



A robust low-cost adaptive filtering technique for phonocardiogram signal denoising



S. Hannah Pauline, Samiappan Dhanalakshmi*

Department of Electronics and Communication Engineering, College of Engineering and Technology, Faculty of Engineering and Technology, SRM Institute of Science and Technology, SRM Nagar, Kattankulathur, Kanchipuram, Chennai, TN, India

ARTICLE INFO

Article history:

Received 8 February 2022

Revised 1 July 2022

Accepted 11 July 2022

Available online 14 July 2022

Keywords:

Adaptive noise cancellation

Least mean square

Signal to noise ratio

Average noise reduction

ABSTRACT

Objective: Phonocardiogram (PCG) represents the recordings of various heart sounds. To diagnose the different ailments of the heart, it is required to analyze these PCG signals. However, recording PCG signals is challenging since it is prone to surrounding noise signals. Therefore, there is a need to denoise the PCG signal before being used for advanced processing. This paper proposes an Adaptive Noise Cancellers-based filter model for effectively denoising and recovering the PCG signal.

Method: This work introduces an optimum adaptive filter structure for estimating a noise-free signal with high accuracy using Least Mean Square (LMS) algorithm. A noisy signal is processed through multiple adaptive filter stages connected in series in the proposed work. Multiple stages are automatically added, and each stage filter's step size process is dynamically changed. The estimate of clean PCG signal approximated using this multistage cascaded adaptive filter architecture is subsequently used in the next module to recover the clean PCG signal with high accuracy.

Results: The proposed robust multistage adaptive filter is evaluated for denoising synthetic and experimental PCG signals corrupted by Gaussian and pink noise of various input Signal to noise (SNR) levels. The experimental data are taken from the physionet database (Classification of Heart Sound Recordings: The PhysioNet/Computing in Cardiology Challenge 2016). The results demonstrate that the robust multistage filter model performs remarkably well.

Discussion: Compared with various filter configurations, the proposed filter structure achieves an 8–50% reduction in MAE values and the 45–87% reduction in MSE values. Further, there is an improved SNR of 15–60%, ANR of 15–65%, and PSNR improvement by 7–25% comparatively. The correlation between the clean signal and its estimate obtained using the proposed filter model is more than 0.99.

Conclusion: Using an LMS adaptive filter in the proposed filter model offers a cost-effective hardware implementation of Adaptive Noise Canceller with high accuracy. In the future, the suggested robust multistage adaptive filter model can be tested for real-time performance when improved convergence speed and accuracy are desired.

© 2022 Elsevier B.V. All rights reserved.

1. Introduction

Diseases related to the heart, commonly referred to as cardiovascular diseases (CVD) cause high death rates worldwide. To record the heart signals the different methods are electrocardiogram (ECG) [1], phonocardiogram (PCG) [2] and photoplethysmogram (PPG) [3]. Among the available methods, only PCG signal provides information about heart signals' acoustic properties, which is useful for advanced processing. The electronic stethoscope is fre-

quently used to record the PCG signals. However, capturing PCG signals and other biological signals [4] is difficult because they are vulnerable to ambient noise [5]. While recording PCG signals, they are corrupted by various noise signals such as lung sounds, environmental noise, stethoscope movement, etc. The noise corrupted signal may be wrongly diagnosed as a pathological heart sound. Further, a noisy pathological PCG signal may lead to errors in identifying the ailment. As a result, PCG signal denoising [6] is a must before analysis and advanced processing. Denoising a PCG signal to improve signal quality by reducing background noise is a difficult challenge. Clinical evaluation [7] is performed to test the signal quality. The denoising algorithms' performance determines the ac-

* Corresponding author.

E-mail address: dhanalas@srmit.edu.in (S. Dhanalakshmi).

curacy of the results, which decreases as the noise level increases [8].

The literature suggests various temporal and frequency domain PCG signal denoising methods [9]. The frequency-domain approaches are preferable because they provide sufficient information about the spectral characteristics of the PCG signal components. Here, the time domain signal is first transformed into the frequency domain using a specific transform function such as Fourier Transform or Wavelet Transform (WT). Then the transformed signal is processed via frequency domain-based denoising techniques. Sanei et al. [10] proposed applying Singular Spectrum Analysis(SSA) to separate the murmurs from the PCG signal. In [11], the Tunable Q Wavelet method removes murmurs from PCG signals. Both algorithms, however, take a long time to compute. In [12] Sujadevi et al. proposed a Variational Mode Decomposition (VMD) based PCG denoising method for mobile phonocardiography. This method decomposes the signal into various modes and is spectrally separated. A combination of EMD and Wavelet-based method of PCG signal denoising is proposed in Figueiredo et al. [13]. Using this method, S1 and S2 components can be filtered and extracted effectively. Although these strategies provide efficient results, they take a long time to compute [14]. Kalman filter [15] is another prominent technique for PCG signal denoising. Recently, an AR-based Kalman method has been proposed in Nazemi et al. [16]. Even though the proposed filter improves SNR values, it requires a pre-processing low-pass whose cut-off frequency must be set according to prior knowledge about the signal's nature. The most popular method for the denoising of the PCG signal is based on the DWT [17–19]. Since the DWT coefficients of the PCG signal components will be large, they will be located at a specific frequency band, while the coefficients for the noise components will have small amplitude and scattered in different frequency bands [20]. Thus, denoising can be achieved by suppressing the small coefficients. However, the performance of the DWT-based denoising algorithm significantly depends on the choice of the parameters, the mother wavelet, number of decomposition levels and the levels to be processed, threshold value, and threshold function [21]. Further, producing excellent SNR values necessitates using a predetermined basis function. In the time domain, denoising algorithms proposed are based on traditional Chebyshev IIR filters [22] and adaptive noise canceller(ANC) [23,24].

In this work, we have investigated the feasibility of adopting the Adaptive Noise Cancelling approach, commonly used for signal denoising in telecommunication, to PCG signal denoising in this study. Adaptive filters evaluate clean signals the best with automatic performance adaption. Adaptive filters [25] vary their weights based on the output error signal and do not require any prior information; as a result, they are used in a wide range of applications, including noise [26] and echo cancellation [27], signal [28], and line enhancement [29]. The filter's efficiency is improved by using a suitable algorithm. There are several adaptive filtering algorithms proposed by several researchers such as LMS [30], NLMS [31], Recursive Least Squares (RLS) [32], Filtered-x LMS (FxLMS) [33], Affine Projection Algorithm(APA) [34] and Fast APA [35]. The convergence speed is improved by using Variable Step Size(VSS) algorithms proposed by several researchers. Some of the recently proposed variable step size algorithms are VSS LMS algorithm [36], VSS NLMS [37] and VSS APA [38] and VSS Pseudo APA [39]. Design of cost-effective hardware for signal denoising requires algorithms with minimum computations. Of all the adaptive filtering algorithms, LMS is one such algorithm that [40] has fewer computations and is more straightforward, but it has a poor convergence rate. Fast converging low complexity algorithms are proposed by several researchers for applications including echo and noise cancellation. In [41] Albu et al. proposed a fast block exact Gauss Seidel Pseudo Affine Projection Algorithm using re-

duced computational complexity with faster convergeing speed. For Acoustic Echo Cancellation a Variable Step Size Dichotomous Co ordinate Descent Affine Projection algorithm with lesser computations and high speed convergence is proposed in Albu et al. [42].

In this study, we propose modifying the filter structure to obtain a higher convergence speed of the steady-state MSE while maintaining a simple computational, algorithmic approach. Several experts have also recommended changing the filter structure to attain a low MSE. The adaptive filter cascade structure [43] has shown to be highly effective and has been effectively implemented in ANC systems. The cascaded adaptive filter construction suggested by Ahmed et al. [44] improves the output SNR of a line enhancer. Prandoni et al. [45] offer a cascaded Finite Impulse Response (FIR) filter for adaptive linear prediction, indicating that a cascaded structure converges faster to an ideal predictor than a single-stage filter. Yu et al. [46] presented a cascade combination of a higher-order LMS filter and lower-order Recursive Least Square (RLS) filter as predictors for lossless audio compression. This cascaded Recursive Least Square-Least Mean Square predictor has a higher prediction gain and faster convergence. The cascaded RLS-LMS predictor achieves the best compression ratio for MPEG-4 lossless audio coding [47]. For Narrowband Active Noise Control, [48] proposes a cascaded adaptive filter model based on the Filtered-x Least Mean Square (FxLMS) algorithm. The cascade model eliminates noise successfully and has a faster convergence speed. The use of a multistage filter for engine noise suppression improves filter speed adaption, according to [49]. A multistage LMS filter is used to remove artifacts from ambulatory Electrocardiogram (ECG) signals efficiently [50]. Using a multistage modified Normalized Least Mean Square (NLMS) method [51], efficient removal of various noise signals from ECG signal is achieved with a high output SNR value and faster convergence speed. In terms of SNR, the proposed cascaded 2-stage [52] and 3-stage [53] LMS filter topologies for Adaptive Noise Cancelling surpass ordinary LMS adaptive filters. The multi-cascaded adaptive filter structure introduced in [54] is used to decrease impulsive noise from speech data with fewer calculations. The LMS filter [55] multistage architecture provides exceptional performance with minimal computational complexity.

Based on the above studies, we infer that a multistage filter architecture outperforms conventional adaptive filters in convergence speed and MSE for various applications, including Active Noise Control, signal enhancement, linear prediction, noise cancellation, and suppression. This paper proposes a new cascaded adaptive filter model for PCG signal denoising based on a traditional LMS adaptive filter. Compared to the existing signal denoising techniques, the primary advantage of the proposed filter model is the reduction in computational complexity. The proposed filter model employs the LMS algorithm for adaptation which requires a minimum number of computations and provides a low-cost and straightforward implementation of a hardware processor for efficient denoising of PCG signals. We use two modules of Adaptive Noise Canceller so that, firstly, a multistage series-connected adaptive filter configuration is used to estimate the noise-free signal in module I. The estimate of the clean signal is obtained from this configuration which is then used as the reference input signal in the subsequent stage adaptive filter to get the replica of the clean signal with minimum noise. The first module of PCG signal enhancement is a multistage series-connected configuration wherein the number of stages and the step-size for each stage is adjusted automatically. The conventional Variable Step-Size algorithms update the step-size of the adaptive filter at each iteration, whereas in the filter model proposed, the step-size is changed at each stage of the filter and not for every iteration, thus reducing the computational complexity. The next module uses an ANC system to attain a

Table 1

Mathematical notations.

Notation	Meaning
Small bold letter, e.g., \mathbf{x}	Vector
n	Discrete time index E
$E[\cdot]$	Expectation operator
$[\cdot]^T$	Transpose operator
$ \cdot $	Magnitude a, b

Table 2

Abbreviations.

Abbreviation	Meaning
ANC	Adaptive Noise Canceller
PCG	Phonocardiogram
LMS	Least Mean Square
SNR	Signal to Noise Ratio
MAE	Mean Absolute Error
MSE	Mean Square Error
ANR	Average Noise Reduction
PSNR	Peak Signal to Noise Ratio
CC	Correlation Coefficient
CVD	Cardiovascular diseases
ECG	Electrocardiogram
PPG	Photoplethysmogram
DWT	Discrete Wavelet Transform
EMD	Empirical Mode Decomposition
VMD	Variable Mode Decomposition
SSA	Singular Spectrum Analysis
FIR	Finite Impulse Response
RLS	Recursive Least Square
FxLMS	Filtered x Least Mean Square
NLMS	Normalised Least Mean Square
APA	Affine Projection Algorithm
VSS	Variable Step Size

clean signal with minimum noise at a higher speed. [Section 2](#) lists the notations and abbreviations used in the study. The materials and methods are described in [Section 3](#). [Section 4](#) contains the MATLAB simulation results, demonstrating the usefulness of the proposed approach. [Section 5](#) presents the discussion of the results comparatively, and [Section 6](#) concludes with future scope.

2. Notations and abbreviations

[Table 1](#) presents the various mathematical notations used throughout this paper and their meanings. Also, [Table 2](#) shows the definitions of abbreviations used throughout this manuscript.

3. Materials and methods

3.1. Data

This study used synthetic PCG signal and experimental PCG raw data available at PhysioNet/PhysioBank to evaluate the proposed filter's denoising capability.

3.1.1. Synthetic PCG signal

The synthetic PCG signal consisting of a sequence of S1-S4 waveforms is used to evaluate the proposed filter's denoising capability. The signal duration is 1s, and the sampling rate is 1k. Gaussian noise of input SNR levels of -1 dB and +4 dB is added, and the noisy signal is passed through the proposed filter.

3.1.2. Experimental data

Experimental PCG data (both normal and pathological) are taken from the physionet database (Classification of Heart Sound Recordings: The PhysioNet/Computing in Cardiology Challenge

2016). The dataset comprises both healthy subjects and pathological patients, including children and adults. Each subject/patient may have contributed between one and six heart sound recordings. The recordings last from several seconds to more than one hundred seconds. All recordings have been resampled to 2000 Hz and provided in.wav format. Each recording contains only one PCG lead. The heart sound recordings were collected from different locations on the body. The typical four locations are the aortic, pulmonic, tricuspid, and mitral, but they could be one of nine different locations. We have used a healthy PCG signal (a0007) and a pathological PCG signal (a0001) of duration 2 s and sampled at 8 kHz to study the performance of the proposed filter model.

3.2. Proposed robust multi-stage adaptive filter configuration

Adaptive Noise Cancellers that use LMS adaptive filters are structurally simple and provide reliable performance, but the steady-state MSE takes a long time to converge. We propose a robust multistage LMS adaptive filter model to obtain a faster convergence speed and minimal MSE value. [Fig. 1](#) shows that the proposed filter architecture has two ANC modules. A multistage LMS adaptive filter structure is employed in the first ANC module with an automatic cascading of stages. Module I is used for estimating the clean signal $\hat{s}(n)$. In the second module, the signal $\hat{s}(n)$ from module I is passed through the conventional LMS adaptive filter to obtain an accurate estimate of $s(n)$. The robust multistage adaptive filter structure is shown in [Fig. 2](#).

The proposed Robust Multi-stage adaptive filter model has several notable features namely

- Module I automatically varies the number of stages to be inserted in series.
- The step size of filter is automatically modified at every stage.
- Module II is a simple Adaptive Noise Canceller that uses the clean signal estimate from module I as the reference input signal to recover the clean signal with high accuracy.

