

# Neural Policy Synthesis and Verification with Guarantees

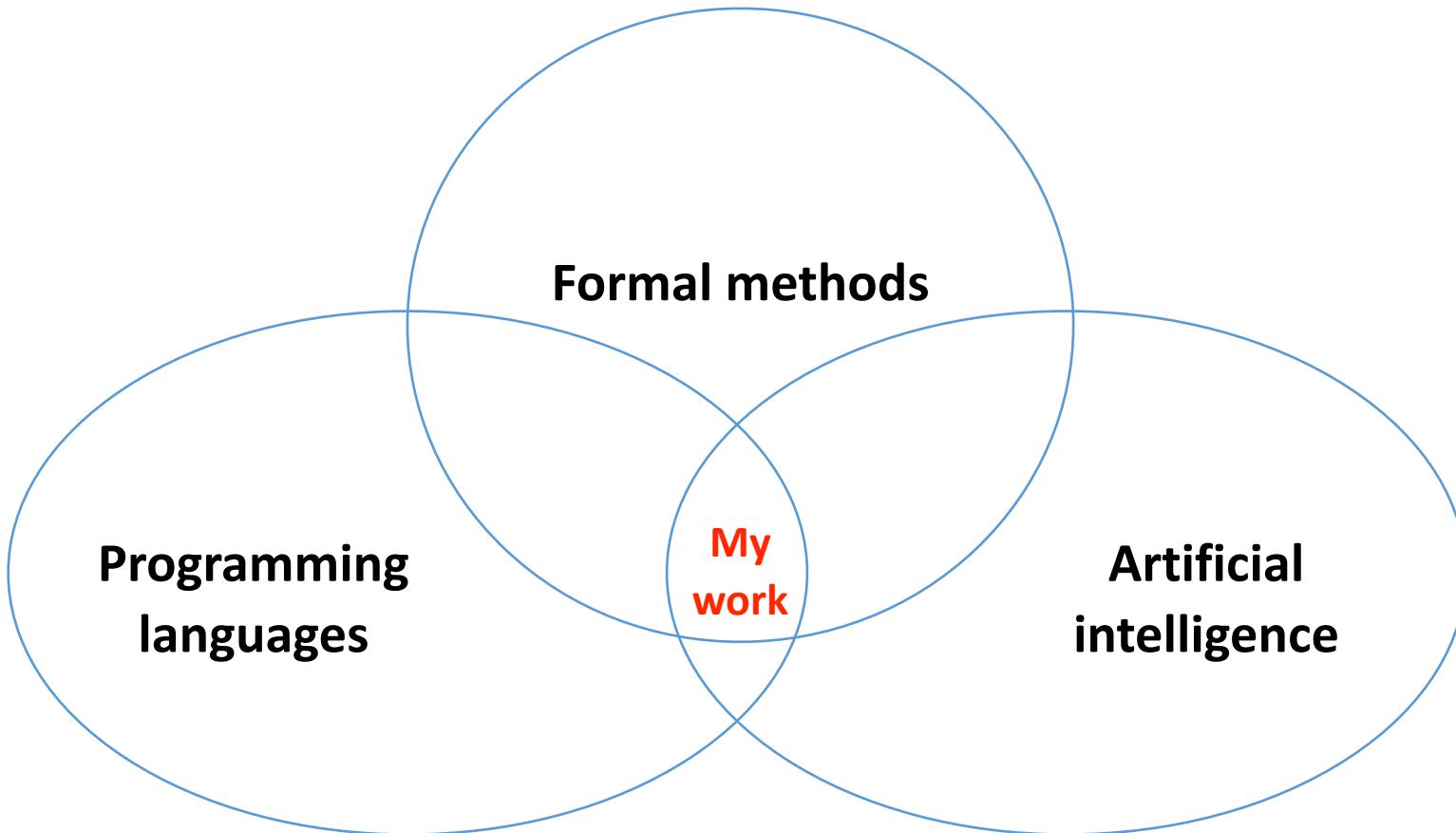
Djordje Zikelic

SCIS Research Cluster Seminar Series, August 2024

