MT451P - Assignment 4

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1. Compute the shape operator and Gaussian and mean curvatures for the round sphere of radius r, the round cylinder of radius r and any flat plane in R^3.

Solution: asd

2. Assume that the surface Σ is described locally near p as the graph of the function $f : \mathbb{R}^2 \to \mathbb{R}$, with $p = (0, 0, 0)$. Further suppose that f satisfies

$$f_x(0,0) = f_y(0,0) = 0$$

Then prove that the shape operator, S, of Σ at $p = (0, 0, 0)$, has matrix (in the standard basis) of the following form:

$$\text{Matrix } S = \begin{bmatrix} f_{xx}(0,0) & f_{xy}(0,0) \\ f_{yx}(0,0) & f_{yy}(0,0) \end{bmatrix}$$

Solution: A local parametrization around around p is given by

$$\phi : (u, v) \to (u, v, f(u, v))$$

Then the normal vector field is given by

$$N = \frac{\phi_u \times \phi_v}{|\phi_u \times \phi_v|} = \frac{(-f_u, -f_v m, 1)}{\sqrt{1 + f_u^2 + f_v^2}}$$

Define $\alpha = \sqrt{1 + f_u^2 + f_v^2}$. Then

$$\nabla_{\vec{e}_1} N = \left(\frac{\partial}{\partial u}\left(\frac{-f_u}{\alpha}\right), \frac{\partial}{\partial u}\left(\frac{-f_v}{\alpha}\right), \frac{\partial}{\partial u}\left(\frac{1}{\alpha}\right) \right)$$
$$\nabla_{\vec{e}_2} N = \left(\frac{\partial}{\partial v}\left(\frac{-f_u}{\alpha}\right), \frac{\partial}{\partial v}\left(\frac{-f_u}{\alpha}\right), \frac{\partial}{\partial v}\left(\frac{1}{\alpha}\right) \right)$$

The derivative will be similar for all of them so we will only work out 1 of them.

$$\frac{\partial}{\partial u}\bigg|_{p=0}\left(\frac{-f_u}{\alpha}\right) = \frac{-\alpha f_{uu} + \alpha' f_u}{\alpha^2}\bigg|_{p=} = -f_{uu}(0,0)$$

Finally we get that

$$-\nabla_{v_1\vec{e}_1+v_2\vec{e}_2}N = v_1\left(f_{uu}(0,0), f_{uv}(0,0), 0\right) + v_2\left(f_{vu}(0,0), f_{vv}(0,0), 0\right)$$

which can be represented as

$$\begin{bmatrix} f_{xx}(0,0) & f_{xy}(0,0) \\ f_{yx}(0,0) & f_{yy}(0,0) \end{bmatrix}$$