Module I (stages I to K). The module I has a multi-stage filter structure that employs LMS adaptation. In this structure, the number of series-connected stages is inserted automatically. The primary and secondary inputs to the first stage ANC are

$$d_1(n) = s(n) + c(n). \quad (1)$$

and

$$x_1(n) = c'(n). \quad (2)$$

The noise-added signal and reference noise are represented by $d_1(n)$ and $x_1(n)$, respectively and the input reference signal is the noise signal $c'(n)$ correlated to $c(n)$. The outputs of the first stage are given as

$$e_1(n) = d_1(n) - y_1(n) \quad (3)$$

where

$$\begin{aligned} y_1(n) &= \mathbf{w}_1^T(n) \mathbf{x}_1(n) \\ &= \mathbf{w}_1^T(n) \mathbf{c}'(n) = \hat{c}(n), \end{aligned} \quad (4)$$

where the filter weights are $\mathbf{w}_1(n) = [w_0, w_1, \dots, w_{L-1}]^T$ and the filter input is $\mathbf{x}_1(n) = [x_0(n), x_1(n-1), \dots, x_{L-1}(n-L+1)]^T$. The filter order is L . The weights are updated by

$$\mathbf{w}_1(n+1) = \mathbf{w}_1(n) + \mu_{1LMS} e_1(n) \mathbf{c}'(n), \quad (5)$$

Substituting [Eqs. \(1\)](#) and [\(4\)](#) in [\(3\)](#), we get

$$\begin{aligned} e_1(n) &= s(n) + c(n) - \hat{c}(n) \\ &= s(n) + \Delta c(n), \end{aligned} \quad (6)$$

The inputs to the second stage ANC are

$$\begin{aligned} d_2(n) &= e_1(n) = d_1(n) - \hat{c}(n) \\ &= s(n) + \Delta c(n) \end{aligned} \quad (7)$$

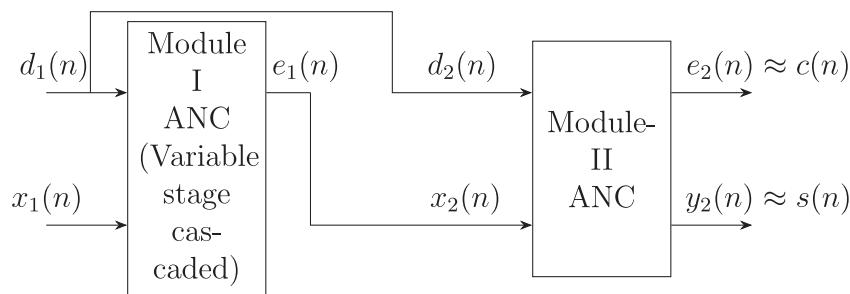


Fig. 1. General block diagram.

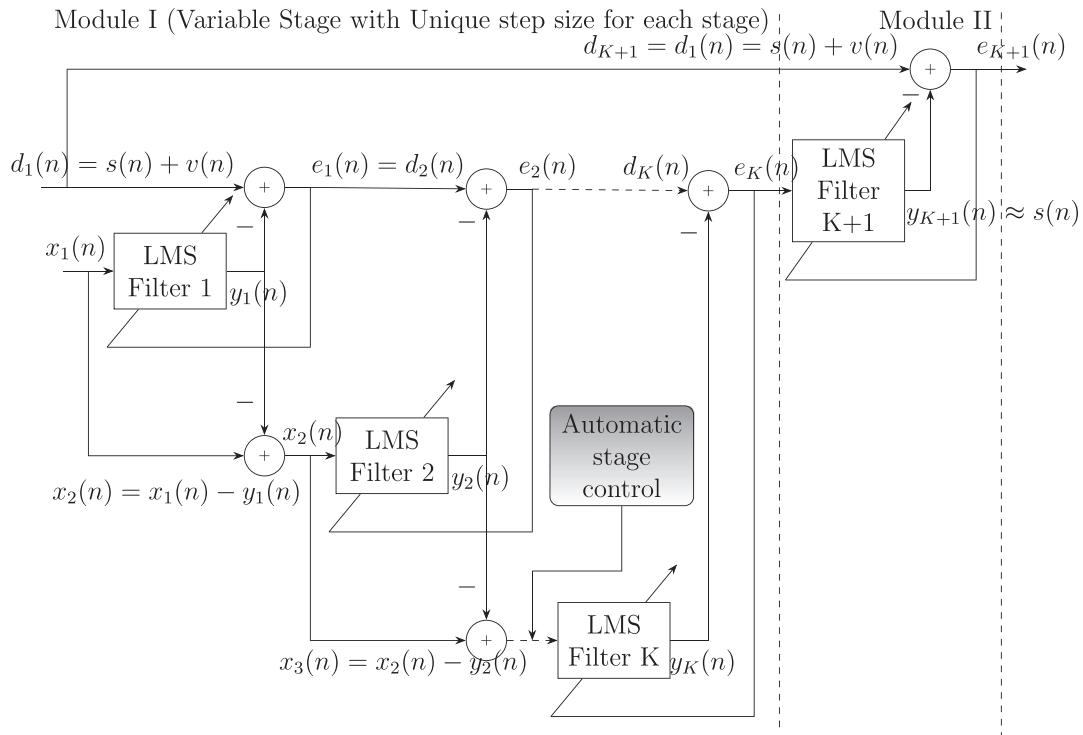


Fig. 2. Proposed robust multi-stage LMS adaptive filter.

The purpose of next stage is to remove the noise $\Delta c(n)$ from $d_2(n)$. As a result, as the secondary or reference input signal to the filter, we must utilize a signal that is correlated to $\Delta c(n)$. This is obtained from input and output of the filter employed in stage I as

$$\begin{aligned} x_2(n) &= x_1(n) - y_1(n) = c'(n) - \hat{c}(n) \\ &= \Delta c'(n) \end{aligned} \quad (8)$$

$\Delta c'(n)$ is correlated to $\Delta c(n)$ and hence is more efficient in removing the noise. The control switch is also used to select a suitable algorithm in the second stage. This process continues, and multiple stages are added; at the final stage K th the inputs are

$$d_K(n) = e_{K-1}(n) = s(n) + \rho c(n) \quad (9)$$

and

$$x_K(n) = x_{K-1}(n) - y_{K-1}(n) = \rho c'(n) \quad (10)$$

The outputs are

$$e_K(n) = d_K(n) - y_K(n) \quad (11)$$

where

$$\begin{aligned} y_K(n) &= \mathbf{w}_K^T(\mathbf{n}) \mathbf{x}_K(n) \\ &= \mathbf{w}_K^T(\mathbf{n}) \rho \mathbf{c}'(n) = \rho \hat{c}(n) \end{aligned} \quad (12)$$

Substituting Eqs. (9) and (12) in (11)

$$e_K(n) = s(n) + \rho c(n) - \rho \hat{c}(n) \approx s(n) \quad (13)$$

if

$$\rho c(n) - \rho \hat{c}(n) \approx 0 \quad (14)$$

This means that the approximate replica of a noiseless signal is acquired at $e_K(n)$, which is the final ANC stage's error output. However, in order to obtain a signal with minimum noise, $\rho c(n) - \rho \hat{c}(n) \approx 0$ is required, which is only attainable at the ANC's optimal stage. The proposed filter design automatically determines the stage at which the term $\rho c(n) - \rho \hat{c}(n)$ is at its minimum.

Module II (stage $K + 1$). The estimate of a clean signal obtained from module I contains minimum noise and is denoted by $e_K(n) = \tilde{s}(n)$. The precise estimate of the clean signal $s(n)$ can be retrieved from the noise-contaminated signal by using $e_K(n)$ as the reference input signal to the next $K + 1$ th stage adaptive filter. ($K + 1$)th stage inputs are

$$d_{K+1}(n) = d_1(n). \quad (15)$$

and

$$x_{K+1}(n) = e_K(n) = \tilde{s}(n). \quad (16)$$

The outputs are

$$y_{K+1}(n) = \mathbf{w}_{K+1}^T(n) \mathbf{x}_{K+1}(n) \approx s(n). \quad (17)$$

where

$$\mathbf{w}_{K+1}(n+1) = \mathbf{w}_{K+1}(n) + \mu e_{K+1}(n) \mathbf{x}_{K+1}(n). \quad (18)$$

and

$$e_{K+1}(n) = d_{K+1}(n) - y_{K+1}(n) = s(n) + c(n) - \tilde{s}(n) \approx c(n). \quad (19)$$

From Eqs. (17), (19) it is proved that the clean signal $s(n)$ is obtained at the filter output $y_{K+1}(n)$ and the noise signal $c(n)$ is obtained at output error signal $e_{K+1}(n)$ of $(K+1)$ th stage ANC. The proposed filter model can be employed in applications where it is necessary to separate two signals with high accuracy.

MSE. Eq. (19) gives the error signal at the $K+1$ th stage, and the MSE is

$$\begin{aligned} E[|e_{K+1}(n)|^2] &= E[|s(n) + c(n) - \tilde{s}(n)|^2] \\ &= E[|s(n) - \tilde{s}(n)|^2] + E[|c(n)|^2] + 2E[|c(n)(s(n) - \tilde{s}(n))|]. \end{aligned} \quad (20)$$

It is assumed that $c(n)$ and $\tilde{s}(n)$ has no correlation with $s(n)$, we can deduce the following

$$2E[|c(n)s(n)|] = 0. \quad (21)$$

and

$$2E[|c(n)\tilde{s}(n)|] = 0. \quad (22)$$

Inserting Eqs. (21) and (22) in (20)

$$E[|e_{K+1}(n)|^2] = E[|s(n) - \tilde{s}(n)|^2] + E[|c(n)|^2]. \quad (23)$$

From Eq. (23) it is noted that if

$$E[|s(n) - \tilde{s}(n)|^2] \approx 0 \quad (24)$$

then the replica of the noise signal is obtained at the output error signal of stage $K+1$ th ANC. When the stage $K+1$ filter's output error signal is close to $c(n)$, the stage $K+1$ filter's output $y_{K+1}(n) \approx s(n)$, i.e. the noise-free signal, is obtained at the stage $K+1$ adaptive filter output. The preceding study shows that the proposed adaptive filter can eliminate noise from the input signal $d(n)$, and the de-noised signal is represented as

$$y_{K+1}(n) \approx s(n). \quad (25)$$

The de-noised signal adaptive filter, based on the above analysis, is closer to $s(n)$.

3.2.1. Automatic control of stages

The ANC stages to be added in series in the switched multistage adaptive filter are calculated using $e_j(n)$ and $c'(n)$. It is important to note that the additive noise $c(n)$ and the clean signal $s(n)$ are unrelated. As a result, we can deduce that the $c'(n)$ reference noise signal, which is similar to $c(n)$, is unrelated to $s(n)$. At each stage, however, the output error signal $e_j(n)$ is closer to $s(n)$. The correlation between $c'(n)$ and $e_j(n)$ at each stage is used to add further stages (i.e., if $c'(n)$ and $e_j(n)$ are highly correlated, the noise component in the output error signal is high, which is undesirable. There is still one more stage that has to be added; otherwise, the optimal stage will be attained. The correlation between $c'(n)$ and $e_j(n)$ is calculated as

$$\rho_{e_j c'} = \frac{\text{Cov}(e_j, c')}{\sigma_{e_j} \sigma_{c'}} \quad (26)$$

where $\rho_{e_j c'}$ denotes the correlation coefficient. The covariance and standard deviation of output error and reference noise are represented by $\text{Cov}(e_j, c')$, σ_{e_j} and $\sigma_{c'}$ respectively.

3.2.2. Automatic step size control

Choosing a step size is the most difficult part of the LMS algorithm. Large step sizes allow for rapid adaptation, which results in a high excess Mean Square Error (excess MSE). There will be a loss of stability if the step size is large. Even if the additional MSE is minimal, a too-small step-size causes delayed convergence. The largest step size necessary to sustain the LMS algorithm is given by the upper bound defined in Bismor et al. [56]

$$0 < \mu < \frac{2}{\lambda_{\max}}, \quad (27)$$

where μ and λ_{\max} represent the step size and the greatest eigenvalue of the input signal's autocorrelation matrix. The suggested filter model feeds the remaining reference noise from the previous phase into each filter stage, resulting in a distinct input signal to the filter at each level. As a result, instead of utilising common value of step size for all filters, the filter's adaption speed is increased by employing unique step sizes for every stage. A step size is fixed for the first stage filter. The maximum value of step size μ_{\max_j} is computed using Eq. (27) at every stage. Depending on the comparison of $\mu_{\max_{j-1}}$ and μ_{\max_j} , the step size is determined at every stage as given below

$$\mu_L = \begin{cases} \mu_{j-1} * (f), & \mu_{\max_j} > \mu_{\max_{j-1}} \\ \mu_{j-1} * (1/f), & \mu_{\max_j} < \mu_{\max_{j-1}} \end{cases} \quad (28)$$

Because 'f' is a constant number ranging from 1 to 2, the value chosen for it is critical for the filter stage's convergence. The proposed robust multistage adaptive filter model converges quicker than the model that uses fixed step sizes at all stages.

Algorithm 1 summarizes the proposed robust multistage LMS adaptive filter model.

3.3. Statistical analysis

The performance of the robust multistage adaptive filter model is analysed on the PCG signal corrupted with Gaussian and pink noise with high and low input SNR levels. The algorithm's performance can be quantitatively evaluated using statistical metrics described in Eqs. (29)–(34).

Mean-Square-Error (MSE) given as

$$MSE = \frac{1}{N} \sum_{i=1}^N (|s(n)| - |y(n)|)^2 \quad (29)$$

Signal to Noise Ratio (SNR)(in dB)

$$SNR = 10 \log_{10} \frac{\sum_{i=1}^N s(n)^2}{\sum_{i=1}^N (s(n) - y(n))^2} \quad (30)$$

Average Noise Reduction(ANR)(in dB)

$$ANR = -10 \log_{10} \frac{E[y^2(n)]}{E[s^2(n) + c^2(n)]} \quad (31)$$

Peak Signal to Noise Ratio (PSNR)(in dB)

$$PSNR = \frac{\max(s^2[n])}{MSE} \quad (32)$$

Mean Absolute Error(MAE)

$$MAE = \frac{\sum_{i=1}^N |y(n) - s(n)|}{N} \quad (33)$$

Correlation Coefficient (CC)

$$CC = \frac{N \left(\sum_{i=1}^N s(n)y(n) \right) - \left(\sum_{i=1}^N s(n) \right) \left(\sum_{i=1}^N y(n) \right)}{\sqrt{\left[N \sum_{i=1}^N s^2(n) - \left(\sum_{i=1}^N s(n) \right)^2 \right] \left[N \sum_{i=1}^N y^2(n) - \left(\sum_{i=1}^N y(n) \right)^2 \right]}} \quad (34)$$

Algorithm 1: Proposed robust cascaded adaptive filter.

```

1 Inputs:
2  $d_1(n) = s(n) + c(n)$ ,  $x_1(n) = c'(n)$ ,  $\mu$ ,  $L$ ,  $\rho_{threshold}$ 
3 Outputs:
4  $K, e, y, w$ 
5 Computations
6  $\mu_{max_1} = \frac{2}{\lambda_{max_1}}$ 
7 First stage parameters
8  $y_1(n) = \mathbf{w}_1^T \mathbf{x}_1$ 
9  $\mathbf{w}_1(n+1) = \mathbf{w}_1(n) + \mu_{1LMS} e_1(n) \mathbf{x}_1(n)$ 
10  $e_1(n) = d_1(n) - y_1(n)$ 
11 Correlation  $\rho_{e_{1LMS}c'}$  between  $e_{1LMS}$  and  $c'$  at Stage I.
12  $j = 1$ 
13 while  $\rho_{e_jc'} > \rho_{threshold}$  do
14    $j = j + 1$ 
15    $x_j(n) = x_{j-1}(n) - y_{j-1}(n)$ 
16    $\mu_{max_j} = \frac{2}{\lambda_{max_j}}$ 
17    $d_j(n) = e_{j-1}(n)$ 
18   if  $\mu_{max_j} < \mu_{max_{j-1}}$  then
19      $\mu_{jLMS} = \mu_{(j-1)LMS} * \frac{1}{f}$  ;
20   else
21      $\mu_{jLMS} = \mu_{(j-1)LMS} * f$ ;
22   end
23    $y_j(n) = \mathbf{w}_j^T \mathbf{x}_j$ 
24    $\mathbf{w}_j(n+1) = \mathbf{w}_j(n) + \mu_{jLMS} e_j(n) \mathbf{x}_j(n)$ 
25    $e_j(n) = d_j(n) - y_j(n)$ 
26    $\rho_{e_jy_j} = corr(e_j, c')$ 
27    $\mu_{max_j} = \frac{2}{\lambda_{max_j}}$ 
28 end
29 Compute Stage K+1 ANC parameters using LMS adaptive
  algorithm
30  $\mathbf{x}_{K+1}(n) = \mathbf{e}_K(n)$ 
31  $\mathbf{d}_{K+1}(n) = \mathbf{d}_1(n)$ 
32  $y_{K+1}(n) = \mathbf{w}_{K+1}^T(n) \mathbf{x}_{K+1}(n)$ 
33  $e_{K+1}(n) = d_{K+1}(n) - y_{K+1}(n)$ 
34  $\mathbf{w}_{K+1}(n+1) = \mathbf{w}_{K+1}(n) + \mu e_{K+1}(n) \mathbf{x}_{K+1}(n)$ 

```

In Eqs. (29)–(34), $s(n)$ represents the clean signal, $y(n)$ is the filtered signal, $c(n)$ denotes the additive noise signal. N is the number of samples.

