RF Heating and Current Drive

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Overview

Heating Mechanism

- $\omega_{LH}^2(x) \equiv \frac{\omega_{pi}^2}{1+\omega_{pe}^2/\Omega_e^2}$
  have to choose the density such that resonance $\omega = \omega_{LH}$ happens at the place required to be heated. Usually for lower hybrid wave, $\Omega_i \ll \omega \ll \Omega_e$, no cyclotron damping in this frequency range
- according to accessibility analyses, resonance happens near the edge, providing off-axis current drive
- electron heating mostly, Landau damping (velocity limit? since the *collisionality* $\sim 1/v_e^3$)
- Rely on asymmetric wave spectrum interacting with electrons on tail of distribution, the gain in electron momentum correspond to a net toroidal current
- Resonance usually happens at $v_{\|} \simeq 3v_{Te}$ (tail of distribution), recall $\nu_{ei} \simeq 1/v_{\|}^3$, there is less friction trying to restore the distribution function to Maxwellian, less power is required to sustain the current drive(i.e., higher efficiency)

Ehst and Karney's Formula for Efficiency

Deficiencies in different units:

$$\eta_I = \frac{\int_A J_{CD}\, dA}{\int_V S_{LH}\, dV} \approx \frac{1}{2\pi R}\left[\frac{J_{CD}}{S_{LH}}\right]_{\rho_J} = \frac{1}{2\pi R}\eta = \frac{1}{2\pi R}\left[\frac{e}{m_e \nu_0 v_{Te}}\right]_{\rho_J} \tilde{\eta}$$

$$\eta = \left[\frac{J_{CD}}{S_H}\right]_{\rho_J}$$

$$\tilde{\eta} = \left[\frac{J_{CD} \,/\, env_{Te}}{S_H \,/\, m_e n \nu_0 v_{Te}^2}\right]_{\rho_J} = \left[\frac{m_e \nu_0 v_{Te}}{e}\right]_{\rho_J} \eta$$

$$v_{Te} = \left(\frac{2T_e}{m_e}\right)^{1/2}$$

$$\nu_0 = \frac{\omega_{pe}^4 \ln \Lambda}{2\pi n_e v_{Te}^3}$$

Ehst and Karney's Formula for Efficiency

$$\tilde{\eta} = CMR\eta_0 \tag{1}$$

$$M = 1$$

$$R = 1 - \frac{\varepsilon^n \rho_J^n (x_r^2 + w^2)^{1/2}}{\varepsilon^n \rho_J^n x_r + w} \qquad n = 0.77 \qquad x_r = 3.5$$

$$C = 1 - \exp(-c^m x_t^{2m}) \qquad m = 1.38 \qquad c = 0.389$$

$$\eta_0 = \frac{K}{w} + D + \frac{4w^2}{5 + Z_{eff}} \qquad K = \frac{3.0}{Z_{eff}} \qquad D = \frac{3.83}{Z_{eff}^{0.707}}$$

Selection for $n_{||}$ and ω

- Accessibility Constraints
- Position of current drive layer

Accessibility Analysis

Given by cold plasma dispersion relation, Mode conversion condition:

$$\frac{\omega_{pi}}{\omega} = N_{\|} Y \pm \sqrt{1 + N_{\|}^2 (Y^2 - 1)} \qquad (2)$$

Where $Y = \frac{\omega^2}{\omega_{ce}\omega_{ci}}, N_{\|} = \frac{ck_z}{\omega}$.

