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P. Kenneth Seidelmann and Alice K.B. Monet

Astrometry with Pan-STARRS and PS1: Pushing the limits of atmospheric refraction, dispersion, and extinction corrections for wide field imaging.

K. C. Chambers

Institute for Astronomy, University of Hawaii, 2680 Woodlawn Rd., Honolulu, HI 96822

Abstract.
The potential of the Pan-STARRS project and the prototype PS1 telescope for new advances in ground based astrometry are discussed. In addition we provide a set of improved analytic expressions for atmospheric refraction and extinction accurate to 1 mas up to zenith angles of 75 degrees using an extended set of meteorological data. The application of these expressions to wide field mosaic CCD imaging surveys at high air mass is discussed. The results suggest that the absolute and relative astrometric performace of such surveys may be significantly better than previously assumed, approaching and perhaps exceeding errors ≤ 1 mas to 21 magnitude.

1. Pan-STARRS, PS1, and the Astrometric and Photmetric Survey

The Institute for Astronomy at the University of Hawaii is developing a large optical synoptic survey telescope system: the Panoramic Survey Telescope and Rapid Response System (Pan-STARRS) (Kaiser 2002). The array will consist of four 1.8m telescopes, each with a 7 square degree field of view. Pan-STARRS will be able to scan the entire visible sky to 24th magnitude in less than a week, and this unique combination of sensitivity and cadence will open up many new avenues of research in the time domain and address a wide range of astrophysical problems in solar system, stellar, galactic, extragalactic, and cosmological studies.

The Pan-STARRS Telescope No. 1 (PS1) is the prototype telescope for the Pan-STARRS project and is scheduled to begin construction on Haleakala shortly, with first light in January 2006. PS1 is a single 1.8 meter telescope whose 7 square degree field will be imaged by a Gigapixel Camera under development by J. Tonry and collaborators.

A major part of the PS1 Reference Mission (Chambers 2004) is an Astrometric and Photometric Survey, designed to cover 3π steradians in *grizy* filters. This survey will provide an astrometric and photometric backbone for the Pan-STARRS project.

CCD mosaic cameras such as UH8K, CFHT12K, and Megacam all demonstrate flexure with movement of the CCD chips relative to one another. On the other hand, Subaru's SuprimeCam and Tonry's Orthogonal Transfer OPTIC camera have both demonstrated relative ground based astrometry with ~ 3 mas accuracy (Vieira 2005). Thus it is prudent to anticipate the possibility of approaching similar accuracy with the PS1 Photometric and Astrometric Survey, and, after 10 years of Pan-STARRS, accuracies ≤ 1 mas.

Therefore, we foresee the need for new Mean-to-Observed Place transformations with precisions and accuracies < 1 mas - something not currently available (e.g. with SLALIB). Mean-to-Apparent transformations can be computed in accordance with the International Earth Rotation Service as agreed upon in IAU 2000 Resolutions B1, B2, and B3, and are provided by the SOFA Fortran library. However, the next step of Apparent-to-Observerved transformations include a treatment of atmospheric refraction, and here greater precision and accuracy are needed to reach the desired levels.

The Pan-STARRS project is in the process of constructing software libraries in C (called PSLib) to meet the astrometric, photometric, and image processing needs of the Pan-STARRS project. These libraries will be released soon for community use and development. Below we indicate some of the new algorithms being included in PSLib, including the implementation of the relevent SOFA routines in C programing language.

Figure 1 shows the cumulative airmass distribution of both an all sky survey and a "sweet spot" hazardous asteroid survey from Hawaii (Kaiser 2004). This shows that refraction up to 75 degrees zenith angle is the step in the transformations that is currently of most concern to reach the desired result. Thus, we revisit atmospheric refraction algorithms in section 3 below.