4. Results

The robust multistage filter model's efficiency is tested by enhancing the Phonocardiogram (PCG) signals taken from the Physionet database [57–59]. Normal and abnormal PCG signals of duration 2 s were taken from the physionet database sampled at 8 kHz. Gaussian and pink noise of different input levels is used to corrupt the signal, and the noisy signal is used to evaluate the proposed filter's denoising capability. The proposed filter's performance is also tested using a synthetic PCG signal [60] of duration 1s and sampling frequency of 1 kHz. The simulation software used is MATLAB. The fixed parameters used for simulation are $\mu = 0.1$ and filter order $L = 2$. The value of parameter ' f ' may be selected between 1 and 2. Values of ' f ' greater than two will cause the filter to diverge at higher cascaded stages. Depending on the value of $\rho_{threshold}$, the number of stages to be connected in series vary automatically. Hence selecting a fixed value of $\rho_{threshold}$ for all input noise levels and signal types may lead to cascading of higher stages. The value of $\rho_{threshold}$ is application-specific and is selected

Table 3
Variation of the correlation function and step-size.

Signal type	Input SNR	Stage	MSE	$\rho_{e_i(n)c'(n)}$
Normal	+4 dB	I	1.61E-04	0.0818
		II	6.98E-05	0.0564
		III	3.40E - 05	0.0412
	-4 dB	I	1.57E-04	0.0551
		II	6.89E-05	0.041
		III	4.23E - 05	0.0318
Abnormal	+1 dB	I	1.55E-04	0.1502
		II	6.74E-05	0.0794
		III	3.25E - 05	0.0422
	-1 dB	I	1.48E-04	0.1767
		II	6.44E-05	0.1022
		III	4.08E - 05	0.0619

appropriately. A different value of $\rho_{threshold}$ is desirable for different noise levels and noise types.

4.1. Qualitative performance assessment

The subjective performance assessment of the robust multistage LMS adaptive filter regarding the quality of the output signal is presented below. We have taken both synthetic and experimental PCG signals to evaluate the subjective performance of the proposed filter. The performance of the proposed filter is noted in the presence of Gaussian noise and pink noise. *Normal PCG signal* Fig. 3(c) represents the signal deteriorated with a Gaussian noise having a +4 dB input level. Fig. 3(d) indicates that the robust multistage filter efficiently removes the noise and restores the clean signal.

In Fig. 4, we have introduced a Gaussian noise of -4 dB input SNR level, and Fig. 4(d) denotes that the proposed filter can remove the higher levels of noise signals. The performance of the proposed filter should be validated in the presence of colored pink noise. Pink noise of input SNR = +5 dB is added to the clean signal as depicted in Fig. 5(c). Restoration of clean signal with minimum noise is attained using the proposed filter model as shown in Fig. 5(d). Fig. 6 depicts the performance of the proposed filter in a high noise environment, where the input pink noise level is -1 dB. From Fig. 6(d), it is noted that the proposed filter model effectively reduces colored noise. The proposed filter exhibits good de-noising properties in pink noise, as indicated in Figs. 5 and 6, and even if the noise levels are high, the proposed filter model performs efficiently well. *Pathological PCG signals* Fig. 7 shows the denoising capability of the proposed filter for the pathological PCG sample (a0001) taken from the physionet database and corrupted with Gaussian noise of +1 dB input signal to the noise level. Fig. 7(d) denotes that the filter removes the noise efficiently. In Fig. 8, we observe that the signal is corrupted by a high level of Gaussian noise with an input SNR of -1 dB, and Fig. 8(d) denotes the denoising capability of the multistage adaptive filter. In Fig. 9, a pink noise of input SNR +5 dB is added to the pathological PCG signal, and the filter's performance is tested using the noisy signal. Fig. 9(d) noted that the filter removes the noise with good efficiency. In Fig. 10, we observe that the pathological PCG signal is corrupted by a high level of pink noise with an input SNR of -1 dB. Fig. 10(d) denotes the denoising capability of the MS adaptive filter.

Synthetic PCG signals Fig. 11(a) depicts the clean synthetic PCG signal with S1-S2-S3-S4 waveforms. The clean synthetic PCG signal is corrupted by Gaussian noise of input SNR = +4 dB, as illustrated in Fig. 11(c). The denoised signal is depicted in Fig. 11(d).

It is noted that the proposed filter reduces the noise at high speed and maximum accuracy. Similarly, the proposed filter efficiently minimizes the noise for an input SNR level of -1 dB, as illustrated in Fig. 12(d). Fig. 13(a) shows the synthetic PCG signals

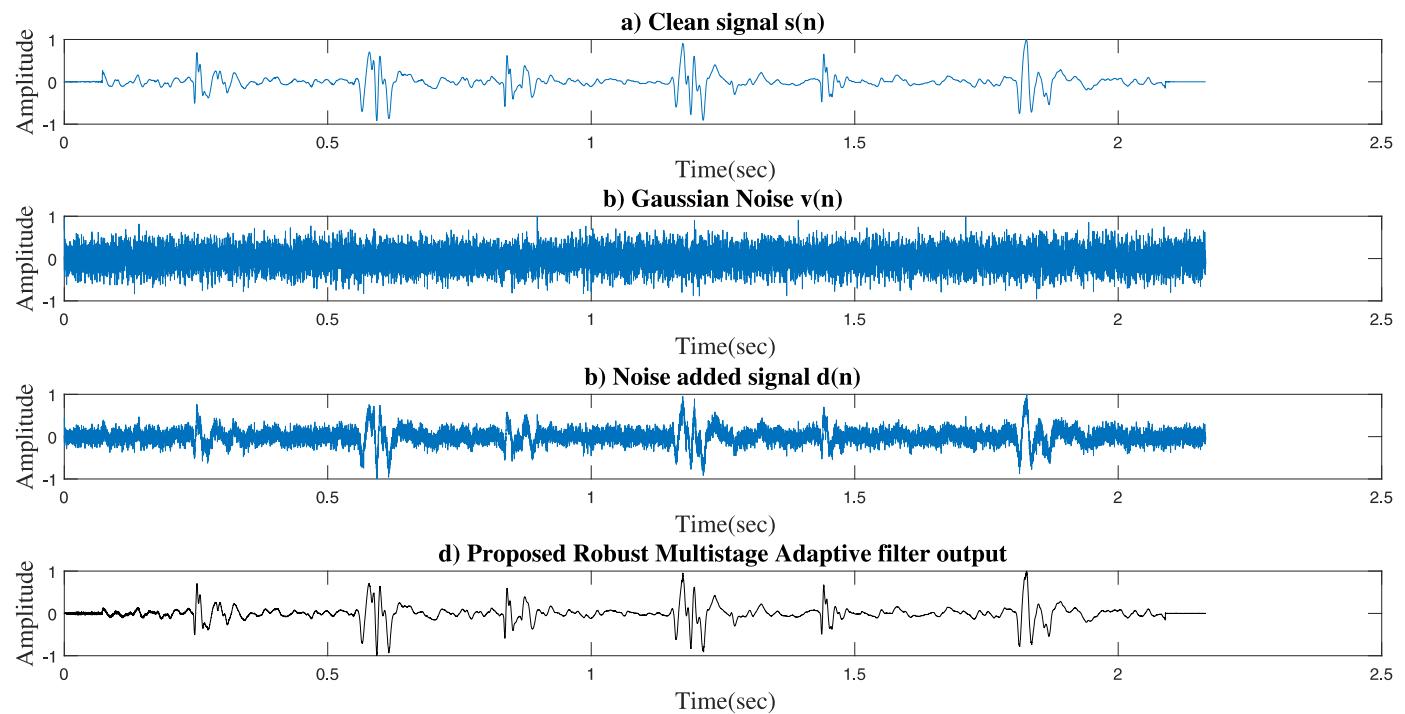


Fig. 3. Proposed robust multistage filter denoising performance for Gaussian noise corrupted normal PCG signal (a) Noise-free signal (b) Additive Gaussian noise input signal to noise level of +4 dB (c) Signal with noise (d) Proposed robust multistage adaptive filter output.

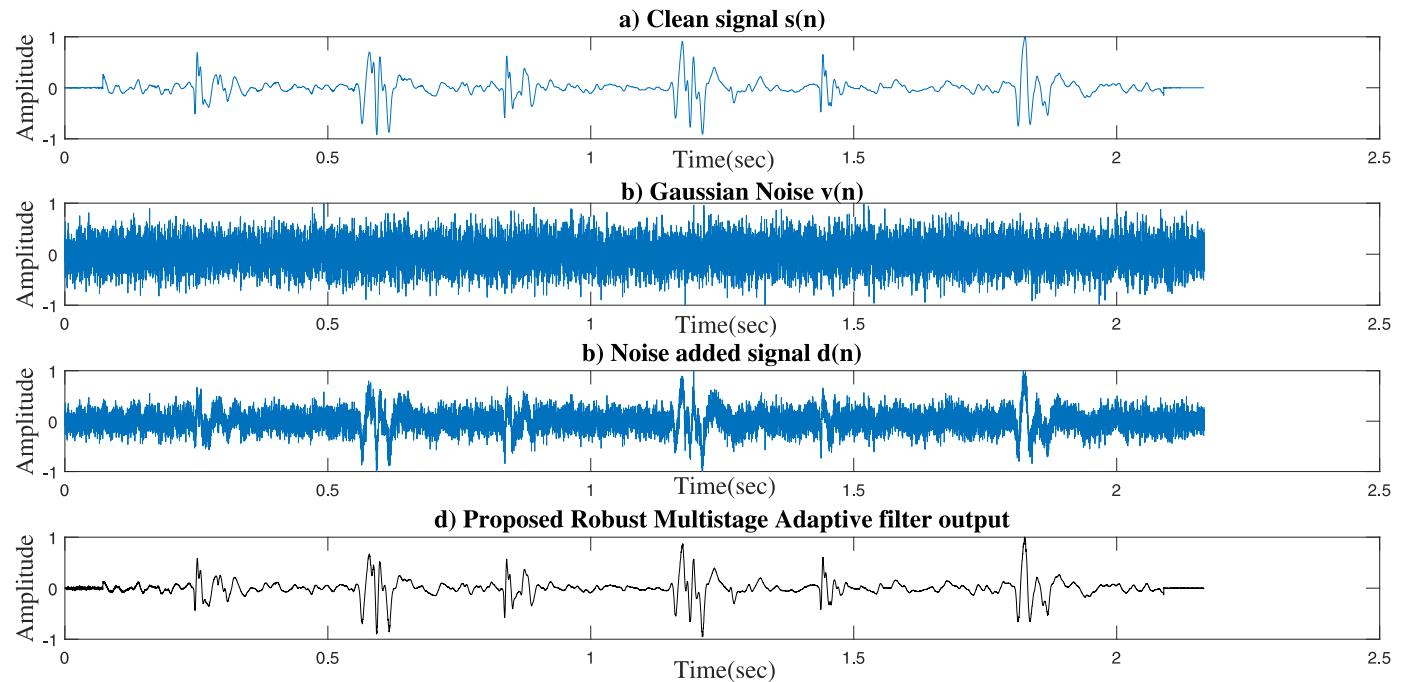


Fig. 4. Proposed robust multistage filter denoising performance for Gaussian noise corrupted normal PCG signal (a) Noise-free signal (b) Additive Gaussian noise input signal to noise level of -4 dB (c) Signal with noise (d) Proposed robust multistage adaptive filter output.

with the sequence of S4-S1-S2-S3 waveforms. The signal is corrupted by Gaussian noise with an input SNR = +4 dB, as shown in Fig. 13(b). Also, it can be noted from Fig. 13(b) that it is not possible to identify the S1 to S4 waveforms from the noisy signal. Fig. 13(c) depicts the denoised signal using the proposed multistage adaptive filter model. Fig. 13(c) shows that the start and end of S4-S1-S2-S3 waveforms are identified in the denoised signal. This proves that the proposed filter can be effectively utilized

for denoising PCG signals at high speed and with greater accuracy using a simple computation algorithm. Further, the denoised signal can be considered helpful for advanced processing due to the minimum noise present.

Automatic control of stages. It is concluded that the proposed Robust Multistage Adaptive Filter has the best noise reduction capability and proves to be very attractive for PCG signal enhancement. The suggested adaptive filter structure has two components.

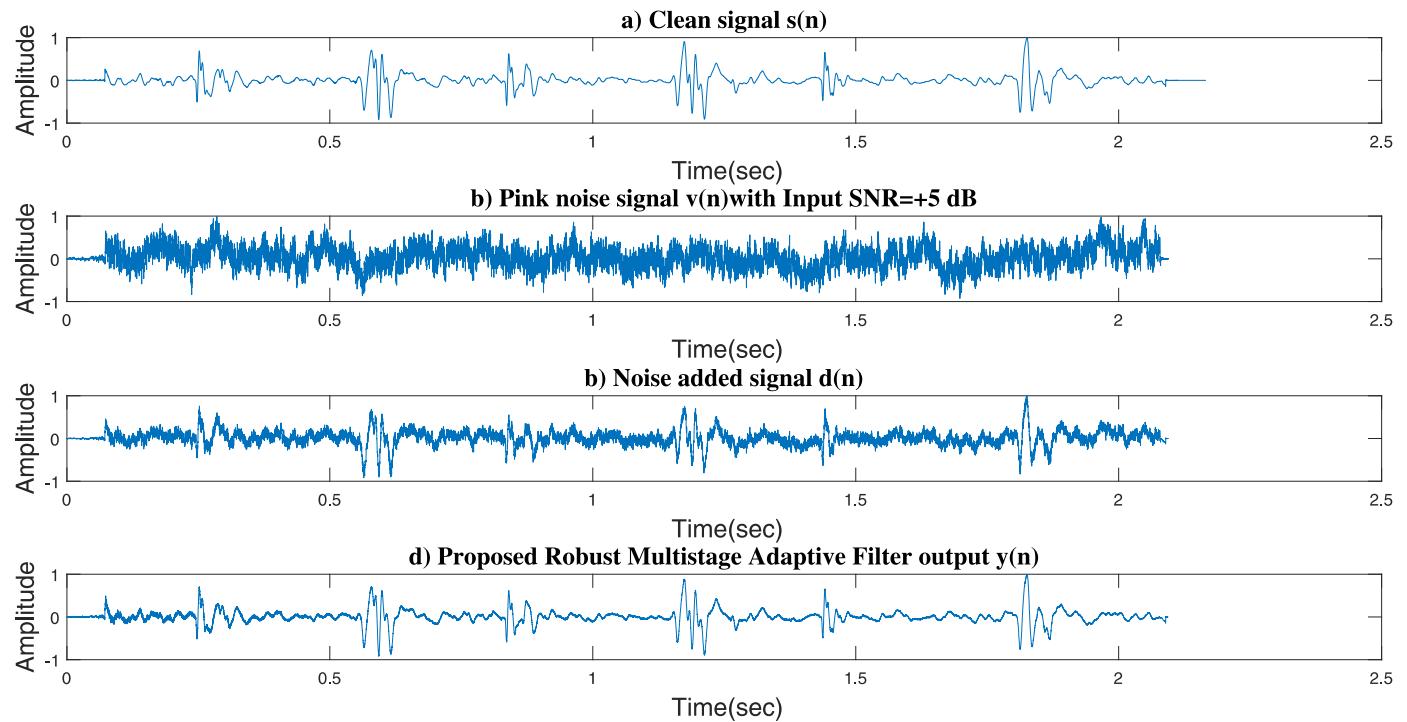


Fig. 5. Proposed robust multistage filter denoising performance for Pink noise corrupted normal PCG signal (a) Noise-free signal (b) Additive pink noise input signal to noise level of +5 dB (c) Signal with noise (d) Proposed robust multistage adaptive filter output.