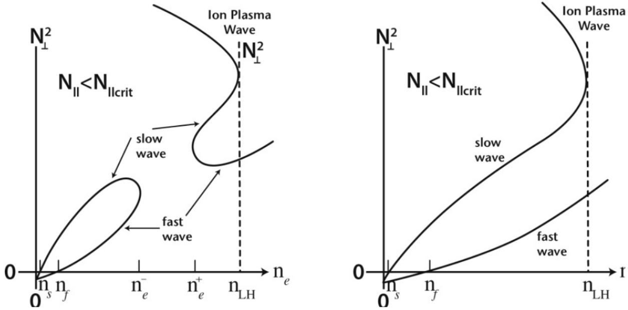


FIG. 6.4. Accessibility diagram of lower hybrid waves for two different values of $N_{\|}$. Left: The $N_{\|}$ is too low for accessibility to the lower hybrid layer; however, accessibility to the slow–fast wave mode conversion layer (n_e^-) is available for electron Landau absorption

Accessibility Analysis

- if $Y^2 > 1$,which means $\omega^2 > \omega_{ce}\omega_{ci}$,there will always be position for slow wave to converse to fast wave.
- if $Y^2 < 1$(small wave frequency), there will be several cases depends on the parallel wave number. The critical value is the following:

$$N^2_{\|critical} = 1 + \frac{\omega^2_{pe}}{\omega^2_{ce}} |_{Resonance} \tag{3}$$

The resonance layer is given by $\omega^2 = \omega^2_{LH}(x_r)$

- if $N^2_{\|} > N^2_{\|critical}$, the wave can access the lower hybrid layer, as the right diagram shows.
- if $N^2_{\|} < N^2_{\|critical}$, mode conversion happens, as the left diagram shows.
- if $N^2_{\|} = N^2_{\|critical}$, two mode conversion positions merge, critical case.

Accessibility Analysis

Collection of Accessibility Constraints

1. No mode conversion before reaching ρ_J
   $n_{\|}^2 \geq \left(\left(1 - \frac{1-\hat{\omega}^2}{\hat{\omega}^2} X\right)^{1/2} + X^{1/2} \right)^2$
   where $X(\rho_J) = \omega_{pe}^2(\rho_J)/\Omega_e^2(\rho_J)$
   $\hat{\omega}^2(\rho_J) = \omega^2/\Omega_e(\rho_J)\Omega_i(\rho_J)$
2. No wave resonance bofore reaching ρ_J
   $\omega^2 > \omega_{LH}^2(\rho_J)$
3. No parametric decay instability before reaching ρ_J
   $\omega^2 > 4\omega_{LH}^2(\rho_J)$
4. No coupling to α particles before reaching ρ_J
   $\omega^2/k_\perp^2(\rho_J) > v_\alpha^2$

No mode conversion and no α particle coupling are the most strict constraints.

Equations

- Solve the following equations simultaneously to determine $n_{\|}^2$, $\hat{\omega}^2$, and ρ_J

$$n_{\|}^2 = \left(\left(1 - \frac{1-\hat{\omega}^2}{\hat{\omega}^2} X\right)^{1/2} + X^{1/2} \right)^2 \tag{4}$$

$$\hat{\omega}^2 = \frac{1}{2}\frac{X}{1+X} + \frac{1}{2}\left[\frac{X^2}{(1+X)^2} + 4\gamma^2 \frac{X^3}{1+X}\right]^{1/2} \tag{5}$$

$$(1+\nu_T)(1-\rho_J^2)^{\nu_T} n_{\|}^2(\rho_J) = \frac{m_e c^2}{2\bar{T}_k} = \frac{28.4}{\bar{T}_k} \tag{6}$$