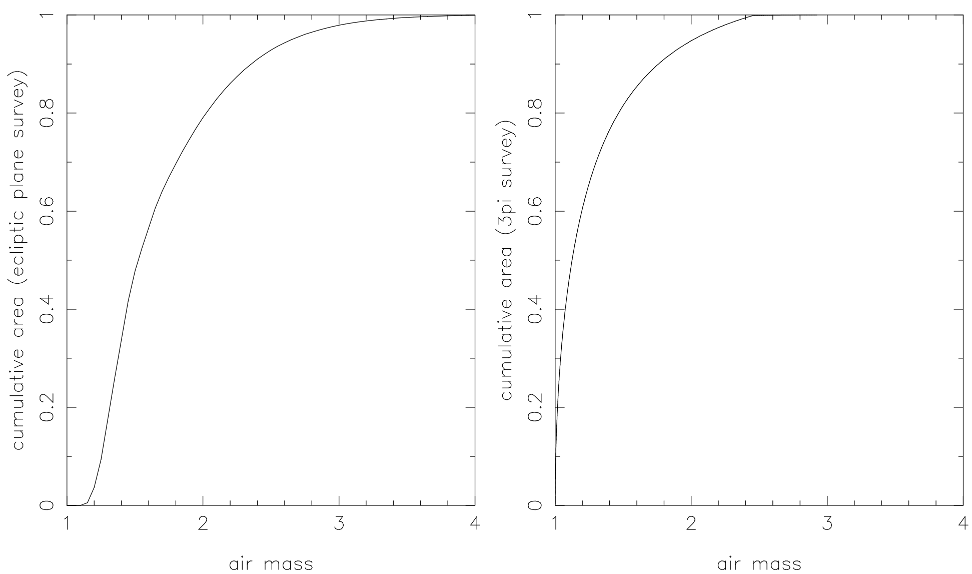

Figure 1. Cumulative area as a function of air mass for (a) an ecliptic plane survey concentrating on the "sweet spots" for potentially hazardous objects, and (b) for a 3π steradian all sky survey from Hawaii. About 20 percent of the all sky survey must be done at an airmass greater than 1.5, and 60 percent of a sweet spot survey. This does not include scheduling, just availability. A scheduled observing program can only be worse.

2. Astrometry with the Pan-STARRS Gigapixel Camera and other CCD mosaics

The basic concept for mapping the sky with PS1 (or any other CCD mosaic camera) is to start with a catalog of mean positions (α, δ) in the ICRS frame;

e.g. UCAC or possibly the Naval Observatory Merged Astrometric Dataset (NOMAD, which is not a compiled catalog, but a merged catalog with a specified rank ordering of input catalogs). Then for every star in UCAC in a given PS1 image the following transformations will be computed:

2.1. Space motion including parallax (SOFA routines STARPV and reverse PVSTAR) to current epoch, geocentric coordinates.

2.2. Light deflection due to gravitational lens effect of the Sun (New PSLib using expressions from e.g. Seidelmann 1992).

2.3. Annual aberration (using SOFA EPV00)

2.4. Precession-nutation from J2000A to current epoch (SOFA Routines)

2.5. Transform from Apparent (α, δ) to (h, δ) using SOFA timescale and Earth rotation routines to compute local apparent sidereal time.

2.6. Transform to Topocentric coordinates (h, δ) correcting for diurnal aberration (New PSLib expressions)

2.7. Transform to (Az,El)

2.8. Apply refraction correction (Section 3)

2.9. Compute Observed positions (α, δ)

2.10. These positions are then distorted by seeing, but if the exposure time is sufficiently long (and 30 seconds has proven sufficient in some cases) then the effect of seeing averages out, and certainly will average out over a great number of observations.

2.11. Apply empirically determined Observed-to-Tangent-Plane optical distortion map. It is likely that this optical distortion is best broken up into separate sections, i.e. optical distortion due to optics that don't rotate with respect to the primary mirror, and those that do, e.g. the dewar window, which is typically a highly aspheric corrector. Steps can be taken to improve the determination of these indepdendent corrections by, for example, rotating the dewar window with respect to the optical axis once or twice per year.

2.12. Linear plate reduction program, with offsets and rotations of solid body chips, or gradient technique to go from distortion corrected tangent plane to chip and pixel coordinates. Degeneracies in the asymmetries in repeated charge transfer in the Orthogonal Charge arrays and or flexure can be determined by ocasional rotation of the camera with respect to the rotator, and small linear orthogonal offsets of the camera with respect to the optical axis.

2.13. From these pixel coordinates we can apply the reverse transforms of each to take the observed positions back to ICRS coordinates.

2.14. Finally the residuals between initial and final positions in the ICRS coordinate system can be reduced by varying the free paramters in the various optical distortion maps as necessary to obtain the best fits.

