Statistics 3080
Homework 10
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Problem 1

```
> set.seed(7311986)
> pop_mean <- 121.8
> pop_sd <- 34.7
> n.8 <- 8
> n.24 <- 24
> n.48 <- 48
> K <- 10000
> samp.8 <- replicate(K, rnorm(n.8, mean=pop_mean, sd=pop_sd))
> samp.24 <- replicate(K, rnorm(n.24, mean=pop_mean, sd=pop_sd))
> samp.48 <- replicate(K, rnorm(n.48, mean=pop_mean, sd=pop_sd))
> wilcox.8 <- apply(samp.8, MARGIN=2, FUN=wilcox.test, alternative="two.sided",
+                   mu=121.8, exact=TRUE)
> wilcox.24 <- apply(samp.24, MARGIN=2, FUN=wilcox.test, alternative="two.sided",
+                   mu=121.8, exact=TRUE)
> wilcox.48 <- apply(samp.48, MARGIN=2, FUN=wilcox.test, alternative="two.sided",
+                   mu=121.8, exact=TRUE)
> p.8 <- do.call(rbind, lapply(wilcox.8, function(z) {z$p.value}))
> p.24 <- do.call(rbind, lapply(wilcox.24, function(z) {z$p.value}))
> p.48 <- do.call(rbind, lapply(wilcox.48, function(z) {z$p.value}))
> reject.8 <- p.8 < 0.05
> reject.24 <- p.24 < 0.05
> reject.48 <- p.48 < 0.05
> norm_t1.8 <- mean(reject.8)
> norm_t1.24 <- mean(reject.24)
> norm_t1.48 <- mean(reject.48)
> norm_t1.8

[1] 0.0362

> norm_t1.24

[1] 0.0484

> norm_t1.48

[1] 0.0492
```

Problem 2

```
> df <- 2
> samp.8 <- replicate(K, rchisq(n.8, df=df))
> samp.24 <- replicate(K, rchisq(n.24, df=df))
> samp.48 <- replicate(K, rchisq(n.48, df=df))
> wilcox.8 <- apply(samp.8, MARGIN=2, FUN=wilcox.test, alternative="two.sided",
+                   mu=1.386294, exact=TRUE)
> wilcox.24 <- apply(samp.24, MARGIN=2, FUN=wilcox.test, alternative="two.sided",
+                    mu=1.386294, exact=TRUE)
> wilcox.48 <- apply(samp.48, MARGIN=2, FUN=wilcox.test, alternative="two.sided",
+                    mu=1.386294, exact=TRUE)
> p.8 <- do.call(rbind, lapply(wilcox.8, function(z) {z$p.value}))
> p.24 <- do.call(rbind, lapply(wilcox.24, function(z) {z$p.value}))
> p.48 <- do.call(rbind, lapply(wilcox.48, function(z) {z$p.value}))
> reject.8 <- p.8 < 0.05
> reject.24 <- p.24 < 0.05
> reject.48 <- p.48 < 0.05
> chi_t1.8 <- mean(reject.8)
> chi_t1.24 <- mean(reject.24)
> chi_t1.48 <- mean(reject.48)
> chi_t1.8

[1] 0.0462

> chi_t1.24

[1] 0.1181

> chi_t1.48

[1] 0.2003
```

