High-dimensional data and dimensionality reduction

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We'll be talking about…

I. Data analysis
II. Properties of high-dimensional data
III. Some vocabulary
IV. Dimensionality reduction methods
V. Examples
VI. Discussion

Era of massive data collection

- Usual data matrix: **D** rows, **N** cols
- Rows give different attributes/measurements
- Columns give different observations

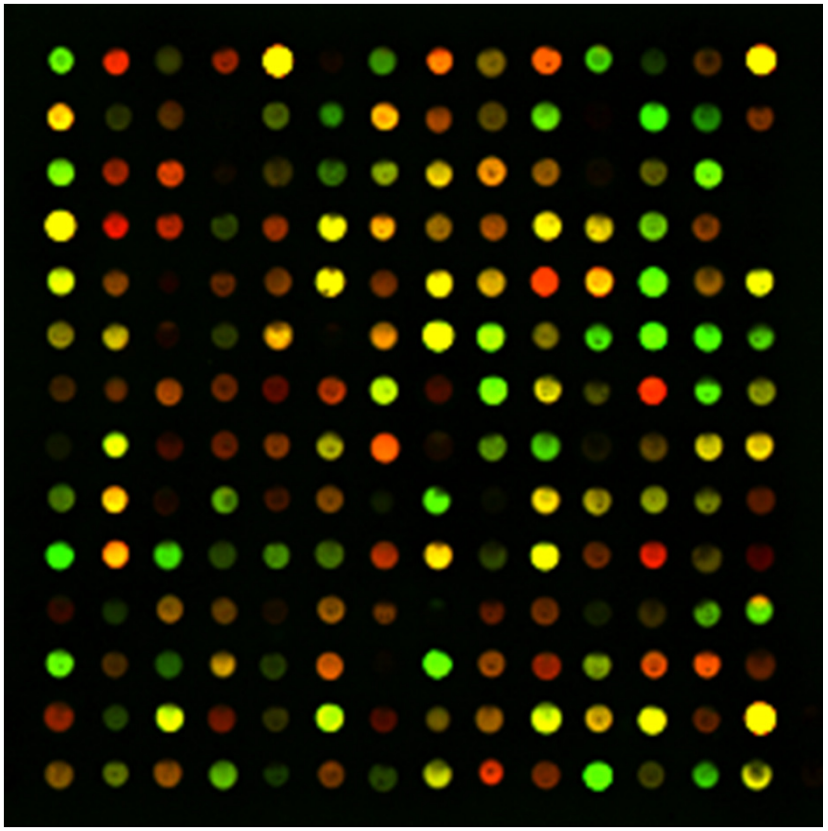
N points in a D-dimensional space

- Term-document data
  - **N** is the number of documents ~ millions
  - **D** is the number of terms ~ thousands
- Consumer preference data (Netflix, Amazon)
  - **N** is the number of individuals ~ millions
  - **D** is the number of products ~ thousands

Problem

- Assumption: **D < N** and **N → ∞**
- Many results fail if **D > N**
- We might have **D → ∞, N** fixed
- Very large number of measurements
  - relatively few instances of the event
- a.k.a. the **large p, small n** problem
  - a.k.a. **High Dimension Low Sample Size (HDLSS)** problem

Example



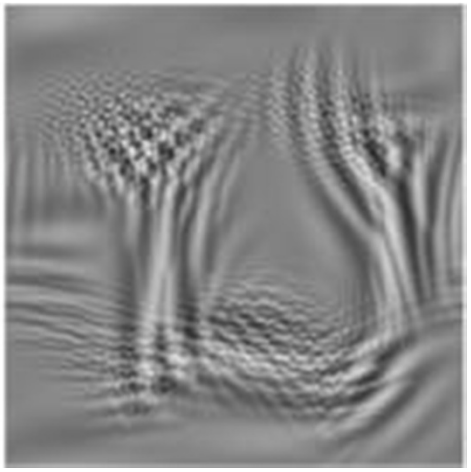
- Breast cancer gene expression data
- Number of measured genes:
  - **D = 6,128**
- Number of tumor samples:
  - **N = 49**

Another example

- Biscuit dough data
- Number of NIR reflectance measures:
  - **D = 23,625**
- Number of dough samples:
  - **N = 39**

Another example: computer vision

- Scene recognition
- Raw Gist feature dimension:
  - **D ~ 300 - 500**
- Number of color image samples:
  - **N ~ 2600**



And another:

- Video concept detection
- Multimedia feature vector:
  - **D ~ 2896**
- Number of video samples:
  - **N ~ 136**

So what?

- **D** is high in our data analysis problems…
- Properties of high dimensional data should be considered
  - Hughes phenomenon
  - Empty space phenomenon
  - Concentration phenomenon

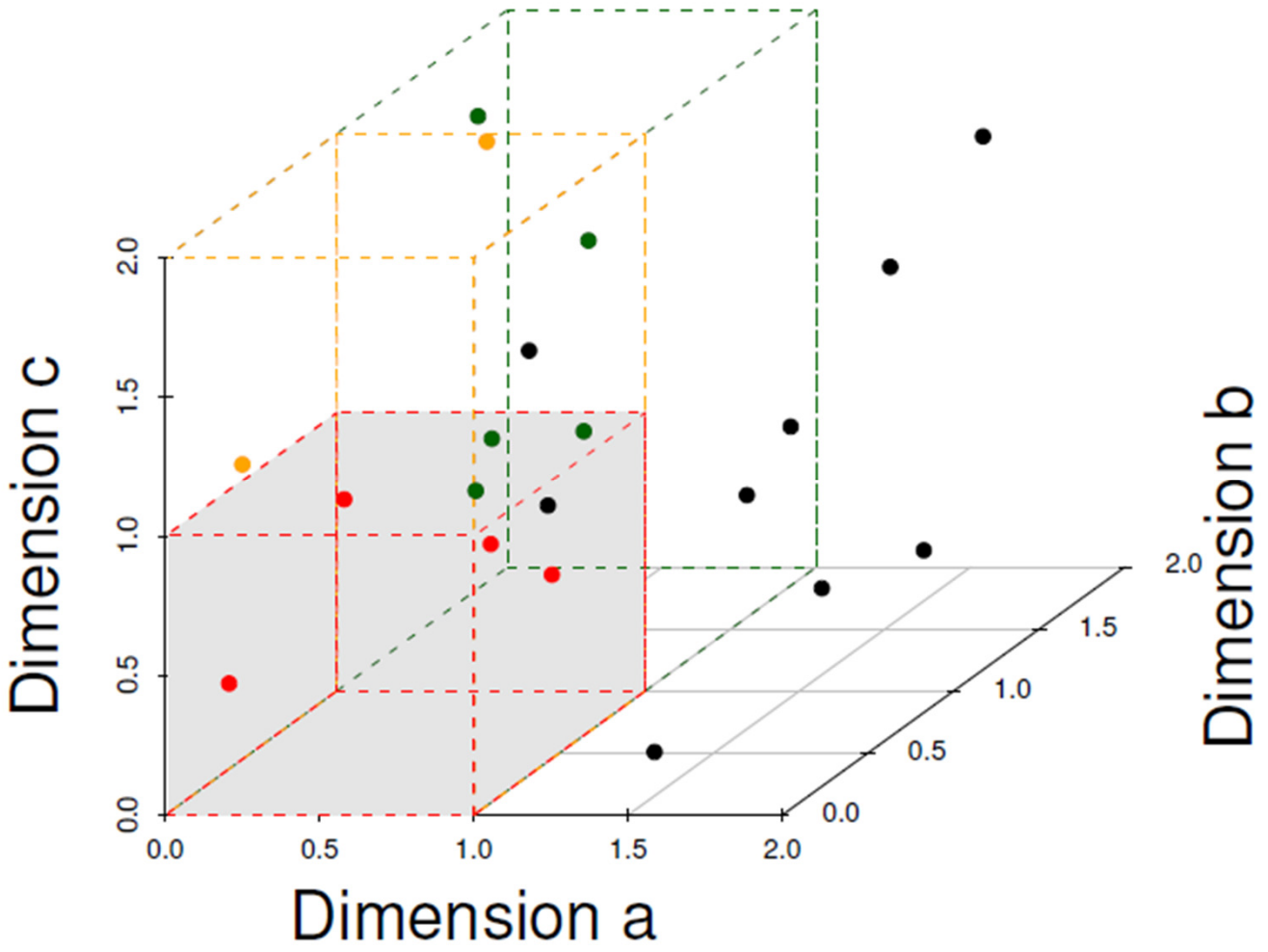
Hughes phenomenon

- a.k.a. **the curse of dimensionality** (R. Bellman, 1961)
- Unit cube in 10 dimensions, discretized with 1/10 spacing → **10^{10}**
- Unit cube in 20 dimensions, same accuracy → **10^{20}** points
- **Number of samples** needed **grows exponentially** with dimension

Empty space phenomenon

- Follows from **COD** and the fact that:
- amount of available data is **limited**
- → high-dimensional space is **sparse**
- You expect an increase in discrimination power (by employing more features)
  - but you lose accuracy
  - due to overfitting

Empty space phenomenon



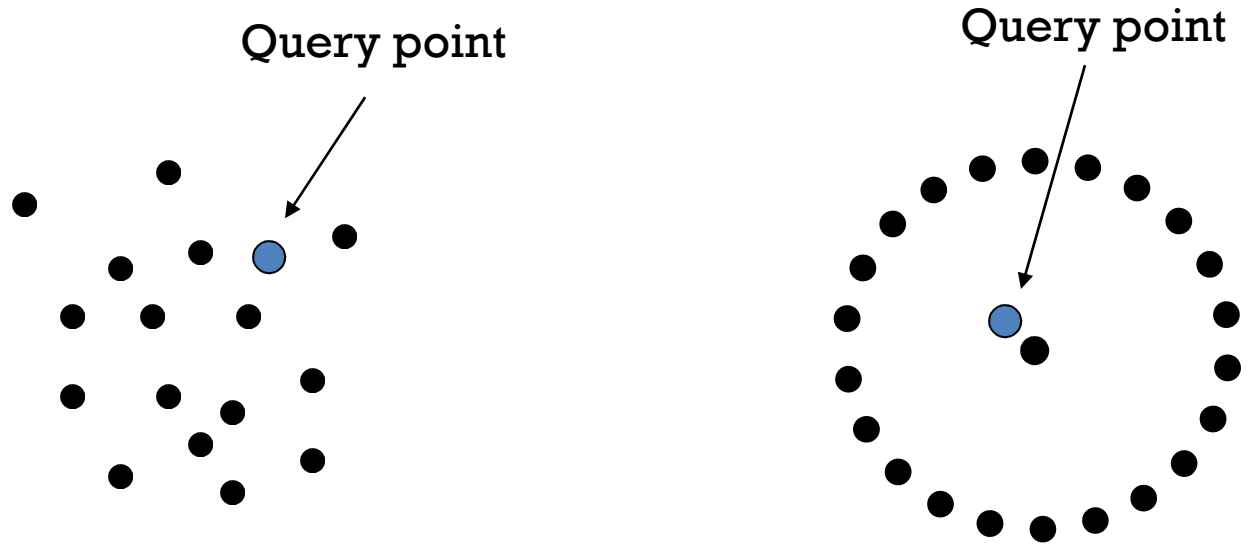
(L. Parsons et al., 2004)

Concentration phenomenon

- "When is nearest neighbor meaningful", (Beyer et al., 1999)
- In high dimensions, under certain conditions,
  - **distance to nearest neighbor** approaches **distance to farthest neighbor**
  - contrast in distances is lost

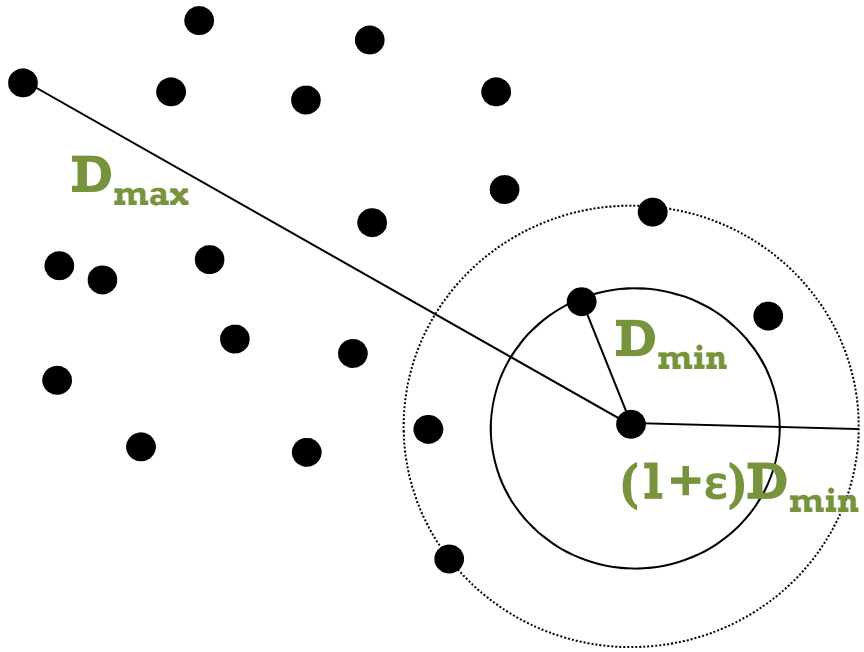
Concentration: continued

- When this happens, there is **no utility** in finding the "nearest neighbor"



Concentration: continued

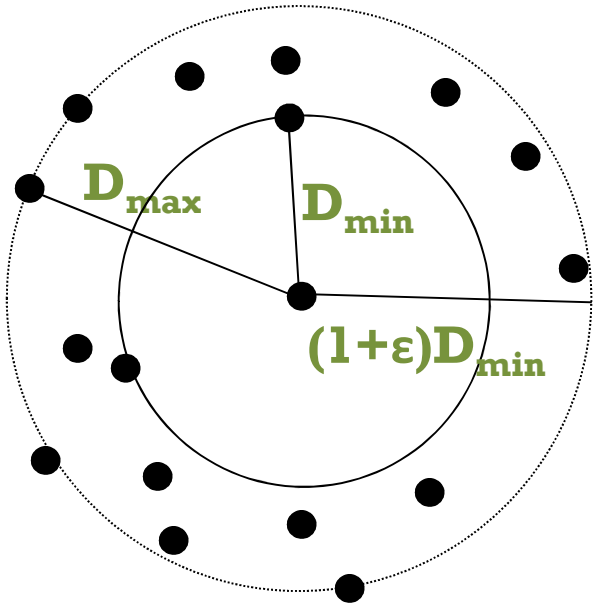
- Definition. **Stable query**



Stable query

Concentration: continued

- Definition. **Unstable query**



$$DMAX \leq (1 + \varepsilon)\ DMIN$$

Concentration: continued

- It is shown that (**under some conditions**), for any fixed $\varepsilon > 0$,
  - as dimensionality rises,
  - the probability that a query is unstable
  - approaches 1

$$\lim_{D \to \infty} \Pr[DMAX_D \leq (1 + \varepsilon)\, DMIN_D] = 1$$

Concentration: i.i.d. case

- Here are some results for i.i.d. dimensions
- Assume:
  - random vector $\mathbf{y} = [y_1, \ldots, y_D]^T$
  - y_i's are i.i.d.
- We'll show:
  - successful drawings of such random vectors yield almost the same norm

Concentration: continued

$$\mu_{\|\mathbf{y}\|} = \sqrt{aD - b} + \mathcal{O}(D^{-1})$$

$$\sigma^2_{\|\mathbf{y}\|} = b + \mathcal{O}(D^{-1/2})$$

The norm of random vectors grows proportionally to $\mathbf{D^{1/2}}$, but the variance remains constant for sufficiently large **D**.