2.15. After accumulating a substantial number of fields it should be possible to determine a consistent distortion model (possibly elevation dependent) and a consistent chip and pixel location model (possibly elevation and rotator angle dependent). These models will be validated, and the new astrometric catalogs produced by the reverse transform will be better than the original input Hipparcos, Tycho, UCAC, NOMAD catalog, as the fields are revisited and the new input catalogs return smaller residuals than the original input catalogs. In short, none of the above transformations should correlate with RA and DEC; any systematic correlation between RA, DEC will ultimately arise from systematic errors in the input catalogs - if the systematic errors in the use of the telescope as a theodolite are sufficiently small. This can be achieved by additional alt/az encoders.

3. Atmospheric Refraction

The hypsometric structure and index of refraction of the Earth's atmosphere produces: (i) atmospheric refraction, resulting in an apparent positional displacement of astronomical objects towards smaller topocentric zenith distances, (ii) chromatic dispersion, along great circles intersecting the topocentric zenith with shorter wavelengths having smaller zenith distances; and (iii) extinction, from scattering and absorption of light by atmospheric gases (including water vapor) and aerosols.

Atmospheric refraction $R(\lambda) = z_{vac} - z_1$ is the difference between the topocentric zenith angle *in vacuo*, where the index of refraction is $n \equiv 1$, and the observed refracted zenith angle at the observatory z_1. (All subscripts "1" in this paper indicate the local values of quantities at a given observatory site, and all units are SI, since this is the system of measurement of the physical quantities discussed in this paper.) There are various ways to express the equation for zenith angle refraction; common ones include the "general equation", e.g. (Smart 1936, 1962), the Auer & Standish transformation (Auer & Standish 1979, 2000), and the Saastamoinen approximation (Saastamoinen 1973a,b,c). All are derived from the "refractive invariant", the fact that along the refracted light path described by the locus of points $r(z, n)$, where r is the radial distance from the center of the Earth and z and n are the local values of the refracted zenith angle and the index of refraction of air respectively, the product $nr\sin z =$ constant remains invariant.

We have revisted the Saastamoinen approximation in search of an improved analytical expression accurate to ~ 1 mas up to zenith angles of 75 degrees for the purposes of the program discussed above. Although different approximations are made at different stages, both the general equation and the Saastamoinen approximation lead to an expression for the refraction $R(\lambda)$ in the form of a series in odd powers of $\tan z_1$:

$$R(\lambda) = R_1 \tan z_1 + R_3 \tan^3 z_1 + R_5 \tan^5 z_1 + R_7 \tan^7 z_1 + R_9 \tan^9 z_1 + \dots \qquad (1)$$

where we have chosen to label the coefficients with a subscript reflecting the exponent of the $\tan z_1$ terms. We find (c.f. Stone 1996)[1]

$$R(\lambda) = \gamma_1 (1 - h_1/r_1) \tan z_1 + [\gamma_1(\gamma_1/2 - h_1/r_1) + \delta_1] \tan^3 z_1 \qquad (2)$$

[1]The R_1 coefficient and the first two terms in the R_2 coefficient in equation 2 above give results similar to that of Stone's equation (4), if it is modified such that the κ factor multiplies only

in radians, where we define for convenience the variable

$$\gamma_1 = n_1 - 1$$

where $n_1(\lambda)$ is the index of refraction of air at the observatory, which is dependent on the observatory's altitude and the meteorological conditions at the time of the observation (see Section 3 below). Each of the other variables in Eq.(2) take detailed discussions which, for the sake of clarity, are divided into separate subsections.

3.1. Observatory height

The height of the observatory from the geometric center of the Earth is

$$r_1 = r_e + r_h, \qquad r_h \approx r_n + r_o, \tag{3}$$

where r_e is the local radius of the reference ellipsoid, r_h is the local height above the reference ellipsoid, r_n is the local geoid height, normal to the reference ellipsoid, r_o is the orthometric height, which is the height above the geoid (or Mean Sea Level) in the direction of normal gravity (i.e. a local plumb line).

