FUSION POWER CALCULATIONS

FUSION POWER GROUP

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1. Temperature and Density Profiles:

$$n = \bar{n}(1+\nu_n)(1-\rho^2)^{\nu_n} \tag{1}$$

$$T = \bar{T}(1+\nu_T)(1-\rho^2)^{\nu_T} \tag{2}$$

2. Plasma Shape Parameters

$$\begin{aligned} x_{\alpha\alpha} &= 0.3(1-\delta^2) \\ c_0 &= -\frac{\delta}{2} \\ c_1 &= \frac{1}{8}[9 - 2\delta - x_{\alpha\alpha}] \\ c_2 &= \frac{\delta}{2} \\ c_3 &= \frac{1}{8}[-1 + 2\delta + x_{\alpha\alpha}] \end{aligned} \tag{3}$$

$$\begin{aligned} x &= \sigma(1-\rho^2) + c_0\rho^4 + c_1\rho\cos(\alpha) + c_2\rho^2\cos(2\alpha) + c_3\rho^3\cos(3\alpha) \\ y &= \kappa\rho\sin(\alpha) \end{aligned} \tag{4}$$

3. Bremmstrahlung Power Losses:

$$S_B(\rho) = \bar{n}^2\bar{T}^{1/2}C_B Z_{eff}(1+\nu_n)^2(1+\nu_T)^{1/2}(1-\rho^2)^{2\nu_n+\frac{\nu_T}{2}} \tag{5}$$

Let:

$$C'_B = \bar{n}^2\bar{T}^{1/2}C_B Z_{eff}(1+\nu_n)^2(1+\nu_T)^{1/2} \tag{6}$$

For a general function $Q(\rho)$.

(7)

$$\begin{aligned} Q_v = & \\ &= 4\pi^2a^2\kappa c_1\int_0^1 Q(\rho)\rho R_0 d\rho + 2\pi^2a^3\kappa\int_0^1 Q(\rho)\rho\big[2c_1(c_2\rho^2 + 3c_0\rho^4 - 2\rho^2\sigma + \sigma) + 3c_2c_3\rho^4\big]d\rho \end{aligned}$$

(8)

$$\begin{aligned} S_B &= C'_B\,(\\ &= 4\pi^2a^2\kappa c_1\left(\frac{R_0}{2+2(2\nu_n+\frac{\nu_T}{2})}\right) + 2\pi^2a^3\kappa\left(\frac{((2\nu_n+\frac{\nu_T}{2})+3)c_1((2\nu_n+\frac{\nu_T}{2})\sigma + c_2) + 6c_0c_1 + 3c_2c_3}{((2\nu_n+\frac{\nu_T}{2})+1)((2\nu_n+\frac{\nu_T}{2})+2)((2\nu_n+\frac{\nu_T}{2})+3)}\right)\,) \end{aligned}$$

4. 5th order polynomial Fit of Thermal Reactivities:

$$
\begin{aligned}
\langle\sigma v\rangle(T(\rho)) = & \\
& l_0 + \\
& l_1\left(\bar{T}(1+\nu_T)(1-\rho^2)^{\nu_T}\right) + \\
& l_2\left(\bar{T}(1+\nu_T)(1-\rho^2)^{\nu_T}\right)^2 + \\
& l_3\left(\bar{T}(1+\nu_T)(1-\rho^2)^{\nu_T}\right)^3 + \\
& l_4\left(\bar{T}(1+\nu_T)(1-\rho^2)^{\nu_T}\right)^4 + \\
& l_5\left(\bar{T}(1+\nu_T)(1-\rho^2)^{\nu_T}\right)^5
\end{aligned} \tag{9}
$$

See attached .mat file for $[l_0....l_5]$ coefficients.

5. Alpha Power:

$$S_\alpha(\rho) = \frac{1}{4}\bar{n}^2 E_\alpha \langle\sigma v\rangle((1+\nu_n)(1-\rho^2)^{\nu_n})^2 \tag{10}$$

For a general function $Q(\rho)$.

(11)
$$
\begin{aligned}
Q_v = & \\
& = 4\pi^2 a^2 \kappa c_1 \int_0^1 Q(\rho)\rho R_0 d\rho + 2\pi^2 a^3 \kappa \int_0^1 Q(\rho)\rho\left[2c_1(c_2\rho^2 + 3c_0\rho^4 - 2\rho^2\sigma + \sigma) + 3c_2c_3\rho^4\right]d\rho
\end{aligned}
$$