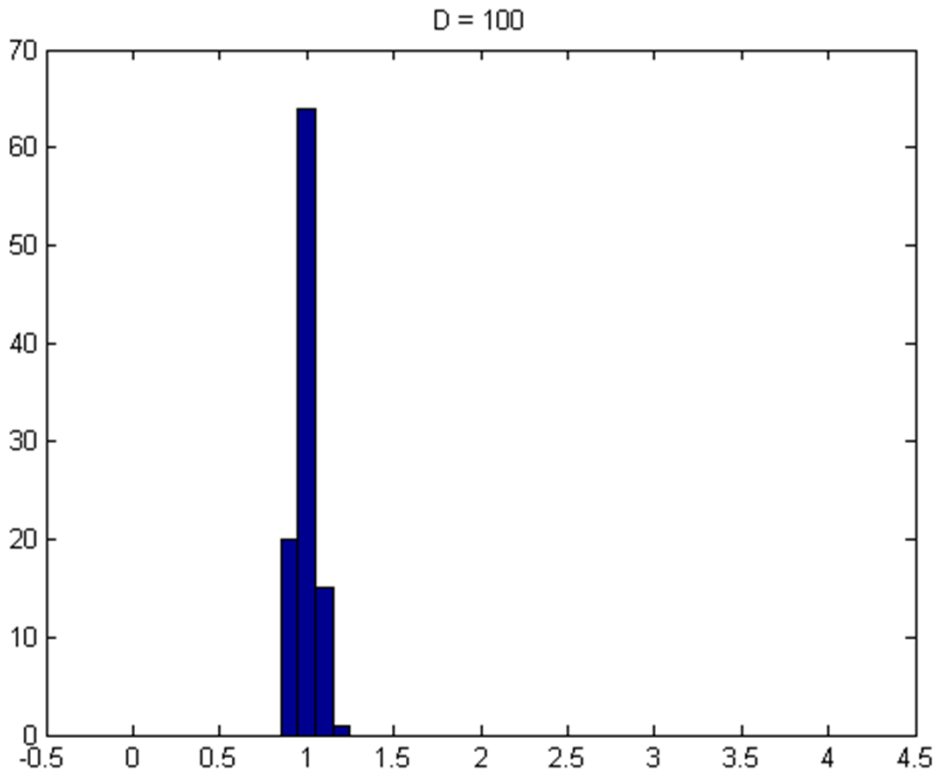
Concentration: continued

$$P\left(\left|\|\mathbf{y}\| - \mu_{\|\mathbf{y}\|}\right| \geq \varepsilon\right) \leq \frac{\sigma^2_{\|\mathbf{y}\|}}{\varepsilon^2}$$

Chebyshev's inequality

(D-1)-sphere

Concentration: simulation results



The relative error tends to zero, meaning that the normalized norm concentrates.

Concentration: continued

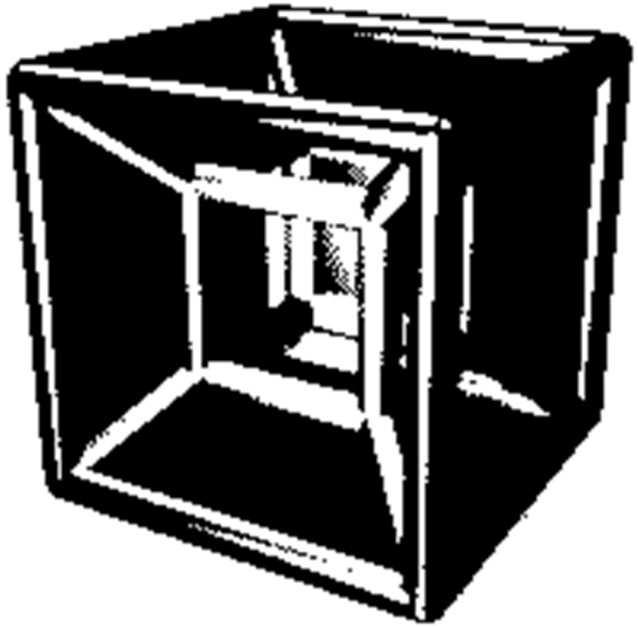
- Where is concentration an issue?
- **NN search**: collection of data points, and query point, find data point closest to query point
  - e.g. used in kNN classification
- Particular interest from vision community
  - each image is approximated with high-dimensional feature vector

Concentration: questions

1. How restrictive are the conditions?
   - sufficient but not necessary
2. When the conditions are satisfied, at what dimensionality do distances become meaningless?
   - about 10-15 (depends on dataset)
3. How can we tell if NN is not meaningful for our dataset?
   - statistical tests? (Casey and Slaney, 2008)
   - how can we fight it? (Houle et al., 2010)

Visualization

- Can be done for up to **4** Ds.



Two approaches to reduce D

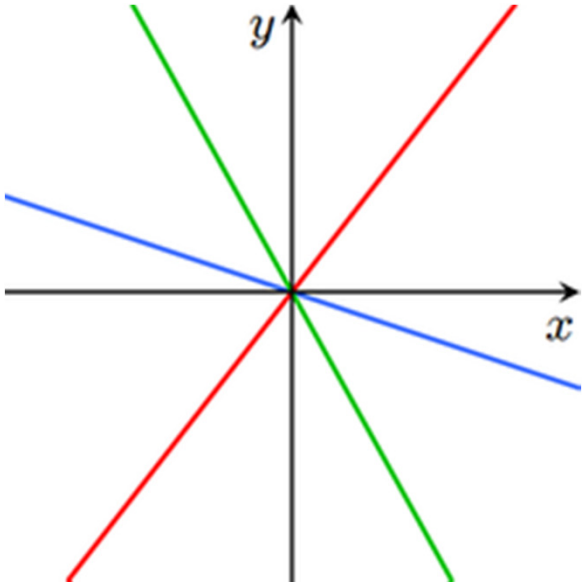
- Feature selection
  - a subset of variables chosen
  - techniques are usually supervised
    - those not correlated with output are eliminated
- Feature extraction
  - even when assuming all variables are relevant
  - detect and eliminate dependencies

the focus of this talk

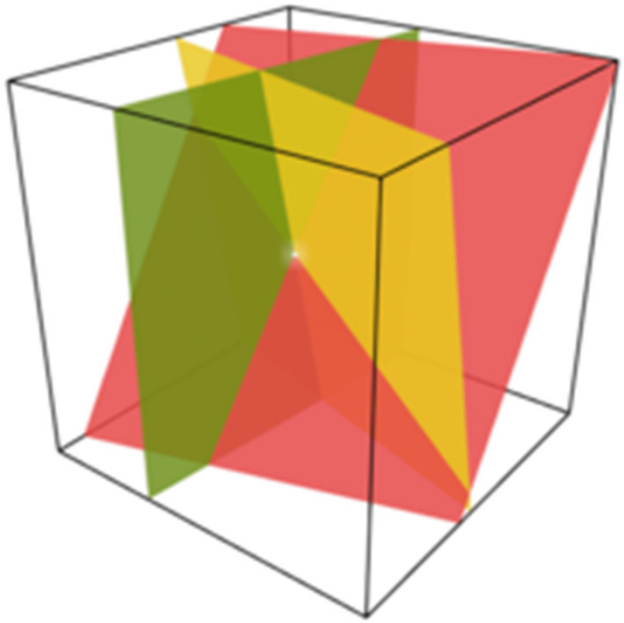
Vocabulary

- So you can develop an intuition about some of the words in the literature
  - Subspace
  - Manifold
  - Embedding
  - Intrinsic dimensionality

Subspace



three 1D subspaces of $\mathbb{R}^2$



three 2D subspaces of $\mathbb{R}^3$

Manifold-- but first some topology

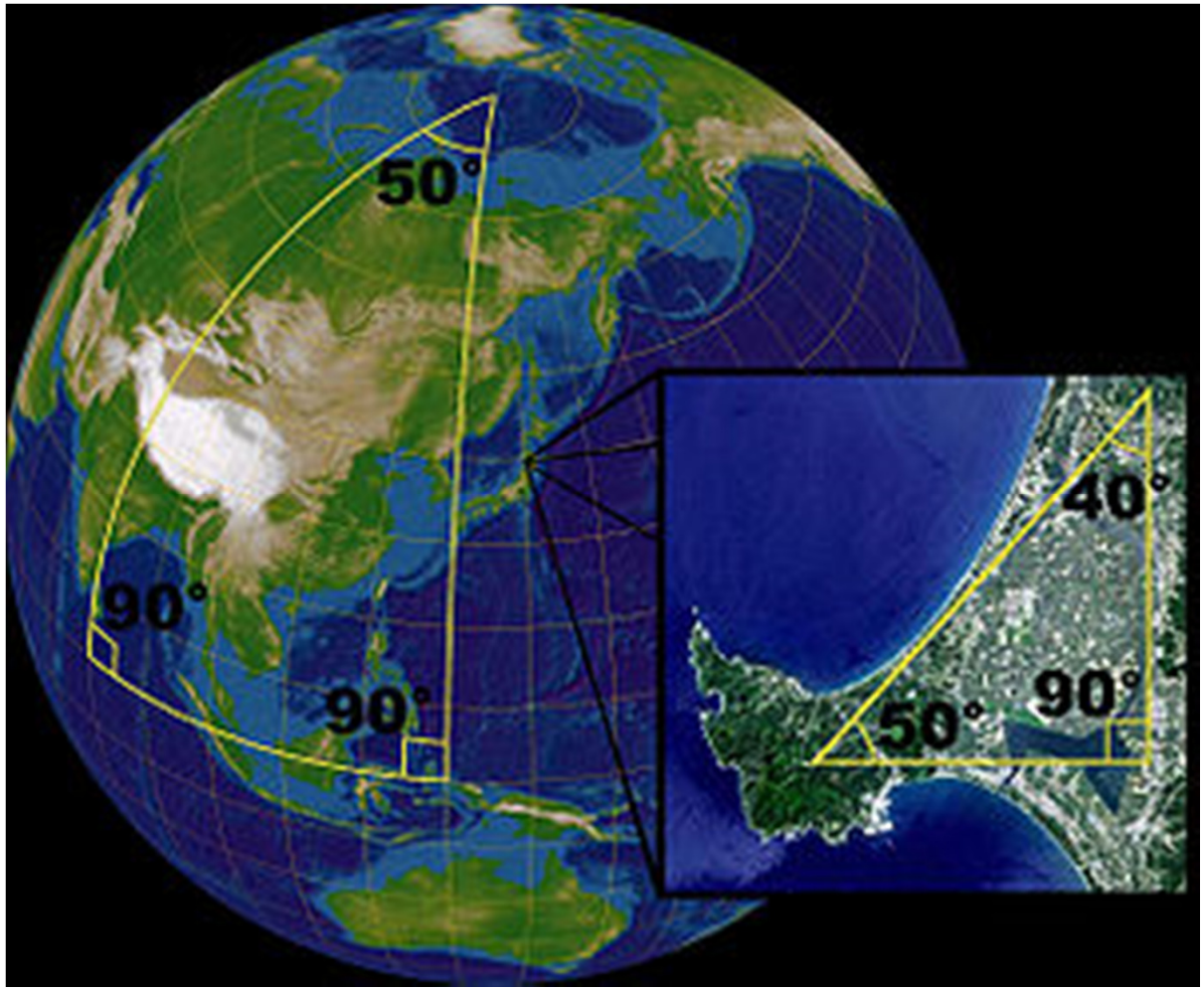
Spatial properties that are preserved under continuous deformations of object

- **Twisting, stretching**
- ~~**Tearing, gluing**~~

"a topologist can't distinguish a coffee mug from a doughnut"