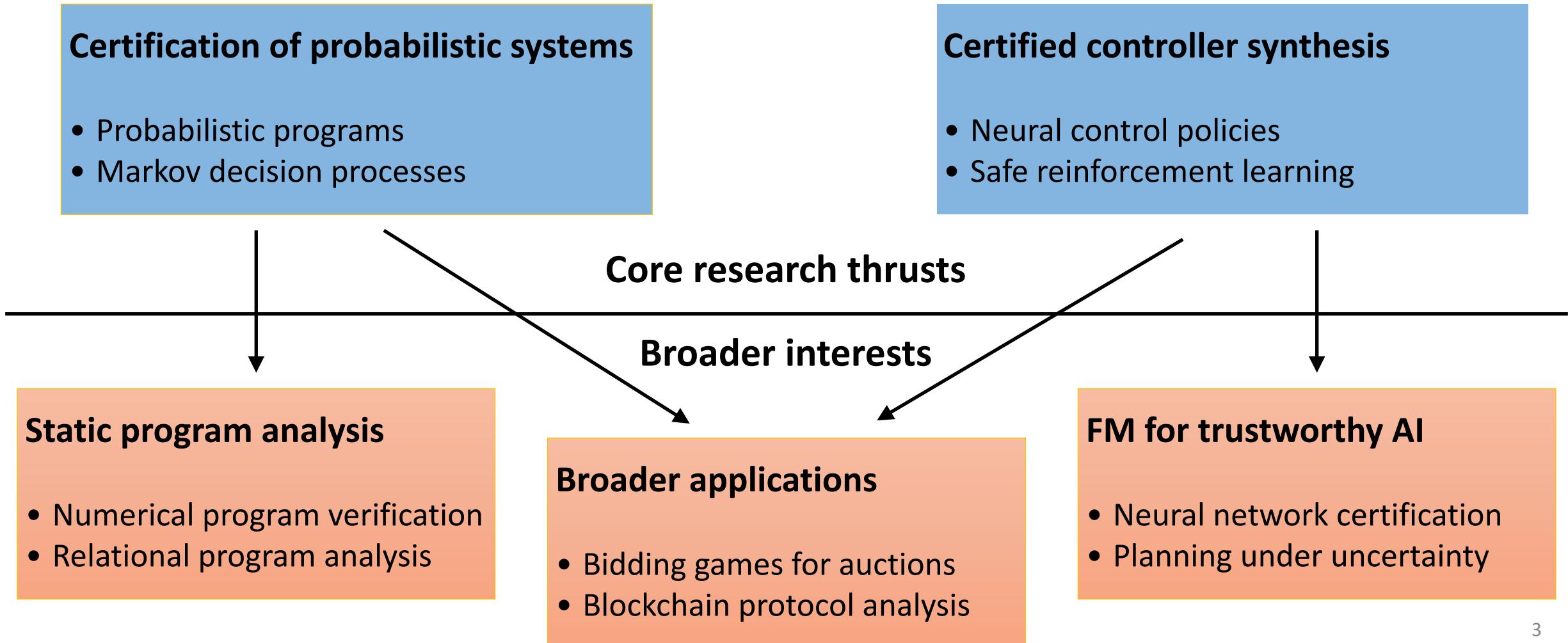


School of  
**Computing and  
Information Systems**

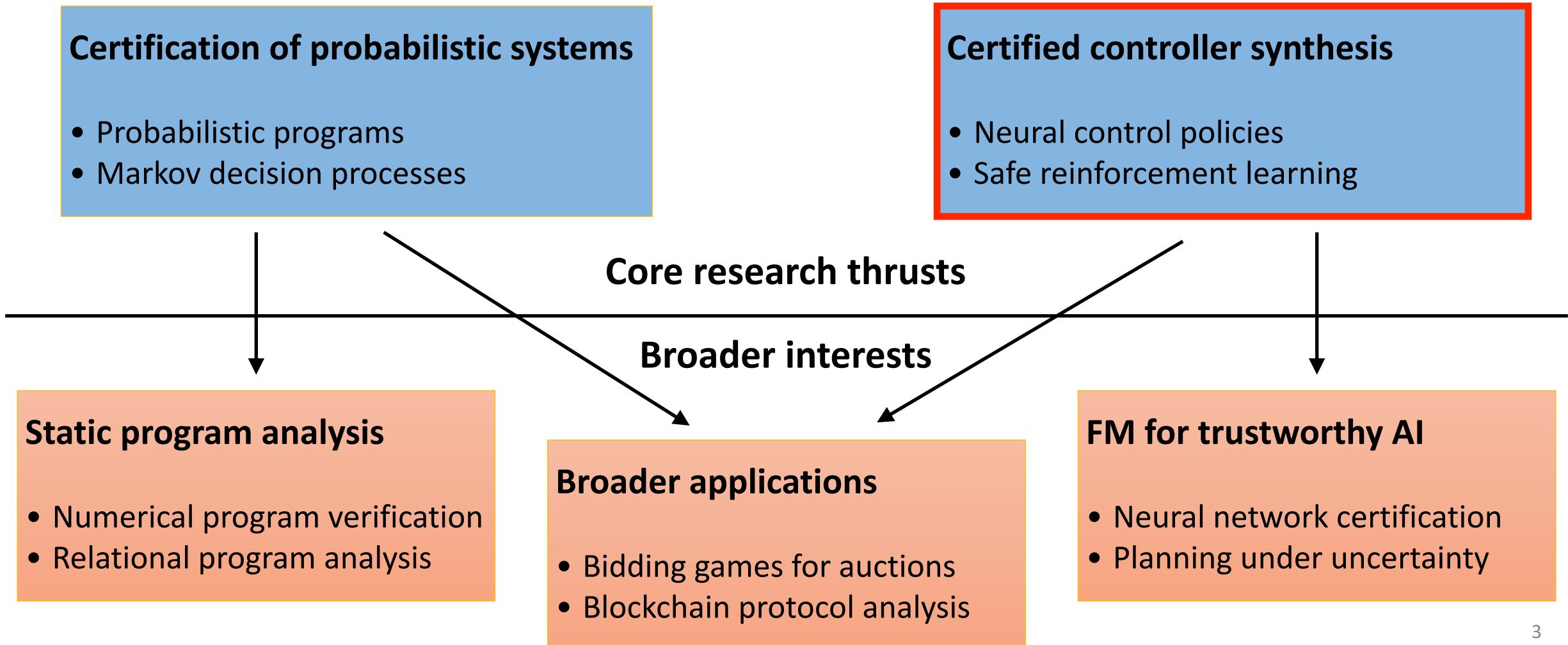
# Short research introduction



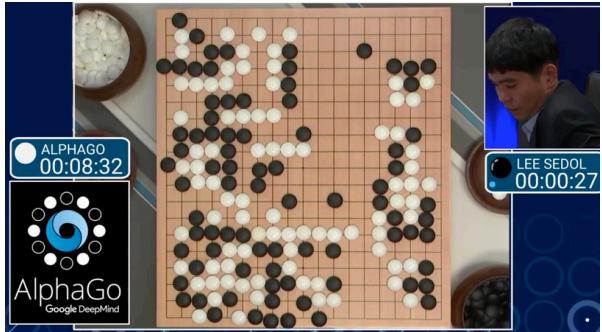
# Certification of software and AI systems in the presence of **uncertainty**



