The effect of visco-elasticity on the stability of inertial cavitation

David Sinden, E. Stride & N. Saffari

Department of Mechanical Engineering
University College London.

9th International Conference on Therapeutic Ultrasound,
Congress Centre, Aix-en-Provence,

September 23rd–26th 2009.

Modelling Cavitation

Modified Rayleigh-Plesset equation[1]

$$\rho \left(\ddot{R} R + \frac{3}{2} \dot{R}^2 \right) = p_g \left(T, c \right) + p_v - p \left(t \right) - \frac{2\sigma}{R} - \frac{4\mu \dot{R}}{R} - \frac{R}{c} \frac{\mathrm{d}}{\mathrm{d}t} \left(p_g - p \right)$$

The equation needs to be modified in a number of ways:

- Incident wave ☑
- Visco-elasticity ☐
- Shape instability ☐
- Temperature & pressure ☐
- Rectified diffusion ☐

[1] Brenner, M.P., Hilgenfeldt, S. and Lohse, D., *Single-bubble sonoluminescence*, Rev. Mod. Phys. (2002) **74**, no. 2, p. 425–484

Visco-elasticity

What is an appropriate model?

A 4-parameter Oldroyd model[1]

$$\frac{4\mu\dot{R}}{R} \mapsto \frac{4\mu\dot{R}}{R} - S_1 - (2\alpha - 1)\, S_2 \quad \text{if and only if} \quad \alpha = 1/2 \ \text{or} \ 1$$

$$\dot{S}_1 = -\left(\frac{1}{\lambda} + \frac{4\alpha\dot{R}}{R}\right) S_1 - \frac{2\mu_p\dot{R}}{\alpha\lambda R}$$

$$\dot{S}_2 = -\left(\frac{1}{\lambda} + \frac{\dot{R}}{R}\right) S_2 - \frac{2\mu_p\dot{R}}{\lambda R}$$

[1] Jiménez-Fernández, J. and Crespo, A., *Bubble oscillation and inertial cavitation in viscoelastic fluids*, Ultrasonics **43** (2005), no. 8, p. 643–651.

Visco-elasticity

What is an appropriate model?

Another model is the linear Voigt model[1]

$$\frac{4\mu\dot{R}}{R} \mapsto \frac{4\mu\dot{R}}{R} - \frac{4G}{3R^3}\left(R^3 - R_0^3\right)$$

Changes the Blake threshold radius and the natural frequency:

$$R_c = \sqrt{\frac{3\tilde{G} + 4R_0^3 G}{2\sigma}}$$

and

$$\omega_0 = \frac{1}{\rho}\left(\frac{3\kappa\left(p_\infty - p_v\right)R_0}{R_0^3 - a^3} + 2\sigma\left(\frac{3\kappa}{R_0^3 - a^3} - \frac{1}{R_0^3}\right) + 4G\right).$$

[1] Yang, X. and Church, C., *A model for the dynamics of gas bubbles in soft tissue*, J. Acoust. Soc. Am. **118** (2005), no. 6, p. 3595–3606.

Visco-elasticity

What is an appropriate model?

Finally the Jefferies model[1]

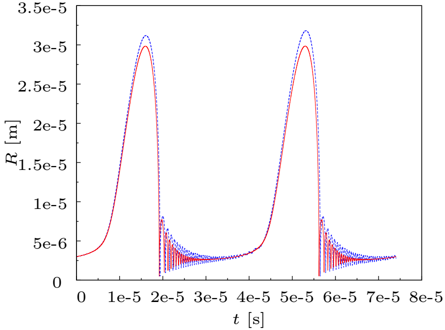
$$\frac{4\mu\dot{R}}{R} \mapsto V$$

$$V + \lambda_1 \left(\frac{\mathrm{d}V}{\mathrm{d}t} + \frac{\dot{R}}{R} \tau_{rr}\left(R\right) \right) = \frac{4}{3}\mu_p \left(\frac{\dot{R}}{R} + \lambda_2 \left(\frac{2\dot{R}^2 + R\ddot{R}}{R^2} \right) \right),$$

$$\tau_{rr}\left(R\right) + \lambda_1 \frac{\mathrm{d}\tau_{rr}\left(R\right)}{\mathrm{d}t} = 4\mu_p \left(\frac{\dot{R}}{R} + \lambda_2 \left(\frac{2\dot{R}^2 + R\ddot{R}}{R^2} \right) \right).$$

[1] Allen, J. S. and Roy, R. A. *Dynamics of gas bubbles in viscoelastic fluids. I. Linear viscoelasticity*, J. Acoust. Soc. Am. **107** (2000), no. 6, p. 3167–3178.

Visco-elasticity



- Maximum amplitude of bubble increases
- Collapse less pronounced
- Afterbounces less damped: increased likelihood of instability.
- How realistic are models compared to observations? Do the models allow for periodic oscillations? *etc.*

Shape oscillations

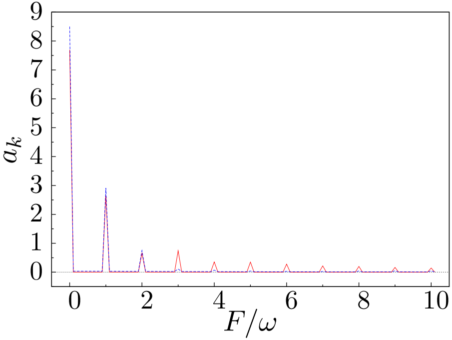
Linear Shape oscillations[1]

$$R(t) \mapsto R(t) + a_n(t)\,Y_n \Rightarrow \ddot{a}_n + B_n(t)\,\dot{a}_n + A_n(t)\,a_n = 0$$

- The model is not robust and relies on assumptions which are not experimentally verifiable: irrotational flow, inviscid and incompressible liquid. Often instability criteria appears arbitary
- What is the effect of visco-elasticity - do bubbles move translationally? Is toroidal component of vorticity non-negligible?
- Truncation ensures Rayleigh-Plesset equation decouples from mode equations: is this valid?

[1] Hao, Y. and Prosperetti, A., *The effect of viscosity on the spherical stability of oscillating gas bubbles*, Phys. Fluids **11** (1999), no. 6, p. 1309–1317.

Visco-elasticity



Can cavitation be controlled?

Whether cavitation can enhance or inhibit treatment is an open question; however it may be an empty question - mathematical techniques exist which can control[1] a reduce version of the Rayleigh-Plesset equation using multiple driving frequencies to either enhance or inhibit resonant frequencies . . .

- Mathematical challenge: incorporation of control techniques into an effective reduced model of the full multi-bubble problem. Can growth of bubble cloud be controlled?
- Practical challenge: transducer capabilities, prediction of small amplitude wave propagation

[1] Ott, E., Grebogi, C. and Yorke, J. A., *Controlling chaos*, Phys. Rev. Lett. **64** (1990), no. 11, p. 1196–1199.

Challenges

- ‘Rectified diffusion’ for both temperature and concentration.
  - Two processes are not equivalent: there exists a saturation concentration c_∞ but for temperature the equilibrium temperature T_∞ may change with time
- Gas and vapour content: effect on boundary layers?
- Incorporate viscous heating into energy equation
- Slowly evolving parameters: $C_v\left(T\right)$, $C_p\left(T\right)$, $\ldots$
- Effect of possible confining surfaces
- Multiple bubble model:
  - Rectified diffusion in multibubble model: complicated concentration profile
  - Primary and secondary Bjerknes force coupled to propagation equations