Problem 3

```
> tnorm_t1.8 <- mean(replicate(K, t.test(rnorm(n.8, mean=pop_mean, sd=pop_sd),
+                    alternative="two.sided", mu=121.8)$p.value) < 0.05)
> tnorm_t1.24 <- mean(replicate(K, t.test(rnorm(n.24, mean=pop_mean, sd=pop_sd),
+                     alternative="two.sided", mu=121.8)$p.value) < 0.05)
> tnorm_t1.48 <- mean(replicate(K, t.test(rnorm(n.48, mean=pop_mean, sd=pop_sd),
+                     alternative="two.sided", mu=121.8)$p.value) < 0.05)
> tchi_t1.8 <- mean(replicate(K, t.test(rchisq(n.8, df=df),
+                   alternative="two.sided", mu=2)$p.value) < 0.05)
> tchi_t1.24 <- mean(replicate(K, t.test(rchisq(n.24, df=df),
+                    alternative="two.sided", mu=2)$p.value) < 0.05)
> tchi_t1.48 <- mean(replicate(K, t.test(rchisq(n.48, df=df),
+                    alternative="two.sided", mu=2)$p.value) < 0.05)
```

```
> table_p <- t(matrix(c(norm_t1.8, norm_t1.24, norm_t1.48, tnorm_t1.8,
+                       tnorm_t1.24, tnorm_t1.48, chi_t1.8, chi_t1.24,
+                       chi_t1.48, tchi_t1.8, tchi_t1.24, tchi_t1.48),
+                       nrow=3, ncol=4))
> colnames(table_p) <- c("n = 8", "n = 24", "n = 48")
> rownames(table_p) <- c("type 1/sign-rank test/normal dist.",
+                          "type 1/t-test/normal dist.",
+                          "type 1/sign-rank test/chi-square dist.",
+                          "type 1/t-test/chi-square dist.")
> table_p

                                        n = 8 n = 24 n = 48
type 1/sign-rank test/normal dist.     0.0362 0.0484 0.0492
type 1/t-test/normal dist.             0.0479 0.0509 0.0484
type 1/sign-rank test/chi-square dist. 0.0462 0.1181 0.2003
type 1/t-test/chi-square dist.         0.1057 0.0775 0.0642

> print("Looking at our table, we see that both types of tests are")

[1] "Looking at our table, we see that both types of tests are"

> print("reliable for a normal distribution, since the type 1 error")

[1] "reliable for a normal distribution, since the type 1 error"

> print("is close to 0.05 for both tests for all values of n, with")

[1] "is close to 0.05 for both tests for all values of n, with"

> print("larger values generally being better. However, for the highly")

[1] "larger values generally being better. However, for the highly"

> print("skewed chi-square distribution, the t-test does much better")

[1] "skewed chi-square distribution, the t-test does much better"

> print("than the sign-rank test, as the type 1 error is much closer")

[1] "than the sign-rank test, as the type 1 error is much closer"

> print("to 0.05 for the former, getting closer as n grows larger, while")

[1] "to 0.05 for the former, getting closer as n grows larger, while"

> print("the type 1 error for the sign-rank test actually gets much worse")

[1] "the type 1 error for the sign-rank test actually gets much worse"
```

```
> print("as n increases. This suggests that parametric tests may")
[1] "as n increases. This suggests that parametric tests may"
> print("perform better on skewed distributions, even though they work")
[1] "perform better on skewed distributions, even though they work"
> print("the best when the distribution is normal. This could be because")
[1] "the best when the distribution is normal. This could be because"
> print("even though nonparametric tests do not assume anything about the")
[1] "even though nonparametric tests do not assume anything about the"
> print("distribution, for highly skewed data, the median is much lower")
[1] "distribution, for highly skewed data, the median is much lower"
> print("than the mean, and so we cannot expect the data to fall equally")
[1] "than the mean, and so we cannot expect the data to fall equally"
> print("on either side of the median.")
[1] "on either side of the median."
```

Problem 4

```
> samp.8 <- replicate(K, rnorm(n.8, mean=pop_mean, sd=pop_sd))
> samp.24 <- replicate(K, rnorm(n.24, mean=pop_mean, sd=pop_sd))
> samp.48 <- replicate(K, rnorm(n.48, mean=pop_mean, sd=pop_sd))
> wilcox.8 <- apply(samp.8, MARGIN=2, FUN=wilcox.test, alternative="two.sided",
+                   mu=115, exact=TRUE)
> wilcox.24 <- apply(samp.24, MARGIN=2, FUN=wilcox.test, alternative="two.sided",
+                    mu=115, exact=TRUE)
> wilcox.48 <- apply(samp.48, MARGIN=2, FUN=wilcox.test, alternative="two.sided",
+                    mu=115, exact=TRUE)
> p.8 <- do.call(rbind, lapply(wilcox.8, function(z) {z$p.value}))
> p.24 <- do.call(rbind, lapply(wilcox.24, function(z) {z$p.value}))
> p.48 <- do.call(rbind, lapply(wilcox.48, function(z) {z$p.value}))
> fail_rej.8 <- p.8 > 0.05
> fail_rej.24 <- p.24 > 0.05
> fail_rej.48 <- p.48 > 0.05
> norm_t2.8 <- mean(fail_rej.8)
> norm_t2.24 <- mean(fail_rej.24)
> norm_t2.48 <- mean(fail_rej.48)
> norm_power.8 <- 1 - norm_t2.8
> norm_power.24 <- 1 - norm_t2.24
> norm_power.48 <- 1 - norm_t2.48
> norm_power.8
```

```
[1] 0.0657
```

```
> norm_power.24
```

```
[1] 0.1551
```

```
> norm_power.48
```

```
[1] 0.2549
```

