                if new_cost < distances[i]:
                    distances[i] = new_cost
                    # Ustalam priorytet jako koszt przejscia + wartosc funkcji heurystyk
                    priority  = new_cost + heuristic(points[i], points[goal])
                    # Dodaje wierzcholek do kolejki
                    queue.enqueue(i, priority)
                    prev[i] = current

    return prev, distances
```

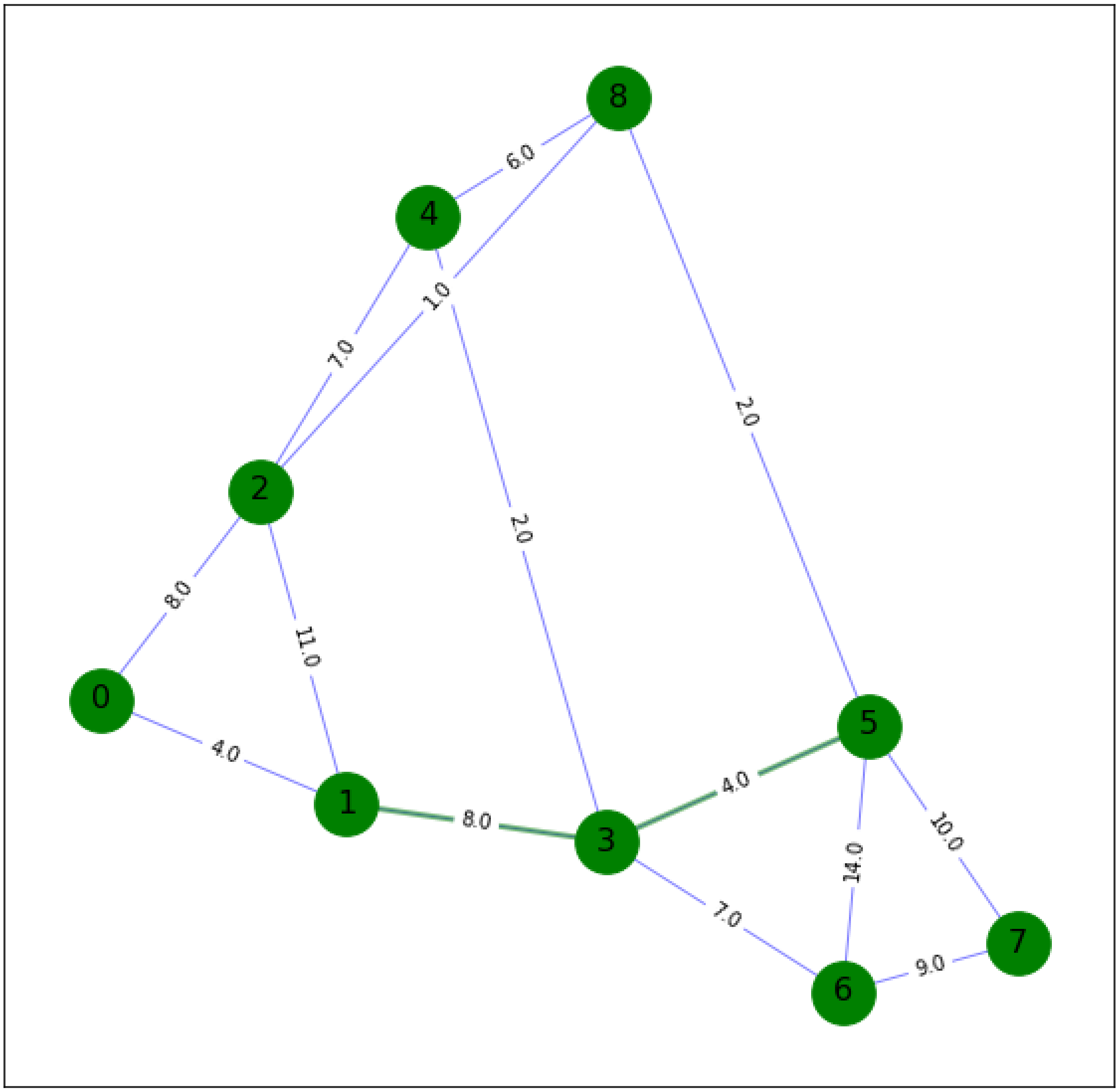
In [34]:

