ESCI654 Project 2

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Problem

Setting and Motivation

I have a love for kimchi, a Korean dish made of fermented vegetables. Among those vegetables is cabbage, which releases the compound sulforaphane—one of the chemicals responsible for kimchi's pungent odor. When I eat the kimchi, I cannot help but to allow its odor to fill the house.

Justification

My girlfriend hates the smell of kimchi, but she is able to tolerate its stench to a certain degree. I would like to eat the kimchi before the odor reaches a concentration that would bother her.

Problem Statement

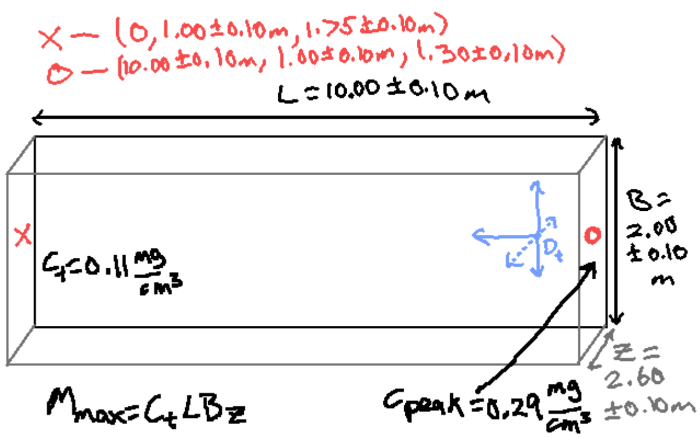

Figure 1: Positions of my girlfriend (X) and I (O) during the kimchi eating event.

My girlfriend and I live in a shipping container tiny house (we don't actually, we live in a real apartment in Portsmouth). I open a rather large fresh jar of kimchi to eat; my girlfriend is working at her standing desk and her head is at the X position in *Figure 1.* I don't want to bother her, and I want as much time as possible, so I position myself (and the jar of kimchi) at the O position in *Figure 1.* The jar releases sulforaphane with a constant center concentration of $C = 0.29\,\frac{mg}{cm^3}$. My girlfriend's tolerance is $C_t = 0.11 \pm 0.01\,\frac{mg}{cm^3}$. The doors and windows are closed because it's winter. How long do I have to eat the prized kimchi before the concentration at her location becomes too much?

Solution

There are two ways to solve this problem; one solution is more considerate than the other solution. The less considerate solution is to calculate the amount of time it would take the concentration at her position to reach the threshold concentration. That solution is less considerate because it does not take into account the fact that, once I close the jar of kimchi, there will still be sulforaphane in the room, and eventually the air will become fully mixed at a concentration greater than the threshold concentration. Therefore, the considerate solution is to figure out how much time I have to eat the kimchi until the mass of sulforaphane present in the air reaches a value such that, once the air mixes, the sulforaphane concentration becomes the threshold concentration.

One thing I need is the turbulent diffusivity of the stagnant air in our tiny house. I calculated the turbulent diffusivity, D_t, experimentally in a process outlined in the Appendix. I found that $D_t = 0.004\,\frac{m^2}{s} \pm 10\%$. I am assuming that the turbulent diffusivity of the room is isotropic and homogeneous.

The next step is to calculate the maximum amount of sulforaphane I can allow to be released into the room, M_{max}. The threshold concentration $C_t = 0.11 \pm 0.01\,\frac{mg}{cm^3}$ and the volume of the room V can be used to figure out what M_{max} equals.

$$M_{max} = C_t V$$

$$V = LBz$$

where L is the length of the room, B is the width of the room, and z is the room's height.

$$V = (10.00 \pm 0.10\,m)(2.00 \pm 0.10\,m)(2.60 \pm 0.10\,m)$$

I will assume that the uncertainties are random and uncorrelated, so I will add the relative uncertainties in quadrature.