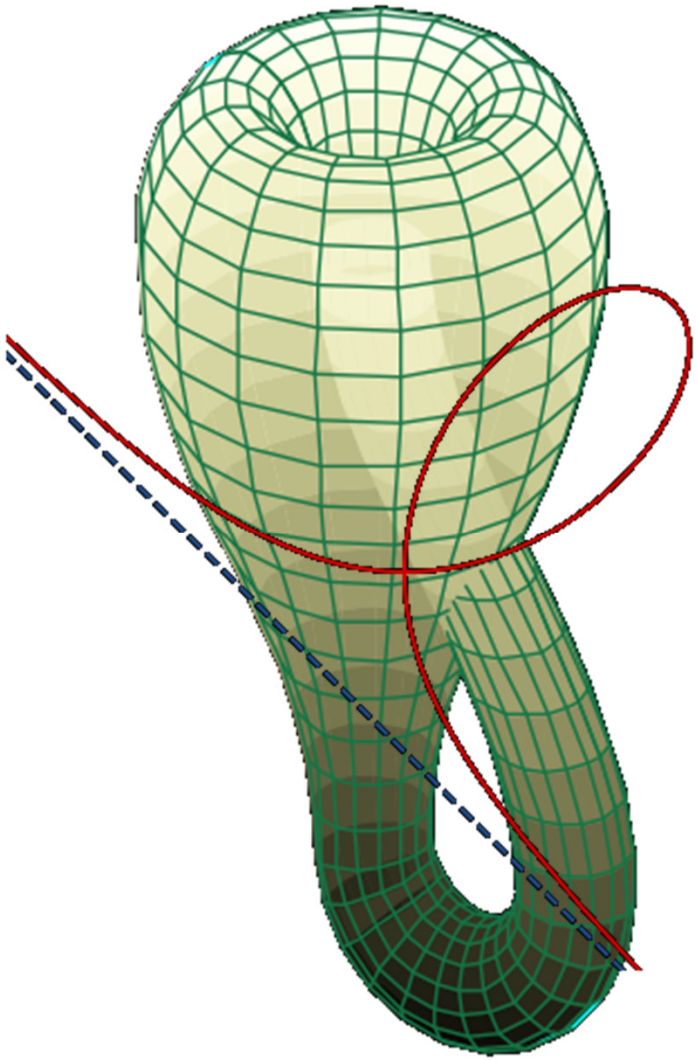
Manifold

locally isomorphic to Euclidean space



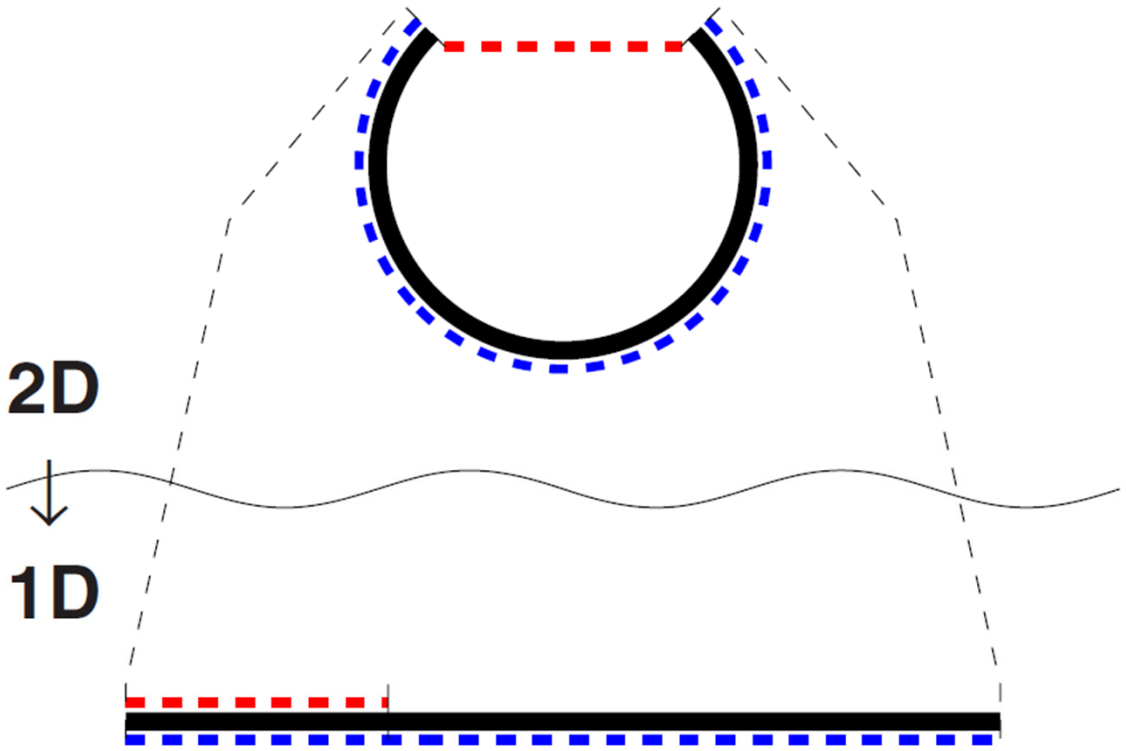
Embedding and p-manifold

- Embedding
- P-manifold
- Every **curve** is a 1-manifold
- Every **surface** is a 2-manifold

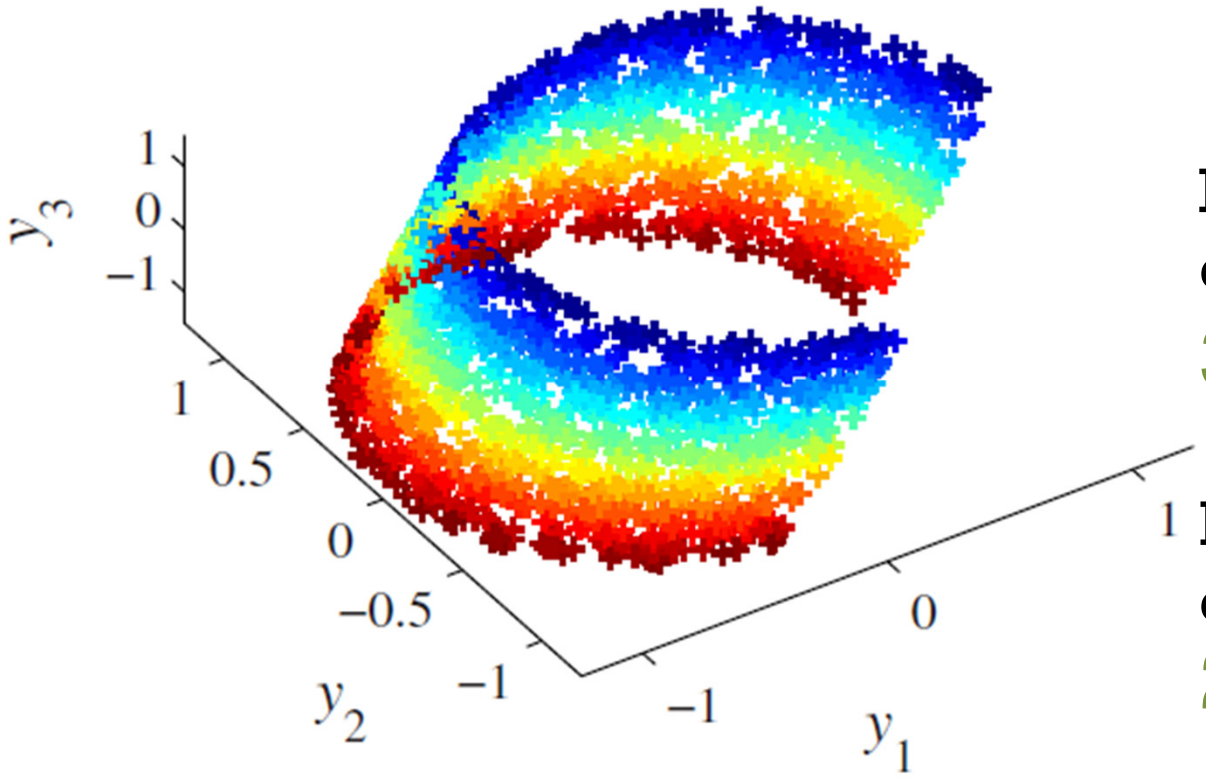


Dimensionality reduction (DR)

- Re-embedding a manifold from a high-dimensional space to a low-dimensional one
- **s.t.** manifold structure is preserved (**connectivity** and **local relationships**)
- one-to-one mapping



Data dimension and intrinsic dimension



Data dimensionality =

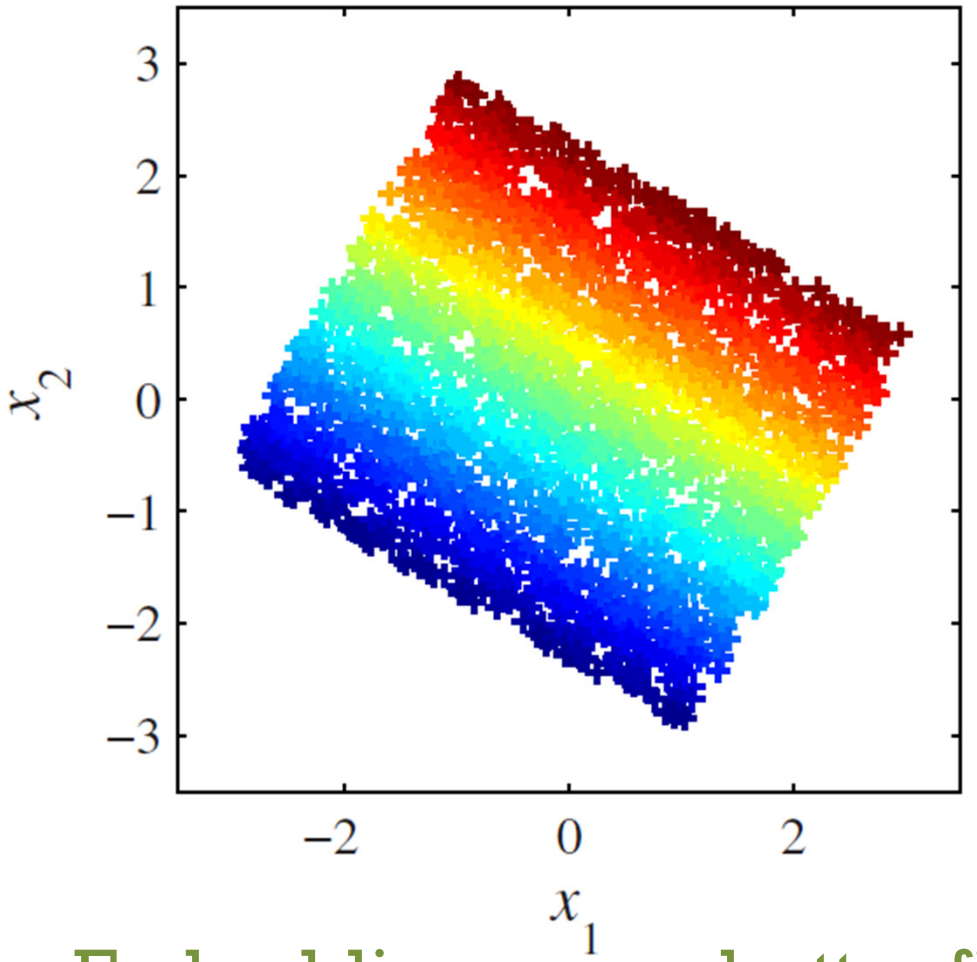
3

Intrinsic dimensionality =

2

Data does not completely fill the embedding space.

A new embedding

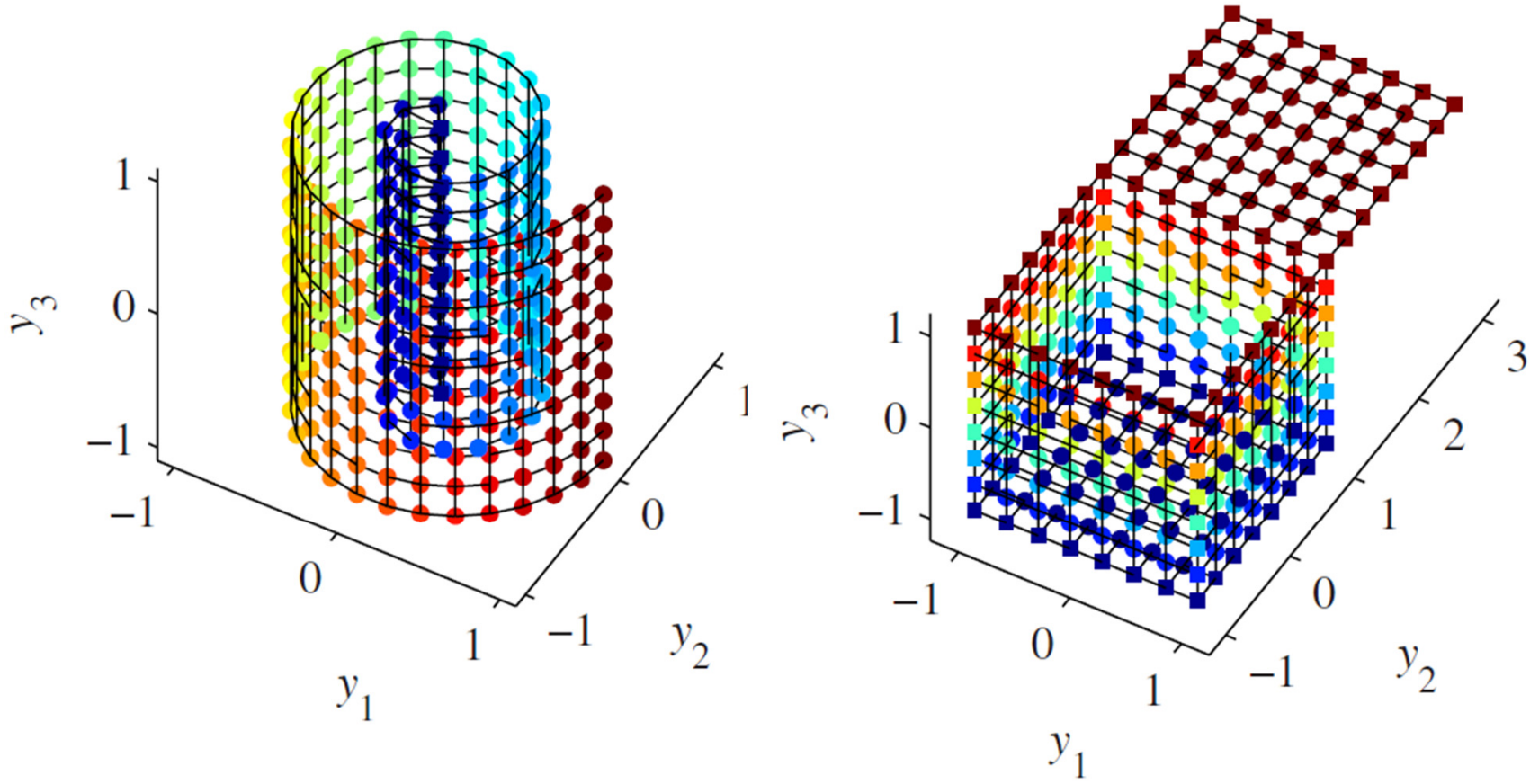


Data dimensionality =
2

Intrinsic dimensionality =
2

Embedding space better filled.

