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2381 Appendix F

2382 Expanding on the Bootstrap Current

2383 The bootstrap current fraction – f_{BS} – is an important parameter that enters in
2384 the design of tokamak reactors. It must be calculated with reasonable accuracy to
2385 determine how much external current drive is required. The value of f_{BS} thus has
2386 a strong impact on the overall fusion energy gain. Obtaining reasonable accuracy
2387 requires a moderate amount of analysis, which is presented in a following section.
2388 The results are summarized below.

2389 F.1 Summarized Results

2390 The analysis is based on an expression for the bootstrap current valid for arbitrary
2391 cross section assuming (1) equal temperature electrons and ions $T_e = T_i = T$, (2) large
2392 aspect ratio $\epsilon \ll 1$, and (3) negligible collisionality $\nu_* \rightarrow 0$. Under these assumptions
2393 the bootstrap current $\mathbf{J}_{BS} \approx J_{BS} \mathbf{e}_\phi$ has the form

$$J_{BS} = -3.32 f_T R_0 n T \left(\frac{1}{n} \frac{dn}{d\psi} + 0.054 \frac{1}{T} \frac{dT}{d\psi} \right) \quad (\text{F.1})$$

2394 Here, $f_T \approx 1.46(r/R_0)^{1/2}$ is an approximate expression for the trapped particle frac-
2395 tion and ψ is the poloidal flux.

The analysis next section shows that Eq. (F.1) leads to an expression for the bootstrap fraction, assuming for simplicity elliptical flux surfaces, that can be written as:

$$f_{BS} = \frac{I_{BS}}{I} = \frac{2\pi a^2 \kappa}{I} \int_0^1 J_{BS} \rho d\rho = \frac{K_{BS}}{K_n} \frac{\bar{n} \bar{T} R_0^2}{I_P^2} \quad (\text{F.2})$$

$$K_{BS} = 4.879 \cdot K_n \cdot \left(\frac{1 + \kappa^2}{2} \right) \cdot \epsilon^{5/2} \cdot H_{BS} \quad (\text{F.3})$$

$$H_{BS} = (1 + \nu_n)(1 + \nu_T)(\nu_n + 0.054\nu_T) \int_0^1 \frac{\rho^{5/2} (1 - \rho^2)^{\nu_n + \nu_T - 1}}{b_p} d\rho \quad (\text{F.4})$$

$$b_p(\rho) = \frac{-e^{\gamma \rho^2} (\gamma \rho^2 - 1 - \gamma) - 1 - \gamma}{\rho (e^\gamma - 1 - \gamma)} \quad (\text{F.5})$$

$$\bar{J}_\phi(\rho) = -\frac{I}{\pi a^2 \kappa} \left[\frac{\gamma^2 (1 - \rho^2) e^{\gamma \rho^2}}{e^\gamma - 1 - \gamma} \right] \quad (\text{F.6})$$

2396 In this expression b_p is a normalized form of the poloidal magnetic field derived from
 2397 a prescribed model for the *total* flux surface averaged current density profile $\bar{J}_\phi(\rho)$.
 2398 The $\bar{J}_\phi(\rho)$ profile, in analogy with the density and temperature profiles, is not self-
 2399 consistent but is chosen to have a plausible experimental shape characterized by the
 2400 parameter γ . The profile can have either an on-axis ($\gamma < 1$) or off-axis peak ($\gamma > 1$).
 2401 The normalized internal inductance l_i and radial location of the current peak ρ_m are
 2402 related to the value of γ by:

$$l_i = \frac{4\kappa}{1 + \kappa^2} \int_0^1 b_p^2 \rho d\rho$$

(F.7)

2403

$$\rho_m = \begin{cases} \left(\frac{\gamma}{\gamma - 1} \right)^{1/2}, & \gamma > 1 \\ 0, & \gamma < 1 \end{cases} \quad (\text{F.8})$$