$$V = 52\,m^3 \pm \sqrt{(\frac{0.10}{10.00})^2 + (\frac{0.10}{2.00})^2 + (\frac{0.10}{2.60})^2}$$

$$V = 52\,m^3 \pm 6.39\%$$

$$M_{max} = (0.11 \pm 0.01\,\frac{mg}{cm^3})(\frac{100^3\,cm^3}{1\,m^3})(52\,m^3 \pm 6.39\%)$$

$$M_{max} = 5720000\,mg \pm \sqrt{(\frac{0.01}{0.11})^2 + 0.0639^2}$$

$$M_{max} = 5720000\,mg \pm 11.11\%$$

The next step is to determine the vertical mixing time scale $\tau_{mix,z}$, the horizontal mixing time scale $\tau_{mix,y}$, and the longitudinal mixing time scale $\tau_{mix,x}$.

$$\tau_{mix,z} = \frac{(\frac{z}{2})^2}{2D}$$

because the sulforaphane is released halfway between the floor and ceiling,

$$\tau_{mix,y} = \frac{(\frac{B}{2})^2}{2D}$$

because the sulforaphane is released halfway between the north and south walls,

$$\tau_{mix,x} = \frac{L^2}{D}$$

because the sulforaphane is released at the eastern edge of the room.

$$\tau_{mix,z} = \frac{(\frac{2.60 \pm 0.10\,m}{2})^2}{2(0.004\,\frac{m^2}{s} \pm 10\%)}$$

$$\tau_{mix,z} = \frac{(1.30 \pm 0.05\,m)^2}{0.008\,\frac{m^2}{s} \pm 10\%}$$

$$\tau_{mix,z} = 211.25\,s \pm \sqrt{(2 * \frac{0.05}{1.30})^2 + 0.010^2}$$

$$\tau_{mix,z} = 211.25\,s \pm 7.76\%$$

$$\tau_{mix,y} = \frac{(\frac{2.00 \pm 0.10\,m}{2})^2}{2(0.004\,\frac{m^2}{s} \pm 10\%)}$$

$$\tau_{mix,y} = \frac{(1.00 \pm 0.05\,m)^2}{0.008\,\frac{m^2}{s} \pm 10\%}$$

$$\tau_{mix,y} = 125\,s \pm \sqrt{(2 * \frac{0.05}{1.00})^2 + 0.010^2}$$

$$\tau_{mix,y} = 125\,s \pm 10.05\%$$

$$\tau_{mix,x} = \frac{(10.00 \pm 0.10\,m)^2}{0.004\,\frac{m^2}{s} \pm 10\%}$$

$$\tau_{mix,x} = 25000\,s \pm \sqrt{(2 * \frac{0.10}{10.00})^2 + 0.010^2}$$

$$\tau_{mix,x} = 25000\,s \pm 2.2\%$$

Therefore I know that the sulforaphane will fully mix vertically and horizontally well before it ever mixes longitudinally. At time $t = 211.25\,s \pm 25.98\%$, the sulforaphane will be fully mixed vertically and horizontally and the rest of the problem becomes one-dimensional.

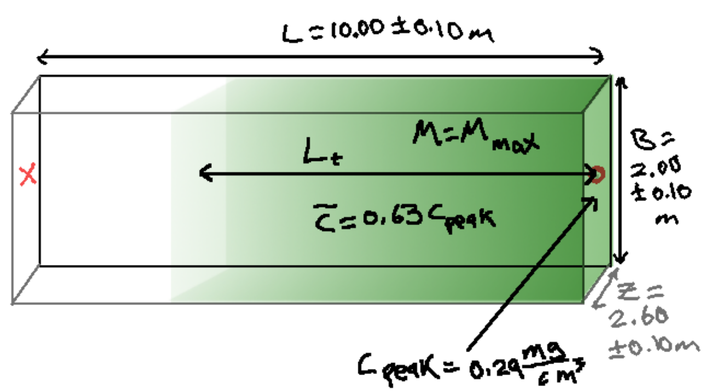

Figure 2: Threshold length of the sulforaphane cloud after vertical and horizontal mixing.