# Certification of software and AI systems in the presence of **uncertainty**



# Learning-enabled Policy (or Controller) Synthesis



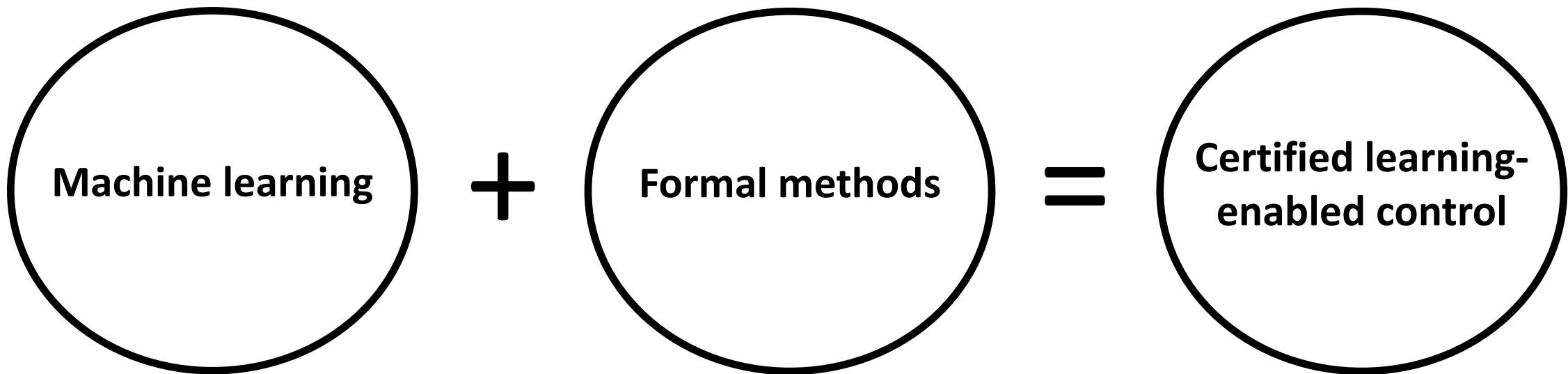
Not safety-critical



Safety-critical

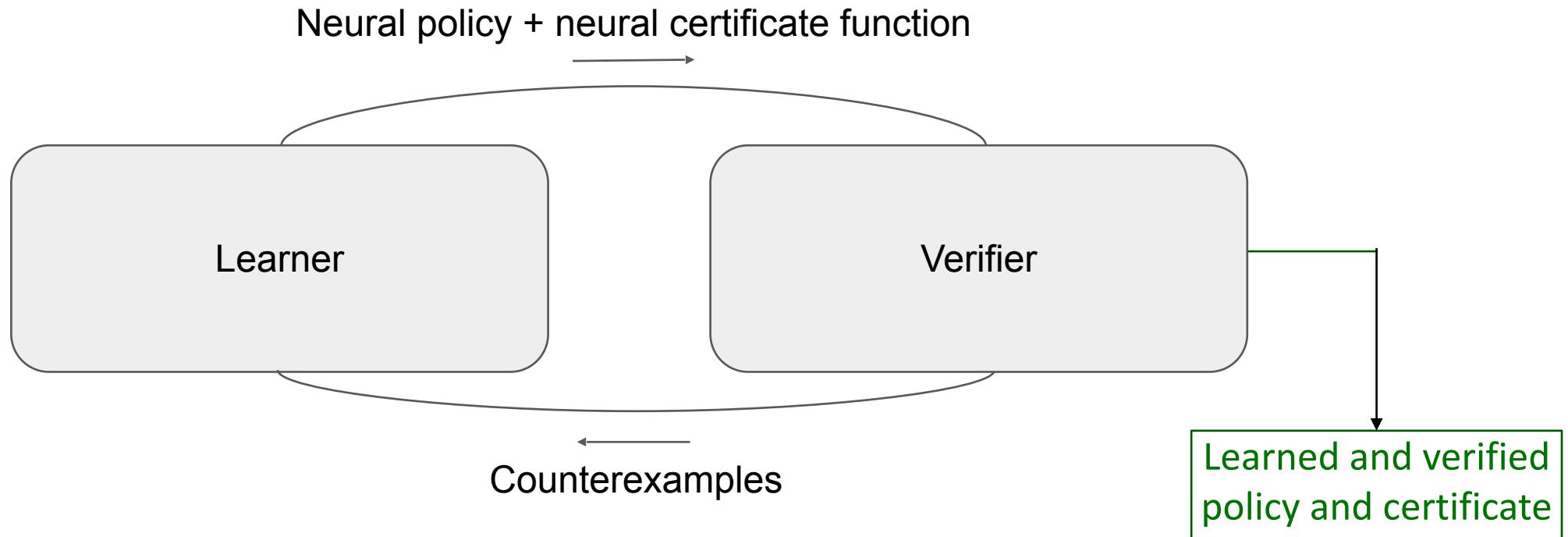
Safety-critical applications require **correctness guarantees**

# New paradigm for certified neural control



**Key idea: Learn neural policy + neural certificate of correctness**

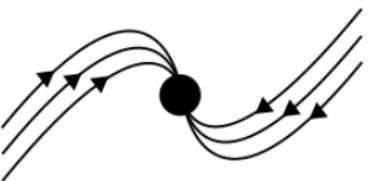
# Certified control through a learner-verifier loop