```python
start = 1
end = 5

fig = plt.figure(figsize=(10, 10))

pos = nx.spring_layout(G)

prev, distance = a_star_search(M, start, end, list(pos.values()))

edgeList = getPath(prev, start, end)

nx.draw_networkx_nodes(G, pos, nodelist=[i for i in range(9)], node_color='g', node_size
```

```
nx.draw_networkx_edges(G, pos, width=1,alpha=0.5,edge_color='b')
nx.draw_networkx_edges(G, pos, edgelist=edgeList, width=3, alpha=0.5,edge_color='g')

nx.draw_networkx_edge_labels(G, pos, font_size=10, edge_labels = nx.get_edge_attributes(
nx.draw_networkx_labels(G, pos, font_size=16)

plt.title(f"Najkrotsza sciezka z wierzcholka {start} do wierzcholka {end} - {distances[e
plt.show()
```

```
/var/folders/sn/9k1r60m16tl0pqjy83j24x940000gn/T/ipykernel_7849/131781978.py:2: RuntimeW
arning: invalid value encountered in double_scalars
  return ((P1[0] - P2[0])**2 + (P1[1] - P2[1]))**(1/2)
```

Najkrotsza sciezka z wierzcholka 1 do wierzcholka 5 - 11.0



Porownanie czasow wykonywania algorytmow

In [35]:
```
import timeit


def bellmanFordSearchTimeIt():
    _, prev = bellmanFordAlgorithm(M, start)
    edgeList = getPath(prev, start, end)
    return edgeList


def aStarSearchTimeIt():
    prev, _ = a_star_search(M, 0, 5, list(pos.values()))
    edgeList = getPath(prev, 0, 5)
    return edgeList
```

```python
# Czas obliczen znalezienia najkrotszej sciezki z jednego punktu do kolejnego
bellmanFordTime = timeit.timeit(bellmanFordSearchTimeIt, number=1000) / 1000
aStarSearchTime = timeit.timeit(aStarSearchTimeIt, number=1000) / 1000

print("Czas wykonywania algorytmu Bellmana Fords", bellmanFordTime)
print("Czas wykonywania algorytmu A star", aStarSearchTime)
```

```
Czas wykonywania algorytmu Bellmana Fords 4.143012500003351e-05
Czas wykonywania algorytmu A star 3.2862208000096874e-05
```

Dodatkowo stworzylem wizualizacje dzialania algorytmu i umiescilem ja na swoim githubie https://djmmatracki.github.io/A_star_algorithm/.

Zadanie 2

Z punktu dzialania algorytmu waznymi wlasnosciami grafu moze byc ilosc krawedzi. Algorytm A - star za priorytet berze sobie odleglosc do koncowego wierzcholka, wiec jezeli bedzie duzo wierzcholkow blisko koncowego, ale nie beda one polaczone z nim moze to zmylic algorytm. Algorytm A-star znajduje sciezke z jednego wierzcholka do celu w przeciwienstwie do algorytmu Dijkstry, ktory znajduje kazde polaczenie.

Zadanie 3

$|V|$ - ilosc wierzcholkow $|E|$ - ilosc krawedzi

Zlozonosc obliczeniowa Algorytmu Bellmana-Forda to $O(|V| \cdot |E|)$

Pesymistyczna zlozonosc obliczeniowa algorytmu A star $O(|E|)$
Pesymistyczna zlozonosc pamieciowa algorytmu A star wynosi $O(|V|)$

Optymistyczna zlozonosc obliczeniowa to $O(1)$, wtedy pierwsza proba znalezienia celu konczy sie powodzeniem.