3.2. The magnitude of normal gravity at the observatory

The local magnitude of normal gravity[2] g_1 is the acceleration due to the combination of the gravity of the ellipsoid, the local mass distribution, and the centripetal acceleration from the Earth's rotation, the vector sum being directed opposite to the local zenith by definition, being close but not identical to the normal to the ellipsoidal, but not intersecting the geometric center of the Earth. i.e. the atmospheric topocentric zenith differs from the geocentric astronomical zenith, the impact of this difference on calculating the atmospheric refraction is minimal, see Seidelmann (1992). The acceleration at the observatory is

$$g_1(r_h, \phi) = g(\phi)\left[1 - 2(1 + f + m_r - 2f\sin^2\phi)\left(\frac{r_h}{a}\right) + 3\left(\frac{r_h}{a}\right)^2\right] \tag{4}$$

where

$$g(\phi) = g_e\left(\frac{1 + k_s\sin^2\phi}{\sqrt{1 - \epsilon^2\sin^2\phi}}\right) \tag{5}$$

and ϕ is the latitude of the observatory and the other constants are given in Table 2.

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his β, rather than the whole coefficient as is done for both coefficients. We suspect this is a typographical error; his equation as written gives much greater residuals when compared to the Pulkovo Refraction Tables than he reports in his Table 2, whereas the agreement is comparable, if his equation is modified as described, and computed for the Pulkovo baseline model of $\lambda = 590$nm, $15°C$, 101325 Pa, zero water vapor, and mean sea level at latitude 45 degrees. An exact comparison is not possible because the residuals in his Table 2 are averages computed from a wide but unspecified range of meteorological conditions.

[2]Stone uses a common (e.g. Allen 1973; Seidelmann 1992; Allen 2001) expression for the normal gravity as a function of latitude and altitude, which apparently first appeared in Lambert (1949). However its derivation is unclear – cited as *"Some notes on the calculation of the geopotential", unpublished manuscript*, Lambert (1949).

Table 1. WGS-84 World Geodetic System Reference Ellipsoid for GPS

Semi-major axis a	6378137.0 m
Flattening $f = (a-b)/a$	0.003352811
Eccentricity $\epsilon = \sqrt{a^2+b^2}/a$	0.081819
Polar gravity g_p	9.8321849378 ms^{-2}
Equatorial gravity g_e	9.7803253359 ms^{-2}
Somigliana's Constant $k_s = ((b/a)(g_p/g_e) - 1)$	0.001931853
Angular velocity of the Earth ω	0.00007292115 rad/s
$GM_{\oplus}$ including atmosphere	3986004.418$\times 10^8 m^3 s^{-2}$
Gravity ratio $m_r = \omega^2 a^2 b/(GM)$	0.003449787

3.3. The scale height above the observatory

The scale height of the atmosphere above the observatory is

$$h_1 = \mathcal{Z}_1 \mathcal{R} T_1 / g_1 M_a [1 - x_w(1 - M_w/M_a)], \tag{6}$$

where T_1 is the local air temperature at the observatory in degrees Kelvin, $\mathcal{R} = 8.314472$J mol^{-1}K^{-1} is the gas constant,

$$\begin{aligned}\mathcal{Z}_1 = 1 &- (P_1/T_1)\left[a_0 + a_1 t_1 + a_2 t_1^2 + (b_0 + b_1 t_1) x_w + (c_0 + c_1 t_1) x_w^2\right] \\ &+ (P_1/T_1)^2 (d + e x_w^2)\end{aligned} \tag{7}$$

is the compressibility of moist air at local air temperature $t_1 = T_1 - 273.15$ in degrees Celsius. The values of the other constants in $\mathcal{Z}_1$ are given in Table 2. The quantity $M_a = 0.0289635 + 1.2001 \times 10^{-8}(x_{c1} - 400)$ kg/mole is the molar mass of dry air with a CO_2 concentration x_{c1} in μmol/mol, $M_w = 0.018015$ kg/mole is the molar mass of water vapor, and x_w is the molar fraction of water vapor in moist air, which depends on the local humidity.