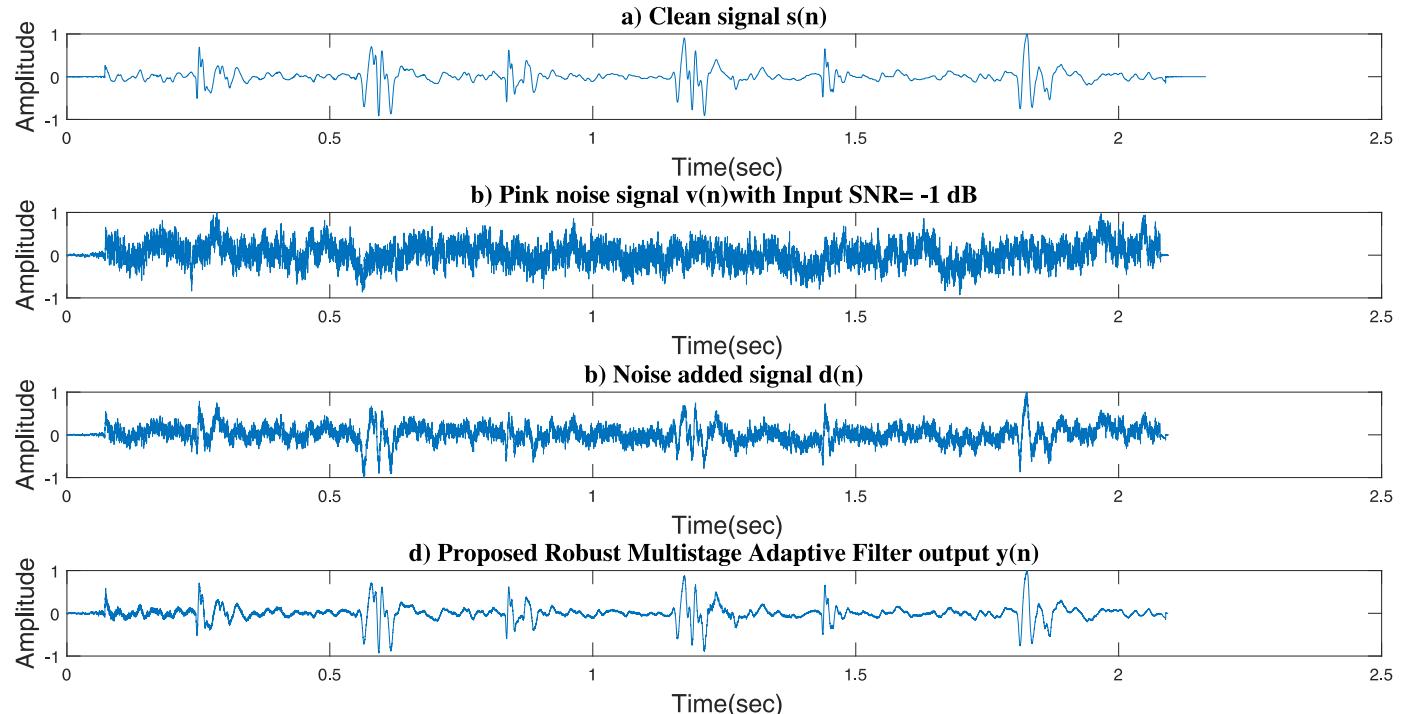


Fig. 6. Proposed robust multistage filter denoising performance for pink noise corrupted normal PCG signal (a) Noise-free signal (b) Additive pink noise input signal to noise level of -1 dB (c) Signal with noise (d) Proposed robust multistage adaptive filter output.

The first is a multi-stage filter with unique step sizes at every stage. Multiple stages inserted are determined automatically based on the correlation between each stage ANC's output error signal and the noise input at stage I ANC. The correlation between $e_1(n)$ and $c'(n)$ is larger than the correlation between $e_3(n)$ and $c'(n)$, as seen in Table 3. As the number of cascaded stages increases, the estimate of a clean signal at the ANC $e_i(n)$ error output con-

tains less noise signal. Consider the case of a normal PCG signal with Gaussian noise of input noise level of -4 dB. As it can be observed from the last column of Table 3, as the correlation $\rho_{e_i(n)c'}$ drops from 0.041 at stage II to 0.0318 at stage III, the MSE value also reaches a minimum value of 4.23E-05 at stage III. Hence it is concluded that the MSE value keeps reducing if more number of stages are cascaded since $\rho_{e_i(n)c'}$ is lesser as the number of stages

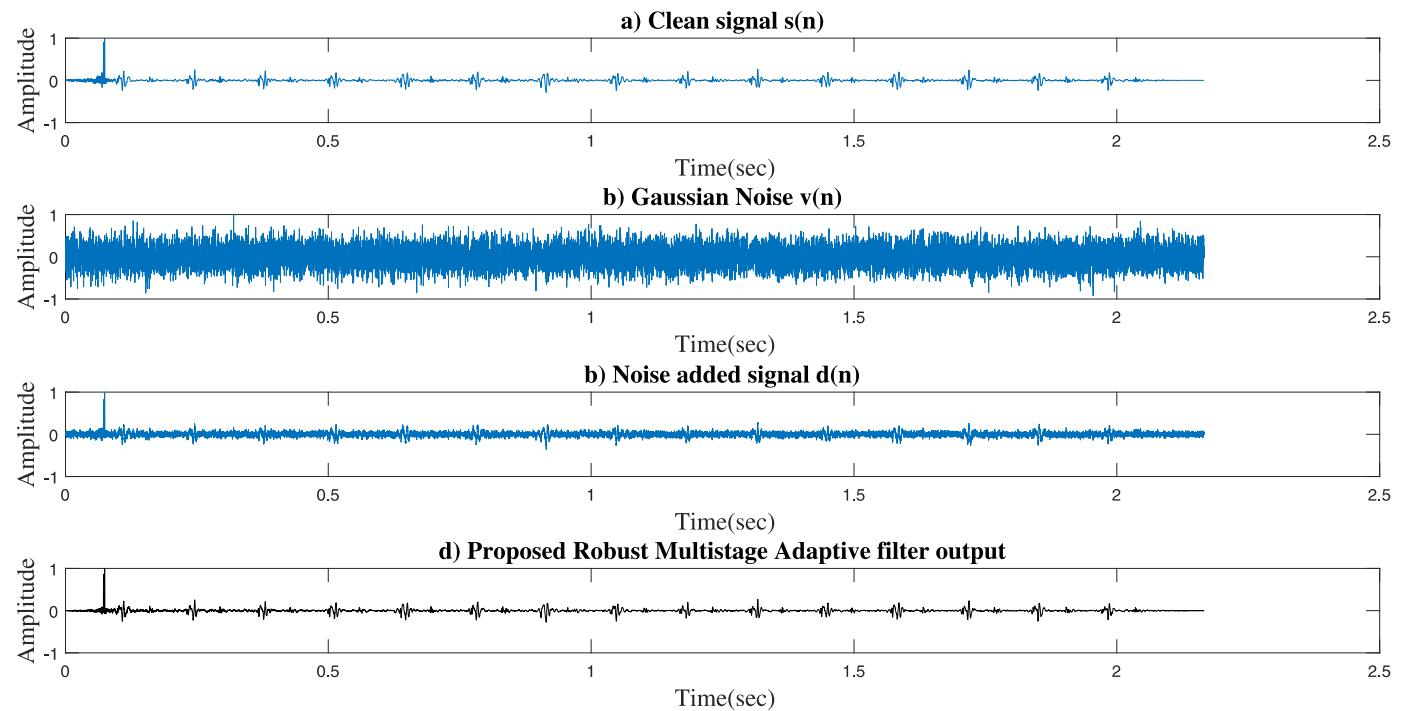


Fig. 7. Proposed robust multistage filter denoising performance for Gaussian noise corrupted abnormal PCG signal (a) Noise-free signal (b) Additive Gaussian noise input signal to noise level of +1 dB (c) Signal with noise (d) Proposed robust multistage adaptive filter output.

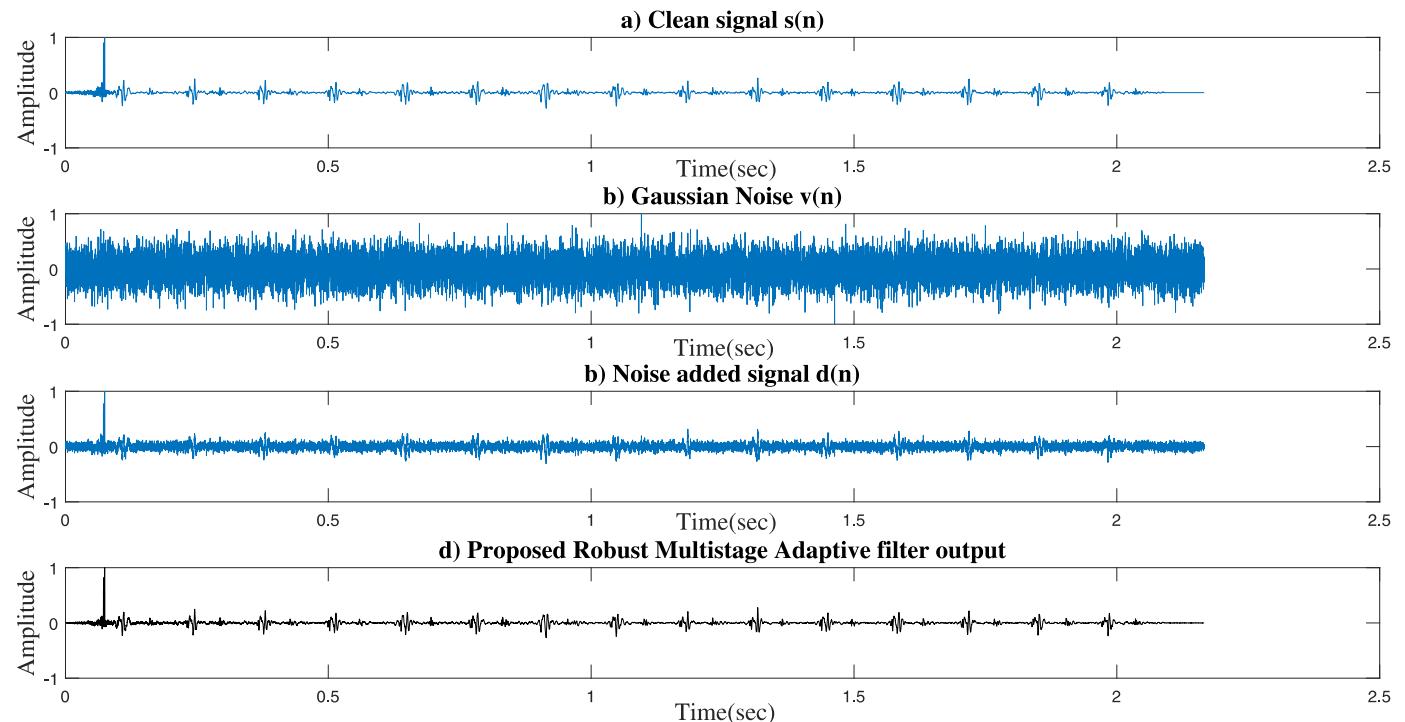


Fig. 8. Proposed robust multistage filter denoising performance for Gaussian noise corrupted abnormal PCG signal (a) Noise-free signal (b) Additive Gaussian noise input signal to noise level of -1 dB (c) Signal with noise (d) Proposed robust multistage adaptive filter output.

i is increased. The value of $\rho_{threshold}$ is chosen based on the type of noise, the input noise level, and the desired output MSE and SNR values. The number of filter stages to be added in series is determined by the value of $\rho_{threshold}$, which can be determined using the trial-and-error approach.

5. Discussion

Qualitative and quantitative comparative study of the robust multistage filter with various state-of-art filters [52,53] are conducted.

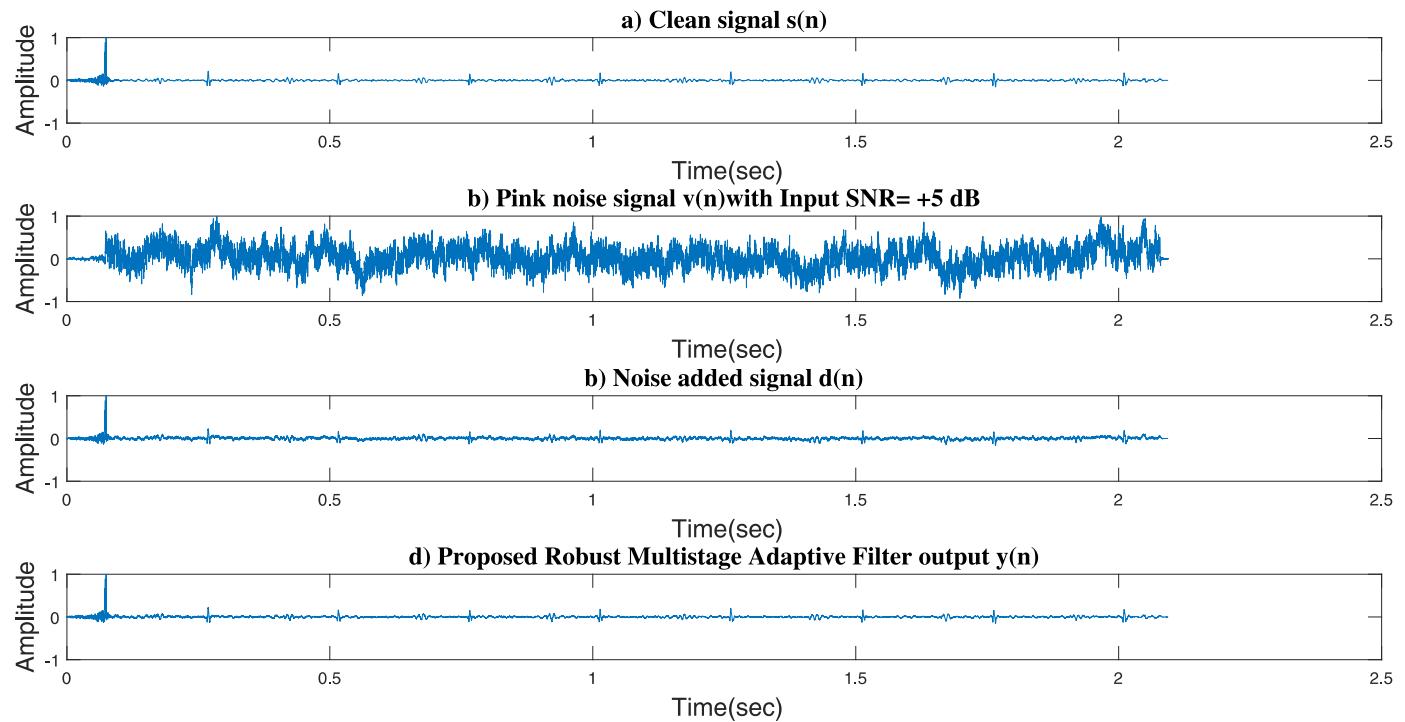


Fig. 9. Proposed robust multistage filter denoising performance for pink noise corrupted abnormal PCG signal (a) Noise-free signal (b) Additive Gaussian noise input signal to noise level of +5 dB (c) Signal with noise (d) Proposed robust multistage adaptive filter output.

