Constructor University **Autumn Semester 2024**

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Calculus and Linear Algebra for Graduate Students MDE-MET-01

Assignment Sheet 6. Released: November 25, 2024
Due: December 5, 2024

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1. **[5 + 5 points]** Determine whether the critical points of the following functions are local maxima or minima
   (a) $f = 2x^3 + 5x^2 - 10x + 1$,
   (b) $f = x^4 - x$.

2. **[5 + 5 points]** Compute the first three terms of the Taylor Series
   (a) evaluated about $(1, 1)$ for the function
   $$f = \begin{pmatrix} xy \\ x\left(x + 3y^2 x\right) \end{pmatrix},$$
   (b) evaluated about $(0, 0)$ for the function
   $$f = \begin{pmatrix} \sin(x + y) \\ x \end{pmatrix}.$$

3. **[5 + 5 points]** For a real number x, one can compute e^x as an infinite sum
   $$\mathrm{e}^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots,$$
   where $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$.
   The matrix exponential, e^A, is defined as
   $$\mathrm{e}^A = 1 + \frac{A}{1!} + \frac{A^2}{2!} + \frac{A^3}{3!} + \frac{A^4}{4!} + \ldots,$$
   where A is a square matrix. Such computations often arise in solutions of certain differential equations.
   (a) Show that for a diagonal matrix $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, we have $\mathrm{e}^A = \begin{bmatrix} \mathrm{e}^a & 0 \\ 0 & \mathrm{e}^b \end{bmatrix}$.
   (b) Compute e^A for the matrix
   $$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

4. **[5 + 5 points]** Classify the critical points of the functions
   (a) $f = x\left(y^2 - 1\right) + y^2$,
   (b) $f = xy^2 + y^2 z + zx^2$.

5. **[5 + 5 points]** Find the directional derivatives of
   (a) $f = x \sin y^2$ in the direction of $\vec{v} = (0, 1)$,
   (b) $f = xy^2 + y^2 z + zx^2$ in the direction of $\vec{v} = \left(0, 1, \sqrt{2}\right)$.