To calculate both the scale height h_1 and the index of refraction of moist air at the observatory n_1 we need to calculate x_{w1}, the molar fraction of water vapor from local measurements of the relative humidity or, much better, the dew point. Simple formulas such as Davis (1992) are only suitable above $0°C$, whereas at Mauna Kea observatories the temperature is often below $0°C$, and occasionally the surrounding ground is covered in snow or ice, which also alters saturation vapor pressure. Thus, we adopt the best available equation for the saturation water vapor pressure p_{sv}, the IAPWS equations (Huang 1998). Haung's equations are:

$$\begin{array}{ll} \Omega = T + K_9/(T - K_{10}) & A = \Omega^2 + K_1\Omega + K_2 \\ B = K_3\Omega^2 + K_4\Omega + K_5 & C = K_6\Omega^2 + K_7\Omega + K_8 \\ X = -B + \sqrt{B^2 - 4AC} & p_{sv}(t) = 10^6(2C/X)^4 \end{array} \tag{8}$$

For saturation vapor pressure over ice or snow, use

$$\begin{array}{ll} \Theta = T/273.16 & Y = A_1(1 - \Theta^{-1.5}) + A_2(1 - \Theta^{-1.25}) \\ p_{sv}(t) = 611.657 e^Y & \end{array} \tag{9}$$

The constants are given in Table 1.

Table 2. Constants for Compressibility and Humidity Equations

$a_0 = 1.58123 \times 10^{-6}\mathrm{KPa}^{-1}$	$K_1 = 1.16705214528E + 03$
$a_1 = -2.9331 \times 10^{-8}\mathrm{Pa}^{-1}$	$K_2 = -7.24213167032E + 05$
$a_2 = 1.1043 \times 10^{-8}\mathrm{K}^{-}1\mathrm{Pa}^{-1}$	$K_3 = -1.70738469401e + 01$
$b_0 = 5.707 \times 10^{-6}\mathrm{KPa}^{-1}$	$K_4 = 1.20208247025E + 04$
$b_1 = -2.051 \times 10^{-8}\mathrm{Pa}^{-1}$	$K_5 = -3.23255503223E + 06$
$c_0 = 1.9898 \times 10^{-4}\mathrm{KPa}^{-1}$	$K_6 = 1.49151086135E + 01$
$c_1 = -2.376 \times 10^{-6}\mathrm{Pa}^{-1}$	$K_7 = -4.82326573616E + 03$
$d = 1.83 \times 10^{-11}\mathrm{K}^2\mathrm{Pa}^{-2}$	$K_8 = 4.05113405421E + 05$
$e = -0.765 \times 10^{-82}\mathrm{K}^2\mathrm{Pa}^{-2}$	$K_9 = -2.38555575678E - 01$
	$K_{10} = 6.50175348448E + 02$
	$A_1 = -13.928169$
	$A_2 = 34.7078238$

Now, to calculate the mole fraction of water vapor x_w, we need the so called "enhancement factor"

$$f(p,t) = a' + b'p + c't^2 \tag{10}$$

where $a' = 1.00062$, $b' = 3.14 \times 10^{-8}$, and $c' = 5.60 \times 10^{-7}$, and where p and t are the air pressure in Pascals and air temperature.

If you have the more precise measurement of the dew point t_d (or frost point), then

$$x_w = f(p,t) \times p_{sv}(t_d)/p. \tag{11}$$

On the other hand, if you only have the relative humidity RH, a less accurate expression is

$$x_w = (RH/100) \times f(p,t) \times p_{sv}(t)/p. \tag{12}$$

3.4. The index of refraction of moist air at the observatory

The Ciddor equation for the index of refraction of moist air (Ciddor 1996) has been adopted by the International Association of Geodesy (IAG) as the standard, as it is believed to provide the most accurate results under the largest range of wavelength, temperature, and humidity conditions (300 to 1690 nm; -40 to 100 C; 0-80% RH). Note for astronomy, the air temperature at the tropopause in the 1976 standard atmosphere is 216.6 K, below the stated range of validity. Astronomical observers often observe up to 90% RH, where water droplets can form and change the effective index of refraction. Note that developments in the equation for the index of refraction of moist air have been poorly tracked in the astronomical literature and it is critical to examine in every application what equation is actually being used. [3]

[3] Elden's (1953) original fit to the available data covered the wavelength range 2752 to 6440 Å with reasonable residuals. His 1953 constants still survive in the astronomical literature in the equations of Allen (1973); Stone (1996); Allen (2001); Roe (2002) – who extrapolates the 1953 equation to K band to correct for dichoric adaptive optics; and in the computer codes SLALIB and ZEEMAX. None discuss the range of validity. However, Elden himself revised them (Elden 1966), and these were further discussed and updated by Peck & Reeder (1972); Birch & Downs (1993, 1994); Ciddor (1996); Bonsch & Potulski (1998). Rueger, (1998) and Stone & Zimmerman (2004) make convincing arguments for the Ciddor equation, and the latter's approach is followed here.

