Constructor University **Autumn Semester 2024**

Dr D. Sinden

Calculus and Linear Algebra for Graduate Students MDE-MET-01

Assignment Sheet 3. Released: October 17, 2024
Due: October 28, 2024

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1. **[5 points]** The rank of a matrix A is the number of its pivot columns or equivalently, the dimension of $\operatorname{Col} A$. Show that the matrix
$$A = \begin{pmatrix} 1 & 4 & -3 \\ 2 & 8 & 6 \\ -3 & -12 & -9 \end{pmatrix}$$
has rank 1.

2. **[10 points]** Fill out these matrices so that they have rank 1:
$$A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & & \\ 4 & & \end{pmatrix}, \qquad B = \begin{pmatrix} & 9 & \\ 1 & & \\ 2 & 6 & -3 \end{pmatrix}, \qquad C = \begin{pmatrix} a & b \\ c & \end{pmatrix}.$$

3. **[5 points]** Let A be a square matrix. Do A^2 and A always have the same nullspace?

4. **[5+5+5 points]** Find a basis of $\operatorname{Nul} A$ and a basis of $\operatorname{Col} A$ for the following matrices:

   (a) $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 3 & 5 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{pmatrix}$

   (c) $\begin{pmatrix} 3 & 4 & 0 & 7 \\ 1 & -5 & 2 & -2 \\ -1 & 4 & 0 & 3 \\ 1 & -1 & 2 & 2 \end{pmatrix}$

5. **[5 points]** An $n \times n$ matrix is invertible, if and only if it has rank n. Explain, why this is true.

6. **[5+5 points]** For a given collection of vectors, determine whether they form a basis or not.

   (a) $\left\{ \begin{pmatrix} 3 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 8 \\ -2 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} -6 \\ 4 \\ -3 \\ 3 \end{pmatrix} \right\}$

   (b) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ -3 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ -6 \\ -8 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}$