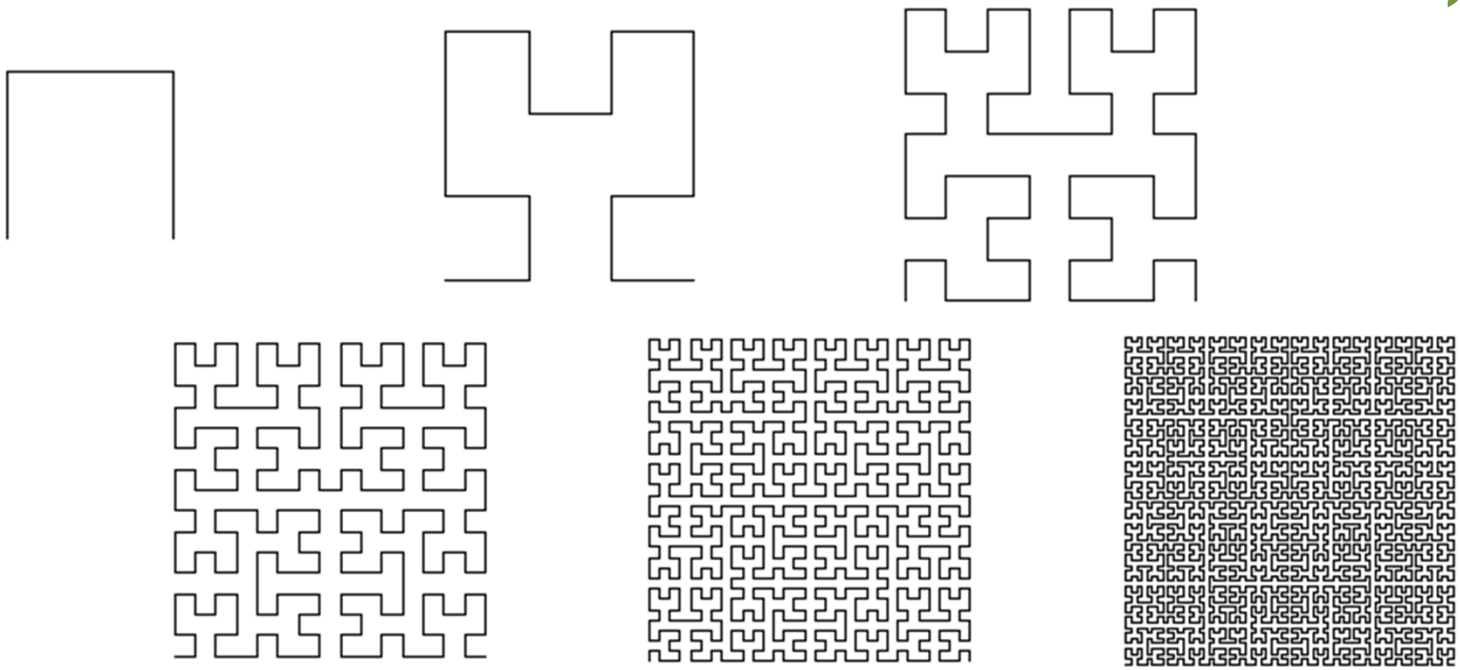
Datasets



(J. A. Lee, and M. Verleysen, 2007)

Datasets: intrinsic dimensionality

- It depends how you define the information content of data
- Several algorithms have been proposed to estimate it (J. A. Lee, and M. Verleysen, 2007)



Subspace learning (linear DR)

- Assume a linear model of data
  - (a linear relation between observed and latent variables)
- We'll look at
  - **PCA (Principal Component Analysis)**
  - **classical metric MDS (Multidimensional Scaling)**
  - **RP (Random Projections)**

PCA (H. Hotelling, 1933)

$$\mathbf{y} = [y_1, \ldots, y_d, \ldots, y_D]^T$$

$$\mathbf{x} = [x_1, \ldots, x_p, \ldots, x_P]^T$$

$$\mathbf{y} = \mathbf{W}\mathbf{x} \qquad \mathbf{W}^T\mathbf{W} = \mathbf{I}_P$$

$$E_{\mathbf{y}}\{\mathbf{y}\} = \mathbf{0}_D$$

$$E_{\mathbf{x}}\{\mathbf{x}\} = \mathbf{0}_P$$

Given: data coordinates

$$\mathbf{Y} = [\mathbf{y}(1), \ldots, \mathbf{y}(n), \ldots, \mathbf{y}(N)]$$

PCA: continued

$$\mathbf{C}_{\mathbf{yy}} = E\{\mathbf{y}\mathbf{y}^T\}$$

$$\mathbf{C}_{\mathbf{yy}} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^T$$

$$\mathbf{W} = \mathbf{V}\mathbf{I}_{D\times P}$$

$$\hat{\mathbf{x}} = \mathbf{I}_{P\times D}\mathbf{V}^T\mathbf{y}$$

MDS (I. Borg and P. Groenen, 1997)

$$s_{\mathbf{y}}(i,j) = s(\mathbf{y}(i), \mathbf{y}(j)) = \langle \mathbf{y}(i) \cdot \mathbf{y}(j) \rangle$$

$$\begin{aligned}
\mathbf{S} = [s_{\mathbf{y}}(i,j)]_{1 \le i,j \le N} &= \mathbf{Y}^T \mathbf{Y} \\
&= (\mathbf{W}\mathbf{X})^T (\mathbf{W}\mathbf{X}) \\
&= \mathbf{X}^T \mathbf{W}^T \mathbf{W} \mathbf{X} \\
&= \mathbf{X}^T \mathbf{X} \ .
\end{aligned}$$

Given: matrix of scalar products

MDS: continued

$$\begin{aligned}\mathbf{S} &= \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T \\ &= (\mathbf{U}\mathbf{\Lambda}^{1/2})(\mathbf{\Lambda}^{1/2}\mathbf{U}^T) \\ &= (\mathbf{\Lambda}^{1/2}\mathbf{U}^T)^T(\mathbf{\Lambda}^{1/2}\mathbf{U}^T)\end{aligned}$$

$$\hat{\mathbf{X}} = \mathbf{I}_{P\times N}\mathbf{\Lambda}^{1/2}\mathbf{U}^T$$

MDS: discussion

- Widely used and developed in human sciences
  - particularly **psychometrics**
- People are asked to give qualitative separation between objects
- So each object is characterized by **distances** to other objects

Neither **S** nor **Y** but **distances** available

A technique called **double centering**

$$
\begin{aligned}
d^2_{\mathbf{y}}(i,j) &= \|\mathbf{y}(i) - \mathbf{y}(j)\|_2^2 \\
&= \langle \mathbf{y}(i) - \mathbf{y}(j) \cdot \mathbf{y}(i) - \mathbf{y}(j) \rangle \\
&= \langle \mathbf{y}(i) \cdot \mathbf{y}(i) \rangle - 2\langle \mathbf{y}(i) \cdot \mathbf{y}(j) \rangle + \langle \mathbf{y}(j) \cdot \mathbf{y}(j) \rangle \\
&= s_{\mathbf{y}}(i,i) - 2s_{\mathbf{y}}(i,j) + s_{\mathbf{y}}(j,j) \ ,
\end{aligned}
$$

squared Euclidean distance

$$
s_{\mathbf{y}}(i,j) = -\frac{1}{2}(d^2_{\mathbf{y}}(i,j) - \langle \mathbf{y}(i) \cdot \mathbf{y}(i) \rangle - \langle \mathbf{y}(j) \cdot \mathbf{y}(j) \rangle)
$$

Double centering: continued

$$\mathbf{S} = -\frac{1}{2}(\mathbf{D} - \frac{1}{N}\mathbf{D}\mathbf{1}_N\mathbf{1}_N^T - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^T\mathbf{D} + \frac{1}{N^2}\mathbf{1}_N\mathbf{1}_N^T\mathbf{D}\mathbf{1}_N\mathbf{1}_N^T)$$

$$\mu_j(d^2_{\mathbf{y}}(i,j)) =$$

$$\langle \mathbf{y}(i) \cdot \mathbf{y}(i) \rangle + \mu_j(\langle \mathbf{y}(j) \cdot \mathbf{y}(j) \rangle)$$

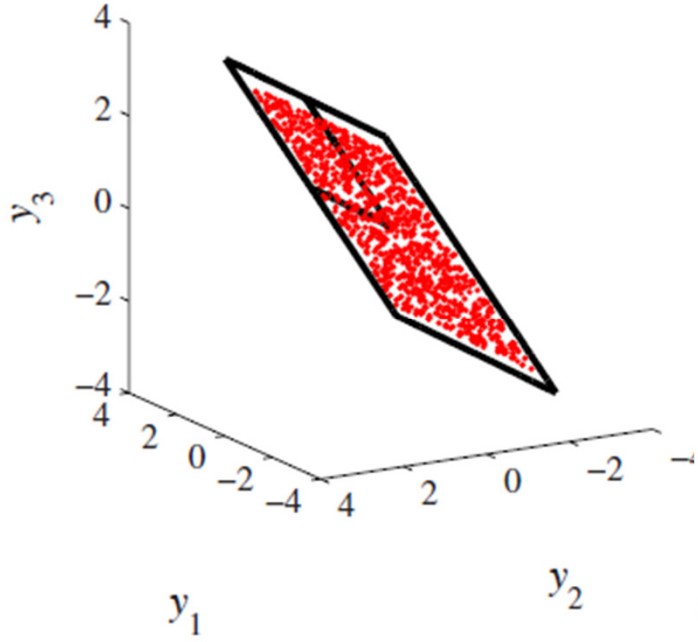
$$\mu_i(d^2_{\mathbf{y}}(i,j)) =$$

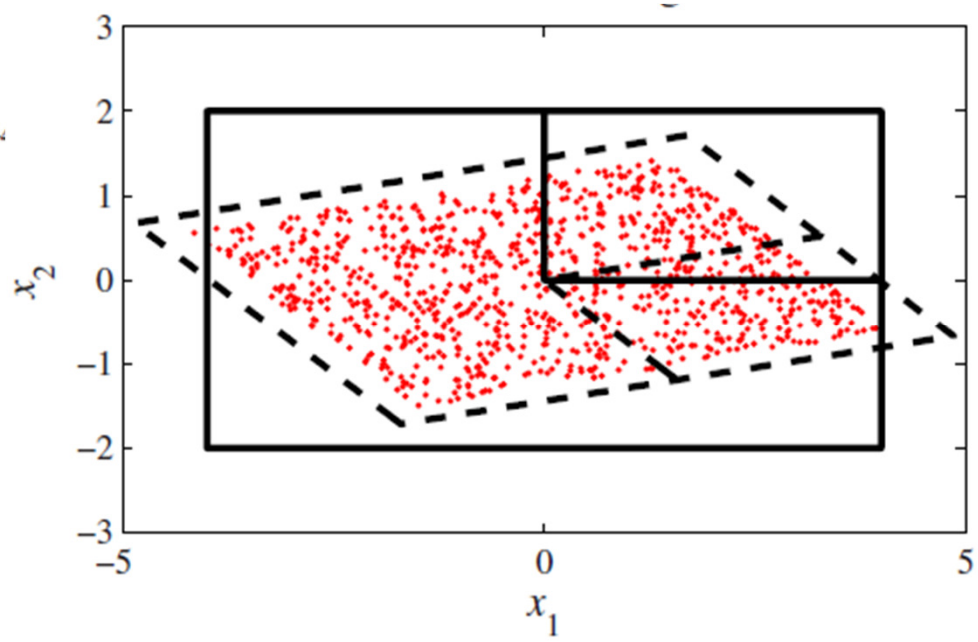
$$\mu_i(\langle \mathbf{y}(i) \cdot \mathbf{y}(i) \rangle) + \langle \mathbf{y}(j) \cdot \mathbf{y}(j) \rangle$$

$$\mu_{i,j}(d^2_{\mathbf{y}}(i,j)) =$$

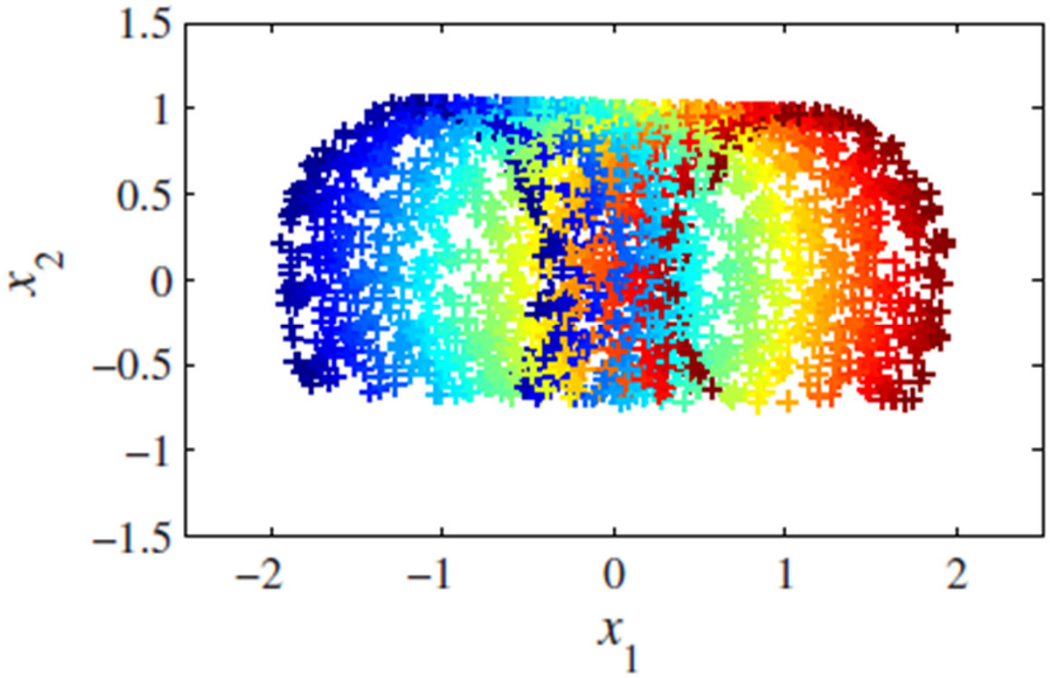
$$\mu_i(\langle \mathbf{y}(i) \cdot \mathbf{y}(i) \rangle) + \mu_j(\langle \mathbf{y}(j) \cdot \mathbf{y}(j) \rangle)$$

PCA/MDS: results





PCA/MDS: results



PCA/MDS: discussion

- Metric MDS and PCA give the same solution
- Both focus mainly on retaining **large pairwise distances**, instead of small pairwise distances which is more important
- Both may consider two points as near points, whereas their distance over the manifold is much larger