The index of refraction of air at standard temperature and pressure is (Ciddor 1996)

$$\gamma_{as} = 10^{-8}\left(\left[\frac{k_1}{k_0-\sigma^2}\right]+\left[\frac{k_3}{k_2-\sigma^2}\right]\right), \tag{13}$$

where $\sigma = 1/\lambda$ is the wavenumber of wavelength of light in microns. Adjusting for the (annually varying and secularly increasing) value of atmospheric CO_2 concentration x_{CO2} in units of μmole/mole, the expression for dry air becomes

$$\gamma_{axs} = \gamma_{as}\left[1+5.34\times10^{-7}(x_{CO2}-450\ \mu\text{mole/mole})\right]. \tag{14}$$

For water vapor under standard conditions, Ciddor finds

$$\gamma_{ws} = 1.022\times10^{-8}\left[\omega_0+\omega_1\sigma^2+\omega_2\sigma^4+\omega_3\sigma^6\right]. \tag{15}$$

Following (Owens 1967), the indicies can be combined in proportion to their densities, thus the index of refraction of moist air at the observatory $n_1 = \gamma_1 + 1$ is given by

$$\gamma_1 = (\rho_a/\rho_{axs})\gamma_{axs} + (\rho_w/\rho_{ws})\gamma_{ws}, \tag{16}$$

where

$$\begin{aligned}\rho_a &= (1-x_w)P_1M_a/(\mathcal{Z}_1\mathcal{R}T_1)\\ \rho_w &= x_wP_1M_w/(\mathcal{Z}_1\mathcal{R}T_1)\\ \rho_{axs} &= P_{STP}M_a/(\mathcal{Z}_a\mathcal{R}T_{STP})\end{aligned} \tag{17}$$

and ρ_{ws} is given in Table 3.

Table 3. Constants in the Ciddor Eq. for index of refraction of moist air

$k_0 = 23.0185\ \mu\text{m}^{-2}$	$\omega_0 = 295.235\ \mu\text{m}^{-2}$	$P_{STP} = 101325$ Pa
$k_1 = 5792105\ \mu\text{m}^{-2}$	$\omega_1 = 2.6422\ \mu\text{m}^{-2}$	$T_{STP} = 288.15$ K
$k_2 = 57.362\ \mu\text{m}^{-2}$	$\omega_2 = -0.03238\ \mu\text{m}^{-4}$	$\mathcal{Z}_a = 0.9995922115$
$k_3 = 167917\ \mu\text{m}^{-2}$	$\omega_3 = 0.004028\ \mu\text{m}^{-6}$	$\rho_{ws} = 0.00985938$ kg m^3

3.5. The tropopause term in the equation of refraction

The final term in the Refraction Equation (2) is δ_1, which comes from our re-derivation of the Saastamoinen approximation:

$$\delta_1 = 5\left(\frac{h_1}{r_1T_1}\right)^2\left[\frac{\gamma_{ft}T_{ft}^2-\gamma_tT_t^2}{1-(h_1\beta/T_1)}+\gamma_tT_t^2\right] \tag{18}$$

where β is the lapse rate of the troposphere, and the index of refraction of the free troposphere is given by

$$\gamma_{ft} = \gamma_t(T_{ft}/T_t)^{-(T_1/h_1\beta)-1} \tag{19}$$

and the index of refraction of the tropopause is given by

$$\gamma_t = \gamma_1\exp\left[\frac{T_1(r_t-r_1)}{T_th_1}\right] \tag{20}$$

where β is the lapse rate in K/m, the temperature of the tropopause T_t, and height of the tropopause r_t are all determined from contemporaneous meterological data (radiosonde or modern forecast models). Then the temperature of the free troposphere is given by

