Calculus and Linear Algebra for Graduate Students MDE-MET-01

Assignment Sheet 1. Released: September 18, 2024
Due: September 28, 2024

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1. [**5 points**] If $v + w = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $v - w = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, compute and draw v and w.

2. [**5+5+5+5 points**] Describe geometrically all linear combinations of the following collections of vectors. In each case your answer should be one of the following: a line, a plane, or all of $\mathbb{R}^3$.

   (a) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$;

   (b) $\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 12 \\ 3 \end{bmatrix}$;

   (c) $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 6 \end{bmatrix}$;

   (d) $\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

3. [**5+5 points**] The vectors $a_1, a_2, \ldots, a_n$ are in an m-dimensional space $\mathbb{R}^m$, and a linear combination $c_1 a_1 + \cdots + c_n a_n$ is the zero vector.

   (a) Write that statement at the matrix level, that is use the matrix A with the a vectors in its columns and use the column vector $c = (c_1, \ldots, c_n)$.

   (b) Write that statement at the scalar level, i.e. use subscripts and summation notation to add up numbers. The column vector a_j has components $a_{1j}, a_{2j}, \ldots, a_{mj}$.

4. [**5+5 points**]

   (a) What is the sum of the twelve vectors that go from the center of a clock to the hours 1:00, 2:00, …, 12:00?

   (b) Assume that the distance from the center of the clock to the hours is equal to 1. What is the sum of the twelve vectors that start at 6:00 (at the bottom) and go to the hours 1:00, 2:00, …, 12:00?

5. [**5 points**] Solve, by any method, the system of linear equations:

$$\begin{aligned} 4x + 12y - 16z &= 1 \\ 12x + 40y + -38z &= 2 \\ -16x - 38y + 90z &= 3. \end{aligned}$$