Random projections
(W. B. Johnson and J. Lindenstrauss, 1984)

- A **linear** method
- Simple yet powerful
- Randomly chosen low-dimensional subspace
  - the projection doesn't depend on the data
  - a "**data-oblivious**" method

RP: algorithm

- Here's how to obtain the **P×D** linear transform **R** (Dasgupta, 2000)
  1. set each entry of **R** to an i.i.d. ~**N(0,1)** value
  2. make the **P** rows of the matrix orthogonal using the **Gram-Schmidt** algorithm
  3. normalize rows to unit length

RP: the theory behind the algorithm

- **JL Thm.** A set of points of size **n** in a high-dimensional Euclidean space, can be mapped into a **q**-dimensional space, $q \geq O(\log(n)/\varepsilon^2)$, such that the distance between any two points changes by only a factor of $1 \pm \varepsilon$.
  - (W. B. Johnson and J. Lindenstrauss, 1984)

RP theory: continued

(S. Dasgupta and A. Gupta, 1999)

- A matrix whose entries are normally distributed represents such a mapping with probability at least **1/n**, therefore doing **O(n)** projections will result in an arbitrarily high probability of preserving distances.
- Tighter bound obtained:
  - $q \geq 4*(\varepsilon^2/2 - \varepsilon^3/3)^{-1} \ln(n)$

RP: discussions

- It is shown that RP **underperforms** PCA as a preprocessing step in classification (but still remains comparable)
  - (D. Fradkin and D. Madigan, 2003)
- **But**, it is **computationally more attractive** than PCA and can replace it
  - e.g. when initial dimension is ~6000
  - PCA is $O(D^3)$ vs. RP which is $O(P^2D)$
  - with some loss of accuracy, even faster versions of RP have been proposed

Manifold learning

- Manifold assumption
  - **"data lies on a low-dimensional manifold in the high-dimensional space"**
- It's an assumption that helps reduce the hypothesis space
  - *A priori* information on **the support** of the **data distribution**

Manifold learning (nonlinear DR)

- First we'll look at
  - **Isomap (Isometric feature map)** and
  - **LLE (Locally Linear Embedding)**
- Easy to understand/explain
- Both build a graph **G** using **K**-rule: $O(N^2)$
  - A discretized approximation of the manifold, sampled by the input
- Both published in **Science** in **2000**, and lead to the rapid development of **spectral methods for NLDR**
- **~3840** and **~3735** citations, respectively

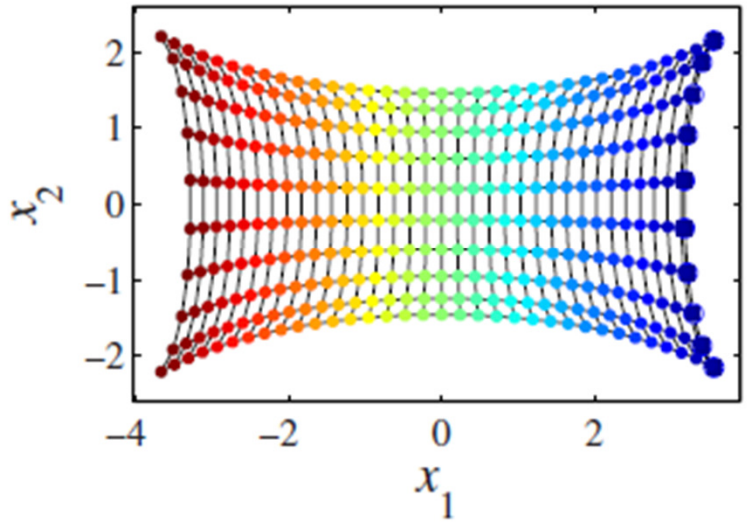
Nonlinear DR: continued

- We'll also be looking at
  - **Autoassociative neural networks** and
  - **Autoencoders**
    - use a new technique for training autoassociative neural nets
- and an overview of some other NLDR algorithms
- so hang on to your seats!

Isomap (J. Tenenbaum et al., 2000)

1) Build graph **G** with **K**-rule
2) Weigh each edge by its Euclidean length **(weighted graph)**
3) Perform **Dijkstra**'s algorithm, store square of pairwise distances in Δ
4) Perform **MDS** on Δ

Isomap: results



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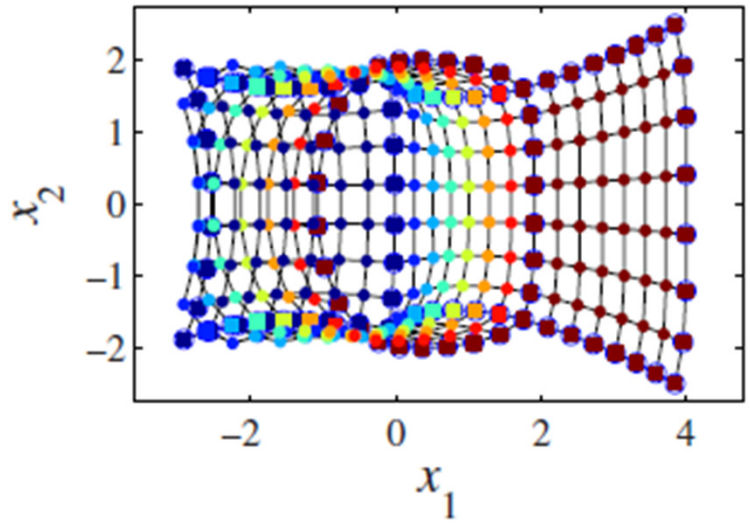
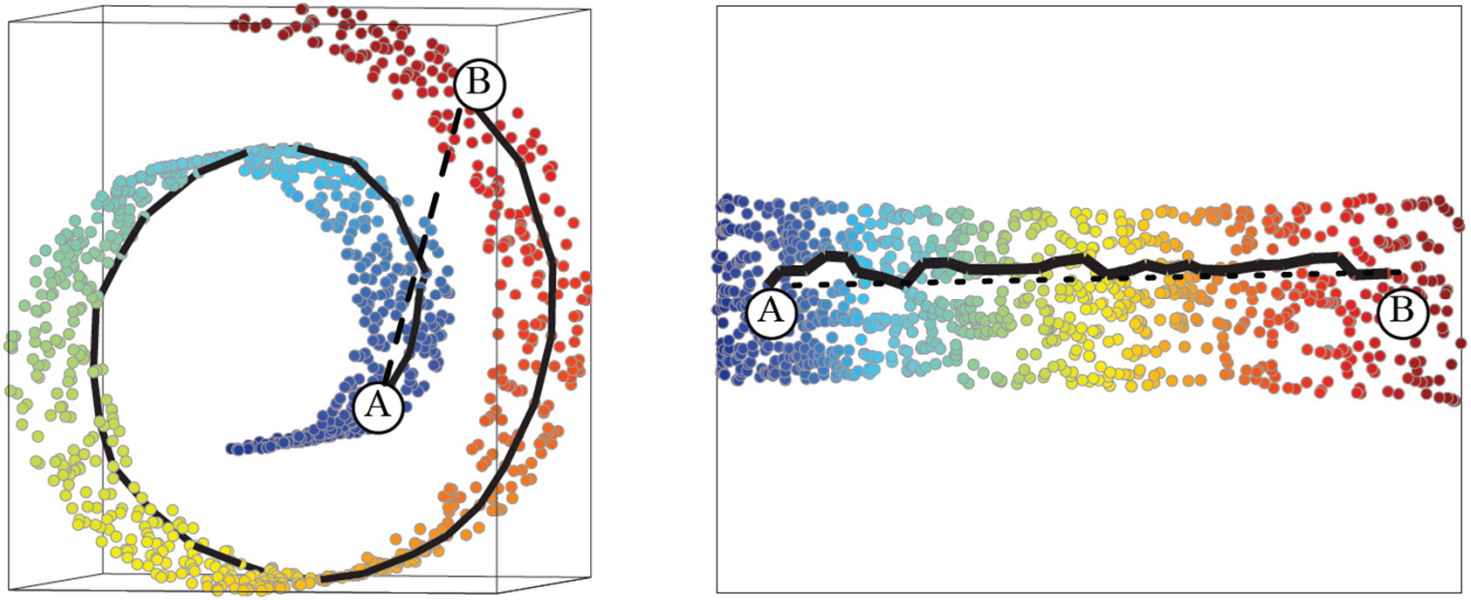


N.A.

Isomap: discussion

- A variant of MDS
  - estimates of **geodesic distances** are substituted for Euclidean distances



(L. K. Saul et al., 2005)

Isomap: discussion

- A variant of MDS
  - Nonlinear capabilities brought by graph distances and **not by** inherent nonlinear models of data
- Computation time dominated by calculation of shortest paths
- Guaranteed convergence for **developable manifolds** only
  - Pairwise geodesic distances computed between points of the P-manifold, can be mapped to pairwise Euclidean distances measured in a P-dimensional Euclidean space

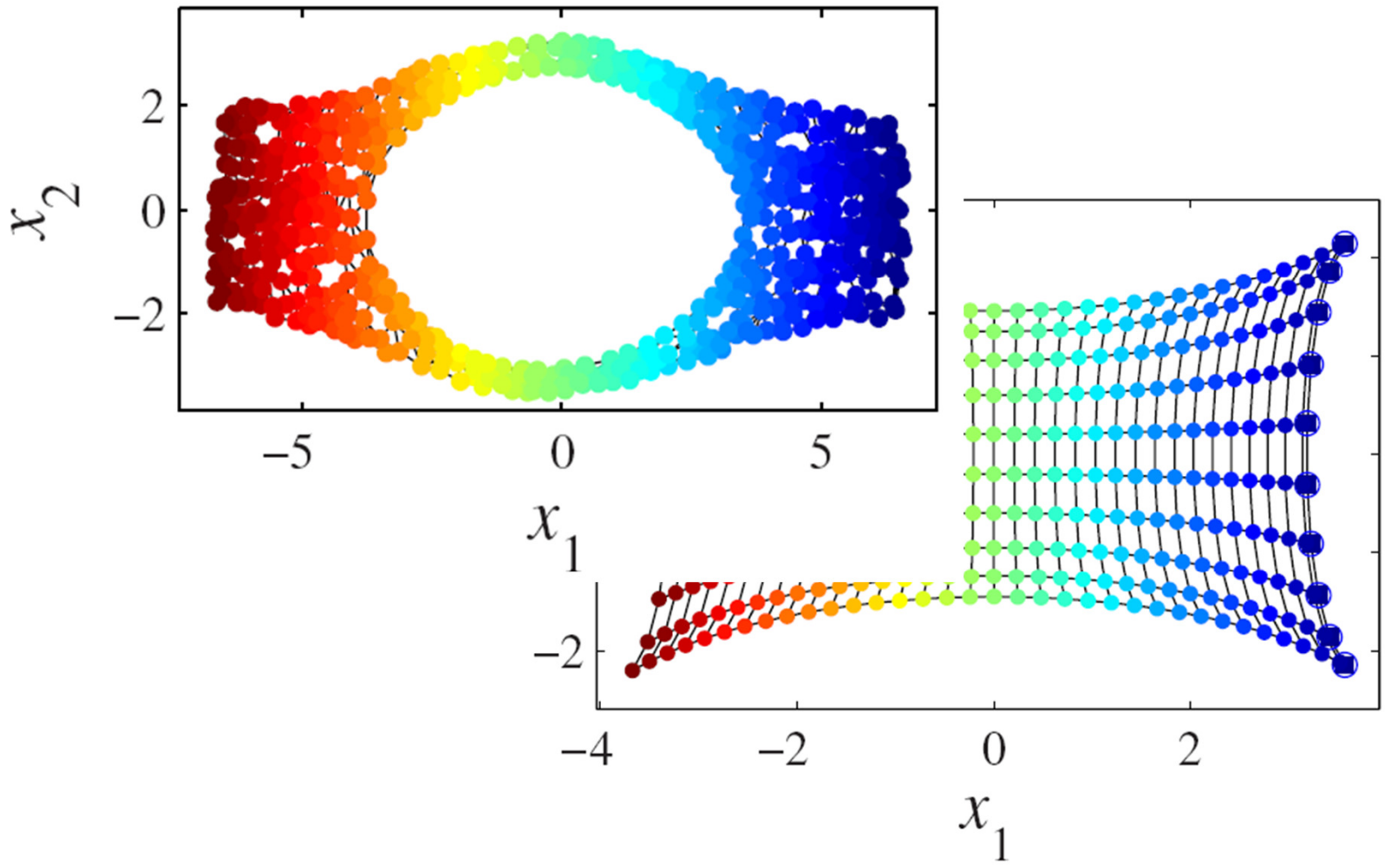
Isomap: discussion

- Dijkstra's algorithm solves the **single-source shortest path** problem
- So we need to run Dijkstra for each vertex
- More efficient than Floyd-Warshall because graph is sparse

Isomap: discussion

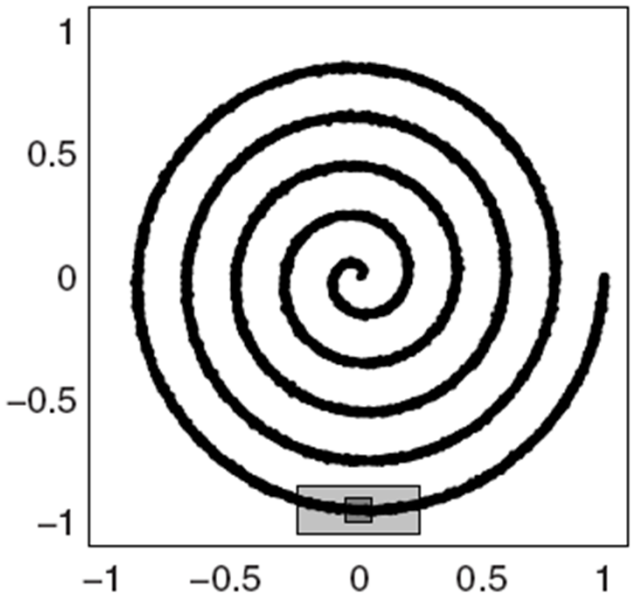
- Results of Isomap strongly depend on the quality of the estimation of geodesic distances
- If **data set is sparse**, (and no shortcuts take place)
  - graph distances are likely to be overestimations
- If **data manifold contains holes**, paths need to go around holes
  - graph distances are overestimations