$$T_{ft} = T_t - \beta(r_t - r_1). \tag{21}$$

3.6. Calculating the atmospheric refraction from both the observed and true zenith angle

The monochromatic refraction can now be calculated for any given wavelength λ_{air} (formally only within the range of validity - 300 to 1670 nm) given the altitude of the observatory h_1; contemporaneous meteorological measurements at the observatory of air temperaure T_1 (K); atmospheric pressure P_1 (Pa); percent relative humidity RH, or preferably dew point temperature t_d (°C); as well as a small set of additional meteorological data: the lapse rate of the troposphere β (K/m); the radius of the tropopause $r_t = r_e + h_t$(m), where h_t is the height above MSL of the tropopause; the temperature of the tropopause T_t(K); and the atmospheric concentration of CO_2 x_{CO2} (μmole/mole). The characterization of the troposphere and tropopause can be determined either by interpolation between radiosonde measurements or from hourly updated meteorological models available at many observatory sites, or, worst case, simply adopting the 1976 Standard Atmosphere values:[4] $\beta = -0.0065$ K/m, $h_t = 11000$m, $T_t = 216.6$ K and assuming $x_{C02} = 375\mu$mole/mole.[5]

If the number of electrons being created from the illumination from the source in the interval of wavelength $d\lambda$ is $N_\lambda d\lambda$, then the mean refraction is

$$\bar{R} = \frac{\int R(\lambda) N_\lambda d\lambda}{\int N_\lambda d\lambda} \tag{22}$$

4. Atmospheric Dispersion

The atmospheric dispersion is then

$$\overline{(R - \bar{R})^2} = \frac{\int (R - \bar{R})^2 N_\lambda d\lambda}{\int N_\lambda d\lambda} \tag{23}$$

5. Air Mass and Extinction

By Laplace's theorem, the monochromatic airmass (mass per unit area along the refracted path) is

$$M(z_1) = (P_1/g_1) R(\lambda)/\sin z_1 \tag{24}$$

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[4]Seidelmann (1992) contains a typographical error quoting this value in units of K/km

[5]Closed rooms have higher CO_2 concentration, thus the STP laboratory measurements have concentrations near 450 μmole/mole (the prefered unit to parts per million per volume). The secular increase of atmospheric CO_2 in the industrial age is well documented, with an annual cycle superimposed due to terrestrial biomass and ocean exchange.

in kg/m^2. Thus

$$M(z_1) = (P_1/g_1)\left(\gamma_1(1 - h_1/r_1)\mathrm{sec}z_1 + [\gamma_1(\gamma_1/2 - h_1/r_1) + \delta_1]\mathrm{sec}^3 z_1\right). \quad (25)$$

This is generally normalized to P_1/g_1, i.e. in spite of the daily changes in barometric pressure, and thus daily changes in the true mass of air over the observatory, the resulting change in extinction is generally treated as a drift in photometric zeropoint. For a survey program like Pan-STARRS, one could instead normalize to a standard barometric pressure (i.e. the altitute pressure), and thus during a low in atmospheric pressure, the airmass would be less than 1 at zenith, and during periods of high pressure the airmass would be greater than 1 at zenith. From the variation with zeropoint and temperature and barometric pressure, this would remove most of the observed variation in zeropoint in the CFHT legacy program. (Magnier, private comunication).

The mean airmass is then

$$\bar{M}(z_1) = \left(\frac{P_1}{g_1}\right)\int\left(\gamma_1(1 - h_1/r_1)\mathrm{sec}z_1 + [\gamma_1(\gamma_1/2 - h_1/r_1) + \delta_1]\mathrm{sec}^3 z_1\right)N_\lambda d\lambda. \quad (26)$$

and depends weakly on the filter bandpass. Use of this more accurate expression for airmass should lead to improved extinction corrections at high airmass.

6. Limits to ground based relative and absolute astrometry

The limits to ground based astrometry may well be our abilitiy to measure the atmospheric profile along the line of sight of a given observation, and the systematic limit of the telescope axes encoders (and sophistication of the telescope mount model.) The refraction model above requires only the additional data of the temperature, height, and pressure of the tropopause, but much more detailed atmospheric information will be available for PS1 from our sky probes which measure atmospheric absorption for each field and even phase drifts of GPS clocks from Rubidium or Cesium standard clocks. These can be converted directly into a nearby line of sight index of refraction at optical wavelengths. Thus we encourage wide field survey telescopes to err on the side of over instrumenting the accuracy and repeatibility of the axes encoders.

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