Transport formulas

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1 Confinement time

For a scaling relation of the form

$$\tau_E = C I^{\alpha_I} B^{\alpha_B} \overline{n}^{\alpha_n} P^{\alpha_P} R^{\alpha_R} \kappa^{\alpha_\kappa} \epsilon^{\alpha_\epsilon} S_{cr}^{\alpha_S} M^{\alpha_M} \text{ [sec]}$$

we have the following popular scaling laws:

Scaling	$C \times 10^3$	α_I	α_B	α_n	α_P	α_R	α_κ	α_ϵ	α_S	α_M
ITERH-98P(y,2)	56.2	0.93	0.15	0.41	-0.69	1.97	0.78	0.58	-	0.19
ITPAH-04P(y,1)	22.8	0.86	0.21	0.40	-0.65	0.32	-	-0.99	0.84	0.08
ITPAH-04P(y,2)	19.8	0.85	0.17	0.26	-0.45	-0.04	-	-1.25	0.82	0.11
ITPAH-04P(y,3)	88.0	0.90	-	0.30	-0.47	1.73	-	0.43	-	-
ITER-89P	38	0.85	0.20	0.10	-0.50	1.50	0.50	0.30	-	0.50
ITERL-96P(th)	23	0.96	0.03	0.40	-0.73	1.83	0.64	-0.06	-	0.20

ITER98y,2 is the most popular choice.

1.1 I-mode

Using α_R from the ITER98y,2 scaling relation, we have that

$$\tau_E = 0.082 \frac{I_M^{0.69} B_0^{0.77} \bar{n}_{20}^{0.017} a^{0.81} R_0^{1.39}}{P_M^{0.29}} \text{ [sec]}$$

Source: J.P. Freidberg's re-derivation of a size scaling for I-mode using the Kadomtsev constraint. I-mode has not been replicated on enough machines to have a robust empirical size scaling.

2 H-mode power threshold

The L-H transition can only occur if the heating power is greater than the threshold value

$$P_{LH} = 1.38 \bar{n}_{20}^{-0.77} B_0^{0.92} R_0^{1.23} a^{0.76} \text{ [MW]}$$

Source: J.P. Freidberg, *Plasma Physics and Fusion Energy* sec. 14.5.2.

3 Density limit

Operating above the Greenwald density results in disruptions. This limit is defined as

$$n_G = \frac{I_p}{\pi a^2}\,[10^{20}/\mathrm{m}^3]$$

Arbitrary limits on the achievable fraction of this denisty can be found by looking at the database of H-mode shots. A reasonable choice for power plant operation is

$$0.35 < \frac{n}{n_G} < 0.7$$

where the lower limit arises from difficulties in entering H-mode.

4 Edge pedestal scaling

$$p_{ped} = 18.3a^{-0.57}\left(\frac{M}{n_{ped}}\right)\left(\frac{I_p}{R}\right)^2\left(\frac{P_{total}}{P_{LH}}\right)^{0.144}\ [\mathrm{kPa}]$$

Source: Sugihara, IOP 2003.

5 Simple model for n and T profiles in H-mode

$$n = \frac{4}{3}\bar{n}\left(1-\rho^2\right)^{1/3}$$
$$T = \frac{5}{3}\bar{T}\left(1-\rho^2\right)^{2/3}$$
$$\rho^2 = \frac{x^2}{a^2} + \frac{y^2}{\kappa^2 a^2}$$

This gives eliptical contours using the maximum, on-axis values of $\bar{T}$ and $\bar{n}$.

Source: J.P. Freidberg, *Plasma Physics and Fusion Energy*.