2404 F.2 Detailed Analysis

2405 The starting point for the analysis is the general expression for the bootstrap current
 2406 in a tokamak with arbitrary cross section.⁴³ This expression can be simplified by
 2407 assuming (1) equal temperature electrons and ions $T_e = T_i = T$, (2) large aspect ratio
 2408 $\epsilon \ll 1$, and (3) negligible collisionality $\nu_* \rightarrow 0$. The bootstrap current $\mathbf{J}_{BS} \approx J_{BS} \mathbf{e}_\phi$
 2409 reduces to

$$J_{BS} = -3.32 f_T R_0 n T \left(\frac{1}{n} \frac{dn}{d\psi} + 0.054 \frac{1}{T} \frac{dT}{d\psi} \right) \quad (\text{F.9})$$

2410 Several values of the trapped particle fraction f_T have been given in the literature.⁴⁴
 2411 For simplicity we use a form valid for large aspect ratio. This is a slightly optimistic
 2412 value but saves a large amount of detailed calculation. It can be written as,

$$f_T \approx 1.46 (r/R_0)^{1/2} = 1.46 \epsilon^{1/2} \rho^{1/2} \quad (\text{F.10})$$

2413 Here, as in the main text, ρ is a radial-like flux surface label that varies between
 2414 $0 \leq \rho \leq 1$. In other words $\psi = \psi(\rho)$. Under these assumptions the bootstrap current
 2415 reduces to:

$$J_{BS} = -4.85 R_0 \epsilon^{1/2} \left(\frac{\rho^{1/2} n T}{d\psi/d\rho} \right) \left(\frac{1}{n} \frac{dn}{d\rho} + 0.054 \frac{1}{T} \frac{dT}{d\rho} \right) \quad (\text{F.11})$$

2416 Since we have specified profiles for $n(\rho)$ and $T(\rho)$ all that remains in order to be able
 2417 to evaluate $J_{BS}(\rho)$ is to determine $\psi' = d\psi/d\rho$. Keep in mind that at this point, in
 2418 spite of the approximations that have been made, the expression for $J_{BS}(\rho)$ is still
 2419 valid for arbitrary cross section.

2420 The analysis that follows shows how to calculate ψ' for an arbitrary cross section
 2421 including finite aspect ratio. As an example an explicit expression for large aspect
 2422 ratio, finite elongation ellipse is obtained. Consider the Grad-Shafranov equation for
 2423 the flux: $\Delta^* \psi = -\mu_0 R J_\psi$. We integrate this equation over the volume of an arbitrary

2424 flux surface making use of Gauss' theorem, which leads to:

$$\int_S \frac{\mathbf{n} \cdot \nabla \psi}{R^2} dS = -\mu_0 \int_V \frac{J_\phi}{R} d\mathbf{r} \quad (\text{F.12})$$

Next, assume that the coordinates of the flux surface can be expressed in terms of ρ and an angular-like parameter α with $0 \leq \alpha \leq 2\pi$. In other words, the flux surface coordinates can be written as $R = R(\rho, \alpha) = R_0 + ax(\rho, \alpha)$ and $Z = Z(\rho, \alpha) = ay(\rho, \alpha)$. The functions $R(\rho, \alpha)$ and $Z(\rho, \alpha)$ are assumed to be known. The term on the left hand side can be evaluated by noting that

$$d\mathbf{l} = dl \mathbf{t} \quad (\text{F.13})$$

$$dl = (R_\alpha^2 + Z_\alpha^2)^{1/2} d\alpha \quad (\text{F.14})$$

$$\mathbf{t} = \frac{R_\alpha \mathbf{e}_R + Z_\alpha \mathbf{e}_Z}{(R_\alpha^2 + Z_\alpha^2)^{1/2}} \quad (\text{F.15})$$

$$\mathbf{n} = \mathbf{e}_\phi \times \mathbf{t} = \frac{Z_\alpha \mathbf{e}_R - R_\alpha \mathbf{e}_Z}{(R_\alpha^2 + Z_\alpha^2)^{1/2}} \quad (\text{F.16})$$

$$dS = R d\phi dl = 2\pi R (R_\alpha^2 + Z_\alpha^2)^{1/2} d\alpha \quad (\text{F.17})$$