# Prior work: Deterministic systems\*

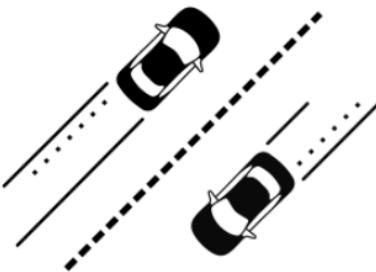
## Lyapunov Function

Certifies stability of a fixed point



## Barrier Function

Certifies invariance of a region



## Contraction Metric

Certifies ability to track arbitrary trajectories



Learning neural policies with classical control theory certificates  
(+ verification by reduction to SMT-solving)

\*Image taken from: Dawson, Gao, Fan. *Safe Control with Learned Certificates: A Survey of Neural Lyapunov, Barrier, and Contraction Methods for Robotics and Control*. IEEE Transactions on Robotics. 2023

Can we guarantee neural policy correctness  
in the presence of **environment uncertainty**?

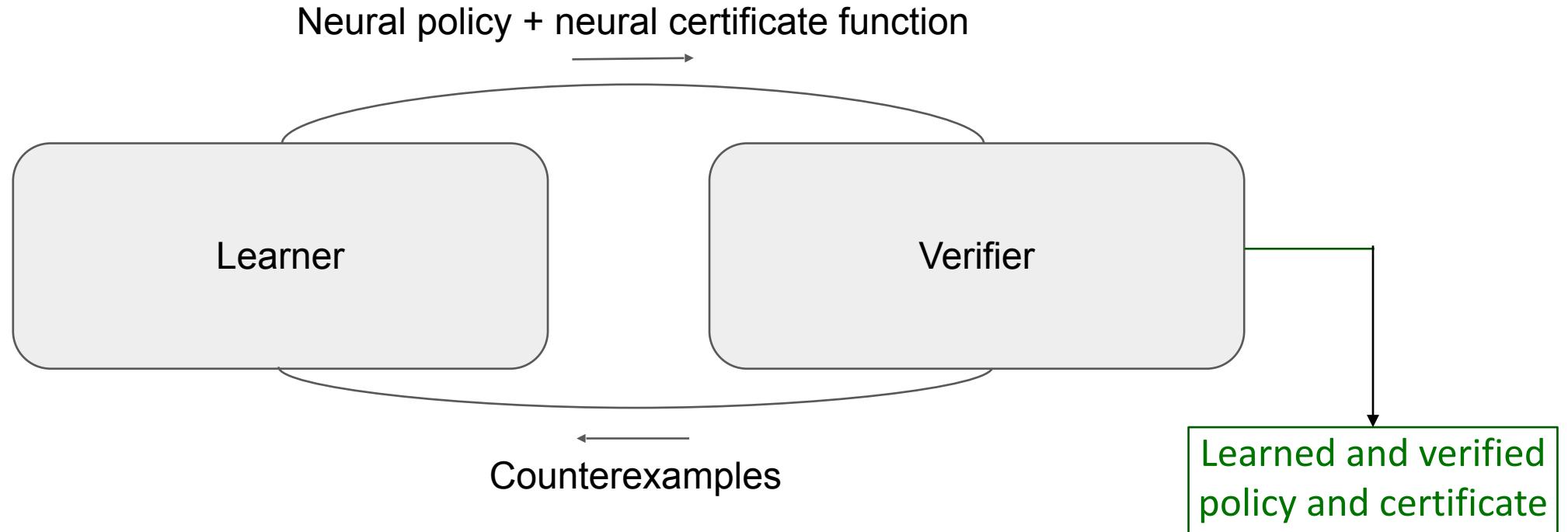


# A Learner-verifier Framework for Neural Stochastic Control and Verification

[NeurIPS23, AAAI23, TACAS23, ATVA23, AAAI22]

Joint with Mathias Lechner (MIT, Liquid AI), Krishnendu Chatterjee (ISTA), Thomas A. Henzinger (ISTA),  
Matin Ansaripour (ATVA'23, EPFL), Abhinav Verma (NeurIPS'23, Penn State)