Isomap: discussion



Isomap: estimation of intrinsic dimension

- A single run of **PCA**, **MDS**, or **Isomap**
- Gap in eigenvalues



-PCA, MDS: **2**

-Isomap: **1**

LLE (S. Roweis and L. Saul, 2000)

1) Build graph **G** with **K**-rule
2) Find the weight matrix **W** for reconstructing each point from its **K** neighbors
3) Find the low-dimensional coordinates **X**, that are reconstructed from weights **W** with minimum error

LLE: step 2)

2) Find the weight matrix **W** for reconstructing each point from its **K** neighbours

$$\mathcal{E}(\mathbf{W}) = \sum_{i=1}^{N} \left\| \mathbf{y}(i) - \sum_{j \in \mathcal{N}(i)} w_{i,j} \mathbf{y}(j) \right\|^2$$

LLE: step 3)

3) Find the low-dimensional coordinates **X**, that are reconstructed from weights **W** with minimum error

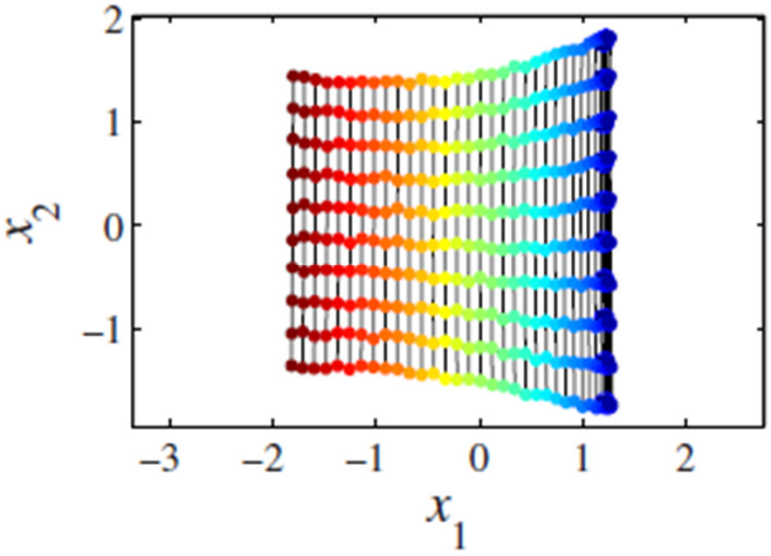
$$\Phi(\hat{\mathbf{X}}) = \sum_{i=1}^{N} \left\| \hat{\mathbf{x}}(i) - \sum_{j \in \mathcal{N}(i)} w_{i,j} \hat{\mathbf{x}}(j) \right\|^2$$

LLE step 3): discussion

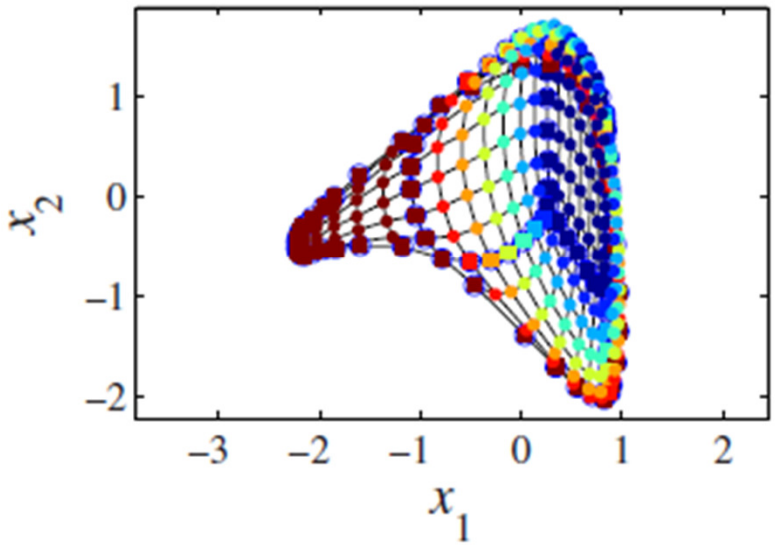
- Optimal embedding is found by computing the bottom **P+1** eigenvectors of **M**

$$\mathbf{M} = (\mathbf{I} - \mathbf{W})^T (\mathbf{I} - \mathbf{W})$$

LLE: results



A.



A.

LLE: discussion

- Like MDS, LLE uses **EVD**, which is purely linear
  - Nonlinear capabilities of LLE come from the computation of nearest neighbors (**thresholding**)
- Unlike MDS, cannot estimate **intrinsic dimensionality** (no telltale gap in **M**)
- Works for non-convex manifolds, but not ones that contain holes
- Very sensitive to its parameter values

Discussion: "local manifold learning" (Y. Bengio and M. Monperrus, 2005)

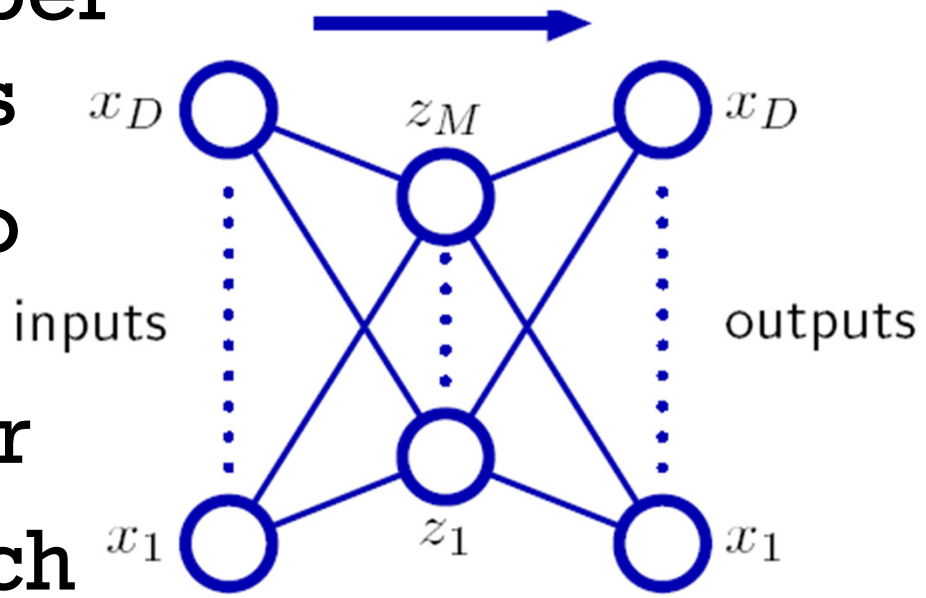
- LLE, Isomap are **local learning** methods
- They could fail when
  - Noise around manifold
  - High curvature of the manifold
  - High intrinsic dimension of the manifold
  - Presence of multiple manifolds with little data per manifold

Autoassociative neural nets
(M. A. Kramer, 1991)

- Nonlinear capabilities of Isomap and LLE were not brought by inherent **nonlinear models of data**
- Also, both methods use **'local'** generalization
- Apart from **supervised** learning for **classification**, neural nets have been used in the context of **unsupervised** learning for **dimensionality reduction**

Autoassociative NN: continued

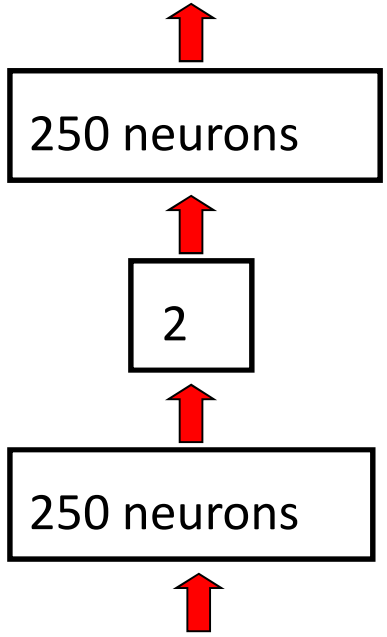
- DR achieved by using net with same number of input and outputs
- Optimize weights to minimize reconstruction error
- Net tries to map each input vector onto itself



(C. M. Bishop, 2006)

Autoassociative NN: the intuition

- Net is trained to reproduce its input at the output
- So it packs as much information as possible into the central bottleneck



Autoassociative NN: continued

- Number of hidden units is smaller than number of inputs
  - there exists a **reconstruction error**
- Determine network weights by minimizing the reconstruction sum-of-squares error:

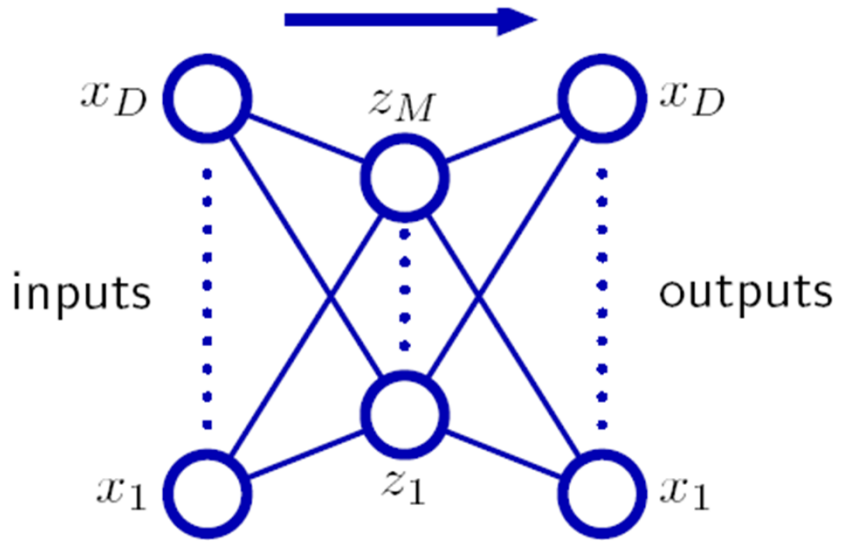
$$E(\mathbf{w}) = \frac{1}{2}\sum_{n=1}^{N} \|\mathbf{y}(\mathbf{x}_n, \mathbf{w}) - \mathbf{x}_n\|^2$$

Autoassociative NN and PCA

- Here's an interesting fact:
- If hidden units have linear activation functions,
- It can be shown that error function has a **unique global minimum**
- At this minimum, the network performs a projection onto an **M**-dimensional subspace
  - spanned by the **first M PCs** of the data!

Autoassociative NN and PCA: continued

- Vector of weights leading into zi's from a basis set which spans the principal subspace
- These vectors need not be orthonormal

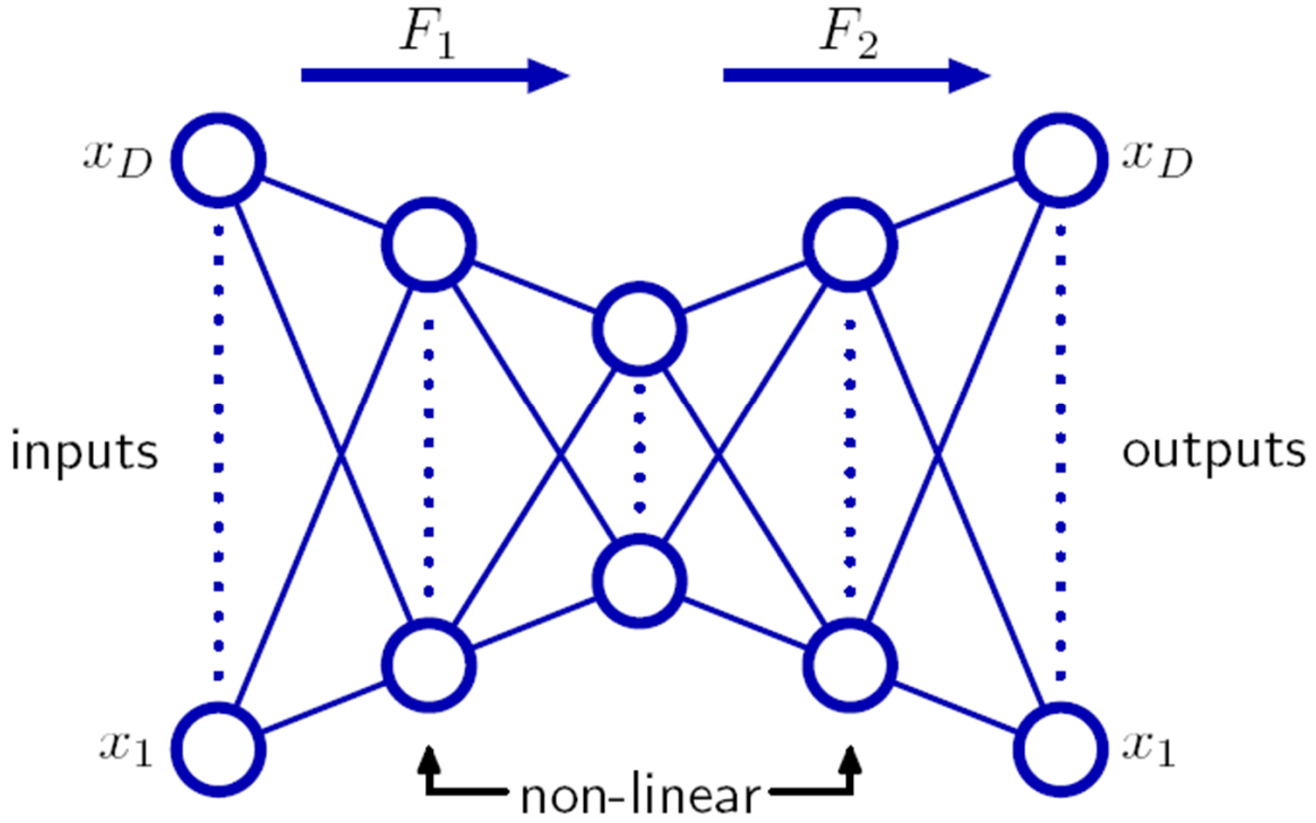


Autoassociative NN and PCA: continued

- Even with nonlinear activation functions for the hidden units,
  - the min error solution is again the projection onto the PC subspace
  - so there is no advantage in using 2-layer NNs to perform DR
  - standard PCA techniques based on SVD are better

