

CTMS-MAT-13: Numerical Methods**Assignment Sheet 3. Released: 14 March 2025****Due: 24 March 2025****Exercise 1 [3+3+3+3 Points]:**

Let $f(x) = 3x^3 - 4x^2 + 4x - 1$, and consider the starting points $x_0 = 0$, and $x_1 = 1$.

- a) Check whether bisection, secant, and Newton's method can be applied.
- b) Apply three steps of the bisection, Newton's and secant method. (For Newton's method start from $x_0 = 0$)
- c) Find the root analytically and compare the errors of the results you computed in **b**).
- d) Which of the above methods are expected to converge and why?

Exercise 2 [8 Points]:

For

$$f(x) = x^4 - 0.45x^2 - 1$$

- a) Draw the function and sketch the secant method
- b) With an guess of $x_0 = 1.5$ and $x_1 = 1.4$, show that the second iterate is 1.2203.

Exercise 4 [7 Points]:

- a) From Newton's method, derive the secant method.
- b) For Newton's method, what happens for the function $f(x) = x^3 - 5x$ with the initial guess $x_0 = 1$?

Exercise 4 [8 Points]:

Starting with $(0, 0)$ apply two iterations of the Newton method for the system of non-linear equations

$$\begin{aligned} -x^2 + x + 4y &= -2 \\ (x-1)^2 + (2y-3)^2 &= 5 \end{aligned}$$