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An Accurate Method for Computing Atmospheric Refraction

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ABSTRACT. This paper describes a method for computing atmospheric refraction that is simple, highly accurate, and usable for wavelengths λλ3000–10,000 Å. When compared with the *Refraction Tables of the Pulkovo Observatory* [Albalakin, V. R. 1985 (Leningrad, Nauka)], the computed and Pulkovo refractions are in extremely good agreement (~10 mas or better) for zenith distances under 65°, and at a zenith distance of 75°, the agreement degrades to about ~150 mas. Moreover, this refraction is easy to compute, since only knowledge of the meteorological conditions at the observing site (e.g., ambient temperature, atmospheric pressure, and dew point or relative humidity), the apparent zenith distance of the object being observed, and the location of the observing site relative to the Earth's geoid are needed for its calculation. Because of its simplicity and high accuracy, the refraction presented in this paper is well suited for programs where huge amounts of data need to be processed quickly to star positions or for accurately setting automated telescopes. In addition, the refraction for a given telescope passband will differ in a way dependent on the spectrum of the object being observed. This paper describes how a mean refraction, weighting the selective refraction over the passband, can be computed.

1. INTRODUCTION

Ideally, atmospheric refraction at a particular observing site should be determined empirically. As discussed by Podobed (1965) and Eichhorn (1974), refraction can be determined from nightly observations of circumpolar stars observed at both upper and lower culminations. These observations are often incomplete, but by combining these data with a general theory for refraction, tables of refraction can be constructed for a wide range of meteorological conditions and zenith distances. The most extensive tabulations are given in the *Refraction Tables of the Pulkovo Observatory* (Abalakin 1985), based on observations made at Pulkovo and the atmospheric model of A. I. Nefed'eva and I. G. Kolchinsky. These tables supersede the previous Pulkovo refraction tables (Orlov 1956), which have been used for many years in meridian astrometry. The empirical observations for atmospheric refraction are difficult to obtain, since the zenith distances of stars observed at both culminations have to be very accurately determined. Historically, these observations have been made with meridian telescopes.

Unfortunately, refraction tables have been determined only for the visual passband, and now there are many needs for computing refraction at other wavelengths. In particular, there are currently several programs for determining billions of star positions, using either photographic or digital data, in passbands differing significantly from the visual (Gunn and Knapp 1993; Lasker 1995). The Pulkovo refraction tables are not suitable for these and similar programs. For example, the Flagstaff Astrometric Scanning Transit Telescope (FASTT) is being used to determine accurate star positions in the J2000 extragalactic reference frame with long strip scans (Stone et al. 1996). Many thousands of stars can be observed each night, and the telescope is using a very broad passband (λλ4700–7300 Å) so that objects as faint as V~17.5 mag can be routinely observed. Figure 1 shows the passband for the FASTT along with the traditional V passband, and as seen, the passband centers are significantly different. If effective wavelengths are used to define their centers, then these centers differ by about 570 Å, and since refraction is a strong function of wavelength (see Fig. 2), the two passbands are expected to have different amounts of refractions. This difference will be 0.3 arcsec at a zenith distance of 45°, which is significant, and at greater zenith distances, this difference will become even larger, reaching a value of 0.7 arcsec at a zenith distance of 75°. If errors in refraction are to be kept under ±50 mas at zenith distances under 70° (which is needed for accurate positional astrometry), then clearly the Pulkovo tables are not adequate for computing refraction in FASTT reductions. Accurate computations for refraction are also needed in spectrophotometry according to Filippenko (1982).

In the absence of good empirical data, atmospheric refraction can be computed theoretically with the refraction models presented by Garfinkel (1967) and others. These models require detailed knowledge about the structure of the atmosphere and are complicated to use. By making several simplifying assumptions (discussed in Sec. 2), the computation for atmospheric refraction can be reduced to a boundary-value problem, where only knowledge of the meteorological conditions at the observing site and the zenith distances of objects being observed are needed. This simplified refraction model is very fast, and accurate to ±40 mas or better at zenith distances under 70°. Because of its simplicity and high accuracy, this refraction is very suitable in programs where large amounts of data have to be rapidly reduced to star positions. The model does fail for zenith distances greater than 75°, which fortunately includes few observations in astronomy. Furthermore, these calculations can be made for arbitrary passbands within the wavelength range

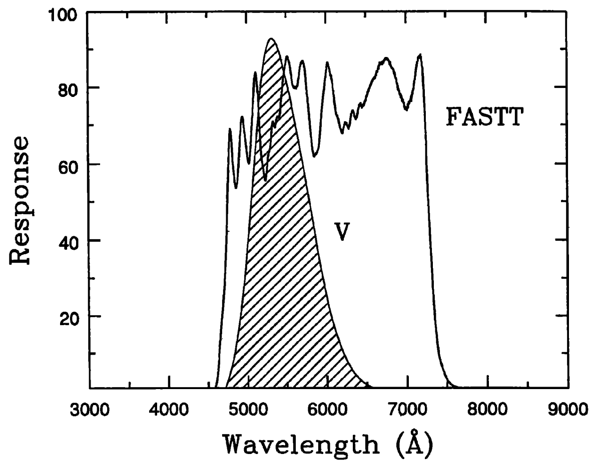

FIG. 1—The passbands for the FASTT transit telescope and the Johnson V passband are shown. The irregular response of the former is caused by the interference filter used to define the passband.