2425 It then follows that

$$\mathbf{n} \cdot \nabla \psi = \frac{1}{(R_\alpha^2 + Z_\alpha^2)^{1/2}} \left(Z_\alpha \frac{\partial \psi}{\partial R} - R_\alpha \frac{\partial \psi}{\partial Z} \right) = \frac{1}{(R_\alpha^2 + Z_\alpha^2)^{1/2}} \frac{d\psi}{d\rho} Z_\alpha \rho_R - R_\alpha \rho_Z \quad (\text{F.18})$$

2426 We can rewrite the last term by noting that

$$\begin{aligned} dR = R_\rho d\rho + R_\alpha d\alpha &\rightarrow d\rho = (Z_\alpha dR - R_\alpha dZ) / (R_\rho Z_\alpha - R_\alpha Z_\rho) \\ dZ = Z_\rho d\rho + Z_\alpha d\alpha &\rightarrow d\alpha = (-Z_\rho dR + R_\rho dZ) / (R_\rho Z_\alpha - R_\alpha Z_\rho) \end{aligned} \quad (\text{F.19})$$

2427 from which follows

$$\begin{aligned}\rho_R &= \frac{Z_\alpha}{(R_\rho Z_\alpha - R_\alpha Z_\rho)} \\ \rho_Z &= -\frac{R_\alpha}{(R_\rho Z_\alpha - R_\alpha Z_\rho)}\end{aligned}\tag{F.20}$$

2428 the normal gradient reduces to

$$\mathbf{n} \cdot \nabla \psi = \frac{R_\alpha^2 + Z_\alpha^2}{(R_\rho Z_\alpha - R_\alpha Z_\rho)} \frac{d\psi}{d\rho}\tag{F.21}$$

2429 Using this relation we see that the left hand side of Eq. (F.12) can now be written

2430 as:

$$\int_S \frac{\mathbf{n} \cdot \nabla \psi}{R^2} dS = 2\pi \frac{d\psi}{d\rho} \int_0^{2\pi} \frac{R_\alpha^2 + Z_\alpha^2}{(R_\rho Z_\alpha - R_\alpha Z_\rho)} \frac{d\alpha}{R}\tag{F.22}$$

2431 Consider now the right hand side of Eq. (F.12). The critical assumption is that the

2432 current density is approximated by its flux surface averaged value, $J_\phi(\rho, \alpha) \approx \bar{J}_\phi(\rho)$.

2433 This is obviously not self-consistent with the Grad-Shafranov equation. Even so, it

2434 should suffice for present purposes where we only need to evaluate global volume

2435 integrals. Also, in the same spirit as prescribing $n(\rho)$ and $T(\rho)$ we assume that $\bar{J}_\phi(\rho)$

2436 is also prescribed. Under these assumptions the right hand side of Eq. (F.12) simplifies

2437 to:

$$\begin{aligned}-\mu_0 \int_V \frac{J_\phi}{R} d\mathbf{r} &= -2\pi\mu_0 \int_A J_\phi dA \\ &= -2\pi\mu_0 \int_0^\rho d\rho \int_0^{2\pi} J_\phi (R_\rho Z_\alpha - R_\alpha Z_\rho) d\alpha \\ &\approx -2\pi\mu_0 \int_0^\rho d\rho \left[\bar{J}_\phi \int_0^{2\pi} (R_\rho Z_\alpha - R_\alpha Z_\rho) d\alpha \right]\end{aligned}\tag{F.23}$$

2438 Combining the results in Eqs. (F.22) and (F.23) leads to the required general expres-

2439 sion for $d\psi/d\rho$,

$$\frac{d\psi}{d\rho} \int_0^{2\pi} \frac{R_\alpha^2 + Z_\alpha^2}{(R_\rho Z_\alpha - R_\alpha Z_\rho)} \frac{d\alpha}{R} = -\mu_0 \int_0^\rho d\rho \left[\bar{J}_\omega \int_0^{2\pi} (R_\rho Z_\alpha - R_\alpha Z_\rho) d\alpha \right]\tag{F.24}$$

2440 Next, to help specify a plausible choice for \bar{J}_ϕ it is useful to define the kink safety

2441 factor and the actual local safety factor. The kink safety factor is defined by

$$q_* = \frac{2\pi a^2 B_0}{\mu_0 R_0 I} \left(\frac{1 + \kappa^2}{2} \right) \quad (\text{F.25})$$