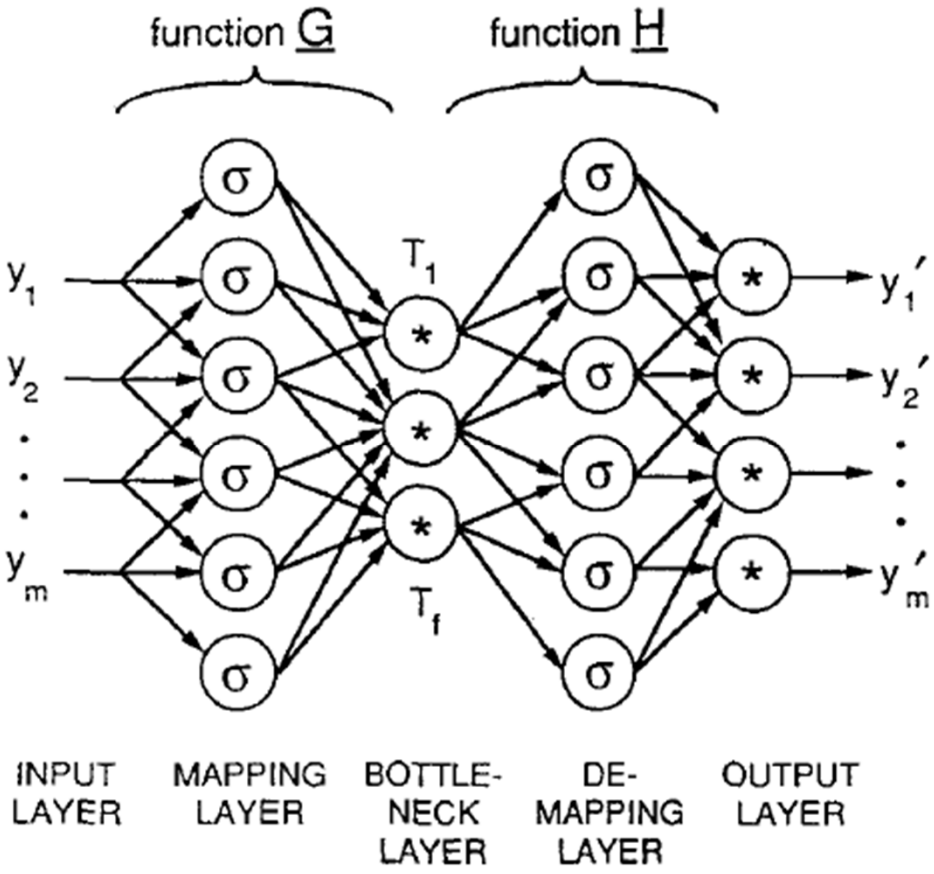
Autoassociative NN: nonlinear PCA

- What we need is additional hidden layers -- consider the 4-layer net below



Autoassociative NN: NLPCA

- Training to learn the identity mapping is called
  - **self-supervised backpropagation** or
  - **Autoassociation**
- After training, the combined net has no utility
  - And is divided into two single-hidden layer nets **G** and **H**



NLPCA: discussion

- Start with random weights, the two nets (**G** and **H**) can be trained together by minimizing the discrepancy between the original data and its reconstruction
- Error function as before (sum-of-squares)
  - but no longer a quadratic function of net params.
  - risk of falling into local minima of err. func.
  - and burdensome computations
- Dimension of subspace must be specified before training

Autoencoder
(G. E. Hinton and R. R. Salakhutdinov, 2006)

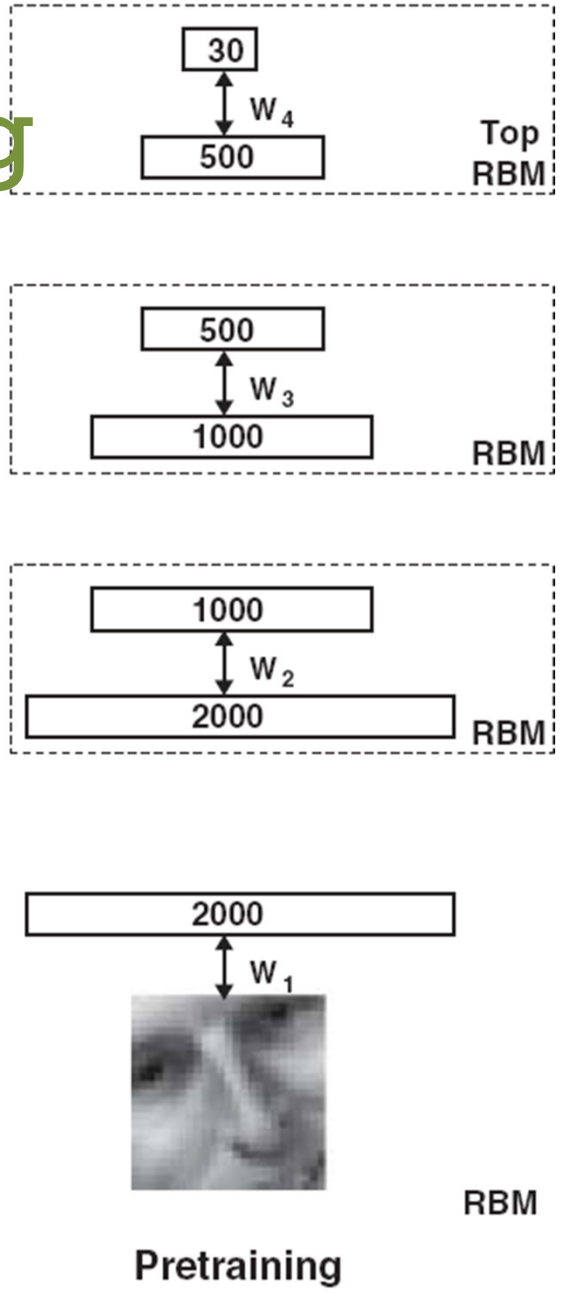
- It was known since the 1980s that **backpropagation** through **deep neural nets** would be very effective for **nonlinear dimensionality reduction** -- subject to:
  - fast computers … OK
  - big data sets … OK
  - **good initial weights** …

Autoencoder: continued

- BP = backpropagation (CG methods, steepest descent, …)
- Fundamental problems in training **nets with many hidden layers** (**"deep" nets**) with BP
  - learning is slow, results are poor
- But, results can be improved significantly if **initial weights** are close to solution

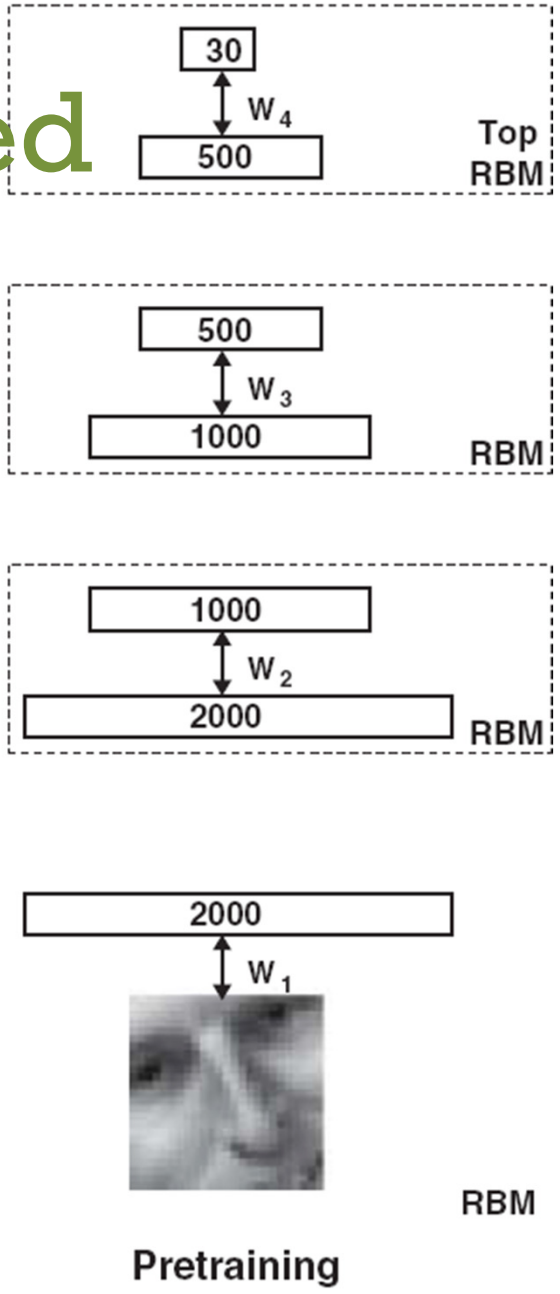
Autoencoder: pretraining

- Treating each neighboring set of layers like an RBM
  - to approximate a good solution
- **RBM = Restricted Boltzmann Machine**
  - will be the topic of an upcoming talk



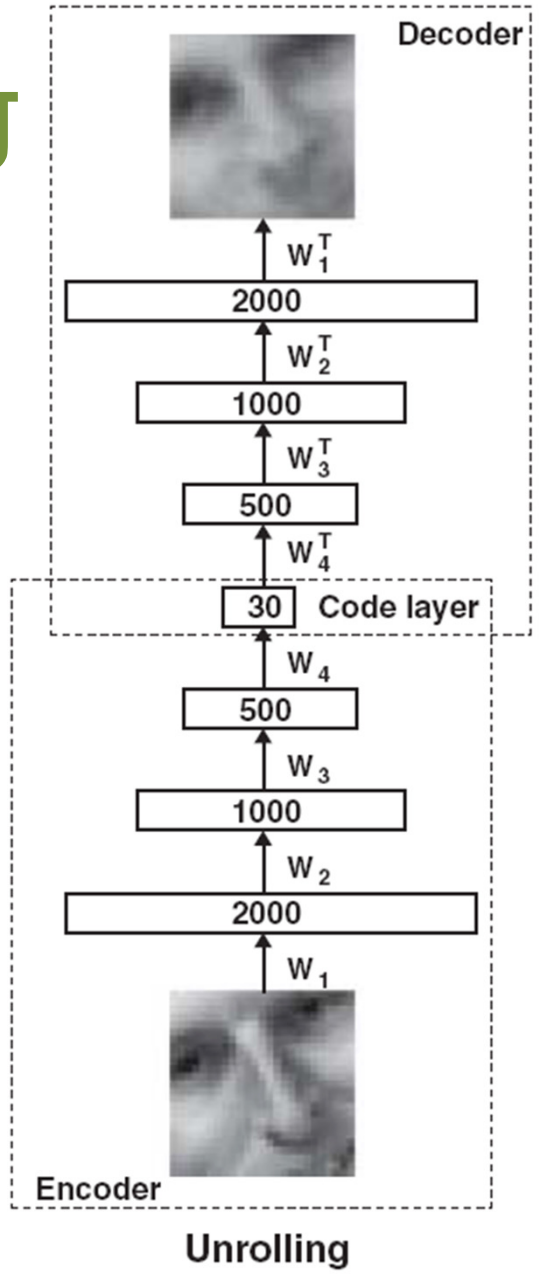
Autoencoder: continued

- The learned features of one RBM are used as data for training the next RBM in the stack
- The learning is unsupervised.



Autoencoder: unrolling

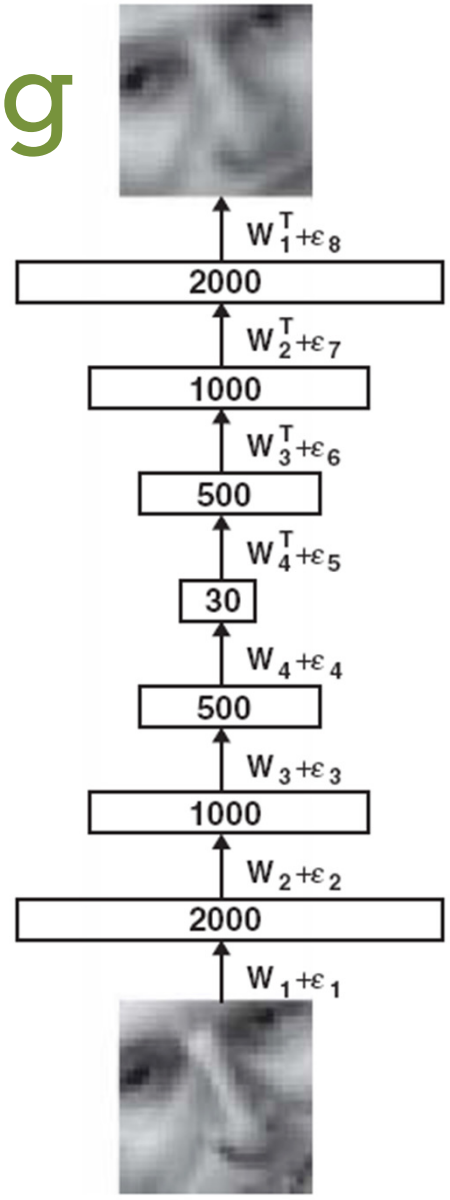
- After pretraining, the model is unfolded
- Produces encoder and decoder networks that use the same weights
- Now, we'll go on to the global fine-tuning stage