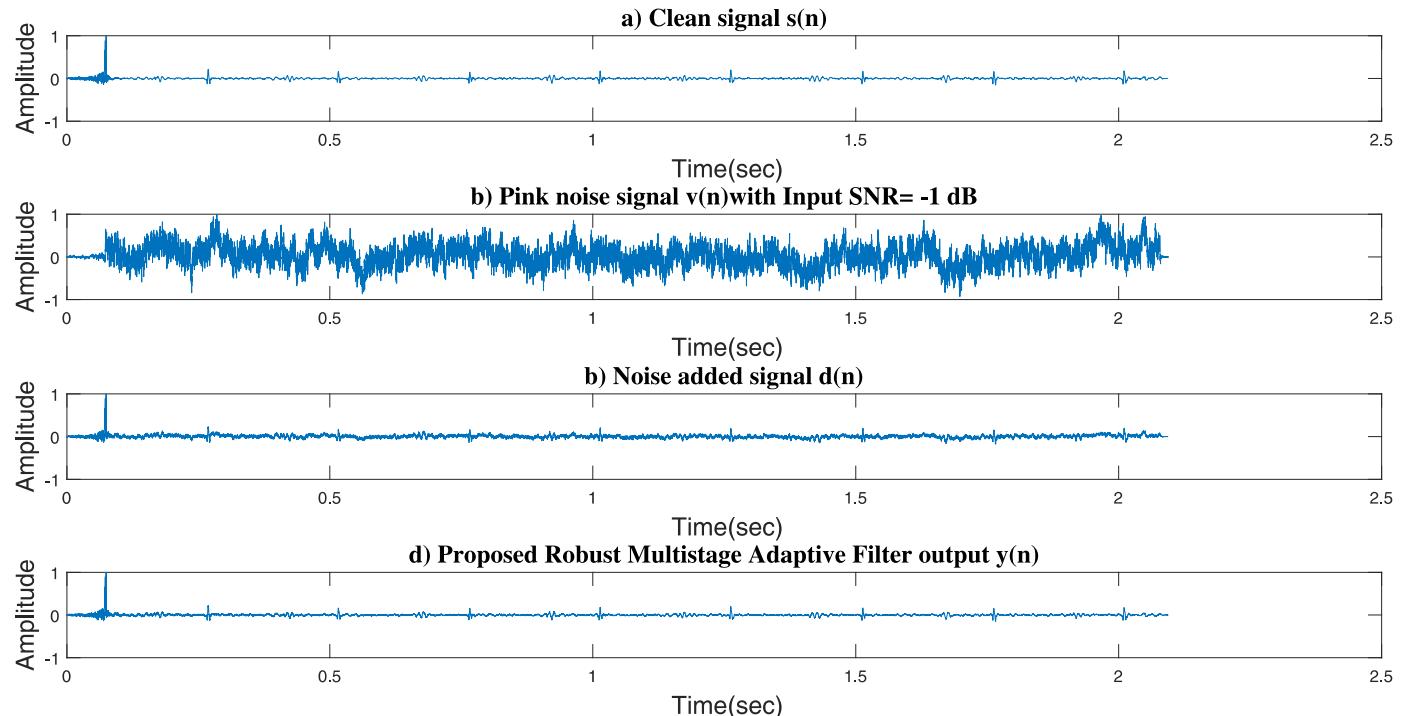


Fig. 10. Proposed robust multistage filter denoising performance for pink noise corrupted abnormal PCG signal (a) Noise-free signal (b) Additive Gaussian noise input signal to noise level of -1 dB (c) Signal with noise (d) Proposed robust multistage adaptive filter output.

5.1. Qualitative performance assessment

Normal PCG signal. Fig. 14 compares the proposed filter's denoised output with other filter models for a Gaussian noise added PCG signal with input SNR = +4 dB. Fig. 14(d) depicts that the signal is denoised effectively even in the initial

time duration. Compared to the other filter models, the output of the proposed filter converges before 0.1 s, as denoted in Fig. 14(d).

The proposed filter has comparatively good denoising capability for the input SNR of -4 dB, as illustrated in Fig. 15(d). It concludes that the proposed robust multistage adaptive filter model has the

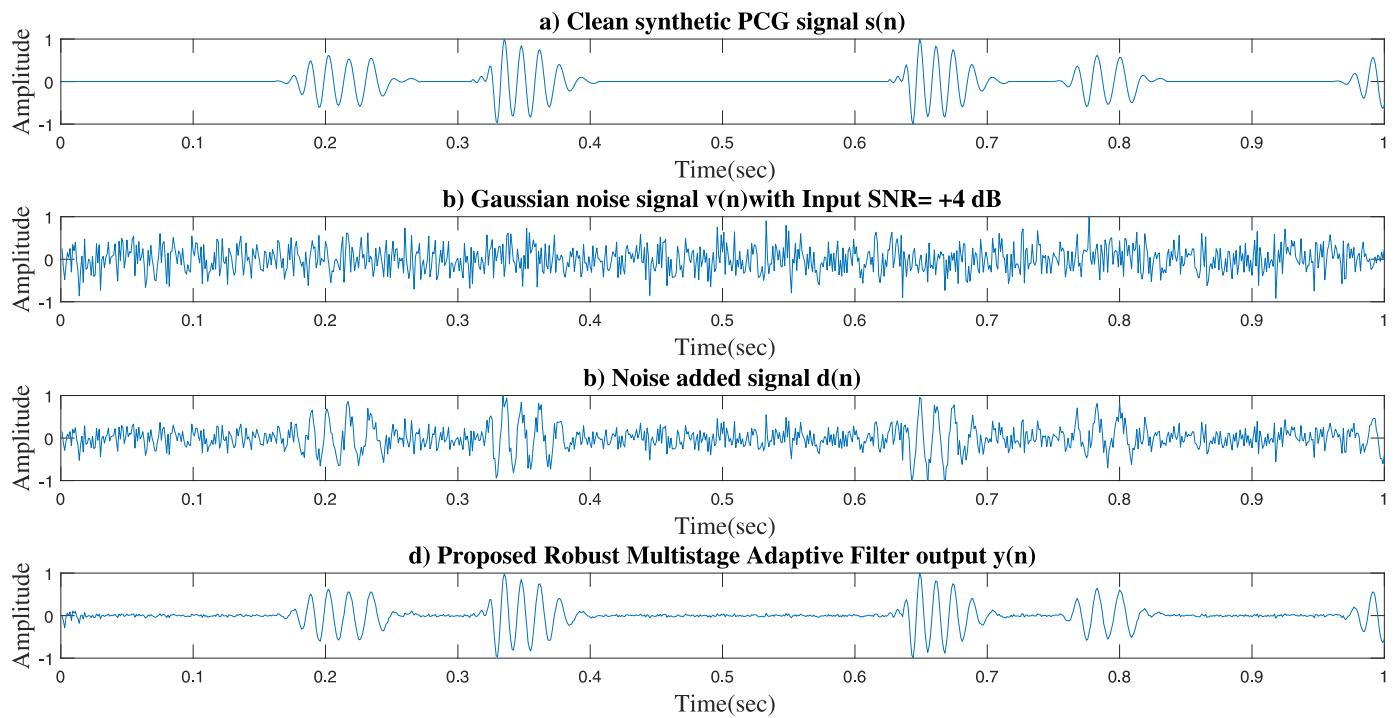


Fig. 11. Proposed robust multistage filter denoising performance for Gaussian noise corrupted synthetic PCG signal (a) Noise-free signal (b) Additive Gaussian noise input signal to noise level of +4 dB (c) Signal with noise (d) Proposed robust multistage adaptive filter output.

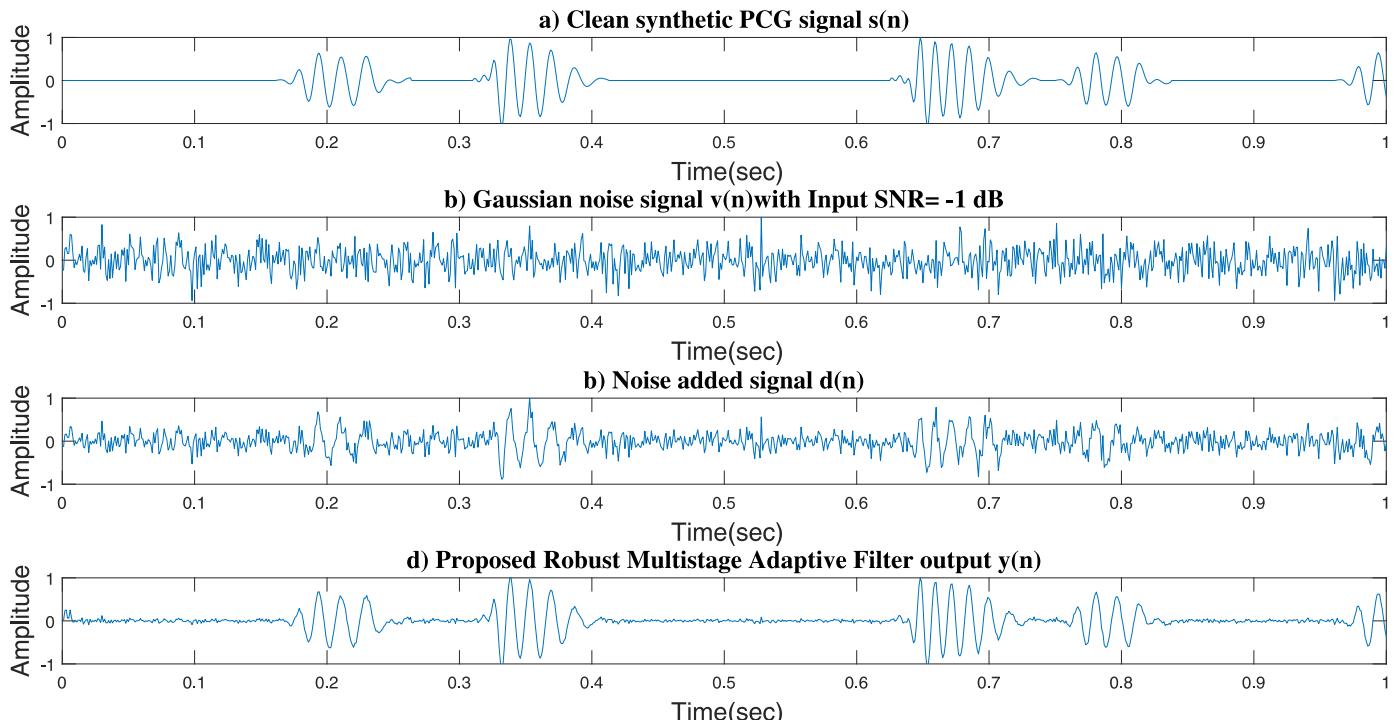


Fig. 12. Proposed robust multistage filter denoising performance of Gaussian noise corrupted synthetic PCG signal (a) Noise-free signal (b) Additive Gaussian noise input signal to noise level of -1 dB (c) Signal with noise (d) Proposed robust multistage adaptive filter output.

best noise reduction capability and proves to be very attractive in biomedical for denoising PCG signals.

Pathological PCG signals. Fig. 16 illustrates the performance of the robust multistage adaptive filter in the presence of Gaussian noise with an input SNR of +1 dB. It is noted from Fig. 16(d) that the output is more refined using the robust multistage adaptive filter.

Similarly, Fig. 17 depicts the comparative denoising performance of the proposed filter. Fig. 17(d) shows that the proposed filter outperforms the other models. We conclude from the above subjective results that the proposed Robust multistage adaptive filter performs faster and provides a high convergence speed.

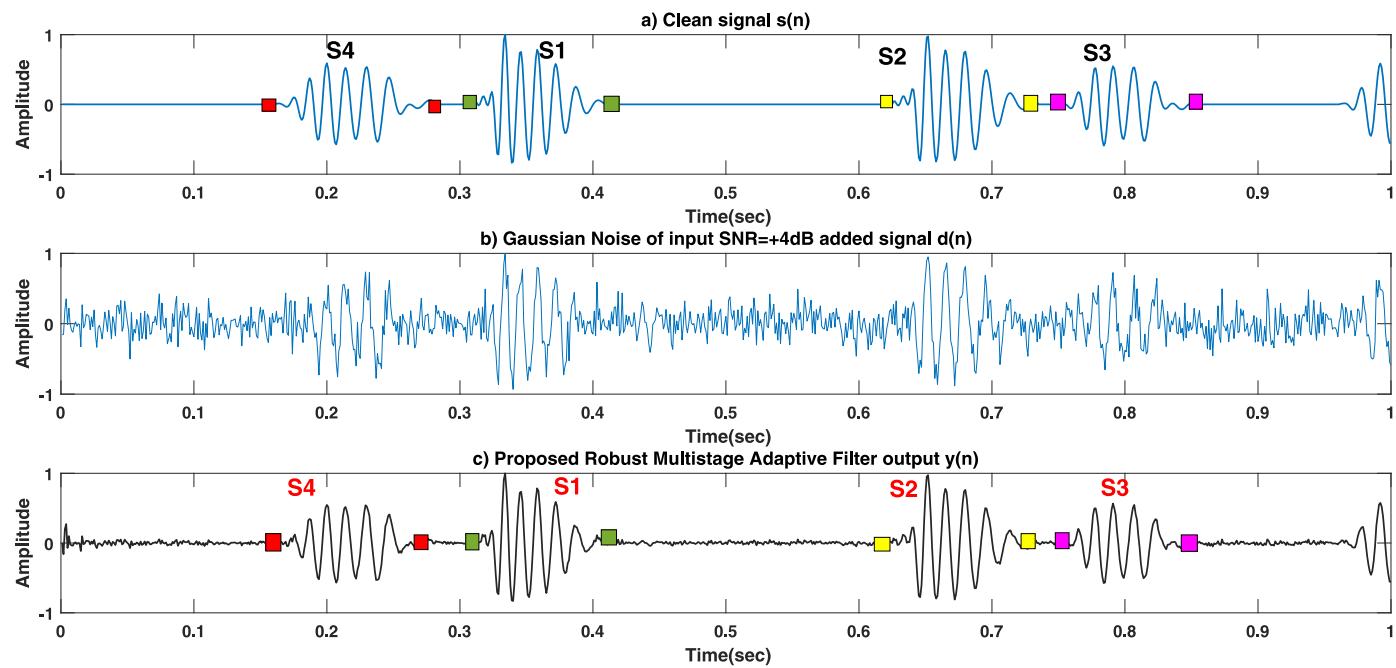


Fig. 13. Denoising performance of proposed robust multistage filter (a) Noise-free signal with S4-S1-S2-S3 waveforms (b) Noisy Signal (d) Proposed robust multistage adaptive filter indicating S4-S1-S2-S3.