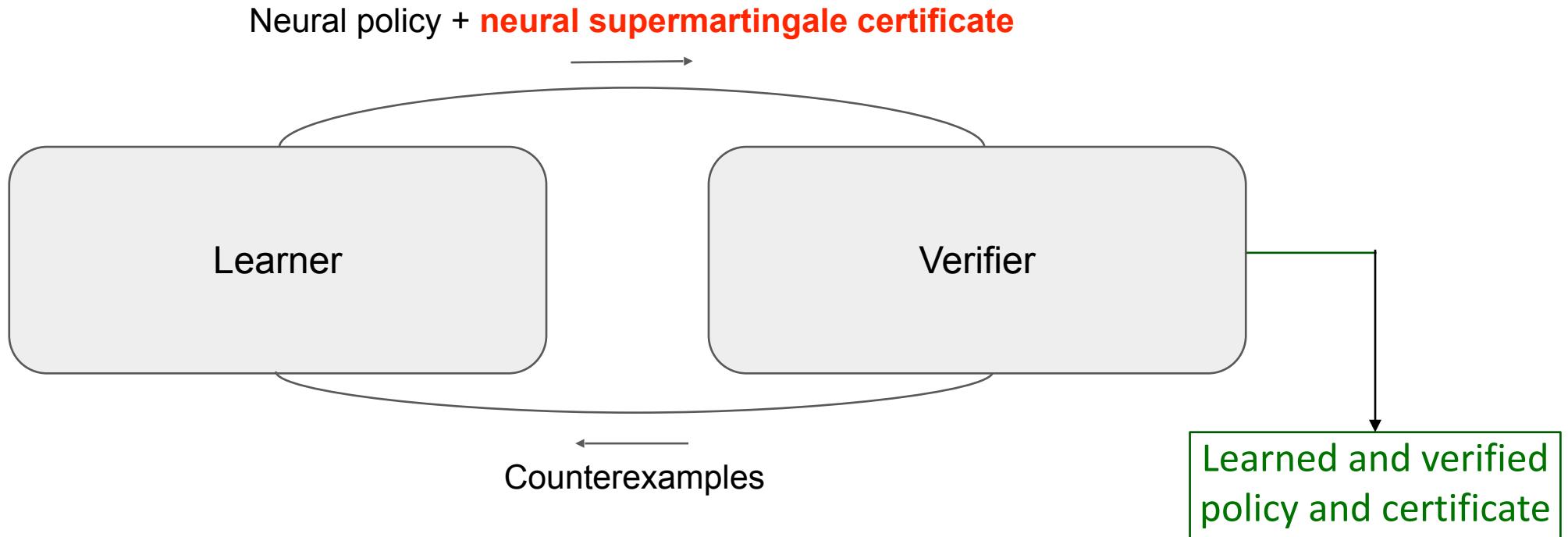
# Certified control through a learner-verifier loop



**Q1: What should be the certificates for stochastic dynamical systems?**

**Q2: How to learn and formally verify neural policies and certificates?**

# How to formally verify learned controllers in stochastic dynamical systems?



# What are supermartingales?

**Supermartingale** – stochastic process decreasing in expectation  $\mathbb{E}[X_{n+1} | X_n] \leq X_n$

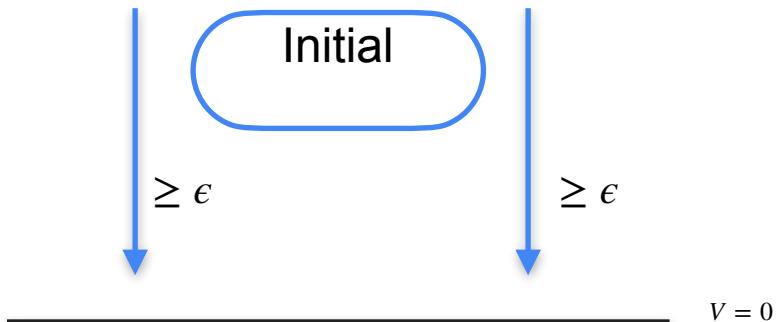
# Supermartingale certificates in stochastic control

## Ranking supermartingales (RSMs) for probability 1 reachability

[Kushner, Transactions on Automatic Control 1966, Chakarov, Sankaranarayanan, CAV 2013]

A measurable function  $V : X \rightarrow \mathbb{R}$  for a target set  $X_t$  such that:

1. Nonnegativity.  $V(x) \geq 0$  for  $x \in X$
2. Strict expected decrease.  $\exists \epsilon > 0$  s.t.  $\mathbb{E}_{w \sim d}[V(f(x, \pi(x), w))] \leq V(x) - \epsilon$  for  $x \in X \setminus X_t$