$\lambda\lambda$3000–10,000 Å, which is important for many applications. Sections 2, 3, and 4 of this paper describe how this refraction can be computed, and in Sec. 5, this refraction is compared with tabulated values of refraction determined at the Pulkovo Observatory.

2. REFRACTION THEORY

In a pure sense, atmospheric refraction should be determined theoretically by tracing the path of light through the Earth's atmosphere, wherein the refraction will be just the difference in the directions of the light before it enters the atmosphere and as seen at the telescope. In order to make this tracing, detailed knowledge of the atmospheric temperature, pressure, and water vapor is needed along this path. Although these aerological data can be obtained from radiosonde, radar, and lidar measurements, it would be impractical to do so on a nightly basis because of the high costs involved. Alternately, a model for the atmosphere can be assumed, and the aerological data assumed from it. The *U.S. Standard Atmosphere* (1976) is often chosen, and there have been several studies using this approach (Garfinkel 1967; Fukaya and Yoshizawa 1985; Yatsenko 1995). Also, refraction can be determined in a very straightforward manner, requiring only knowledge of the meteorological conditions (ambient temperature, atmospheric pressure, and water vapor) recorded at the observing site with each observation. Besides being very simple (only analytic expressions are used) and fast, this approach is also very accurate for zenith distances under 75°. The theory is described by Smart (1965) and Green (1985) and will be briefly discussed in this paper.

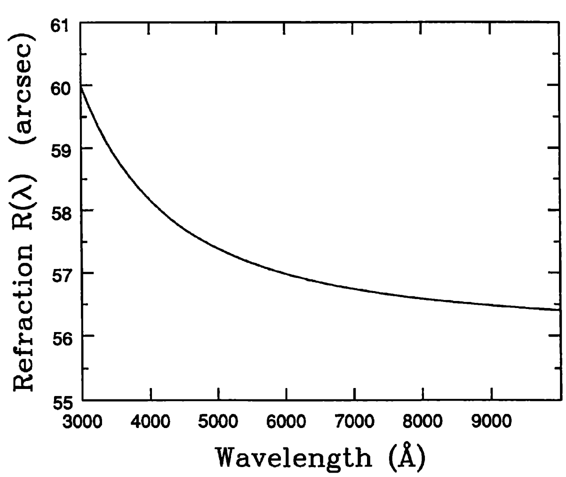

FIG. 2—Atmospheric refraction as a function of wavelength is shown over the wavelength interval $\lambda\lambda$3000–10,000 Å, at a zenith distance of 45°, and ambient conditions characterized by temperature 15 °C, atmospheric pressure 760 mm, and no water-vapor pressure. Refraction is stronger at shorter wavelengths.

A good approximation is that the Earth's atmosphere is spherically symmetric, in which case, the atmospheric refraction $R(\lambda)$ at wavelength λ is given exactly by the refraction integral

$$R(\lambda)=r_0 n_0 \sin z_0 \int_1^{n_0} \frac{dn}{n(r^2n^2-r_0^2n_0^2\sin^2 z_0)^{1/2}}, \tag{1}$$

tracing the path of light from above the Earth's atmosphere to the observing site, where z_0 is the apparent zenith distance of a star (as seen by an observer at the observing site); n_0 is the index of refraction of air at the observing site; n is the index of refraction at some arbitrary point along the incoming path of light; r_0 is the geocentric distance of the observing site; and r is the geocentric distance at the arbitrary point. The subscript (0) corresponds to conditions at the Earth's surface. According to the Gladstone–Dale Law, the index of refraction n at an arbitrary point in the Earth's atmosphere can be expressed in terms of the densities of air at the arbitrary point ρ and at the Earth's surface ρ_0 with the expression

$$n=1+(n_0-1)\frac{\rho}{\rho_0}. \tag{2}$$

Equation (2) can be substituted into Eq. (1), and the latter expanded into a power series of the form

$$R(\lambda)=B_1 \tan z_0+B_2 \tan^3 z_0+B_3 \tan^5 z_0+\cdots, \tag{3}$$

where the first two terms in the expansion are given by

$$R(\lambda)=\kappa\gamma(1-\beta)\tan z_0-\kappa\gamma(\beta-\gamma/2)\tan^3 z_0 \tag{4}$$

with the definitions

$$\gamma=n_0-1, \tag{5}$$

$$\beta=\frac{H_0}{r_0}. \tag{6}$$

By retaining only the first two terms in Eq. (3) (which is a very good approximation for zenith distances $z_0<75°$), the refraction can be easily computed with Eq. (4), which requires only knowledge of the meteorological conditions at the observing site and the apparent zenith distance of the object being observed. For a spherical Earth, the value of parameter κ is $\kappa=1.0$.