The next step is to determine what the threshold length of the cloud, $L_t = 2\sigma$, where σ is the cloud length scale, will be where the average concentration $\bar{C}$ is such that there is M_{max} amount of sulforaphane present. At 2σ, the average concentration within the cloud is always

$$\bar{C} = 0.63C = 0.63(0.29\,\frac{mg}{cm^3}) = 0.1827\,\frac{mg}{cm^3}$$

Since the air is fully mixed horizontally and vertically, we can calculate what threshold volume, V_t is necessary for the average concentration to produce a mass of M_{max}, and we can use the height and width of the room to do so.

$$M_{max} = \bar{C}V_t$$

$$V_t = \frac{M_{max}}{\bar{C}}$$

$$V_t = L_t Bz$$

$$L_t Bz = \frac{M_{max}}{\bar{C}}$$

$$L_t = \frac{M_{max}}{Bz\bar{C}}$$

$$L_t = \frac{5720000\, mg \pm 11.11\%}{(2.00 \pm 0.10\, m)(2.60 \pm 0.10\, m)(0.l827\, \frac{mg}{cm^3})(\frac{100^3\, cm^3}{1\, m^3})}$$

$$L_t = 6.031\, m \pm \sqrt{(\frac{0.10}{2.00})^2 + (\frac{0.10}{2.60})^2 + 0.1111^2}$$

$$L_t = 6.031\, m \pm 12.78\%$$

Once the cloud is that long, enough sulforaphane will have been emitted into the room such that even if I stop eating kimchi, once the air fully mixes my girlfriend's tolerance level will have been reached.

The final step is to figure out how long I have, t, until the cloud becomes that long.

$$L_t = 2\sigma = 2\sqrt{2Dt}$$

$$L_t^2 = 4(2Dt)$$

$$L_t^2 = 8Dt$$

$$\frac{L_t^2}{8D} = t$$

$$\frac{(6.031\, m \pm 12.78\%)^2}{8(0.004\, \frac{m^2}{s}) \pm 10\%} = t$$

$$t = 1136.66\, s \pm \sqrt{(2 * 0.1278)^2 + 0.010^2}$$

$$t = 1136.66\, s \pm 25.58\%$$

$$t = 1136.66 \pm 290.76\, s$$

$$t \approx 1100 \pm 300\, s$$

I have about 1100 seconds to eat from my prized kimchi before the amount of sulforaphane released will bother my girlfriend. However, if I want to be safe, I should err on the side of 800 seconds (in case of error).

Model Assumptions

Turbulent diffusion is isotropic and homogeneous

I am assuming that the vertical, horizontal, and longitudinal diffusivity are the same. If it were not the same, then the amount of sulforaphane released by the jar of kimchi would change; it would either be more or less than I calculated. If they were not the same then I would also expect the threshold length, L_t, to be different. Additionally, I am assuming that the jar of kimchi is releasing sulforaphane radially (i.e., there is negligible upward motion caused by releasing from the opening of a jar, rather than a point-source) such that the concentration is the same at the center across time and space.

Indoor air is stagnant

I am assuming that the kimchi eating event occurs at a time where there is not much air motion; it is occurring in what is essentially a sealed room without much air current. If this were not true then I would expect to see greater turbulence which would increase the diffusion rate of the sulforaphane. There also may be advection if there is an air current going in a clear and consistent direction (such as from window to window).