Table 4

Comparison of MSE, SNR, ANR, PSNR, MAE, CC performance of the proposed robust multistage adaptive filter in Gaussian noise environment with the various existing filter models.

Signal type	Input SNR	Filter configuration	MSE	SNR dB	ANR dB	PSNR dB	CC	MAE
Normal	+4 dB	LMS filter	1.51E-04	34.0135	39.2582	38.2234	0.0052	0.9837
		Two stage LMS filter	9.46E-05	38.6545	38.9807	40.239	0.0037	0.9897
		Three stage filter	6.75E-05	42.0289	42.3528	41.7045	0.0026	0.9926
		Proposed Robust MS Filter	1.86E - 05	54.8989	60.0165	47.2939	0.0023	0.998
	-4 dB	LMS filter	1.52E-04	33.9242	42.7819	38.1847	0.0038	0.9836
		Two stage LMS filter	9.59E-05	38.5267	38.8619	40.1835	0.0027	0.9896
		Three stage LMS filter	7.05E-05	41.6034	41.9353	41.5197	0.0025	0.9923
		Proposed Robust MS Filter	2.99E - 05	50.1655	58.9507	45.2382	0.0029	0.9968
Abnormal	+1 dB	LMS filter	1.50E-04	24.2206	30.5574	38.2422	0.0062	0.9583
		Two stage LMS filter	9.61E-05	28.6636	29.4896	40.1718	0.0046	0.9726
		Three stage LMS filter	6.62E-05	32.3982	33.2322	41.7937	0.0028	0.981
		Proposed Robust MS Filter	2.90E - 05	40.644	46.9007	45.3748	0.003	0.9914
	-1 dB	LMS filter	1.55E-04	23.8578	31.5459	38.0846	0.0057	0.9566
		Two stage LMS filter	9.95E-05	28.3134	29.1124	40.0197	0.0043	0.9716
		Three stage LMS filter	6.84E-05	32.0636	32.8918	41.6484	0.0025	0.9803
		Proposed Robust MS Filter	2.36E - 05	42.7007	50.7404	46.268	0.0026	0.9935

5.2. Objective performance evaluation

Quantitative comparison of robust multistage adaptive filter with various filter models. Objective evaluation [61,62] determines the proposed filter's performance accuracy. The objective performance of the proposed filter is validated in the presence of Gaussian noise of different input noise levels. Table 4 compares the MSE, SNR, ANR, PSNR, CC, and MAE of the proposed filter configuration in the presence of Gaussian noise with state-of-art filter models. We have employed the PCG signals, both normal and abnormal, to study the proposed filter's characteristics. As observed from Table 4, columns 4 and 9, the robust multistage filter comparatively achieves minimum values of MSE and MAE. Similarly, columns 5–8 of Table 4 denote that the SNR, ANR, PSNR, and CC values are higher. Compared with various filter configurations, the proposed filter structure achieves a 10–50% reduction in MAE values and the 65–87% reduction in MSE values. Further, there is an improved SNR by 30–60%, ANR of 55–65%, and PSNR improvement

by 15–25% comparatively. The correlation between the clean signal and its estimate obtained using the proposed filter model is more than 0.99. Based on the results obtained, it is stated that the proposed robust multistage filter model denoises a noisy signal with high speed and accuracy using a simple algorithm with less number of computations.

Table 5 compares the MSE, SNR, ANR, PSNR, CC, and MAE of the Multistage adaptive filter configuration in the presence of colored pink noise with other cascaded filter models and traditional LMS adaptive filter. It is noted from Table 5 that the proposed filter gives an improvement of SNR values by 15–40%, ANR values by 15–30%, and PSNR values by 7–16%. The MSE value is reduced by 45–70%, and MAE is reduced by 20–45%. In Table 6, the output of the proposed filter model is compared with the other cascaded filter models and traditional LMS adaptive filter in terms of MSE, SNR, ANR, PSNR, CC, and MAE in the presence of Gaussian noise. From Table 6, it is inferred that the proposed filter improves SNR values by 20–50%, ANR values by 18–30%, and PSNR values by 13–

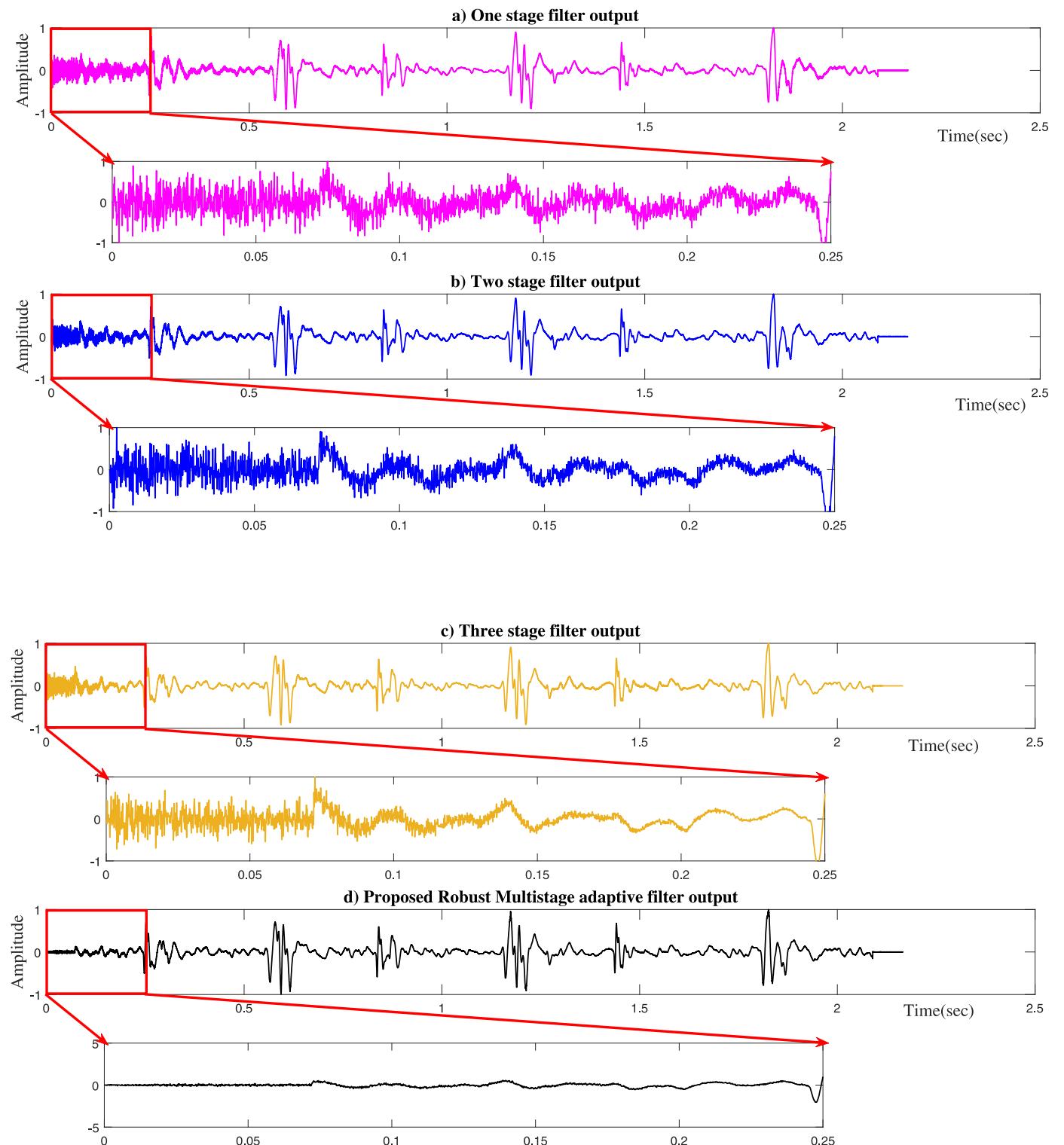


Fig. 14. Proposed robust multistage filter output performance comparison for normal PCG signal corrupted with Gaussian noise of input signal to noise level of +4 dB, (a) Output of LMS filter (b) Output of two-stage filter (c) output of three-stage LMS filter (d) output of robust multistage adaptive filter.

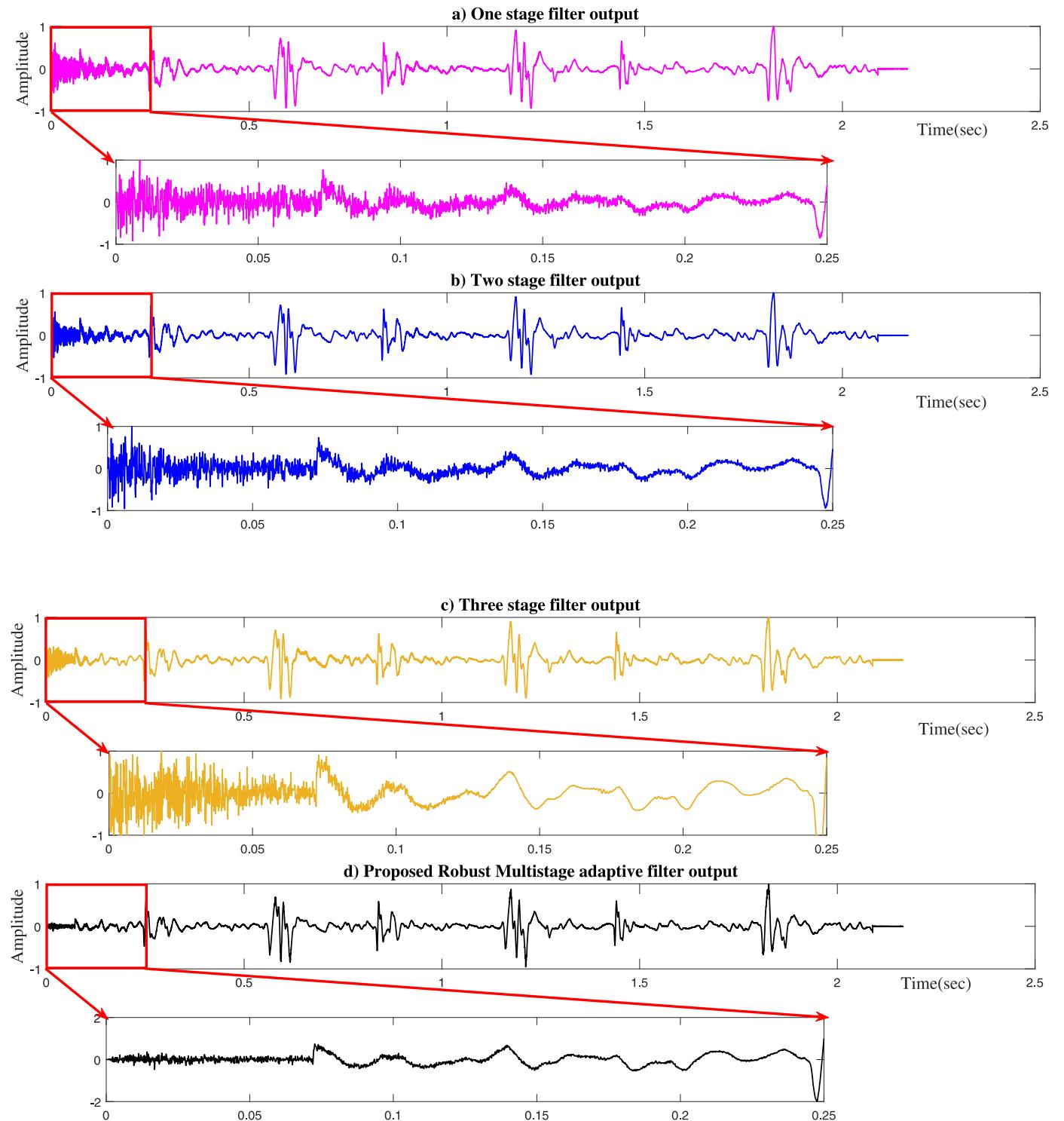


Fig. 15. Proposed robust multistage filter output performance comparison for normal PCG signal corrupted with Gaussian noise of input signal to noise level of -4 dB, (a) Output of LMS filter (b) Output of two-stage filter (c) output of three-stage LMS filter (d) output of robust multistage adaptive filter.

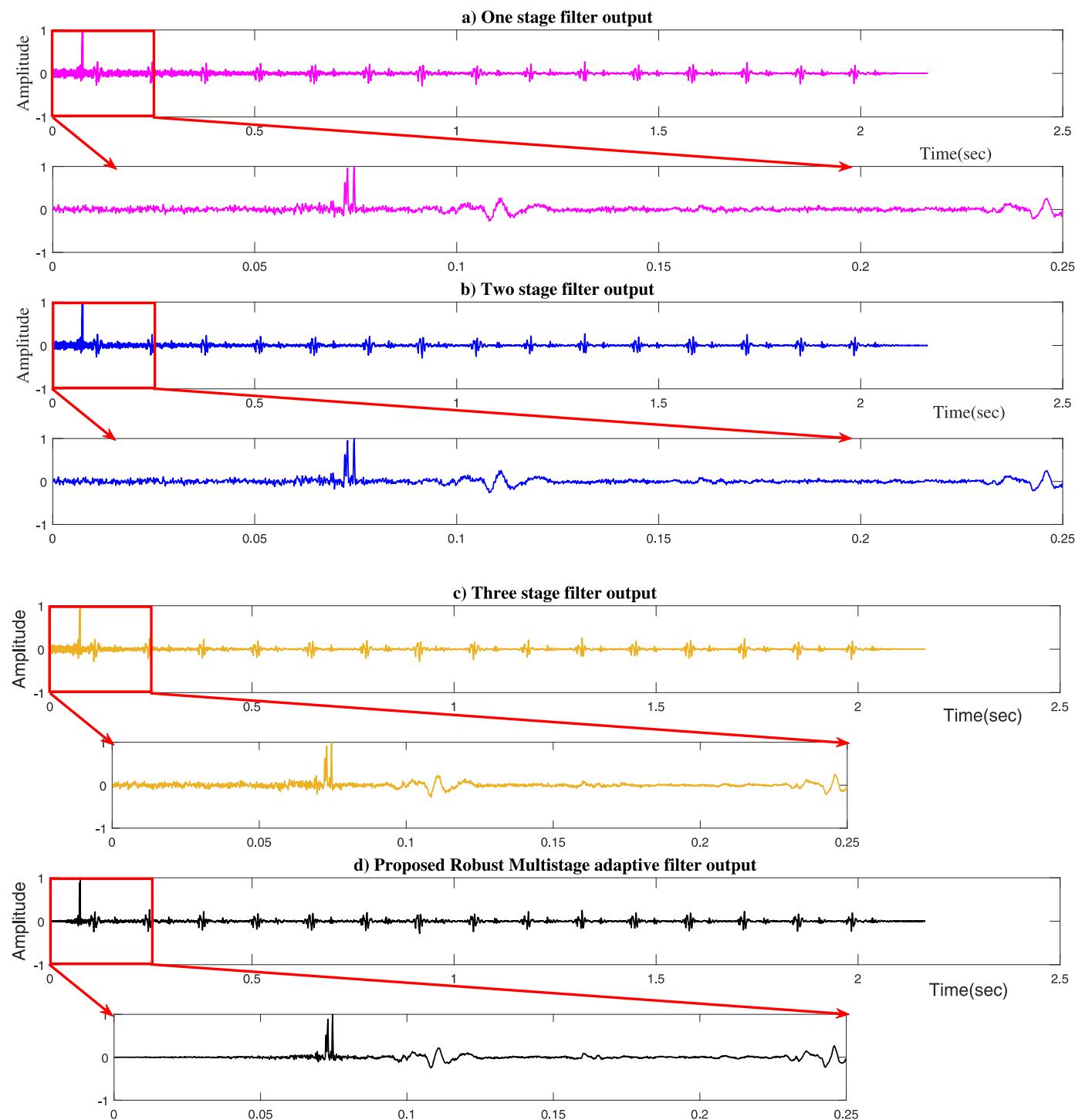


Fig. 16. Proposed robust multistage adaptive filter output performance comparison for abnormal PCG signal corrupted with Gaussian noise of input signal to noise level of +1 dB, (a) Output of LMS filter (b) Output of two-stage filter (c) output of three-stage LMS filter (d) output of robust multistage adaptive filter.