Hence, the computation for refraction is reduced to a boundary-value problem with this formulation. At larger zenith distances, additional terms in the expansion given by Eq. (3) are needed, whose computation requires detailed knowledge of the aerological conditions at all levels in the atmosphere. Notwithstanding this limitation, the refraction presented in this paper is still very useful, since most observations in astronomy (requiring corrections for refraction) are not made at such large zenith distances. The parameter H_0 is defined in the literature as the *height of an equivalent homogeneous atmosphere* and is defined by the following integration:

$$H_0=\frac{1}{\rho_0}\int_0^\infty \rho\, dh. \tag{7}$$

In the above integration, parameter h is the height above the Earth's surface. If the radial density dependence of the atmosphere is assumed to be exponential and of the following form:

$$\rho=\rho_0 e^{-h/h_0}, \tag{8}$$

then H_0 will be just the scale height h_0 of the atmosphere. Numerical approximations for H_0 are known, and when substituted into Eq. (6) and simplified, the equation can be rewritten as

$$\beta=0.001254\left(\frac{273.15+t}{273.15}\right). \tag{9}$$

The above formulation can be modified to include the apparent shape of the Earth, herein assumed to the Earth's geoid or equipotential surface at sea level. The resulting refraction is given by Eq. (4) where parameter κ in the equation is defined as the ratio of the gravity g_0 at the observing site to the sea-level gravity g at the Earth's equator. This ratio is given accurately by

$$\kappa=g_0/g=1+0.005302\sin^2\phi-0.00000583\sin^2(2\phi)$$
$$-0.000000315h, \tag{10}$$

where ϕ is the astronomical latitude of the observing site in degrees, and h is the elevation of the site above sea level in meters.

According to the above theory, the refraction $R(\lambda)$ at the observing site can be computed easily by first computing the index of refraction of air n_0 at the site (see Sec. 3) and then evaluating the algebraic expressions given by Eqs. (5), (9), (10), and (4), in that order.

3. COMPUTATION FOR THE INDEX OF REFRACTION

3.1 The Computation

The index of refraction of air at wavelength λ(Å) can be computed accurately from the known air temperature t (°C), atmospheric pressure p_s (mm), and water vapor pressure p_w (mm) using the empirical approximations given by Owens (1967). As discussed in Owens, these approximations are for standard air (including 0.03% CO_2) and are usable over the following broad range of conditions: -23 °C$<t<47$ °C, $0<p_s<4$ atm, relative humidity $0<\mathrm{RH}<100\%$, and wavelengths 2302 Å$<\lambda<$20,586 Å. The units chosen for the above meteorological variables (t, p_s, and p_w) are those given by most measuring devices. These units need the following conversions to be used in Owens' formulae:

$$T=273.15+t, \tag{11}$$

$$P_s=1.333224(p_s-p_w), \tag{12}$$

$$P_w=1.333224p_w, \tag{13}$$

where the temperature T is now in K and the pressures (P_s, P_w) in millibars. Pressure P_s is the pressure of dry air, which is atmospheric pressure corrected for water vapor.

According to Owens, the index of refraction of air n can be computed very accurately with the following relations:

$$(n-1)\times10^8=\left[2371.34+\frac{683939.7}{(130-\sigma^2)}+\frac{4547.3}{(38.9-\sigma^2)}\right]D_s$$
$$+(6487.31+58.058\sigma^2-0.71150\sigma^4$$
$$+0.08851\sigma^6)D_w, \tag{14}$$

$$D_s=\left[1+P_s\left(57.90\times10^{-8}-\frac{9.3250\times10^{-4}}{T}+\frac{0.25844}{T^2}\right)\right]\frac{P_s}{T}, \tag{15}$$

$$D_w=\left[1+P_w(1+3.7\times10^{-4}P_w)\left(-2.37321\times10^{-3}+\frac{2.23366}{T}\right.\right.$$
$$\left.\left.-\frac{710.792}{T^2}+\frac{7.75141\times10^4}{T^3}\right)\right]\frac{P_w}{T}, \tag{16}$$

where σ is the wave number of monochromatic light of wavelength λ

$$\sigma=10^4/\lambda\ (\mu\mathrm{m}^{-1}). \tag{17}$$

Equations (14), (15), and (16) and the conversion given by Eqs. (11), (12), and (13) enable the index of refraction n at wavelength λ to be computed for air with the meteorological conditions t, p_s, and p_w. If these conditions are known at the observing site (preferably near the telescope), then the local value for the index of refraction of air $n_0(\lambda)$ can be computed with the above formulae, and in turn, the atmospheric refraction $R(\lambda)$ computed with Eq. (4).