Let:

$$I_1 = \int_0^1 Q(\rho)\rho R_0 d\rho \tag{12}$$

$$
\begin{aligned}
I_1 = & \\
& = \frac{l_0(\nu_n+1)^2 R_0}{4\nu_n+2} \\
& + \frac{l_1(\nu_n+1)^2(\nu_T+1)R_0\bar{T}}{4\nu_n+2\nu_T+2} \\
& + \frac{l_2(\nu_n+1)^2(\nu_T+1)^2 R_0\bar{T}^2}{4\nu_n+4\nu_T+2} \\
& + \frac{l_3(\nu_n+1)^2(\nu_T+1)^3 R_0\bar{T}^3}{4\nu_n+6\nu_T+2} \\
& + \frac{l_4(\nu_n+1)^2(\nu_T+1)^4 R_0\bar{T}^4}{4\nu_n+8\nu_T+2} \\
& + \frac{l_5(\nu_n+1)^2(\nu_T+1)^5 R_0\bar{T}^5}{4\nu_n+10\nu_T+2}
\end{aligned} \tag{13}
$$

Let:

$$
I_2 = \int_0^1 Q(\rho)\rho\big[2c_1(c_2\rho^2 + 3c_0\rho^4 - 2\rho^2\sigma + \sigma) + 3c_2c_3\rho^4\big]\,d\rho \tag{14}
$$

(15)

$$
\begin{aligned}
& I_2 = \\
& = \frac{l_0(\nu_n+1)(6c_0c_1 + c_1(2\nu_n+3)(c_2+2\nu_n\sigma) + 3c_2c_3)}{2(2\nu_n+1)(2\nu_n+3)} \\
& + \frac{l_1(\nu_n+1)^2(\nu_T+1)\bar{T}(6c_0c_1 + c_1(2\nu_n+\nu_T+3)(c_2+2\nu_n\sigma+\nu_T\sigma) + 3c_2c_3)}{(2\nu_n+\nu_T+1)(2\nu_n+\nu_T+2)(2\nu_n+\nu_T+3)} \\
& + \frac{l_2(\nu_n+1)^2(\nu_T+1)^2\bar{T}^2(6c_0c_1 + c_1(2\nu_n+2\nu_T+3)(c_2+2\sigma(\nu_n+\nu_T)) + 3c_2c_3)}{2(\nu_n+\nu_T+1)(2\nu_n+2\nu_T+1)(2\nu_n+2\nu_T+3)} \\
& + \frac{l_3(\nu_n+1)^2(\nu_T+1)^3\bar{T}^3(6c_0c_1 + c_1(2\nu_n+3\nu_T+3)(c_2+2\nu_n\sigma+3\nu_T\sigma) + 3c_2c_3)}{(2\nu_n+3\nu_T+1)(2\nu_n+3\nu_T+2)(2\nu_n+3\nu_T+3)} \\
& + \frac{l_4(\nu_n+1)^2(\nu_T+1)^4\bar{T}^4(6c_0c_1 + c_1(2\nu_n+4\nu_T+3)(c_2+2\nu_n\sigma+4\nu_T\sigma) + 3c_2c_3)}{2(\nu_n+2\nu_T+1)(2\nu_n+4\nu_T+1)(2\nu_n+4\nu_T+3)} \\
& + \frac{l_5(\nu_n+1)^2(\nu_T+1)^5\bar{T}^5(6c_0c_1 + c_1(2\nu_n+5\nu_T+3)(c_2+2\nu_n\sigma+5\nu_T\sigma) + 3c_2c_3)}{(2\nu_n+5\nu_T+1)(2\nu_n+5\nu_T+2)(2\nu_n+5\nu_T+3)}
\end{aligned} \tag{15}
$$

$$
S_\alpha = \frac{1}{4}\bar{n}^2 E_\alpha \left(4\pi^2(R_0\epsilon)^2\kappa c_1 I_1 + 2\pi^2(R_0\epsilon)^3 I_2\right) \tag{16}
$$

$$
S_F = S_\alpha * (17.6/3.5) \tag{17}
$$

6. Test Case - Fusion Power

name: fusion power (S_F)
inputs: $S_F(\nu_T, \nu_n, \bar{T}, \bar{n}, \epsilon, \kappa, \delta, R_0)$
test-values:

Parameter	Value
ν_T	1.3
ν_n	0.5
$\bar{T}$	15 (KeV)
$\bar{n}$	1e20
ϵ	0.25
κ	1.8
δ	0.45
R_0	8
S_F	4.0984e9

7. Test Case - Bremsstrahlung

name: bremsstrahlung power(S_B)
inputs: $S_B(\nu_T, \nu_n, \bar{T}_k, \bar{n}_{20}, \epsilon, \kappa, \delta, R_0)$
test values:

Parameter	Value
ν_T	1.3
ν_N	0.5
$\bar{T}_k$	15
$\bar{n}_{20}$	1
ϵ	0.25
κ	1.8
δ	0.45
R_0	8
S_F	3.0544e7