CTMS-MAT-13: Numerical Methods

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Exercise 1 [3+3+3+3 Points]:

Let $f(x) = 3x^3 - 4x^2 + 4x - 1$, and consider the starting points $x_0 = 0$, and $x_1 = 1$.

   a) Check whether bisection, secant, and Newton's method can be applied.

   b) Apply three steps of the bisection, Newton's and secant method. (For Newton's method start from $x_0 = 0$)

   c) Find the root analytically and compare the errors of the results you computed in **b**).

   d) Which of the above methods are expected to converge and why?

Exercise 2 [8 Points]:

For
$$f(x) = x^4 - 0.45x^2 - 1$$

   a) Draw the function and sketch the secant method

   b) With an guess of $x_0 = 1.5$ and $x_1 = 1.4$, show that the second iterate is 1.2203.

Exercise 3 [7 Points]:

   a) From Newton's method, derive the secant method.

   b) For Newton's method, what happens for the function $f(x) = x^3 - 5x$ with the initial guess $x_0 = 1$?

Exercise 4 [8 Points]:

Starting with $(0, 0)$ apply two iterations of the Newton method for the system of non-linear equations

$$-x^2 + x + 4y = -2$$
$$(x - 1)^2 + (2y - 3)^2 = 5$$