3.2 Choices for Meteorological Sensors

If high accuracy is required (i.e., the error in the computed refraction is ±50 mas or less at zenith distances $z_0<70°$), readings for the meteorological conditions (t, p_s, and p_w) should be taken near the telescope and accurately known at the time of each observation. In particular, the local ambient temperature t should be accurate to ±0.1 °C. The FASTT uses resistance temperature detector (RTD) platinum probes that are located near the telescope objective and have been calibrated to ±0.05 °C or better. These probes are very linear and stable and are preferable to other types of probes (thermocouples and thermisters) for measuring temperatures. The atmospheric pressure p_s should be known to ±0.3 mm or better. For the FASTT, atmospheric pressure is measured with Setra digital barometers that have been calibrated to ±0.1 mm. The water-vapor pressure p_w cannot be directly measured. Rather, it can be computed from either a dew-point or relative-humidity reading taken near the telescope, which should be accurate to ±10 °F and ±12%, respectively. Dew-point readings are more accurate and can be measured with either chilled mirror or LiCl sensors. Considering these two kinds of sensors, chilled mirrors are more accurate and do not require frequent recalibrations. The dew point sensors used with the FASTT are accurate to ±2 °F.

The water-vapor pressure (mm) can be determined from a dew point reading t_d (°C) with the following approximation:

$$p_w = 4.50874 + 0.341724 t_d + 0.0106778 t_d^2 + 0.184889 \times 10^{-3} t_d^3 + 0.238294 \times 10^{-5} t_d^4 + 0.203447 \times 10^{-7} t_d^5, \tag{18}$$

which is a numerical approximation adapted from the *Smithsonian Meteorological Tables* (1951) relating water-vapor pressure to the dew point. The appropriate table is given also in Hughes et al. (1992). Often, the dew point t_d is measured in °F which can be converted to °C, needed for Eq. (18), with the following equation:

$$t_d(°C) = \frac{t_d(°F) - 32}{1.8}. \tag{19}$$

The water-vapor pressure can be computed also, albeit less accurately, from the known ambient temperature t (°C) and relative humidity RH (%) by first calculating the corresponding dew point with the following approximation:

$$t_d(°C) = 238.3 \left[\frac{(t+238.3)x + 17.2694t}{(t+238.3)(17.2694-x) - 17.2694t} \right], \tag{20}$$

with the definition

$$x = \ln(\mathrm{RH}/100), \tag{21}$$

and then calculating the water-vapor pressure with Eq. (18). Equation (20) is usable for relative humidities in the range $0 < \mathrm{RH} < 100\%$. The accuracy of the meteorological conditions (t, p_s, and p_w) can be further improved by having redundancy in the measuring devices and averaging the individual readings. For example, the FASTT uses three temperature probes, barometers, and dew-point sensors for computing these quantities.

For astrometry in a small field (i.e., differential reductions made with reference objects in the field), these accuracies can be much worse. For accuracies of ±10 mas or less in a 5° field in declination, the required accuracies are only ±0.5 °C and ±1 mm, respectively, for temperature and pressure readings. The correction for water vapor can be ignored altogether.

4. COMPUTATION FOR REFRACTION

4.1 Mean Refraction

The previously discussed refraction $R(\lambda)$ is only for monochromatic light of wavelength λ. In reality, most observations in astronomy are made through passbands with spectral widths ranging from ~200 to ~ 2000 Å. As a first approximation for the refraction, the effective wavelength of a passband could be determined and then used to compute the refraction with the precepts discussed in the previous sections. Unfortunately, the effective wavelength for a given passband is not a constant, but rather a variable that depends on the nature of the incoming light. For example, Fig. 3 shows the passband for the FASTT along with the normalized spectral energy distributions for a B5 and M0-type star. As shown in the figure, the spectral flux for the B5 star is greatest at the blue edge of the passband, while the opposite is true for the M0 star. Because of this effect, the effective wavelength of the FASTT passband, when observing the B5 star, will be shifted about 160 Å to the blue of the corresponding wavelength for the M0 star. Furthermore, since refraction is stronger at shorter wavelengths, the B5 star will experience more refraction. At a zenith distance of $z_0 = 75°$, the difference in refraction will be 0.13 arcsec, which is significant.

