MECH1010 : Modelling and Analysis in Engineering I: Integration

Test : November 5$^{\text{th}}$ 2009

Time allowed : **50** minutes.

This is an open book test; you may your lecture notes, exercise sheets and any reference books but no electronic aides such as calculators or laptops.

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1. (i) By using integration by parts or otherwise, find the indefinite integral [3]

$$I = \int 2t \cos(t/4)\,\mathrm{d}t.$$

(ii) Evaluate the definite integral

$$I = \int_2^3 \frac{-2x}{x^2-1}\,\mathrm{d}x$$ [3]

to two decimal places.

2. (i) Sketch the region defined by $y = x^2 + 2x$ and the lines $x = 0$, $x = 2$ and the x-axis. [1]
(ii) Find the first moment of area about the zy-plane for a shape defined by $y = x^2 + 2x$ and the lines $x = 0$, $x = 2$ and the x-axis. [5]

3. (i) Use the subtitution $x = a\tan\theta$ to prove the identity

$$I = \int \frac{\mathrm{d}x}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c.$$ [3]

(ii) Find, using partial fractions or otherwise, the indefinite integral

$$I = \int \frac{x^2+1}{x\,(x^2-2x+1)}\,\mathrm{d}x.$$ [4]

(iii) Use the substitution $u = \tan x$ to find the indefinite integral

$$I = \int \frac{\mathrm{d}x}{2 - 7\sin^2 x + 3\cos^2 x}.$$ [5]

4. By dividing the domain into 5 equal sections, use the trapezium rule to approximate

$$I = \int_0^5 \left(x^2+1\right)\,\mathrm{d}x$$

to two decimal places. [7]