25%. The MSE value is reduced by 60–80%, and MAE is reduced by 8–35%. The CC values are above 0.99 for the proposed filter in both cases.

Computational complexity. The proposed work has employed an adaptive filter-based ANC system for PCG signal denoising. Adaptive noise cancellers are primarily used to remove noise from speech and audio signals, and we have explored their usage for denoising PCG signals. The main idea is to reduce the computational time and complexity of building cost-effective hardware for recording heart signals without noise. The number of multiplica-

tions and additions required in one algorithm iteration decides the computational complexity. We have compared the computational complexity of the LMS adaptive filter used in the proposed filter model with state-of-art filter models for adaptive noise cancellation in various fields in Table 7.

From Table 7, we infer that the conventional LMS filter requires a minimum number of computations. Therefore, we have employed an LMS adaptation algorithm for the filters in all cascaded ANC stages. Each stage's multiplications and additions are $2L + 1$ and $2L$, respectively, where L is the filter order. The num-

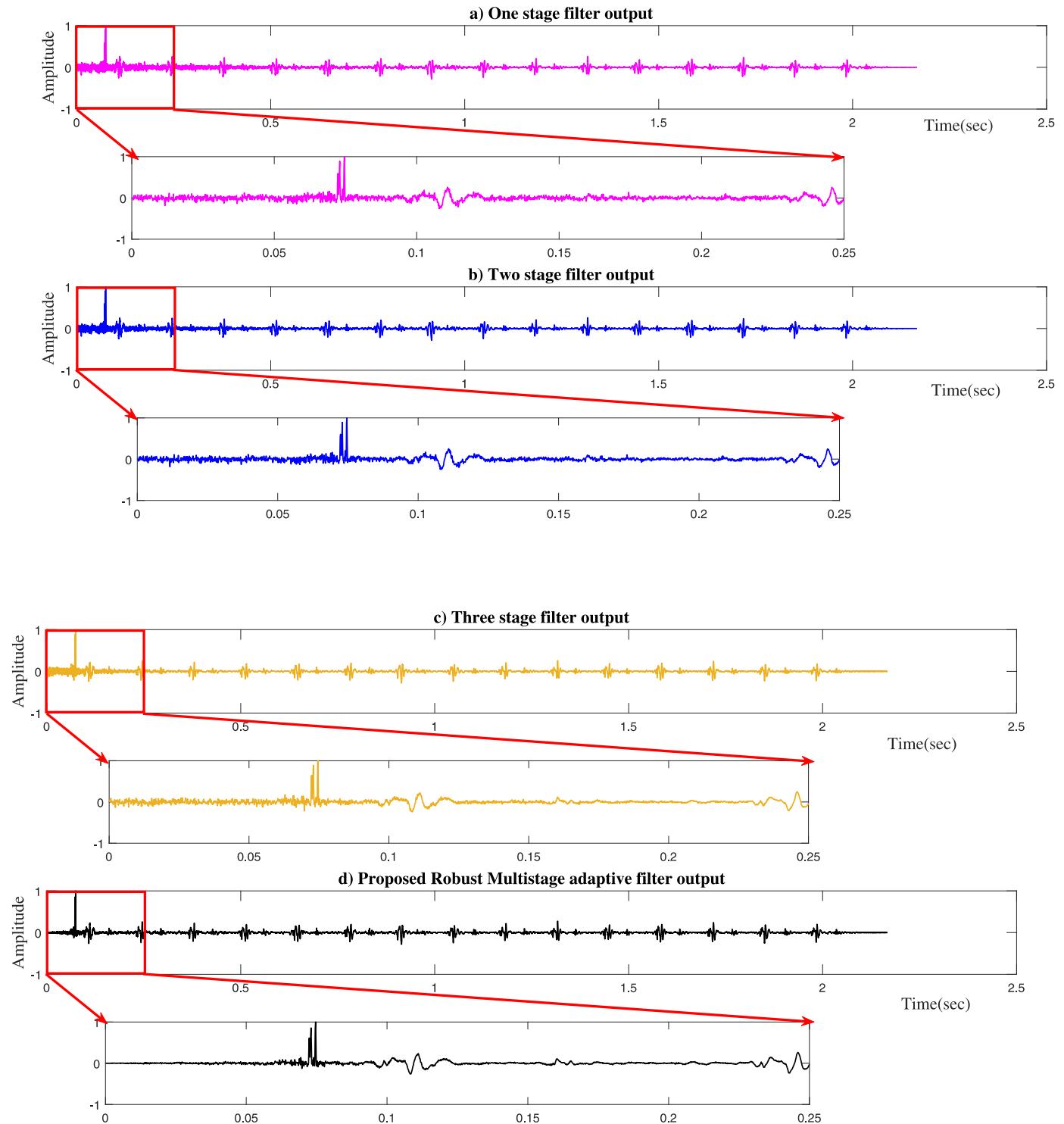


Fig. 17. Proposed robust multistage adaptive filter output performance comparison for abnormal PCG signal corrupted with Gaussian noise of input signal to noise level of -1 dB, (a) Output of LMS filter (b) Output of two-stage filter (c) output of three-stage LMS filter (d) output of robust MS adaptive filter.

ber of computations required for the proposed filter model depends on the number of stages used depending on the application. For removing Gaussian and pink noise from the experimental PCG signal, we required three filter stages in module I and an additional filter stage in module II. So the total multiplications are $4(L+1)$, and additions are $4(2L)$. The proposed filter model introduces additional computations to automatically select

the number of cascaded stages (based on the correlation coefficient) and different step sizes for each stage (based on the autocorrelation matrix). Further, the memory consumption for LMS algorithm is $2L$ as compared to RLS algorithm which has a memory consumption of $L^2 + 3L$ [63]. Since we have emphasized that a cascaded filter structure is very efficient for an ANC system, the LMS adaptive algorithm is best suited compared to the other

Table 5

Comparison of MSE, SNR, ANR, PSNR, MAE, CC performance of the proposed robust multistage adaptive filter in pink noise environment with the various existing filter models.

Signal type	Input SNR	Filter configuration	MSE	SNR dB	ANR dB	PSNR dB	CC	MAE
Normal	+5 dB	LMS filter	2.10E-04	31.0458	34.7486	36.7878	0.9782	0.0099
		Two stage LMS filter	1.34E-04	35.4846	35.7925	38.7155	0.986	0.0078
		Three stage LMS filter	1.01E-04	38.3222	38.6301	39.9479	0.9893	0.0065
		Proposed Robust MS Filter	5.64E - 05	44.171	44.3147	42.488	0.9941	0.0053
	-1 dB	LMS filter	2.53E-04	29.1497	36.2593	35.9643	0.9734	0.01
		Two stage LMS filter	1.58E-04	33.8392	34.0962	38.0009	0.9833	0.008
		Three stage LMS filter	1.33E-04	35.5869	35.8439	38.76	0.9859	0.0072
		Proposed Robust MS Filter	8.07E - 05	40.59	40.6989	40.9327	0.9915	0.0063
Abnormal	+5 dB	LMS filter	1.40E-04	15.4946	23.2387	38.5368	0.9107	0.0085
		Two stage LMS filter	9.43E-05	19.449	21.492	40.2542	0.9372	0.0069
		Three stage LMS filter	6.86E-05	22.6291	24.672	41.6352	0.9531	0.0055
		Proposed Robust MS Filter	3.12E - 05	30.5048	31.439	45.0556	0.9777	0.004
	-1 dB	LMS filter	1.17E-04	17.2679	21.6496	39.3069	0.9231	0.0082
		Two stage LMS filter	8.05E-05	21.036	22.7327	40.9434	0.9454	0.0066
		Three stage LMS filter	6.15E-05	23.7249	25.4217	42.1112	0.9576	0.0055
		Proposed Robust MS Filter	3.66E - 05	28.9023	30.0118	44.3597	0.9741	0.0042

Table 6

Comparison of MSE, SNR, ANR, PSNR, MAE, CC performance of the proposed robust multistage adaptive filter with the various existing filter models for synthetic PCG signal.

Input SNR	Filter model	MSE	SNR dB	ANR dB	PSNR dB	CC	MAE
+4 dB	LMS filter	0.0017	30.6969	35.4969	27.7418	0.9777	0.0205
	2 stage LMS filter	0.0012	34.3479	34.8685	29.3274	0.9843	0.0181
	3 stage LMS filter	8.23E-04	37.8391	38.3867	30.8486	0.9889	0.0144
	Proposed Robust MS Filter	3.48E - 04	45.6264	45.6378	34.5787	0.9948	0.0132
-1 dB	LMS filter	0.0021	29.1061	36.4644	26.8149	0.9739	0.0196
	2 stage LMS filter	0.0014	33.1775	33.7236	28.5831	0.9823	0.0187
	3 stage LMS filter	8.88E-04	36.4372	37.0454	30.5177	0.9873	0.0163
	Proposed Robust MS Filter	5.15E - 04	42.6545	42.8146	32.8849	0.9931	0.0145

Table 7

Computational cost of LMS filter in comparison with the other adaptive filtering algorithms.

Filter Structure	“*” or “/”	‘+’ or ‘-’
LMS adaptive algorithm	$2L + 2$	$2L$
NLMS adaptive algorithm [31]	$3L + 3$	$3L$
FxLMS adaptive algorithm [33]	$3L + 1$	$3L - 2$
Affine Projection Algorithm [34]	$2P^2 + 2PL + L$	$2P^2L + PL - P^2$
RLS Algorithm [32]	$3L^2 + 4L + 1$	$3L^2 + 4L$

adaptive filtering algorithms. Using other filtering techniques apart from LMS in a cascaded filter model will lead to a complex structure. Thus, we can conclude that the proposed Robust Multi-stage Adaptive Filter model provides a cost-effective and straightforward solution for PCG signal denoising in recording heart signals.

6. Conclusion

A Robust Multi-stage Adaptive Filter Model with the automatic addition of multiple stages is proposed in this study. The Robust Multi-stage Adaptive Filter architecture used in Adaptive Noise Cancellation systems successfully denoises the Gaussian noise of different input signals to noise levels from normal and abnormal PCG signals by dynamically inserting additional filter stages in sequence. As a result, the suggested filter model offers a better solution with a faster convergence time and lower MSE. According to the simulation, the suggested Robust Multi-stage Adaptive Filter outperforms conventional adaptive filter architectures, resulting in cost-effective noise reduction hardware. In the future, the suggested robust MS adaptive filter model can be tested for real-time implementation when improved convergence speed and accuracy are desired.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

S. Hannah Pauline: Data curation, Writing – original draft. **Samiappan Dhanalakshmi:** Conceptualization, Methodology, Supervision, Writing – review & editing.

References

- [1] M.A. Talbi, A new ECG denoising technique based on IWT and TVM, Circuits Syst. Signal Process. 40 (2021) 6284–6300.
- [2] A. K. Abbas, R. Bassam, Phonocardiography Signal Processing, vol. 4, 2009.
- [3] A. Alian, K. Shelley, Photoplethysmography, Best Pract. Res. Clin. Anaesthesiol. 28 (2014) 395–406.
- [4] H. Kuresan, D. Samiappan, S. Masunda, Fusion of WPT and MFCC feature extraction in Parkinson’s disease diagnosis, Technol. Health Care 27 (2019) 1–10.
- [5] D. Kumar, P. de Carvalho, M. Antunes, R.P. Paiva, J. Henriques, Noise detection during heart sound recording using periodicity signatures, Physiol. Meas. 32 (2011) 599–618.

