Constructor University Spring Semester 2024

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# CTMS-MAT-13: Numerical Methods

**Assignment Sheet 1. Due: February 28, 2024**

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**Exercise 1 [5+5+3+5 Points]**: Let $f(x) = e^{i\omega x}$ with some real number $\omega$.

**a)** Compute the Taylor series for $f$ around $c = \frac{\pi}{2}$.

**b)** Use the Taylor series truncated after the $n$-th term to compute $f(\pi)$ for $n = 1, \ldots, 5$ and an arbitrary $\omega$.

**c)** Compare values calculated in b) with the actual value of $f(\pi)$ for $\omega = 1$ and create a plot for the errors of the real part and imaginary part as a function of $n$. (Hint: Use Euler's formula)

**d)** Show that the Taylor series for $f(x) = e^{i\omega x}$ around $c = \frac{\pi}{2}$ converges to $f$ for $x \in \left[\frac{\pi}{2}, \pi\right]$.

**Exercise 2 [5+5+2 Points]**:

**a)** Compute the Taylor series for $f(x) = \sin\left(3x^2\right)$ around $c = 0$. (Hint: compute for $\sin(x)$ then substitute).

**b)** The Taylor series for $f(x) = \frac{\sqrt{x+1}}{2}$ around $c = 0$ represents the function for $|x| \leq 1$. What is the Taylor expansion for $n = 1$ and what is the remainder term? Calculate the number of correct digits for $x = 0.0001$ and $x = -0.0001$.

**b)** Convert the following from one base to another and write down you calculations as an expansion:

i) $(530)_{10}$ to $(\ldots)_2$

ii) $(1.1011)_2$ to $(\ldots)_8$