Unrolling

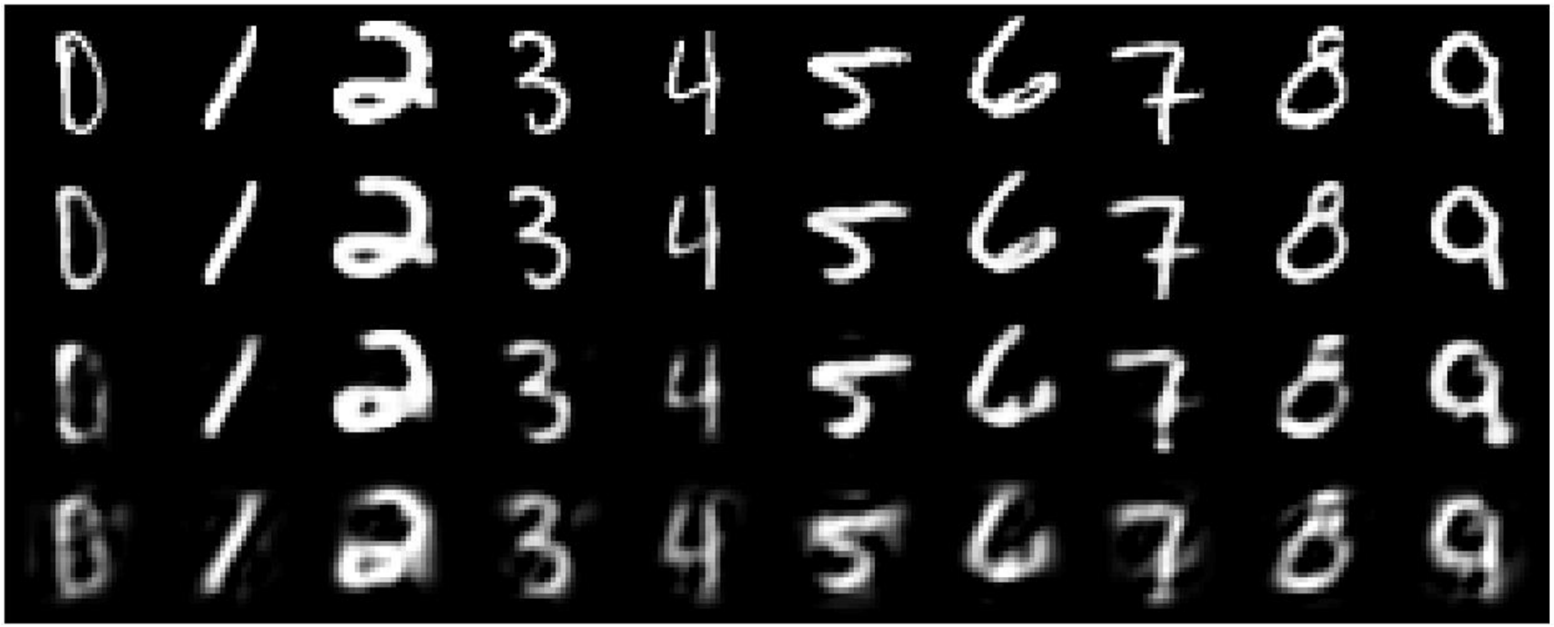
Autoencoder: fine-tuning

- Now use BP of error derivatives to fine-tune ☺
- So we don't run BP until we have good initial weights
- With good initial weights, BP need only perform local search



Fine-tuning

Autoencoder: results



real data

30-D deep auto

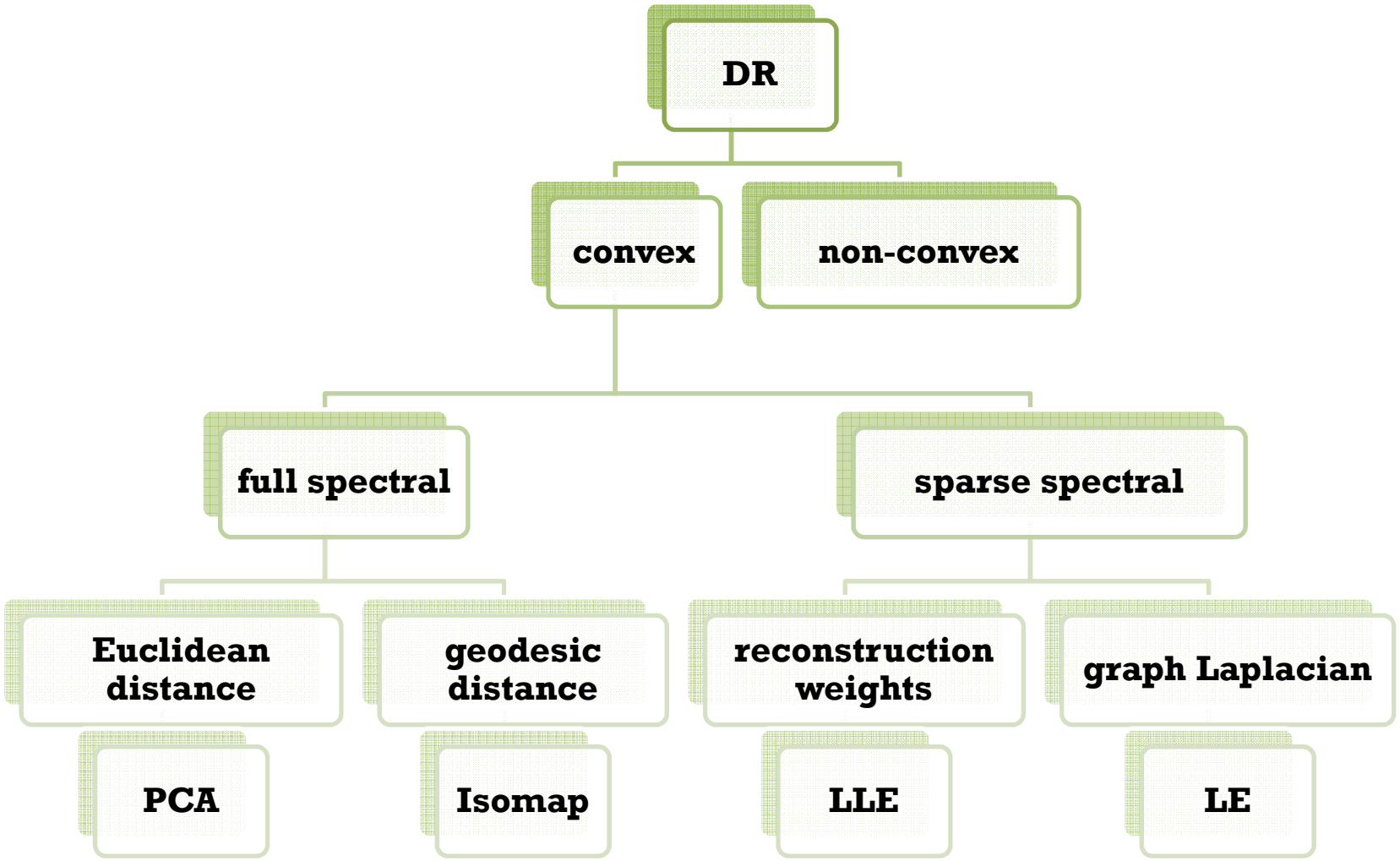
30-D logistic PCA

30-D PCA

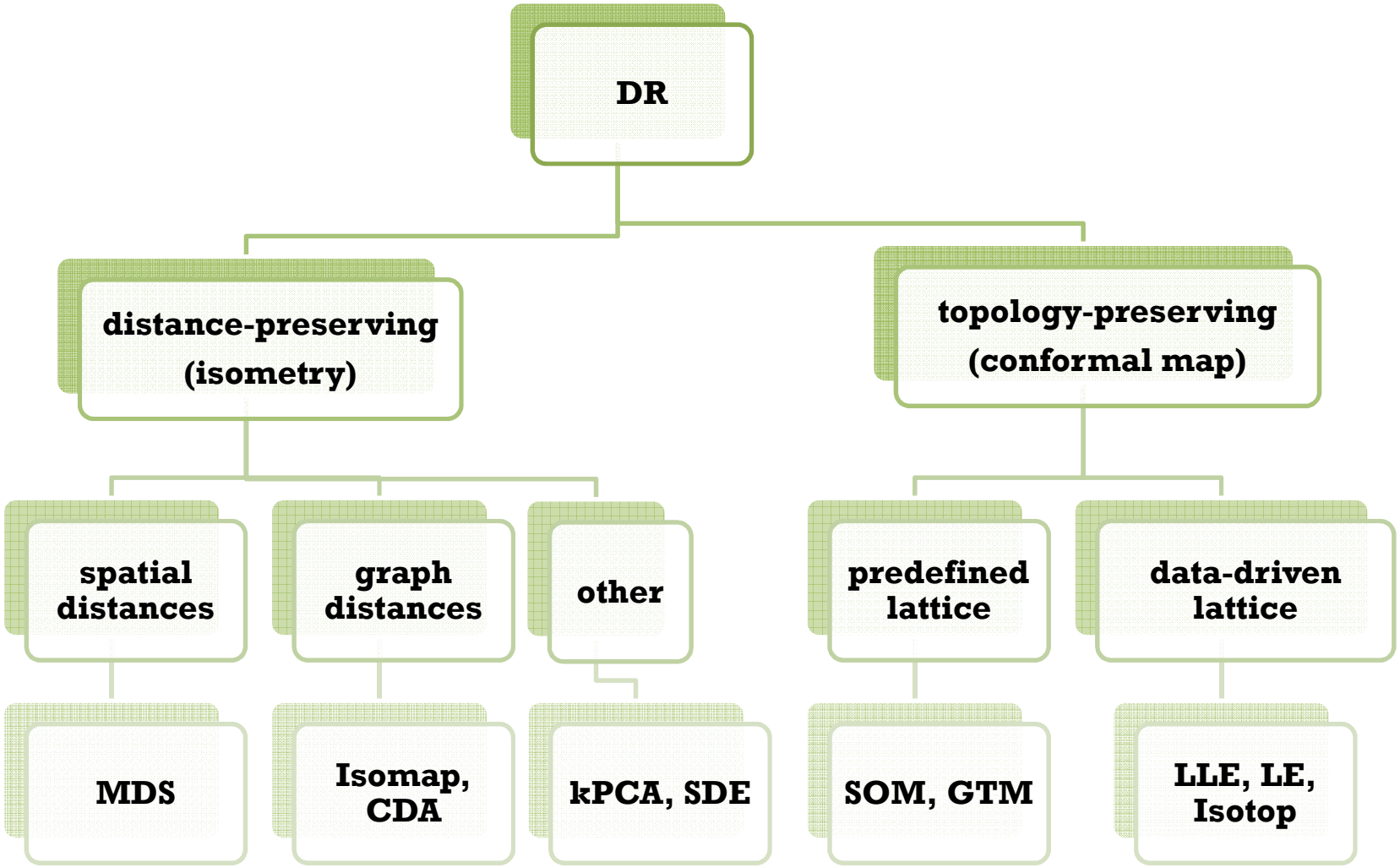
DR: taxonomy

- Here we considered
  - **linear** vs. **nonlinear** (model of data)
- There are many other possible categorizations, to name a few:
  - **local** vs. **non-local** (generalization)
  - **single** vs. **multiple** (coordinate system)
  - **unsupervised** vs. **supervised**
  - **data-aware** vs. **data-oblivious**
  - **exact** vs. **approximate** (optimization)

DR: taxonomy (L. van der Maaten, 2009)



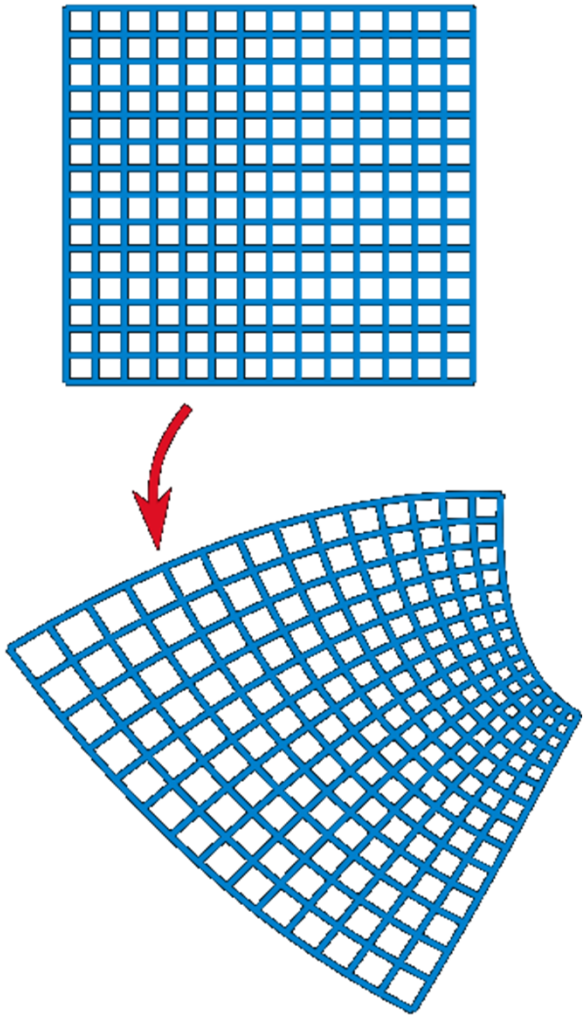
DR: taxonomy (J. A. Lee, and M. Verleysen, 2007)



Note: conformal map (Wikipedia)

A function that preserves angles.

Pairs of lines intersecting at 90° to pairs of curves intersecting at 90°.



Discussion: out-of-sample generalization (Y. Bengio et al., 2003)

- The model of PCA is continuous
  - An implicit mapping is defined:
    - $\mathbf{X} = \mathbf{W}^T\mathbf{Y}$
  - → generalization to new points is easy
- But, MDS, Isomap and LLE provide an explicit mapping
  - $(\mathbf{x}_n, \mathbf{y}_n)$

Discussion: dataset size

- Large datasets: **N>2000**
  - Time and space complexity of NLDR methods at least $O(N^2)$
  - Need to resample available data
    - using k-means for example
- Medium: **200<N≤2000**
  - OK
- Small: **N≤200**
  - Insufficient to identify parameters
  - Use PCA/MDS

Discussion: dataset dimensionality

- Very high: **D>50**
  - NLDR fails b/c of COD
  - First apply PCA/MDS/RP for hard DR
    - can provide robustness to noise
- High: **5<D≤50**
  - COD still exists, use at your own risk
- Low: **D≤5**
  - Apply with confidence

Discussion: dataset intrinsic dimensionality

- Target dim >> intrinsic dim
  - PCA/MDS/RP perform well
- Target dim ≥ intrinsic dim
  - NLDR provides good results
- Target dim < intrinsic dim
  - Use NLDR at your own risk
    - results are meaningless b/c forced
  - Nonspectral methods don't converge
    - spectral methods solve an eigenproblem irrespective of target dimensionality

Discussion: goal of DR

- DR is a **preprocessing** step
  - and some information is lost
- You want to preserve what is important for the next step
  - whether it's classification or clustering
- The **method** and **metric** you use should be in line with the next task

One final note

- Motivation behind DR was to remove COD
- But the mentioned NLDR methods fall prey to COD themselves
  - when intrinsic dimensionality is higher than **4** or **5**

Looking ahead: future sessions

- We'll be talking about
  - **kernel methods**
  - **SVM**
    - (sparse kernel machines)
  - **statistical learning theory**
    - (PAC learning and VC dimension)
- And after that, we'll talk about
  - **deep learning methods**
    - **as a feature extraction method that allows us to deal with the curse of dimensionality**
- We'll try to put it all in the context of information retrieval
  - specifically **multimedia information retrieval**
  - e.g. CBIR, MIR

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Thank you for your attention.