2442 where

$$I = \int J_o dA = \int_0^1 d\rho \left[\bar{J}_o \int_0^{2\pi} (R_\rho Z_\alpha - R_\alpha Z_\rho) d\alpha \right] \quad (\text{F.26})$$

2443 This leads to

$$\frac{1}{q_*} = \frac{\mu_0 R_0}{2\pi a^2 B_0} \left(\frac{2}{1 + \kappa^2} \right) \int_0^1 d\rho \left[\bar{J}_\phi \int_0^{2\pi} (R_\rho Z_\alpha - R_\alpha Z_\rho) d\alpha \right] \quad (\text{F.27})$$

2444 Similarly, the local safety factor can be expressed as

$$q(\rho) = \frac{F(\rho)}{2\pi} \int \frac{dl}{RB_p} \quad (\text{F.28})$$

2445 Here, $F(\rho) = RB_o$. Substituting $RB_p = \mathbf{n} \cdot \nabla \psi$ then yields

$$q(\rho) = \frac{F(\rho)}{2\pi\psi'} \int_0^{2\pi} \frac{1}{R} (R_\rho Z_\alpha - R_\alpha Z_\rho) d\alpha \quad (\text{F.29})$$

2446 with $\psi' = d\psi/d\rho$.

2447 For present purposes we can obtain relatively simple analytic expressions for all the
 2448 quantities of interest by assuming the flux surfaces are concentric ellipses, character-
 2449 ized by $R = R_0 + a\rho \cos \alpha$ and $Z = \kappa a\rho \sin \alpha$. We assume low β so that $F(\rho) \approx R_0 B_0$.
 2450 This model accounts for elongation but neglects the effects of triangularity and finite
 2451 aspect ratio. The derivatives in Eqs. (F.24), (F.27) and (F.29) can now be easily
 2452 evaluated. Also, after some trial and error we chose $\bar{J}_\phi(\rho)$ to be a plausible profile
 2453 which is peaked off-axis at $\rho = \rho_m$.

$$\bar{J}_\phi(\rho) = -\frac{I}{\pi a^2 \kappa} \left[\frac{\gamma^2 (1 - \rho^2) e^{\gamma \rho^2}}{e^\gamma - 1 - \gamma} \right] \quad (\text{F.30})$$

2454 Here, $\gamma = 1/(1 - \rho_m^2)$.

2455 These profiles are substituted into Eq. (F.24) after which each of the integrals can be
 2456 evaluated analytically. A straightforward calculation yields:

$$\begin{aligned}\rho \frac{d\psi}{d\rho} &= -2\mu_0 R_0 a^2 \left(\frac{\kappa^2}{1 + \kappa^2} \right) \int_0^\rho \bar{J}_\phi \rho d\rho \\ &= \frac{\mu_0 R_0 I}{\pi} \left(\frac{\kappa}{1 + \kappa^2} \right) \frac{(1 + \gamma - \gamma \rho^2) e^{\gamma \rho^2} - 1 - \gamma}{e^\gamma - 1 - \gamma}\end{aligned}\tag{F.31}$$

2457 The safety factors are given by

$$\begin{aligned}\frac{1}{q_*} &= \frac{\psi'(1)}{\kappa a^2 B_0} \\ \frac{q(\rho)}{q_*} &= \frac{\rho \psi'(1)}{\psi'(\rho)}\end{aligned}\tag{F.32}$$