Results in Parameter Space

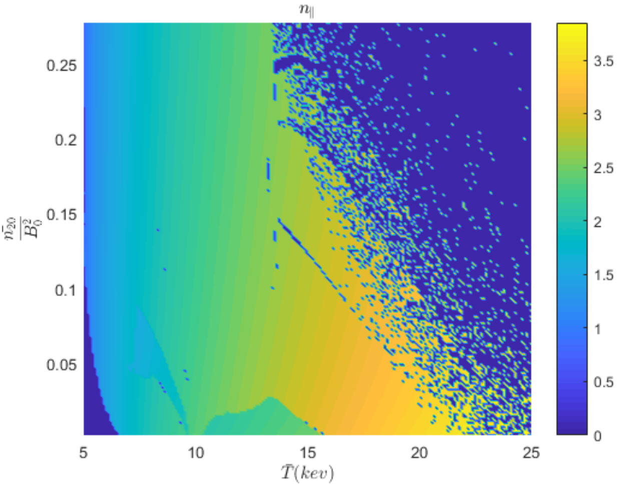


Figure: $n_{||}$ in parameter space

Results in Parameter Space

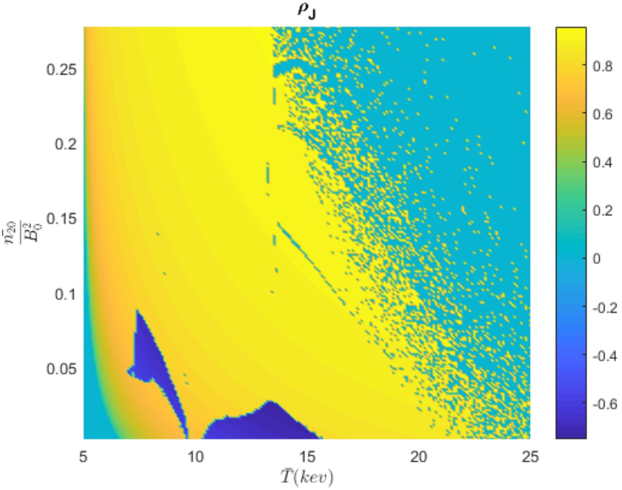


Figure: ρ_J in parameter space

Results in Parameter Space

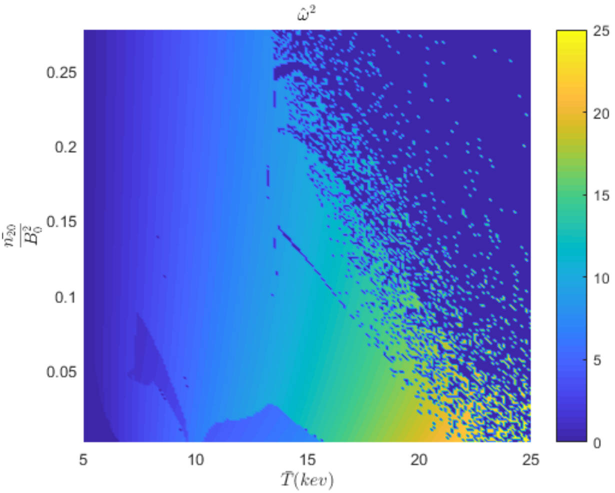


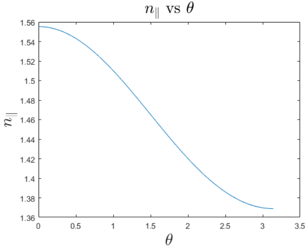
Figure: $\hat{\omega}^2$ in parameter space

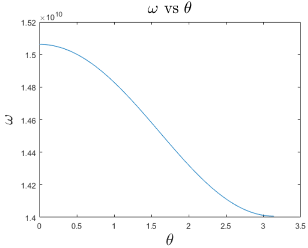
Test Cases

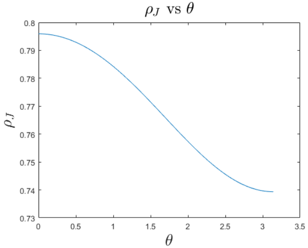
$\bar{T}_k$	10.0	10.0	5.0	5.0
$\bar{n}_{20}$	2.5	1.0	1.0	1.0
B_0	5.0	10.0	10.0	5.0
$n_{\|\|}$	2.5	1.32	1.57	1.96
ρ_J	0.86	0.47	0.04	0.53
$\hat{\omega}^2$	8.08	0.53	0.71	3.29
$\omega(e10)$	3.15	1.72	2.17	2.13
η_{CD}	0.168	0.71	0.59	0.31
η_I	0.016	0.12	0.090	0.053