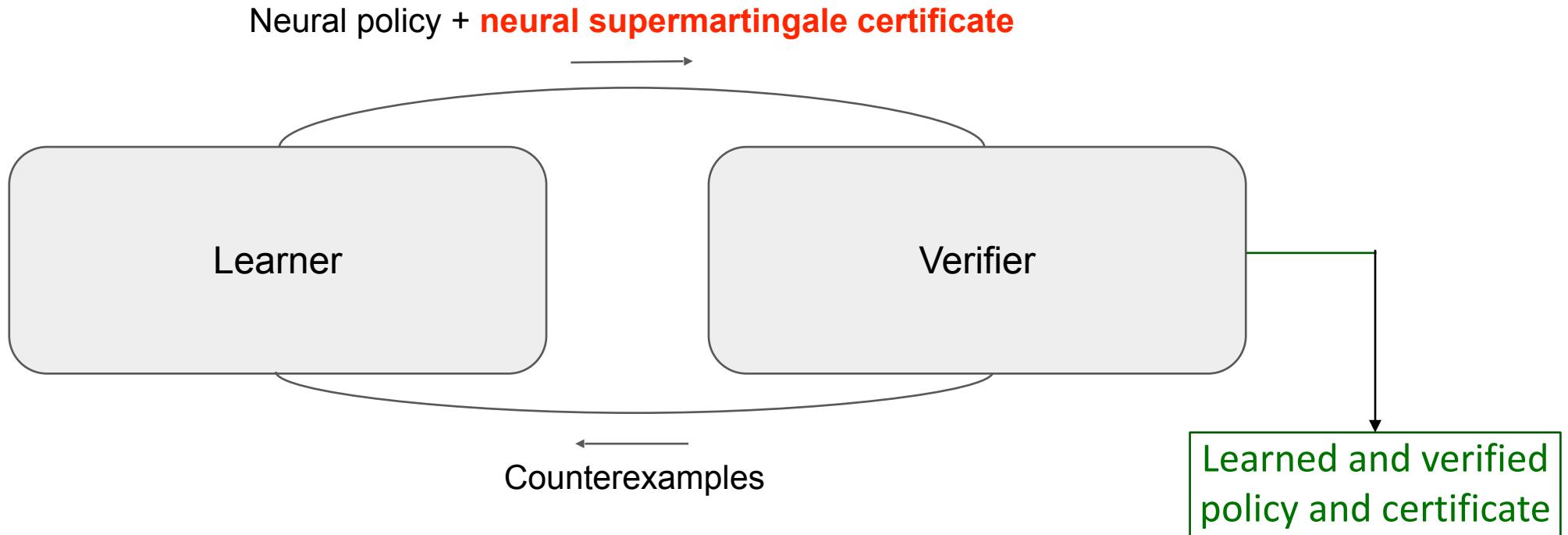
## Stochastic barrier functions for probability $p \in [0,1]$ safety

[Prajna, Jadbabaie, Pappas. CDC 2004]

# Contribution 1: Supermartingale certificates for more general properties

- Reach-avoidance [AAAI23]
- Sequential and boolean compositions of reach-avoidance [NeurIPS23]
- Stability (a.k.a. reach-and-stay) [ATVA23]
- Linear-time temporal logic (LTL) — work in progress!

# Contribution 2: The first learner-verifier loop for stochastic control systems



$$\mathcal{L}(\theta, \nu) = \mathcal{L}_{\text{Init}}(\nu) + \mathcal{L}_{\text{Unsafe}}(\nu) + \mathcal{L}_{\text{Decrease}}(\theta, \nu)$$

Loss encodes defining properties of supermartingale certificates

Discretization  
+ Lipschitz analysis  
+ abstract interpretation

# Key features of the framework

1. Applicable to deterministic and stochastic control systems
2. Both formal synthesis and verification of neural policies
3. Hard formal guarantees

# Experiments

Environment	Reach-avoidance probability
2D system	93.3%
Inverted pendulum	92.1%
Collision avoidance	90.4%

Experiments on reach-avoid tasks



Formally certified neural policies

! Scalability bottleneck, low dimensional environments

# Some ongoing and future directions

**More general properties**

**Black-box setting & safe RL**

**Scalability**

**Multi-agent systems**

# Conclusion

