B9120-001 Dynamic Programming Winter 2020

Prof. Daniel Russo

Homework Assignment 2: Due Monday February 10

Trajectory Tracking with Linear Quadratic control.

This problem treats a variant of the linear quadratic control problem where the goal is to accurately follow a given trajectory rather than keep the system state close to zero. For a fixed sequence $y_0, \cdots y_N \in \mathbb{R}^n$ and symmetric $Q \succ 0$ and $R \succ 0$, consider the problem of minimizing the cumulative expected cost

$$\mathbb{E}\left[\sum_{k=0}^{N-1}\Big((x_k - y_k)^\top Q(x_k - y_k) + u_k^\top R u_k\Big) + (x_N - y_N)^\top Q(x_N - y_N)\right],$$

subject to the linear dynamics

$$x_{k+1} = Ax_k + Bu_k + w_k \qquad k = 0, 1, \cdots, N-1.$$

In this problem, we show that the optimal cost-to-go functions have the form

$$J_t^*(x) = x^\top K_t x + 2q_t^\top x + r_t$$

and the optimal policy has the form $\pi^* = (\mu_0^*, \cdots, \mu_{N_1}^*)$ where

$$\mu_t^*(x) = L_t x + m_t.$$

That is, the optimal policy is an affine function of state.

I am not trying to force you to do highly burdensome algebra. Make sure you understand how to argue the policy and value function take this form. Your solution does not necessarily require a recursive formula for K_t, q_t, r_t, L_t and m_t.

Inventory Control with Forecasts.

Do problem 3.3 parts (a) and (b) from Bertsekas Volume I. Formulate the problem precisely (i.e. clearly specify the cost functions, state dynamics etc.). It is helpful to refer to section 1.4, pages 40-41.