- [6] S.H. Pauline, D. Samiappan, R. Kumar, R. Narayananamoothi, W.L. Khin, A low-cost multistage cascaded adaptive filter configuration for noise reduction in phonocardiogram signal, *J. Healthc. Eng.* 2022 (2022) 24, doi:[10.1155/2022/3039624](https://doi.org/10.1155/2022/3039624).
- [7] S. Tomassini, A. Strazza, A. Sbrollini, I. Marcantoni, M. Morettini, S. Fioretti, L. Burattini, Wavelet filtering of fetal phonocardiography: a comparative analysis, *Math. Biosci. Eng.* 16 (5) (2019) 6034–6046, doi:[10.3934/mbe.2019302](https://doi.org/10.3934/mbe.2019302).
- [8] A.H. Salman, N. Ahmadi, R. Mengko, A.Z.R. Langi, T.L.R. Mengko, Performance comparison of denoising methods for heart sound signal, in: 2015 International Symposium on Intelligent Signal Processing and Communication Systems (ISPACS), 2015, pp. 435–440, doi:[10.1109/ISPACS.2015.7432811](https://doi.org/10.1109/ISPACS.2015.7432811).
- [9] S. Ghosh, R.N. Ponnallagu, R. Tripathy, Heart sound data acquisition and pre-processing techniques: a review, 2020.
- [10] S. Sanei, M. Ghodsi, H. Hassani, An adaptive singular spectrum analysis approach to murmur detection from heart sounds, *Med. Eng. Phys.* 33 (3) (2011) 362–367, doi:[10.1016/j.medengphy.2010.11.004](https://doi.org/10.1016/j.medengphy.2010.11.004).
- [11] S. Patidar, R.B. Pachori, Segmentation of cardiac sound signals by removing murmurs using constrained tunable-Q wavelet transform, *Biomed. Signal Process. Control* 8 (6) (2013) 559–567, doi:[10.1016/j.bspc.2013.05.004](https://doi.org/10.1016/j.bspc.2013.05.004).
- [12] V.G. Sujadevi, K.P. Soman, S.S. Kumar, N. Mohan, A.S. Arunjith, Denoising of phonocardiogram signals using variational mode decomposition, in: 2017 International Conference on Advances in Computing, Communications and Informatics (ICACCI), 2017, pp. 1443–1446, doi:[10.1109/ICACCI.2017.8126043](https://doi.org/10.1109/ICACCI.2017.8126043).
- [13] M.B. Figueiredo, A. de Almeida, B. Ribeiro, Wavelet decomposition and singular spectrum analysis for electrical signal denoising, in: 2011 IEEE International Conference on Systems, Man, and Cybernetics, 2011, pp. 3329–3334, doi:[10.1109/ICSMC.2011.6084183](https://doi.org/10.1109/ICSMC.2011.6084183).
- [14] T. Omari, F. Berekshi-Reguig, An automatic wavelet denoising scheme for heart sounds, *Int. J. Wavelets, Multiresolut. Inf. Process.* 13 (03) (2015) 1550016, doi:[10.1142/S0219691315500162](https://doi.org/10.1142/S0219691315500162).
- [15] A. Almasi, M.-B. Shamsollahi, L. Senhadji, Bayesian denoising framework of phonocardiogram based on a new dynamical model, *Innov. Res. Biomed. Eng.* 34 (3) (2013) 214–225, doi:[10.1016/j.irbm.2013.01.017](https://doi.org/10.1016/j.irbm.2013.01.017).
- [16] M.S. Nazemi, H. Hakimnejad, Z. Azimifar, PCG denoising using AR-based Kalman filter, in: 2021 29th Iranian Conference on Electrical Engineering (ICEE), 2021, pp. 902–906, doi:[10.1109/ICEE52715.2021.9544365](https://doi.org/10.1109/ICEE52715.2021.9544365).
- [17] S.M. Debbal, F. Berekshi-Reguig, Filtering and classification of phonocardiogram signals using wavelet transform, *J. Med. Eng. Technol.* 32(1) (2008) 7521–7532, doi:[10.1080/03091900600750348](https://doi.org/10.1080/03091900600750348).
- [18] S.K. Ghosh, R.K. Tripathy, R.N. Ponnallagu, Evaluation of performance metrics and denoising of PCG signal using Wavelet Based Decomposition, in: 2020 IEEE 17th India Council International Conference (INDICON), 2020, pp. 1–6, doi:[10.1109/INDICON49873.2020.9342464](https://doi.org/10.1109/INDICON49873.2020.9342464).
- [19] R. Potdar, D. Meshram, D. Kumar, Optimal parameter selection for DWT based PCG denoising, *Turkish J. Comput. Math. Educ. (TURCOMAT)* 12 (2021) 7521–7532.
- [20] D.L. Donoho, I.M. Johnstone, Ideal spatial adaptation by wavelet shrinkage, *Biometrika* 81 (3) (1994) 425–455, doi:[10.1093/biomet/81.3.425](https://doi.org/10.1093/biomet/81.3.425).
- [21] D. Gradowski, G. Redlarski, Wavelet-based denoising method for real phonocardiography signal recorded by mobile devices in noisy environment, *Comput. Biol. Med.* 52 (2014) 119–129, doi:[10.1016/j.combimed.2014.06.011](https://doi.org/10.1016/j.combimed.2014.06.011).
- [22] Y.-W. Bai, C.-L. Lu, The embedded digital stethoscope uses the adaptive noise cancellation filter and the type I Chebyshev IIR bandpass filter to reduce the noise of the heart sound, in: Proceedings of 7th International Workshop on Enterprise networking and Computing in Healthcare Industry, 2005. HEALTHCOM 2005., 2005, pp. 278–281, doi:[10.1109/HEALTH.2005.1500459](https://doi.org/10.1109/HEALTH.2005.1500459).
- [23] D. Song, L. Jia, Y. Lu, L. Tao, Heart sounds monitor and analysis in noisy environments, in: 2012 International Conference on Systems and Informatics (IC-SAI2012), 2012, pp. 1677–1681, doi:[10.1109/ICSAI.2012.6223364](https://doi.org/10.1109/ICSAI.2012.6223364).
- [24] Z. Tan, J. Ma, B. Fu, M. Dong, Extract qualified heart sound in varying environment using parallel-training LMS algorithm, in: 2015 IEEE International Conference on Digital Signal Processing (DSP), 2015, pp. 407–411, doi:[10.1109/ICDSP.2015.7251903](https://doi.org/10.1109/ICDSP.2015.7251903).
- [25] A.H. Sayed, *Fundamentals of Adaptive Filtering*, first ed., Wiley Interscience, 2003.
- [26] A. Frech, M. Klügel, P. Russer, Adaptive filtering for noise cancellation and signal analysis in real-time, in: 2013 European Microwave Conference, 2013, pp. 1123–1126, doi:[10.23919/EuMC.2013.6686859](https://doi.org/10.23919/EuMC.2013.6686859).
- [27] H. Pauline, D. Samiappan, R. Kumar, A. Anand, D.A. Kar, Variable tap-length non-parametric variable step-size NLMS adaptive filtering algorithm for acoustic echo cancellation, *Appl. Acoust.* 159 (2020) 107074, doi:[10.1016/j.apacoust.2019.107074](https://doi.org/10.1016/j.apacoust.2019.107074).
- [28] J.D.K. Abel, D. Samiappan, R. Kumar, P. Kumar, Multiple sub-filter adaptive noise canceller for fetal ECG extraction, in: International Conference on recent trends in advanced computing, Procedia, Computer Science, vol.165, Elsevier, 2019, pp. 182–188.
- [29] R.G. Soumya, N. Naveen, M.J. Lal, Application of adaptive filter using adaptive line enhancer techniques, in: 2013 Third International Conference on Advances in Computing and Communications, 2013, pp. 165–168, doi:[10.1109/ICACC.2013.39](https://doi.org/10.1109/ICACC.2013.39).
- [30] Q. Ling, M.A. Iqbal, P. Kumar, Optimized LMS algorithm for system identification and noise cancellation, *J. Intell. Syst.* 30 (1) (2021) 487–498, doi:[10.1515/jisy-s-2020-0081](https://doi.org/10.1515/jisy-s-2020-0081).
- [31] M. Salah, M. Dessouky, B. Abdelhamid, Design and implementation of an improved variable step-size NLMS-based algorithm for acoustic noise cancellation, *Circuits Syst. Signal Process.* 41 (2022) 551–578, doi:[10.1007/s00034-021-01796-5](https://doi.org/10.1007/s00034-021-01796-5).
- [32] V. Tejaswi, A. Surendar, N. Srikantha, Simulink implementation of RLS algorithm for resilient artefacts removal in ECG signal, *Int. J. Adv. Intell. Paradig.* 16 (3–4) (2020), doi:[10.1504/ijap.2020.107529](https://doi.org/10.1504/ijap.2020.107529).
- [33] M.W. Munir, W.H. Abdulla, On FxLMS scheme for active noise control at remote location, *IEEE Access* 8 (2020) 214071–214086, doi:[10.1109/ACCESS.2020.3040718](https://doi.org/10.1109/ACCESS.2020.3040718).
- [34] A. Gonzalez, M. Ferrer, F. Albu, M. de Diego, Affine projection algorithms: evolution to smart and fast algorithms and applications, in: 2012 Proceedings of the 20th European Signal Processing Conference (EUSIPCO), 2012, pp. 1965–1969.
- [35] Y. Feiran, J. Yang, A comparative survey of fast affine projection algorithms, *Digit. Signal Process.* 83 (2018) 297–322, doi:[10.1016/j.dsp.2018.09.004](https://doi.org/10.1016/j.dsp.2018.09.004).
- [36] R.H. Kwong, E.W. Johnston, A variable step size LMS algorithm, *IEEE Trans. Signal Process.* 40 (7) (1992) 1633–1642, doi:[10.1109/78.143435](https://doi.org/10.1109/78.143435).
- [37] A.-G. Rusu, C. Paleologu, J. Benesty, S. Ciocină, A variable step size normalized least-mean-square algorithm based on data reuse, *Algorithms* 15 (4) (2022), doi:[10.3390/a15040111](https://doi.org/10.3390/a15040111).
- [38] C. Paleologu, J. Benesty, S. Ciocină, Robust variable step-size affine projection algorithm suitable for acoustic echo cancellation, in: 2008 16th European Signal Processing Conference, 2008, pp. 1–5.
- [39] Z. Yong-Feng, S. Fang-Fang, Z. Jun, W. Zhen, Optimal step-size of pseudo affine projection algorithm, *Appl. Math. Comput.* 273 (2016) 82–88, doi:[10.1016/j.amc.2015.09.059](https://doi.org/10.1016/j.amc.2015.09.059).
- [40] S. Haykin, B. Widrow, *Least-Mean-Square Adaptive Filters*, first ed., Wiley, Newyork, 2003.
- [41] F. Albu, H.K. Kwan, Fast block exact Gauss-Seidel pseudo affine projection algorithm, *Electron. Lett.* 40 (2004) 1451–1453, doi:[10.1049/el:20046320](https://doi.org/10.1049/el:20046320).
- [42] F. Albu, C. Paleologu, J. Benesty, Y.V. Zakharov, Variable step size dichotomous coordinate descent affine projection algorithm, in: IEEE EUROCON 2009, 2009, pp. 1364–1369, doi:[10.1109/EURCON.2009.5167817](https://doi.org/10.1109/EURCON.2009.5167817).
- [43] A.D. Pouliarikas, *Adaptive Filtering: Fundamentals of Least Mean Squares with MATLAB®*, first ed., CRC Press, Taylor and Francis Group, 2014.
- [44] N. Ahmed, D. Hush, G. Elliott, R. Fogler, Detection of multiple sinusoids using an adaptive cascaded structure, in: ICASSP '84. IEEE International Conference on Acoustics, Speech, and Signal Processing, vol. 9, 1984, pp. 199–202, doi:[10.1109/ICASSP.1984.1172529](https://doi.org/10.1109/ICASSP.1984.1172529).
- [45] P. Prandoni, M. Vetterli, An FIR cascade structure for adaptive linear prediction, *IEEE Trans. Signal Process.* 46 (9) (1998) 2566–2571, doi:[10.1109/78.709548](https://doi.org/10.1109/78.709548).
- [46] R. Yu, C.C. Ko, Lossless compression of digital audio using cascaded RLS-LMS prediction, *IEEE Trans. Speech Audio Process.* 11 (6) (2003) 532–537, doi:[10.1109/TSA.2003.818111](https://doi.org/10.1109/TSA.2003.818111).
- [47] H. Huang, S. Rahardja, X. Lin, R. Yu, P. Franti, Cascaded RLS-LMS prediction in MPEG-4 lossless audio coding, in: 2006 IEEE International Conference on Acoustics Speech and Signal Processing Proceedings, vol. 5, 2006, p. V, doi:[10.1109/ICASSP.2006.1661242](https://doi.org/10.1109/ICASSP.2006.1661242).
- [48] X. Sun, S.M. Kuo, Active narrowband noise control systems using cascading adaptive filters, *IEEE Trans. Audio, Speech Lang. Process.* 15 (2) (2007) 586–592, doi:[10.1109/TASL.2006.881680](https://doi.org/10.1109/TASL.2006.881680).
- [49] J. Freudenberg, S. Stenzel, Suppression of engine noise harmonics using cascaded LMS filters, in: *Speech Communication; 10. ITG Symposium*, 2012, pp. 1–4.
- [50] H. Kim, S. Kim, N. Van Helleputte, T. Berset, D. Geng, I. Romero, J. Penders, C. Van Hoof, R.F. Yazicioglu, Motion artifact removal using cascade adaptive filtering for ambulatory ECG monitoring system, in: 2012 IEEE Biomedical Circuits and Systems Conference (BioCAS), 2012, pp. 160–163, doi:[10.1109/BioCAS.2012.6418472](https://doi.org/10.1109/BioCAS.2012.6418472).
- [51] A. Mehmood, M.I. Baig, E. ul Haq, L. Aslam, Artifacts removal from ECG signal using a multistage MNLMS adaptive algorithm, *Int. J. Signal Process., Image Process. Pattern Recognit.* 10 (11) (2017) 13–22.
- [52] S. Dixit, D. Nagaria, Design and analysis of cascaded LMS adaptive filters for noise cancellation, *Circuits, Syst., Signal Process.* 36 (2017), doi:[10.1007/s00034-016-0332-5](https://doi.org/10.1007/s00034-016-0332-5).
- [53] A. Maurya, Cascade-cascade least mean square (LMS) adaptive noise cancellation, *Circuits, Syst., Signal Process.* 37 (2018), doi:[10.1007/s00034-017-0731-2](https://doi.org/10.1007/s00034-017-0731-2).
- [54] A. Awad, Impulse noise reduction in audio signal through multi-stage technique, *Eng. Sci. Technol., Int. J.* 22 (2018), doi:[10.1016/j.estch.2018.10.008](https://doi.org/10.1016/j.estch.2018.10.008).
- [55] H. Pauline, D. Samiappan, R. Kumar, Variable-stage cascaded adaptive filter technique for signal de-noising application, *Circuits, Syst., Signal Process.* (2021), doi:[10.1007/s00034-021-01868-6](https://doi.org/10.1007/s00034-021-01868-6).
- [56] D. Bismor, K. Czyz, Z. Ogonowski, Review and comparison of variable step-size LMS algorithms, *Int. J. Acoust. Vib.* 21 (2016) 24–39, doi:[10.20855/ijav.2016.1392](https://doi.org/10.20855/ijav.2016.1392).
- [57] C. Liu, D. Springer, Q. Li, B. Moody, R. Abad, F. Chorro, F. Castells Ramon, J. Roig, I. Silva, A. Johnson, Z. Syed, S. Schmidt, C. Papadaniil, L. Hadjileontiadis, H. Naseri, A. Moukadem, A. Dieterlen, C. Brandt, H. Tang, G. Clifford, An open access database for the evaluation of heart sound algorithms, *Physiol. Meas.* 37 (2016) 2181–2213, doi:[10.1088/0967-3334/37/12/2181](https://doi.org/10.1088/0967-3334/37/12/2181).
- [58] A. Goldberger, L. Amaral, L. Glass, S. Havlin, J. Hausdorff, P. Ivanov, R. Mark, J. Mietus, G. Moody, C.-K. Peng, H. Stanley, P. Physiobank, Components of a new research resource for complex physiologic signals, *PhysioNet* 101 (2000) 215–220.

- [59] G. Clifford, C. Liu, B. Moody, D. Springer, I. Silva, Q. Li, R. Mark, Classification of normal/abnormal heart sound recordings: the physionet/computing in cardiology challenge 2016, 2016. 10.22489/CinC.2016.179-154
- [60] M. Homaéinezhad, P. Sabetan, A. Feizollahi, A. Ghaffari, R. Rahmani, Parametric modelling of cardiac system multiple measurement signals: an open-source computer framework for performance evaluation of ECG, PCG and ABP event detectors, *J. Med. Eng. Technol.* 36 (2012) 117–134, doi:[10.3109/03091902.2011.645945](https://doi.org/10.3109/03091902.2011.645945).
- [61] D. Samiappan, V. Chakrapani, Classification of ultrasound carotid artery images using texture features, *Int. Rev. Comput. Softw.* 8 (2013) 933–940.
- [62] D. Samiappan, S. Arunachalam, V. Chakrapani, Classification of multi-category abnormalities in ultrasound carotid artery images using an extreme learning machine, *Int. J. Appl. Eng. Res.* 9 (2014) 5106–5112.
- [63] S. Ciolino, M. Ghavami, A. Aghvami, On the use of wavelet packets in ultra wideband pulse shape modulation systems, *IEICE Trans.* 88-A (2005) 2310–2317, doi:[10.1093/ietsfc/e88-a.9.2310](https://doi.org/10.1093/ietsfc/e88-a.9.2310).