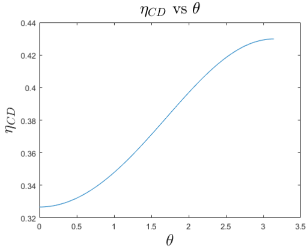
Optimize θ

- $\bar{T}_k = 17.8,\ \bar{n_{20}} = 0.86,\ a/R_0 = 0.25,\ B_0 = 8.75$



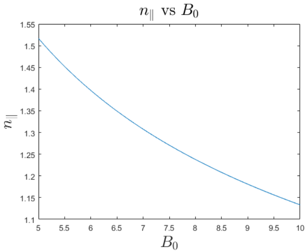


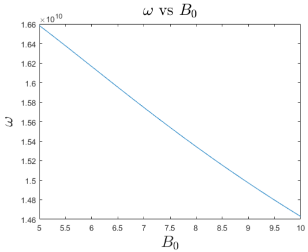


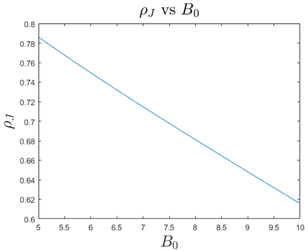


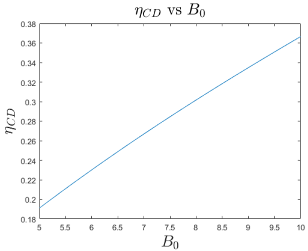
Optimize B_0

- $\bar{T}_k = 17.8,\ \bar{n_{20}} = 0.86,\ a/R_0 = 0.25,\ \theta = 0$



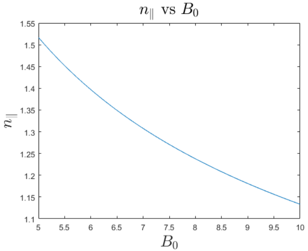


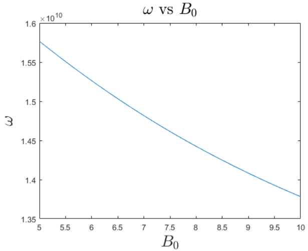


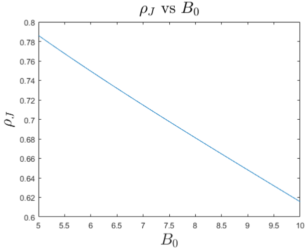


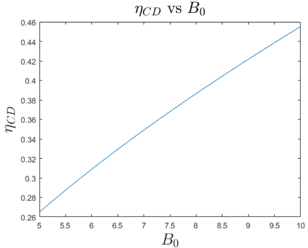
Optimize B_0

- $\bar{T}_k = 17.8,\ \bar{n_{20}} = 0.86,\ a/R_0 = 0.25,\ \theta = \frac{3}{4}\pi$









Freidberg's Single Particle Approach

Assuming the following poloidal density and temperature profile:

$$n = \overline{n}(1+\nu_n)(1-\rho^2)^{\nu_n} \tag{7}$$

$$T = \overline{T}(1+\nu_T)(1-\rho^2)^{\nu_T} \tag{8}$$

ρ is the radial flux label.
Spatial damping coefficient is defined as $\lambda(x) = 2\int_{-a}^{x} k_{\perp i} dx$, where $k_{\perp i}$ is calculated using cold plasma dispersion relation.
Using the normalized density and temperature profile:

$$n_{nor}(\rho) = \frac{n(\rho)}{n_{max}} \tag{9}$$

$$T_{nor}(\rho) = \frac{T(\rho)}{T_{max}} \tag{10}$$

Freidberg's Single Particle Approach

$k_{\perp i}$ is written by:

$$k_{\perp i} = \pi^{1/2}\frac{\omega_{ce}}{c}\frac{\hat{\omega}^3}{1-\hat{\omega}^2}\frac{\xi^3 e^{-\xi^2}}{(1-n_{nor})D} \tag{11}$$

$$D = 1 + (1-\hat{\omega}^4)n_{nor} + (1-\hat{\omega}^2)^2 n_{nor}^2 \tag{12}$$

where $\hat{\omega} = \omega/(\omega_{ce}\omega_{ci})^{1/2}$ and $\xi = \omega/(k_{||}v_{Te})$
Unlike what has been done in Chap15, here we keep ω_{ce} and ω_{ci} as spatial functions.

Freidberg's Single Particle Approach

The power absorbed per unit volume by the resonant particles has already been calculated and for a Maxwellian is given by:

$$S_L = \pi^{(1/2)} \epsilon_0 E_{\parallel}^2 (\frac{\omega_{pe}^2}{\omega}) \xi^3 e^{-\lambda - \xi^2} \quad (13)$$

The current drive can be determined by the momentum balance. The momentum gained by resonant electrons mush be balanced by the momentum lost due to the collisional drag with the ions. The expression of J_{CD} can be finally written as:

$$J_{CD} = \pi^{1/2} \epsilon_0 E_{\parallel}^2 (\frac{\omega_{pe}^2}{\omega}) (\frac{e}{m_e \nu_0 v_{Te}}) e^{-\lambda - \xi^2} G(\xi) \quad (14)$$

$$G(\xi) = \xi^2 \int_0^{\infty} (u_{\perp}^2 + \xi^2)^{1/2} (2u_{\perp}^2 + 2\xi^2 - 3) e^{-u_{\perp}^2} u_{\perp} du_{\perp} \quad (15)$$

And $G(\xi) \approx \xi^5$ is quite a good approximation after testing.

Freidberg's Single Particle Approach

The current drive efficiency is defined by:

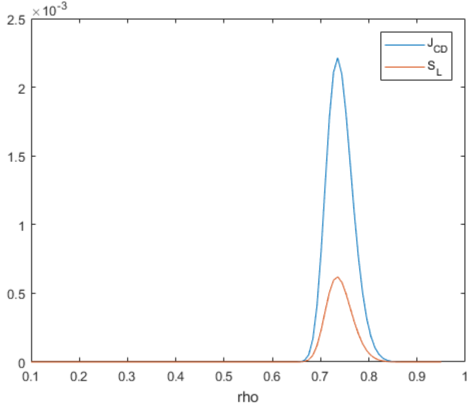
$$\eta_{CD} = \frac{I_{CD}}{P_{CD}} = \frac{\int J_{CD} dA}{2\pi R \int S_L dA} \tag{16}$$

Since all above is derived from a 1-D model, there is no way for us to do this integral. However, based on the fact that the peak of J_{CD} and S_L overlap well with each other. We assume that

$$\eta_{CD} = \frac{J_{CD}(x_J)}{2\pi R S_L(x_J)} \tag{17}$$

Results from Single Particle Approach

- $\bar{T}_k = 17.8,\ \bar{n_{20}} = 0.86,\ a/R_0 = 0.25,\ \theta = 0,\ B_0 = 10,$ $\omega = 1.46 \times 10^{10},\ n_{||} = 1.27$



Comparing Results

Methods	Karney	SingleParticle
$\bar{T}_k$	17.8	17.8
$\bar{n_{20}}$	0.86	0.86
B_0	10.0	10.0
$n_{\|\|}$	1.27	1.27
ρ_J	0.70	0.74
$\omega(e10)$	1.46	1.46
η_{CD}	0.672	0.509
η_I	0.184	0.120

Bootstrap Current

Next Steps

1. Choose $n_{||}$, ω and θ as wave parameter and provide working range of $\bar{T}_k$ and $\bar{n}$
2. Might need to be iterate several times and hopefully it will converge.