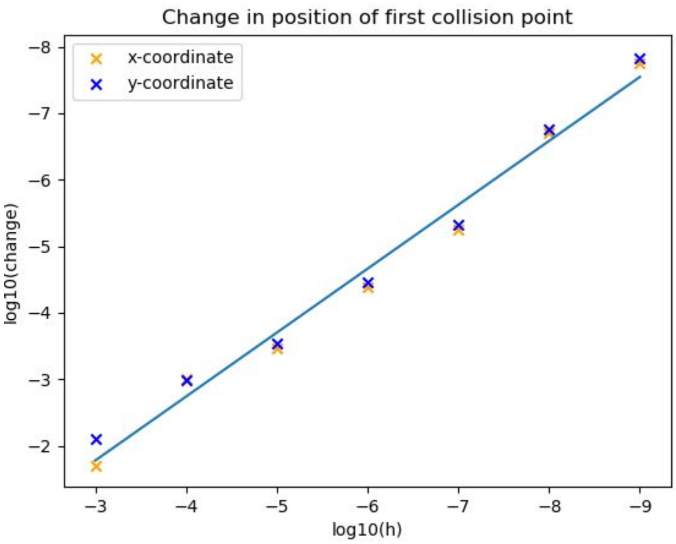
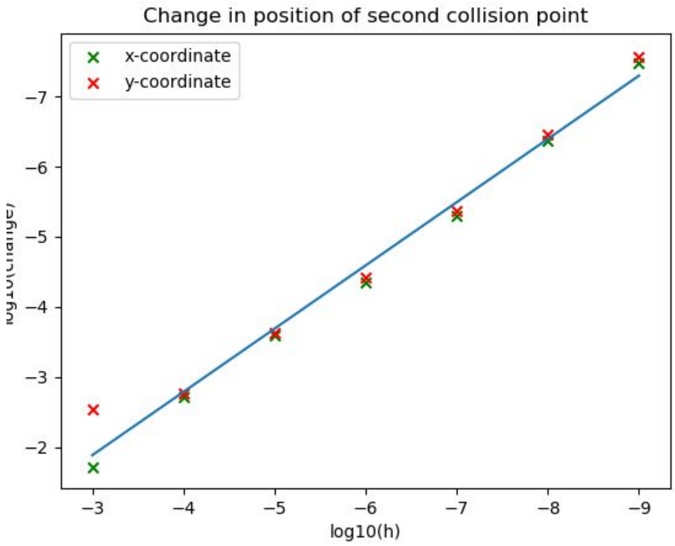
Approach

After the new velocities of each object have been calculated, continuous collision detection is performed between every pair of objects. This has the advantage over discrete collision detection in that it handles the case where two bodies would collide between timesteps, and gives a more accurate value for the collision positions since the exact time of the collision is calculated. Once all the collision points have been found, each pair of colliding bodies is fused into one and the other is removed from the simulation. To simplify the process, if a body collides with more than one other body, only the first collision found leads to a fusion. All the bodies which don't fuse have their position updated normally.

Results





The simulation was run on the setup of the 3 bodies as described in the assignment instructions. The positions of the first and second collisions were recorded for various time step sizes, from 10^{-2} to 10^{-9}. The two plots show the log (base 10) of the consecutive changes in the x- and y-coordinates of the collision points as the time step size, h, is decreased in magnitudes of 10. You can observe that the change between collision points is monotonically decreasing as h gets smaller, so the simulation converges to a solution and the convergence order for each collision point can be estimated from the data.

For the first collision, from straight line of best fit,
$log_{10}(y_h - y_{10h}) = 0.9618\, log_{10} h + 1.11$, therefore
$log_{10}(y_{h/10} - y_h) = 0.9618\, log_{10} h + 0.1482$, and
$y_{h/10} - y_h = 1.41 \cdot h^{0.9618}$.

Assuming the pattern holds for infinitely small values of h,
$$y - y_h = \sum_{n=0}^{\infty} 1.41(h \cdot 10^{-n})^{0.9618} = 1.41 \cdot h^{0.9618} \cdot \sum_{n=0}^{\infty} 10^{-0.9618} = 1.41 \cdot h^{0.9618} \cdot 1.12 = 1.58 \cdot h^{0.9618},$$
so the convergence order $p \approx 0.9618$.

For the second collision, the line of best fit is $log_{10}(y_h - y_{10h}) = 0.9029\, log_{10} h + 0.8279$ so the convergence order $p \approx 0.9029$ by the same deduction. One possible reason for the convergence order being lower for the second collision is that it depends heavily on the position of the first collision, therefore changes in the position of the first collision will cause a change in the position of the second collision.