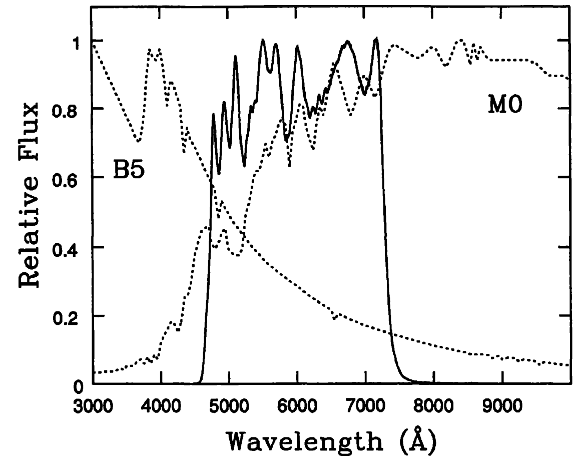

FIG. 3—The passband for the FASTT transit telescope is shown along with normalized spectral energy distributions for two stars with widely differing spectral types. Refraction for the B5 star will be greater, since the passband contains more blue light (see text). This is an example of differential color refraction.

As discussed in Stone (1984), a better method for computing refraction consists of calculating a mean refraction R_m by weighting the individual selective refractions $R(\lambda)$ with the apparent stellar flux at wavelength λ and averaging across the passband. The mean refraction is given then by

$$R_m = \frac{\int_0^\infty S(\lambda)E(\lambda)A(\lambda)L(\lambda)F(\lambda)D(\lambda)R(\lambda)d\lambda}{\int_0^\infty S(\lambda)E(\lambda)A(\lambda)L(\lambda)F(\lambda)D(\lambda)d\lambda}, \tag{22}$$

where $S(\lambda)$ is the spectral energy distribution for the star being observed; $E(\lambda)$ is the transmittance of interstellar dust along the line of sight; $A(\lambda)$ is the transmission of the atmosphere at the airmass being observed; $L(\lambda)$ is the transmission of the telescope optics; $F(\lambda)$ is the filter transmission; $D(\lambda)$ is the quantum efficiency of the detector being used; and $R(\lambda)$ is the selective refraction discussed in the previous sections.

For simplicity, a blackbody function could be used for the spectral energy function $S(\lambda)$; however, this can be a poor assumption, if prominent spectral features are present within the passband. These features can reduce or increase the amount of refraction, depending on their prominence and placement within the passband. For example, a narrow passband centered on ~5000 Å will be strongly affected by TiO absorption (see Fig. 3) when observing an M0-type star. It would be better to use the spectral energy distribution of the star being observed. If the true distribution is not known, which is usually the case, then a distribution can be chosen that matches the spectral type of the star. Tabulations of spectral energy are given for all spectral types by Straizys and Sviderskiene (1972), Gunn and Stryker (1983), and Jacoby et al. (1984). In FASTT computations for refraction, the spectra $S(\lambda)$ given by Straizys and Sviderskiene are used.

The function $E(\lambda)$ in Eq. (22) is given by

$$E(\lambda)=10^{-0.4RE(B-V)a(\lambda)}, \tag{23}$$

where $a(\lambda)$ is the interstellar absorption in magnitudes at wavelength λ, parameter R is defined as the absorption ratio ($R\sim3$), and $E(B-V)$ is the color excess of the star being observed. The absorption $a(\lambda)$ has been normalized to a reddening of $RE(B-V)=1.0$, is tabulated by Allen (1973), and a numerical approximation is given below:

$$a(\lambda)=-0.544472+\frac{0.995958}{\lambda}-\frac{0.0805414}{\lambda^2}. \tag{24}$$

The atmospheric transmission $A(\lambda)$ at zenith distance z_0 is given by

$$A(\lambda)=a'(\lambda)^{\sec z_0}, \tag{25}$$

where

$$a'(\lambda)=1.33425-\frac{0.584170}{\lambda}+\frac{0.290928}{\lambda^2}-\frac{0.0676255}{\lambda^3} \tag{26}$$

is the atmospheric transmission at the zenith, as adapted from the tabulation given in Allen. The wavelengths λ in Eqs. (24) and (26) are in μm. Moreover, the instrumental response functions $L(\lambda)$, $F(\lambda)$, and $D(\lambda)$ in Eq. (22) should be known for a particular telescope/detector system.

Once the individual response functions in Eq. (22) are known, the refraction $R(\lambda)$ at wavelength λ can be computed with the technique discussed previously, and a mean refraction R_m computed by numerically integrating Eq. (22) across the passband. In FASTT reductions, each response function has been tabulated in increments of 50 Å in the wavelength range 3000–10,000 Å, and Eq. (22) integrated over this range with Simpson's Rule. The increment was chosen small enough to resolve most spectral line features. As discussed in Stone (1984), each response function will affect the computed mean refraction in a particular way.

