B9120-001 Dynamic Programming Winter 2020

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Homework Assignment 4: Due Monday February 24

Termination States and Discounted Problems

Do problem 1.2. of Bertsekas Volume II.

This problem shows that some problems, despite being formulated with an undiscounted objective, have an intrinsic form of discounting because any action could result in a termination state being reached. Imagine interacing with a customer who may leave the system at any time. Stochastic shortest path problems, covered in Chapter 3 of the textbook, substantially generalize this idea.

Continuous Time Stopping

Do problem 1.11. of Bertsekas Volume II.

A brief solution suffices. Write down the Bellman operator and identify the optimal policy.

Bonus Problem: Gradient Descent as a Fixed Point Iteration

You do not need to submit this problem.

Consider the simple optimization problem

$$\min_{x \in \mathbb{R}^n} \quad \frac{1}{2} x^\top Q x - b^\top x,$$

where Q is symmetric and positive definite. Let $f(x) = \frac{1}{2} x^\top Q x + b^\top x$ and consider gradient descent with constant stepsize $\alpha > 0$:

$$x_{k+1} = x_k - \alpha \nabla f(x_k) \qquad k = 0, 1, 2, \cdots$$

Define the corresponding operator $H : \mathbb{R}^n \to \mathbb{R}^n$ by $H(x) = x - \alpha \nabla f(x)$ ($\ldots$i.e. $H = I - \alpha \nabla f$). We recast the problem of minimizing f as that of finding a fixed point of H. Show that if α is chosen to be sufficiently small, then H is a contraction mapping with respect to the Euclidean norm $\|x\|_2 = \sqrt{\sum_1^n x_i}$.

You may read about the extension to smooth an strongly convex objectives (i.e. $\mu I \preceq \nabla^2 f(x) \preceq LI$) here: `https://web.stanford.edu/class/msande318/notes/notes-first-order-smooth.pdf`