MHD LIMIT CALCULATIONS

MHD GROUP

Date: April 18, 2017.

1. Temperature and Density Profiles:

$$n = \bar{n}(1+\nu_n)(1-\rho^2)^{\nu_n} \tag{1}$$

$$T = \bar{T}(1+\nu_T)(1-\rho^2)^{\nu_T} \tag{2}$$

2. Model for Plasma Shape

$$\begin{aligned} x &= \sigma(1-\rho^2) + c_0\rho^4 + c_1\rho\cos(\alpha) + c_2\rho^2\cos(2\alpha) + c_3\rho^3\cos(3\alpha) \\ y &= \kappa\rho\sin(\alpha) \end{aligned} \tag{3}$$

$$\begin{aligned} x_{\alpha\alpha} &= 0.3(1-\delta^2) \\ c_0 &= -\frac{\delta}{2} \\ c_1 &= \frac{1}{8}[9 - 2\delta - x_{\alpha\alpha}] \\ c_2 &= \frac{\delta}{2} \\ c_3 &= \frac{1}{8}[-1 + 2\delta + x_{\alpha\alpha}] \end{aligned} \tag{4}$$

3. Elongation κ and triangularity δ

	ITER	ARC	Ignitor	ARIES	FIRE	**Mean**
Triangularity (δ)	0.33	0.45	0.4	0.5	0.5	**0.42**
Elongation (κ)	1.7	1.77	1.83	1.7	1.81	**1.76**

Table 1. Triangularity and elongation values for the 5 fusion machines at the 95% flux surface

Chosen parameters:

- $\kappa = 1.8$
- $\delta = 0.45$

4. Troyon β Limit Calculation

$$\beta = \frac{2\mu_0\bar{p}}{B_p^2 + B_0^2} \approx \frac{2\mu_0\bar{p}}{B_0^2} \tag{5}$$

$$\bar{p} = \frac{\int 2n(\mathbf{r})T(\mathbf{r})d\mathbf{r}}{V} \tag{6}$$

$$\beta_{crit} = \frac{\beta_N I_{MA}}{a B_0} \tag{7}$$

$$\frac{2\mu_0 \bar{p}}{B_0^2} \leq \frac{\beta_N I_{MA}}{a B_0} \tag{8}$$

For a general function $Q(\rho)$.

(9)

$$Q_v = 4\pi^2 a^2 \kappa c_1 \int_0^1 Q(\rho)\rho R_0 d\rho + 2\pi^2 a^3 \kappa \int_0^1 Q(\rho)\rho \left[2c_1(c_2\rho^2 + 3c_0\rho^4 - 2\rho^2\sigma + \sigma) + 3c_2c_3\rho^4\right] d\rho$$

Evaluating the integral for pressure, we have:

(10)

$$\bar{p} = 2\bar{n}\bar{T}\frac{(1+\nu_n)(1+\nu_T)}{1+\nu_n+\nu_T}\left(c_1 + \frac{a}{R_0}\frac{6c_0c_1 + 3c_2c_3 + c_1(3+\nu_n+\nu_T)(c_2+\sigma(\nu_n+\nu_T))}{(2+\nu_n+\nu_T)(3+\nu_n+\nu_T)}\right)$$

$$\begin{aligned}\bar{n}\bar{T} \leq \frac{\beta_N I_{MA} B_0}{4a\mu_0}\frac{(1+\nu_n+\nu_T)}{(1+\nu_n)(1+\nu_T)}\times \\ \left[c_1 + \frac{a}{R_0}\frac{6c_0c_1 + 3c_2c_3 + c_1(3+\nu_n+\nu_T)(c_2+\sigma(\nu_n+\nu_T))}{(2+\nu_n+\nu_T)(3+\nu_n+\nu_T)}\right]^{-1}\end{aligned} \tag{11}$$

(12)

$$\begin{aligned}\bar{n}_{20}\bar{T}_k \leq 12.44\frac{\beta_N I_{MA} B_0}{a}\frac{(1+\nu_n+\nu_T)}{(1+\nu_n)(1+\nu_T)}\times \\ \left[c_1 + \frac{a}{R_0}\frac{6c_0c_1 + 3c_2c_3 + c_1(3+\nu_n+\nu_T)(c_2+\sigma(\nu_n+\nu_T))}{(2+\nu_n+\nu_T)(3+\nu_n+\nu_T)}\right]^{-1}\end{aligned}$$

5. Kink stability limit

Assuming as limit $q_\star \geq 2$, the kink stability limit becomes

$$I \leq I_{max} = \frac{\pi a^2 B_0 \kappa}{\mu_0 R_0} \tag{13}$$