4.2 Practical Refraction

For many stars in observing programs, neither the spectral type nor the color excess are known from spectroscopy. If multiband photometry is available (see, for example, Gunn and Knapp 1993), then a spectral type and color excess can be inferred. If only a color index is known, then a crude spectral type can be determined. When there is neither spectral nor photometric data available, which is often the case, an assumed spectral type and color excess can be adopted. In FASTT observing programs, these anonymous stars are assumed to have a spectral type of K0 and a color excess of $E(B-V)=0.3$ mag, which are representative values for most faint stars. These are working assumptions, bearing in mind the refraction for these stars can be later corrected, should these parameters eventually become known. By assuming these parameters, the functions $S(\lambda)$ and $E(\lambda)$ in Eq. (22) become functions only of wavelength, and the computation for mean refraction, given by Eq. (22), can be simplified to

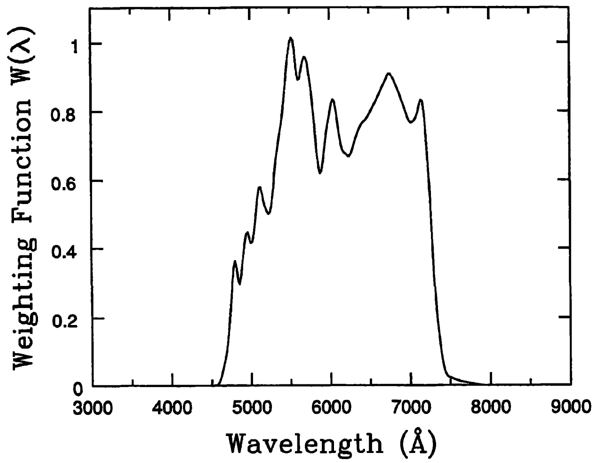

FIG. 4—Weighting function $W(\lambda)$ used to compute refraction in FASTT transit telescope observing programs.

$$R_t=\frac{\int_0^\infty W(\lambda)A(\lambda)R(\lambda)d\lambda}{\int_0^\infty W(\lambda)A(\lambda)d\lambda}, \tag{27}$$

where the weighting function

$$W(\lambda)=S(\lambda)E(\lambda)L(\lambda)F(\lambda)D(\lambda) \tag{28}$$

is only a function of wavelength. In FASTT observing programs, the weighting function $W(\lambda)$ is tabulated in increments of 50 Å, and Eq. (27) is used to compute numerically the mean refraction using this function, the atmospheric response function $A(\lambda)$ given by Eq. (25), and the selective refraction $R(\lambda)$ given by Eq. (4). The adopted weighting function $W(\lambda)$ for the FASTT is shown in Fig. 4. With this function, the calculation for refraction can be made very rapidly and simply (the FASTT subroutine for computing refraction has only 60 lines of computer code).

Adopting a chosen spectral type for anonymous stars will cause a systematic error in the computed refraction at other spectral types. This is illustrated in Table 1, where the refraction for differing spectral types has been computed at a zenith distance of 45°, referenced to the refraction for an F-type star, and given for various commonly used passbands. Several conclusions can be drawn from the differential color refraction given in the table. Namely, the bluer a passband center is, the more pronounced the color refraction will be. This is to be expected considering the selective nature of atmospheric refraction (see Fig. 2). Furthermore, color refraction is a quasilinear function that decreases with later

TABLE 1
Differential Color Refraction at a Zenith Distance ZD=45°

Passband	Spectral Type O	B (mas)	A	F (mas)	G	K (mas)	M
U	101	36	−29	0	8	−4	−42
B	88	58	12	0	−28	−100	−115
V	26	17	12	0	−8	−35	−42
R	22	18	12	0	−6	−21	−35
I	6	5	2	0	−1	−4	−6

TABLE 2
(Calculated–Pulkovo) Refraction Differences

Z.D. (°)	Ambient Temperature (°C) −30° (mas)	−15°	0° (mas)	15°	40° (mas)
0	0	0	0	0	0
15	−1	0	0	0	1
30	1	1	1	1	2
40	6	4	2	2	2
50	6	5	2	2	1
60	4	5	1	0	−3
65	−7	−5	−8	−8	−10
70	−36	−28	−30	−29	−37
72.5	−66	−56	−61	−65	−79
75	−156	−141	−147	−148	−173
77.5	38	−104	−247	−379	−587
80	−107	−1073	−620	−1121	−1229
82.5	−4188	−4170	−4246	−4377	−4697
85	−25129	−25560	−26203	−26973	−28470

spectral type. Differential color refraction is discussed in more detail by Stone (1984).