Eq. (F.31) is now substituted into the expression for the bootstrap current given by Eq. (F.11). The resulting expression can then be integrated over the plasma cross section to yield the bootstrap fraction. A straightforward calculation leads to:

$$f_{BS} = \frac{I_{BS}}{I} = \frac{2\pi a^2 \kappa}{I} \int_0^1 J_{BS} \rho d\rho = \frac{K_{BS}}{K_n} \frac{\bar{n} \bar{T} R_0^2}{I_P^2}\tag{F.33}$$

$$K_{BS} = 4.879 \cdot K_n \cdot \left(\frac{1 + \kappa^2}{2} \right) \cdot \epsilon^{5/2} \cdot H_{BS}\tag{F.34}$$

$$H_{BS} = (1 + \nu_n)(1 + \nu_T)(\nu_n + 0.054\nu_T) \int_0^1 \frac{\rho^{5/2} (1 - \rho^2)^{\nu_n + \nu_T - 1}}{b_p} d\rho\tag{F.35}$$

$$b_p(\rho) = \frac{-e^{\gamma \rho^2} (\gamma \rho^2 - 1 - \gamma) - 1 - \gamma}{\rho (e^\gamma - 1 - \gamma)}\tag{F.36}$$

2458 This is the desired result.

2459 Appendix G

2460 Elaborating on the Current Drive

2461 The driven current fraction – f_{CD} – is an important parameter that enters in the
2462 design of steady-state tokamak reactors. It must be calculated with reasonable ac-
2463 curacy to determine how much bootstrap current is required. The value of f_{CD} thus
2464 has a strong impact on the overall fusion energy gain. Obtaining reasonable accuracy
2465 requires a moderate amount of analysis, which is presented in a following section.
2466 The results are summarized below.

2467 G.1 Summarized Results

2468 We assume that current drive is provided by lower hybrid waves because of the cor-
2469 responding relatively high efficiency and naturally occurring off-axis peaking which
2470 aligns with the bootstrap current. The externally driven lower hybrid current (I_{CD})
2471 is given in terms of the current drive efficiency, η_{CD} , defined as follows:⁴⁵

$$I_{CD} = \eta_{CD} \frac{P_H}{\bar{n}_{20} R_0} = \frac{\eta_{CD}}{Q} \frac{P_F}{\bar{n}_{20} R_0} \quad (\text{G.1})$$

2472 Here, for simplicity and slightly optimistically, we assume that 100% of the klystron
2473 RF power, P_H , is absorbed in the plasma.

2474 The current drive fraction – $f_{CD} = I_{CD}/I_P$ – can then be written as,

$$\begin{aligned} f_{CD} &= K_{CD} \frac{\eta_{CD} \bar{n}_{20} R_0^2 (\hat{\sigma} v)}{I_M} \\ K_{CD} &= 278 \frac{f_D^2 \varepsilon^2 \kappa}{Q} = 0.634 \end{aligned} \quad (\text{G.2})$$

2475 Typical values for η_{CD} are around 0.3.⁴⁵ However, this current drive efficiency is
 2476 actually a function of: \bar{n} , \bar{T} , and B_0 . This dependence must be included in the
 2477 design to obtain reliable results. A self consistent calculation of $\eta_{CD} = \eta_{CD}(\bar{n}, \bar{T}, B_0)$
 2478 requires considerable analysis, the details of which are presented next section.

2479 G.2 Detailed Analysis

2480 To design a steady state fusion reactor, it is necessary to calculate η_{CD} for lower
 2481 hybrid current drive (LHCD). Recall that the driven lower hybrid current I_{CD} is
 2482 related to the lower hybrid RF klystron power absorbed by the plasma P_H by the
 2483 relation:

$$I_{CD} = \eta_{CD} \frac{P_H}{n_{20} R} \quad (\text{G.3})$$

2484 Here, $P_H = \eta_{RF} P_{RF}$, with P_{RF} equal to the total wall power used for current drive
 2485 (plus heating) and $\eta_{RF} \approx 0.5$ is the conversion efficiency from wall power to RF
 2486 absorbed power. Also, $n_{20} = n_{20}(\rho_j)$ and $R = R(\rho_j, \theta)$ are the density and major
 2487 radius evaluated at the minor radius $\rho = \rho_j$ and launch angle θ with ρ_j corresponding
 2488 to the location of the peak driven current density: $J_{max} = J_{CD}(\rho_j, \theta)$. The angle
 2489 θ is a known quantity set by the experimental configuration while ρ_j is yet to be
 2490 determined.