Problem 5

```
> tnorm_t2.8 <- mean(replicate(K, t.test(rnorm(n.8, mean=pop_mean, sd=pop_sd),
+                     alternative="two.sided", mu=115)$p.value) > 0.05)
> tnorm_t2.24 <- mean(replicate(K, t.test(rnorm(n.24, mean=pop_mean, sd=pop_sd),
+                     alternative="two.sided", mu=115)$p.value) > 0.05)
> tnorm_t2.48 <- mean(replicate(K, t.test(rnorm(n.48, mean=pop_mean, sd=pop_sd),
+                     alternative="two.sided", mu=115)$p.value) > 0.05)
> tnorm_power.8 <- 1 - tnorm_t2.8
> tnorm_power.24 <- 1 - tnorm_t2.24
> tnorm_power.48 <- 1 - tnorm_t2.48
> tnorm_power.8
```

```
[1] 0.0771
```

```
> tnorm_power.24
```

```
[1] 0.1521
```

```
> tnorm_power.48
```

```
[1] 0.2674
```

Problem 6

```
> table_p <- t(matrix(c(norm_t1.8, norm_t1.24, norm_t1.48, tnorm_t1.8,
+                       tnorm_t1.24, tnorm_t1.48, chi_t1.8, chi_t1.24,
+                       chi_t1.48, tchi_t1.8, tchi_t1.24, tchi_t1.48,
+                       norm_power.8, norm_power.24, norm_power.48,
+                       tnorm_power.8, tnorm_power.24, tnorm_power.48),
+                       nrow=3, ncol=6))
> colnames(table_p) <- c("n = 8", "n = 24", "n = 48")
> rownames(table_p) <- c("type 1/sign-rank test/normal dist.",
+                        "type 1/t-test/normal dist.",
+                        "type 1/sign-rank test/chi-square dist.",
+                        "type 1/t-test/chi-square dist.",
+                        "power/sign-rank test/normal dist.",
+                        "power/t-test/normal dist.")
> table_p
```

```
                                        n = 8 n = 24 n = 48
type 1/sign-rank test/normal dist.     0.0362 0.0484 0.0492
type 1/t-test/normal dist.             0.0479 0.0509 0.0484
type 1/sign-rank test/chi-square dist. 0.0462 0.1181 0.2003
type 1/t-test/chi-square dist.         0.1057 0.0775 0.0642
power/sign-rank test/normal dist.      0.0657 0.1551 0.2549
power/t-test/normal dist.              0.0771 0.1521 0.2674
```

```
> print("We see that the power is fairly similar for both tests, low")
[1] "We see that the power is fairly similar for both tests, low"
> print("because the standard deviation is high and the null value is")
[1] "because the standard deviation is high and the null value is"
> print("close to the true value. Since the power is only low due to")
[1] "close to the true value. Since the power is only low due to"
> print("that reason, the conclusion that both tests work equally well")
[1] "that reason, the conclusion that both tests work equally well"
> print("for normal data still holds. For both tests, the power increases")
[1] "for normal data still holds. For both tests, the power increases"
> print("with the sample size, which makes sense.")
[1] "with the sample size, which makes sense."
```

Problem 7

```
> print("When the data follows a chi-square distribution, power values")
[1] "When the data follows a chi-square distribution, power values"
> print("for the two tests are not comparable since for highly skewed data,")
[1] "for the two tests are not comparable since for highly skewed data,"
> print("the median is much lower than the mean, and thus using the same")
[1] "the median is much lower than the mean, and thus using the same"
> print("null value for both tests in order to compare them doesn't make sense.")
[1] "null value for both tests in order to compare them doesn't make sense."
```

References:

- https://stackoverflow.com/questions/36100515/p-value-extraction-from-an-apply-function