4.3 Refraction in Equatorial Coordinates

According to the refraction theory discussed in Sec. 2, all the refractions will be along vertical circles. In meridian astrometry, corrections for refraction are used only in computing declinations. However, when observations are made off the meridian, there will be components of refraction in both right ascension and declination. If the equatorial coordinates of an observed star are (α,δ), and (α',δ') are the same coordinates corrected for atmospheric refraction, then these coordinates are related by

$$\alpha'-\alpha=-R_m \sec \delta \sin \psi, \tag{29}$$

$$\delta'-\delta=-R_m \cos \psi \tag{30}$$

where R_m is the mean refraction, discussed previously, and ψ is the parallactic angle between the celestial pole and zenith as seen from the position of the star being observed. The trigonometric functions for this angle, used in the above equations, are given by

$$\sin \psi=\frac{\cos \phi \sin \mathrm{HA}}{\sin z_0}, \tag{31}$$

$$\cos \psi=\frac{\sin \phi-\sin \delta \cos z_0}{\cos \delta \sin z_0}, \tag{32}$$

and the hour angle HA and apparent zenith distance z_0 of the observation, used in the above equations, can be calculated with the following relations:

$$\mathrm{HA}=LST-\alpha, \tag{33}$$

$$\cos z_0=\arccos(\sin \delta \sin \phi+\cos \delta \cos \phi \cos \mathrm{HA}). \tag{34}$$

Consequently, after the mean refraction R_m is computed, the components of refraction in both right ascension and declination can be easily computed by calculating the zenith distance and hour angle of the observation with Eqs. (33) and (34), the parallactic trigonometric functions with Eqs. (31) and (32), and finally, the equatorial components of refraction with Eqs. (29) and (30).

5. COMPARISON WITH OBSERVATION

The numerical refraction discussed in the previous sections can be compared with the *Refraction Tables of the Pulkovo Observatory* (Abalakin 1985), which are based on empirical data and the refraction theory developed by A. I. Nefed'eva and I. G. Kolchinsky. The tables give values of refraction for a wide range of zenith distances (zenith to horizon) and meteorological conditions at a nominal wavelength of 5900 Å. Moreover, these tables are considered the standards for correcting astrometric observations, made in the visual, for refraction and are extensively used in meridian telescope programs.

Numerical refraction at 5900 Å was computed with the precepts discussed in previous sections and compared with the corresponding values taken from Abalakin (1985). The refractions were computed over wide ranges of zenith distance and meteorological conditions (temperature, pressure, and water-vapor pressure at the observing site), and differences in the sense (calculated–Pulkovo) were then formed and compared. In general, there was excellent agreement between the two refractions for zenith distances $z_0<65°$ and good agreement at zenith distances $65°<z_0<75°$. As an illustration, Table 2 gives differences in refraction for various ambient temperatures, and the results for a temperature of 15 °C are shown in Fig. 5. As seen in the figure, there is excellent agreement (~10 mas) between the calculated and Pulkovo refractions at zenith distances $z_0<65°$ and good agreement (~150 mas) for computed refractions down to $z_0=75°$.

As discussed in Sec. 2, the refraction computed in this paper should be only accurate for zenith distances $z_0<75°$, which is highly consistent with the differences given in Table 2. Calculations for refraction at greater zenith distances require detailed modeling of the atmosphere and numerical ray tracing. Fortunately, most observations in astronomy are not made at such large zenith distances, and consequently, the simpler refraction discussed in this paper can be used.

Another comparison between calculated and true refraction is possible. In FASTT observing programs, observations are corrected with the numerical refraction discussed in this paper. As a part of these programs, nightly observations are made of stars and extragalactic radio sources with known very accurate positions. If the FASTT computations for refraction were in serious error, then systematic differences

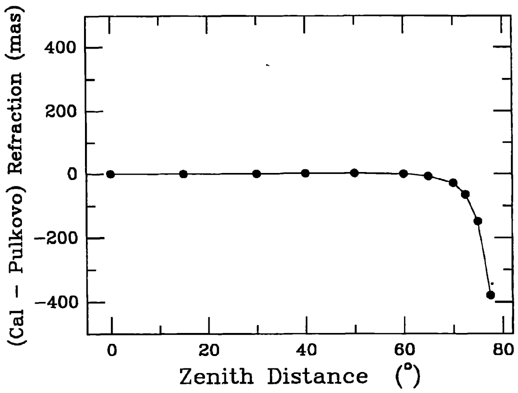

FIG. 5—Differences in refraction are shown in the sense (calculated–Pulkovo Refraction Tables) for an ambient temperature of 15 °C and at various zenith distances. Except at very large zenith distances $z_0 > 75°$ (where the calculated values are expected to fail), there is good agreement between the two refractions. The agreement is excellent (~10 mas or less) at zenith distances $z_0 < 65°$.

should exist between the observed and known declinations of these objects. Such a comparison was made (after applying a correction for telescope tube flexure), and the error in the FASTT declinations was found to be $\Delta R_m = -37 \pm 42$ (s.e.) sec z_0 (mas), which is small and not significantly larger than its formal error. Hence, the computation for refraction used in FASTT reductions seems to be quite accurate.

