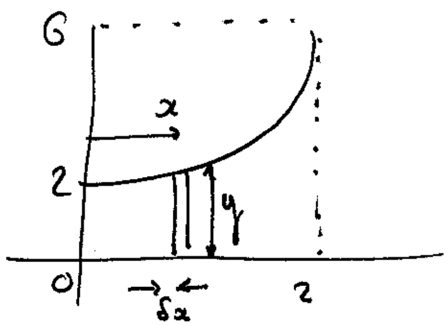
MECH1010: INTEGRATION

Quiz

1(i) $$I = \int \frac{dx}{\sqrt{a^2 - 1 - x^2 + x^2}} = \int \frac{dx}{\sqrt{a^2-(x-1)^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$$

(ii) Note that $$\int_2^3 \frac{2x}{1-x^2}\,dx = -\int_2^3 \frac{-2x}{1-x^2}\,dx = \int_2^3 \frac{f'(x)}{f(x)}\,dx$$
$$= \Big[\ln|1-x^2|\Big]_2^3 = \ln(8/3) \cancel{+c}$$

2/



(i) $$\delta m = \rho y \delta x = \rho(x^2+2)\delta x$$

$$M = \rho\int_0^2 x(x^2+2)\,dx = \rho\int_0^2 x^3 + 2x\,dx$$
$$= \rho\left[\frac{x^4}{4} + x^2\right]_0^2 = 8\rho$$

(ii) $$\delta m = \rho y \delta x, \qquad \delta I_y = x^2 \delta m = \rho x^2(x^2+2)\delta x$$

$$\Rightarrow I_y = \rho\int_0^2 x^2(x^2+2)\,dx$$
$$= \rho\int_0^2 x^4 + x^2\,dx$$
$$= \rho\left[\frac{x^5}{5} + \frac{x^3}{3}\right]_0^2 = \frac{32}{5} + \frac{8}{3}$$
$$= 136/15$$

3 (i) when $x = a\tan\theta$ and $\frac{x}{a} = \tan\theta$, $\theta = \tan^{-1}\left(\frac{x}{a}\right)$

$$\frac{dx}{d\theta} = a\sec^2\theta \Rightarrow dx = a\sec^2\theta \cdot d\theta$$

$$\int \frac{dx}{a^2+x^2} = \int \frac{a\sec^2\theta\, d\theta}{a^2+(a\tan\theta)^2} = \int \frac{a\sec^2\theta}{a^2(1+\tan^2\theta)}\, d\theta$$

$$= \frac{1}{a}\int \frac{\sec^2\theta}{\sec^2\theta}\cdot d\theta = \frac{1}{a}\int d\theta = \frac{1}{a}\theta + c$$

$$= \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c.$$

(ii) Let $\frac{x^2+1}{x(x^2-2x+1)} = \frac{x^2+1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$

$$\Rightarrow A(x^2-1) + B(x^2+x) + C(x^2-x) = x^2+1$$

$$\Rightarrow \left\{\begin{array}{l} A+B+C=1 \\ B=C \\ A=-1 \end{array}\right\} \Rightarrow \quad -1+2B=1 \Rightarrow B=1,\ C=1,\ A=-1$$

$$\int\left(\frac{1}{x+1} + \frac{1}{x-1} - \frac{1}{x}\right) dx = \ln\left|\frac{x^2-1}{x}\right| + c$$

(iii) $u = \tan x$, $\cos^2 x = \frac{1}{1+u^2}$, $\sin^2 x = \frac{u^2}{1+u^2}$, $\frac{du}{1+u^2} = dx$

$$\int \frac{dx}{2 - 7\sin^2 x + 3\cos^2 x} = \int \frac{du/(1+u^2)}{2 - 7u^2/(1+u^2) + 3/(1+u^2)}$$

$$= \int \frac{du}{2(1+u^2) - 7u^2 + 3} = \frac{1}{-5}\int \frac{du}{u^2-1}$$

$$= -\frac{1}{5}\cdot\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + c$$

(iv) $\int_0^{\pi/2} x^2\cos x\, dx$

let $u = x^2$, $du = 2x$; $dv = \cos x$, $v = \sin x$ $\Big\}$ $\int_0^{\pi/2} x^2\cos x\, dx = \left[x^2\sin x\right]_0^{\pi/2} - 2\int_0^{\pi/2} x\sin x\, dx$

$\int_0^{\pi/2} x\sin x\, dx$, let $u = x$, $du = 1$, $dv = \sin x$, $v = -\cos x$

$$\int_0^{\pi/2} x\sin x\, dx = \left[-x\cos x\right]_0^{\pi/2} + \int_0^{\pi/2}\cos x\, dx = 0 + \left[\sin x\right]_0^{\pi/2} = 1$$

$$\Rightarrow \int_0^{\pi/2} x^2\cos x\, dx = \left[x^2\sin x\right]_0^{\pi/2} - 2 = \frac{\pi^2}{4} - 2$$

4/

x_1	0	$f(x_1) = \sqrt{2}$		
x_2	$\pi/10$		1.3968	
x_3	$2\pi/10$			1.3450
x_4	$3\pi/10$		1.2601	
x_5	$4\pi/10$			1.1441
x_6	$5\pi/10$		1	
x_7	$6\pi/10$			0.8313
x_8	$7\pi/10$		0.6420	
x_9	$8\pi/10$			0.4370
x_{10}	$9\pi/10$		0.2212	
x_{11}	π	0		
		$\sqrt{2}$	4.5201	3.7574

$d = 1/10.$

Trapezium: $(\sqrt{2} + 2 \times (4.5201 + 3.7574))/20$

$= (1.4142 + 2(8.2775))/20$

$= (1.4142 + 16.555)/20$

$= 17.9692/20 = 0.8985$

Simpson's Rule: $(1.4142 + 4.(4.5201) + 2.3.7574)/30$

$= (1.4142 + 18.0804 + 7.5148)/30$

$= 27.0094/30$

$= 0.9003$