Prized kimchi acts as a constant source of sulforaphane

I am assuming that the prized kimchi releases sulforaphane at a constant rate and there are no sources or sinks other than the kimchi. A sink that could possibly exist is sorption of sulforaphane to furniture and clothing. If there were sorption occurring then I would have more time to eat my kimchi. However, once the air is clear there may be desorption that would release sulforaphane at a later time, possibly bothering my girlfriend. If the kimchi did not release sulforaphane at a constant rate then I would need to consider a first-order decay function. It would be first-order probably because the amount of sulforaphane released would be dependent on the amount left in the jar, which would decrease with time. Additionally, in reality the kimchi would probably release a puff of sulforaphane when it is first opened and then a first-order decaying amount of sulforaphane thereafter. I am assuming that the time scale for decay is larger than the amount of time in question (because it is a rather large jar of prized kimchi).

Coupled with this is the assumption that my eating of the prized kimchi is not fast enough to change its output. This is realistic because I want to savor the kimchi and enjoy it.

Appendix

Determining the turbulent diffusivity of stagnant air in a home.

Materials and Procedure

Figure 3: Materials used to determine turbulent diffusivity of a stagnant room

A jar of kimchi, a stool, a stopwatch (Casio F-91W), and a measuring tape were used to perform this experiment.

Uncertainty for the measuring tape was to the nearest centimeter because, although I have good judgment, I had to eyeball the measurement a little. Uncertainty for the stopwatch grew with distance because it required me to subjectively determine if I could smell the kimchi, which grew more difficult with distance.

The jar of kimchi was placed on a stool in the middle of a small kitchen, with other furniture moved to the edges to prevent interference. At three different distances (three trials), the jar of kimchi was opened and the stopwatch started. Once I was able to smell the kimchi at that distance I stopped the watch and recorded the time in seconds. Then the jar was sealed again, and the kitchen window opened to clear the air. After ten minutes the window was shut again and the air left to stagnate for five minutes. Then a new trial using the same procedure was performed at a different distance.

Results

Table 1: Touch time measurements for kimchi at various distances.

Trial	Length, L (m)	Time, τ_{touch} (s)
A	2.00 ± 0.01	153 ± 10
B	0.50 ± 0.01	6 ± 1
C	1.00 ± 0.01	35 ± 1

$$\tau_{touch} = \frac{L^2}{8D_t}$$

where D_t is the turbulent diffusivity of the room. Solving for D_t,

$$D_t = \frac{L^2}{8\tau_{touch}}$$

then plugging in Trial A,

$$D_t = \frac{(2.00 \pm 0.01\,m)^2}{8(153 \pm 10\,s)}$$

Uncertainty is correlated in squared values, so the relative uncertainties are added together.

$$D_t = \frac{4.00\,m^2 \pm 1\%}{1224 \pm 80\,s}$$

$$D_t = 0.00327\,\frac{m^2}{s} \pm \sqrt{0.01^2 + (\frac{80}{1224})^2}$$

$$D_t = 0.00327\,\frac{m^2}{s} \pm 6.61\%$$

The same thing is repeated for Trial B and Trial C.

Table 2: Turbulent diffusivity for each trial from *Table 1*.

Trial	Turbulent Diffusivity, D_t ($\frac{m^2}{s}$)
A	$0.00327 \pm 6.61\%$
B	$0.00521 \pm 17.14\%$
C	$0.00357 \pm 3.49\%$

The average diffusivity, D_t is calculated by adding all of them and dividing by 3.

$$D_t = \frac{(0.00327\,\frac{m^2}{s} \pm 6.61\%) + (0.00521\,\frac{m^2}{s} \pm 17.14\%) + (0.00357\,\frac{m^2}{s} \pm 3.49\%)}{3}$$

$$D_t = \frac{(0.00327 \pm 0.000216\,\frac{m^2}{s}) + (0.00521 \pm 0.000893\,\frac{m^2}{s}) + (0.00357 \pm 0.000125\,\frac{m^2}{s})}{3}$$

$$D_t = \frac{0.01205 \pm 0.001234\,\frac{m^2}{s}}{3}$$

$$D_t = 0.00402 \pm 0.0004\,\frac{m^2}{s}$$

$$D_t \approx 0.004\,\frac{m^2}{s} \pm 10\%$$