Centrality and Specialization

I was thinking about what to do and this is something I came up with:

1. Keep your definitions in the paper
2. Add a definition of centrality transfer matrix
3. Revise the results slightly

Here are my pseudo definitions:

Definition (Centrality Transfer Matrix)

Let $\beta = \{v_i, e_0, C_1, ..., C_n, e_n v_j\} \in \mathcal{B}_B(G)$. Then for each $1 < i < n$, the centrality transfer matrix of C_i is defined to be,

$$P(\beta, C_i) = (\rho I - C_i)^{-1} Y_{n-1} (\rho I - C_{i-1})^{-1} \cdots (\rho I - C_1)^{-1} Y_0$$

where we figure out a clean way to define each Y_j.

Result 1

Let $G = (V, E, \omega)$ be strongly connected and let $B \subset V$. Assume Z is a strongly connected component of $G|_{\overline{B}}$. If Z_i is a copy of Z in $\mathcal{S}_B(G)$ with corresponding component branch β, then

$$\mathbf{v}_{Z_i} = P(\beta, Z)\mathbf{v}_B$$

Corollary

Let Z_i and Z_j be copies of Z in $\mathcal{S}_B(G)$ with corresponding component branches α and β. If $In(\alpha, Z_i) = In(\beta, Z_j)$, then $\mathbf{v}_{Z_i} = \mathbf{v}_{Z_j}$

Result 2

Let $G = (V, E, \omega)$ be strongly connected and let $B \subset V$. Assume Z is a strongly connected component of $G|_{\overline{B}}$. Let $D = \{\beta \in \mathcal{B}_B(G) \,|\, In(\beta, Z) \text{ is distinct}\}$. Then,

$$\mathbf{v}_Z = \sum_{\beta \in D} P(\beta, Z)\, \mathbf{v}_B$$

This seems fairly clean. We might not need the C_{in} and C_{out} definitions. We just need a definition for the set D in result 2.Let me know what you think of these. Also, what letter do you think I should use for the centrality transfer matrix?