2491 The value of η_{CD} is related to a normalized quantity $\tilde{\eta}$, the efficiency usually calculated
 2492 in the literature, by a series of connecting formulas. The inter-relations start with

$$\eta_I = \frac{\int_A J_{CD} dA}{\int_V S_H dV} \approx \frac{1}{2\pi} \left[\frac{J_{CD}}{R S_H} \right]_{\rho_j, \theta} = \frac{\eta_{LH}}{2\pi R} \left[\frac{J_{CD}}{R S_{LH}} \right]_{\rho_j, \theta} \quad (\text{G.4})$$

2493 where $\eta_I = I_{CD} = P_H A/W$ is the overall current drive efficiency measuring how many

2494 delivered watts of klystron RF power are required to drive one ampere of current. For
 2495 simplicity and slightly optimistically all delivered power is assumed to be absorbed
 2496 by the plasma. Also, $S_H(\rho, \theta)$ is the klystron power density delivered to the plasma,
 2497 whose absorption is localized around $\rho = \rho_j$.

2498 Due to various losses, only a fraction of the absorbed klystron power, $\eta_{LH} \approx 0.75$,
 2499 actually drives current. These losses have to do with the fact that the power spectrum
 2500 arising from a realistic waveguide array has both positive and negative lobes – it
 2501 is not an ideal positive delta function. The combination of finite spectral width
 2502 plus oppositely driven current from the negative lobe implies that only a portion of
 2503 the total absorbed power actually drives a net positive current. The result of this
 2504 discussion is that the power density, S_{LH} , driving lower hybrid current is related to
 2505 the delivered klystron power density by $S_{LH} = \eta_{LH} S_H$.

Now, the efficiency, $\tilde{\eta}$ usually calculated in the literature is defined by:

$$\tilde{\eta}(\rho_J, \theta) = \left[\frac{J_{CD}/en v_{Te}}{S_{LH}/m_e n \nu_0 v_{Te}^2} \right]_{\rho_j, \theta} \quad (\text{G.5})$$

$$v_{Te}(\rho_J) = \left[\frac{2T_e}{m_e} \right]_{\rho_J}^{1/2} \quad (\text{G.6})$$

$$\nu_0(\rho_J) = \left[\frac{\omega_{pe}^4 \ln \Lambda}{2\pi n_e v_{Te}^3} \right]_{\rho_J} \quad (\text{G.7})$$

2506 It then follows that

$$\eta_I = \frac{\eta_{LH}}{2\pi} \left[\frac{e}{R m_e \nu_0 v_{Te}} \right]_{\rho_j, \theta} \tilde{\eta}(\rho_J, \theta) \quad (\text{G.8})$$

2507 From Eq. (G.3), we see that $\eta_{CD} = \eta_I [n_{20} R]_{\rho_j, \theta}$, which leads to the desired conversion
 2508 relation:

$$\eta_{CD} = \frac{\eta_{LH}}{2\pi} \left[\frac{e n_{20}}{m_e \nu_0 v_{Te}} \right]_{\rho_j} \tilde{\eta}(\rho_J, \theta) = 0.06108 \frac{\eta_{LH}}{\ln \Lambda} T_k \tilde{\eta} \quad (\text{G.9})$$

2509 **An expression for $\tilde{\eta}$**

2510 Needed for the design code is an expression for $\tilde{\eta}(\rho_j, \theta)$. Such an expression, valid for
 2511 arbitrary ρ , has been determined by Ehst and Karney³⁶ – based on a sophisticated
 2512 theoretical analysis combined with extensive numerical results. Once ρ_j is determined
 2513 we set $\rho = \rho_j$ in the expression for $\tilde{\eta}(\rho, \theta)$. Ehst and Karney find that a good fit for
 2514 $\tilde{\eta}(\rho, \theta)$ can be written as:

$$\tilde{\eta} = CMR\eta_0 \quad (\text{G.10})$$

For LHCD, the parameters appearing in Eq. (G.10) have the form:

$$M = 1 \quad (\text{G.11})$$

$$R(\rho, \theta) = 1 - \frac{\varepsilon^n \rho^n (x_r^2 + w^2)^{1/2}}{\varepsilon^n \rho^n x_r + w} \quad n = 0.77 \quad x_r = 2.47 \quad (\text{G.12})$$

$$C(\rho, \theta) = 1 - \exp(-c^m x_t^{2m}) \quad m = 1.38 \quad c = 0.778 \quad (\text{G.13})$$

$$\eta_0(\rho, \theta) = \frac{K}{w} + D + \frac{8w^2}{5 + Z_{eff}} \quad K = \frac{2.12}{Z_{eff}} \quad D = \frac{3.83}{Z_{eff}^{0.707}} \quad (\text{G.14})$$

