

JTMS-MAT-13: Numerical Methods

Assignment Sheet 5. Released: May 02, 2024

Due: May 14, 2024

Exercise 1 [2+2+3 Points]:

For the function $f(x) = x^2 \cos(\pi x)$, evaluate the integral

$$\int_0^1 f(x) dx$$

using five subintervals for

- (a) Trapezium rule,
- (b) Simpson's rule and
- (c) Compute the error in each case against the exact solution.

Exercise 2 [2+3+3+2 Points]:

Given the following system of linear equations:

$$5x_1 - 2x_2 + 1x_3 = -1$$

$$-3x_1 + 9x_2 + x_3 = 2$$

$$2x_1 - x_2 - 7x_3 = 3$$

- (a) Given the starting point $\vec{x}_0 = \vec{0}$, compute the next two iterates \vec{x}_1 and \vec{x}_2 of the Jacobi method.
- (b) Given the starting point $\vec{x}_0 = \vec{0}$, compute the next two iterates \vec{x}_1 and \vec{x}_2 of the Gauss-Seidel method.
- (c) Given the starting point $\vec{x}_0 = \vec{0}$, compute the next two iterates \vec{x}_1 and \vec{x}_2 of the method of successive over-relaxation with $\omega = 1.2$.
- (d) Will the Jacobi and the Gauss-Seidel methods converge? Explain the answer.

Exercise 3 [2+3+2+3 Points]:

Let $A = L + D + U$ be a non-singular matrix with L being a lower triangular, D being a diagonal, and U being an upper triangular matrix.

- (a) Show that for a linear system of equations $A\vec{x} = \vec{b}$ with solution \vec{x} , an iterative solver at iteration step k with an approximation \vec{x}_k has an error $\vec{e}_k = \vec{x} - \vec{x}_k$ which can be written as

$$\vec{e}_k = (I - Q^{-1}A)^k \vec{e}_0$$

with initial error $\vec{e}_0 = \vec{x} - \vec{x}_0$ for the initial guess \vec{x}_0 .

- (b) Show that the Jacobi iteration for solving $A\vec{x} = \vec{b}$ can be written as

$$\vec{x}_{k+1} = D^{-1} \left(\vec{b} - (L + U)\vec{x}_k \right)$$

- (c) Show that the Gauss-Seidel iteration for solving $A\vec{x} = \vec{b}$ can be written as

$$\vec{x}_{k+1} = (D + L)^{-1} \left(\vec{b} - U\vec{x}_k \right)$$

- (d) Derive how the successive over-relaxation iteration for solving $A\vec{x} = \vec{b}$ with weight ω can be rewritten in a similar form to the one in (c).