6. SUMMARY

By assuming that the structure of the Earth's atmosphere is spherically symmetric and only retaining the first two terms in the expansion of the refraction integral (see Sec. 2), a model for atmospheric refraction can be formulated that is simple, fast, and accurate. In particular, only knowledge of the meteorological conditions at the observing site, the apparent zenith distance of the object being observed, and the location of the site relative to the Earth's geoid are needed for computing this refraction. There is good agreement when the computed refraction is compared with the *Refraction Tables of the Pulkovo Observatory* (Abalakin 1985). For apparent zenith distances under $z_0 < 65°$, the error in the computed refraction is ~10 mas or smaller, and at a zenith distance of $z_0 = 75°$, the error increases to ~150 mas. At greater zenith distances, the model for refraction discussed in this paper is not adequate, since, as previously discussed, these computations for refractions require detailed knowledge about the structure of the atmosphere. Calculations for refraction at such great zenith distances are complicated, but can be done (Garfinkel 1967; Fukaya and Yoshizawa 1985; Yatsenko 1995). Fortunately, most observations in astronomy are made at smaller zenith distances.

Refraction tables combining empirical data and theory, such as those given in the Pulkovo Tables (Abalakin 1985), have been determined only for visual light, and there is a growing need for computing refraction at other wavelengths. This paper describes how refraction can be computed accurately for passbands within the wavelength interval 3000–10,000 Å and at zenith distances $z_0 < 75°$. As discussed in Sec. 4, the refraction is computed with Eq. (22), which computes a mean refraction R_m by weighting the selective refraction $R(\lambda)$ with various response functions. As to the nature of these functions, the spectral energy distribution $S(\lambda)$ for the star being observed can be taken from Straizys and Sviderskiene (1972). Gunn and Stryker (1983), or Jacoby et al. (1984); interstellar absorption affecting this energy distribution $E(\lambda)$ is computed with Eqs. (23) and (24) for a chosen reddening $E(B-V)$; atmospheric absorption $A(\lambda)$ is computed with Eqs. (25) and (26) at the observed air mass; $L(\lambda)$ is the known transmission of the optics of the telescope; $F(\lambda)$ is the known filter transmission; $D(\lambda)$ is the quantum efficiency of the detector; and $R(\lambda)$ is the computed refraction at wavelength λ. Once the mean refraction is computed, refraction away from the meridian in right ascension and declination can be calculated with Eqs. (29) and (30).

The selective refraction $R(\lambda)$ can be computed easily if the meteorological conditions (ambient temperature, atmospheric pressure, and water-vapor pressure) are known at the time of the observation and taken near the telescope. Once these parameters are known, the selective refraction $R(\lambda)$ can be computed with Eqs. (4), (5), (9), and (10), after the index of refraction at the observing site is computed with Owens' (1967) approximations given by Eqs. (14), (15), and (16). In order to keep refraction errors under ±50 mas at zenith distances $z_0 < 70°$, the meteorological readings should be accurately known at the time of each observation and taken near the telescope. In particular, these accuracies should be at least ±0.1 °C for the ambient temperature, ±0.3 mm for the atmospheric pressure, and for computing water-vapor pressure, ±10 °F for the dew point and ±12% for relative humidity. The required accuracies of meteorological readings are much less for differential astrometry. The water vapor pressure can be computed with Eq. (18), using either a dew-point reading or a psychrometric measurement converted to a dew point with Eq. (20). Section 3 of this paper describes sensors for measuring the meteorological data.

Often, neither the spectral type nor interstellar reddening is known for a star being observed, in which case, representative values can be adopted and used in the calculations for refraction until these values become later known. This is a common practice used in meridian astrometry. A spectral type of K0 and a color excess of $E(B-V)=0.3$ mag are typical for many faint stars, and these quantities have been adopted in FASTT computations for refraction. By adopting these quantities, the refraction can be computed easily with Eq. (27), using the weighting function $W(\lambda)$ given by Eq. (28), atmospheric absorption $A(\lambda)$, and selective refraction $R(\lambda)$ discussed previously (see Sec. 4.2). Nonetheless, when assuming a single spectral type for all stars in an observing program, there will be an error in the computed refraction at other spectral types. Table 1 gives this error for various passbands, and in most cases, the resulting differential color refraction is small. Stone (1984) discusses differential color refraction in more detail.

Actually, the night-to-night refraction can vary by ~±50 sec z_0 mas because of inhomogeneities and tilts in the overlying atmosphere. This anomalous refraction tends to be stochastic and is ameliorated with longer exposure times (see

Høg 1968; Lindegren 1980; Han 1989; and Stone et al. 1996 for more details). Since these effects are extremely hard to model, night-to-night variations in the observed refraction are expected which can be greatly reduced by averaging data taken over many nights.

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