

JTMS-MAT-13: Numerical Methods

Assignment Sheet 1. Released: February 13, 2024

Due: February 23, 2024

Exercise 1 [5+5 Points]:

Let $f(x) = e^{i\omega x}$ with some positive, real number ω .

- a) Show the Taylor series for f around $c = \frac{\pi}{2}$ is given by

$$f(x) = e^{i\omega \frac{\pi}{2}} \sum_{n=0}^{\infty} \frac{(i\omega)^n \left(x - \frac{\pi}{2}\right)^n}{n!}$$

- b) Use the Taylor series truncated after the n -th term to compute approximations of $f(\pi)$ for $n = 1, \dots, 4$ for $\omega = 1$.

Exercise 3 [5+5 Points]:

- a) Show that the Taylor series, with remainder, for $\ln(1+x)$ about $x = 0$ can be written as

$$\ln(1+x) = \sum_{k=1}^n \frac{(-1)^{k+1}}{k} x^k + \frac{(-1)^n}{n+1} \frac{x^{n+1}}{(1+\xi_x)^{n+1}}.$$

- b) When $\xi_x \in (0, x)$, consider the behaviour of the remainder term in the limit of $n \rightarrow \infty$, and derive a bound on x such that the remainder term vanishes in the limit of $n \rightarrow \infty$.

Exercise 3 [5+5+3 Points]:

- a) Compute the Taylor series for $f(x) = \sin(2x^2)$ around $c = 0$. (Hint: compute for $\sin(x)$ then substitute).
- b) The Taylor series for $f(x) = \frac{\sqrt{x+1}}{2}$ around $c = 0$ represents the function for $|x| \leq 1$. What is the Taylor expansion for $n = 1$ and what is the remainder term? Calculate the number of correct digits for $x = 0.0001$ and $x = -0.0001$.
- c) Convert the following from one base to another and write down you calculations as an expansion:
- $(530)_{10}$ to $(\dots)_2$
 - $(2.05)_{10}$ to $(\dots)_2$
 - $(1.1011)_2$ to $(\dots)_8$ (Hint: consider $(1)_2 + (101)_2 + (100)_2$ to get a three digit representation in base 10, then convert each digit from base 10 to base 8).

Exercise 4 [0.5+0.5+0.5+0.5 Points]: Webcolors can be expressed with six base-16 (hexadecimal) digits (two each for the red, green and blue components, in that order) prefixed with #. The hexadecimal format uses sixteen distinct symbols, most often the symbols 0-9 to represent values 0 to 9, and A-F (or alternatively a-f) to represent values from ten to fifteen.

- How many separate shades are there in each channel of an RGB triplet and in total?
- How are black and white written in this format?
- Convert the hexadecimal colour #00b0ff into an RGB triplet.
- CMYK colours encode four channels (cyan, magenta, yellow and black), taking values between 0-100 (inclusive). Are there more possible colours in the CMYK scheme than hexadecimal?