2515 All quantities have been defined except for $x_t^2(\rho, \theta)$ and $w(\rho, \theta)$. The quantity w is a
 2516 normalized form of the resonant particle velocity which absorbs energy and momen-
 2517 tum from the lower hybrid wave,

$$w(\rho, \theta) = \frac{\omega}{k_{\parallel} v_{Te}} = \frac{c}{v_{Te}} \frac{1}{n_{\parallel}} \quad (\text{G.15})$$

2518 with n_{\parallel} the parallel index of refraction. The value of $n_{\parallel}(\rho, \theta)$ will be discussed shortly.

2519 The quantity x_t^2 is a toroidal correction associated with the fact that trapped particles
 2520 cannot contribute to toroidal current flow. It can be expressed in terms of the local
 2521 mirror ratio by

$$x_t^2(\rho, \theta) = w^2 \left(\frac{B}{B_M - B} \right) \quad (\text{G.16})$$

where from simple guiding center theory assuming that $B \approx B_\phi$

$$B_M = \frac{B_0}{1 - \varepsilon \rho} \quad (\text{G.17})$$

$$B = \frac{B_0}{1 + \varepsilon \rho \cos \theta} \quad (\text{G.18})$$

2522 Calculation of $n_\parallel^2(\rho, \theta)$

2523 The next step in the evaluation of η_{CD} is the calculation of $n_\parallel^2(\rho, \theta)$. Its value is deter-
 2524 mined by the requirements for accessibility from the plasma edge into the absorption
 2525 layer. The relevant physics follows from an analysis of the cold plasma dispersion
 2526 relation given by

$$n_\perp^2(\rho, \theta) = -\frac{K_\parallel}{2K_\perp} \left\{ n_\parallel^2 - K_\perp + \frac{K_A^2}{K_\parallel} \pm \left[\left(n_\parallel^2 - K_\perp + \frac{K_A^2}{K_\parallel} \right)^2 + \frac{4K_\perp K_A^2}{K_\parallel} \right]^{1/2} \right\} \quad (\text{G.19})$$

2527 The plus sign corresponds to the desired root and is often referred to as the slow
 2528 wave.

2529 In the lower hybrid regime the relevant ordering of parameters is

$$\begin{aligned} \omega_{pe}/\Omega_e &\sim \omega_{pi}/\omega \sim n_\parallel \sim 1 \\ \omega_{pi}/\Omega_i &\sim \omega/\Omega_i \sim \Omega_e/\omega \sim n_\perp \sim m_i/m_e^{1/2} \gg 1 \end{aligned} \quad (\text{G.20})$$

2530 leading to the following simple forms for the elements of the dielectric tensor

$$\begin{aligned} K_\perp(\rho, \theta) &= 1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2} \sim 1 \\ K_A(\rho, \theta) &= \frac{\omega_{pe}^2}{\omega \Omega_e} \sim m_i/m_e^{1/2} \\ K_\parallel(\rho) &= -\frac{\omega_{pe}^2}{\omega^2} \sim m_i/m_e \end{aligned} \quad (\text{G.21})$$

2531 The first requirement for accessibility is that the function under the square root
 2532 be positive. When this function passes through zero there is a double root for n_\perp^2
 2533 causing a mode conversion from the slow wave to the fast wave. The fast wave does

not propagate into the plasma. It is reflected back out through the plasma surface, obviously an undesirable result. Avoiding mode conversion requires a sufficiently large value of n_{\parallel}^2 to keep the function under the square root positive. This value must satisfy

$$n_{\parallel}^2(\rho, \theta) \geq \left[K_{\perp}^{1/2} + \left(-\frac{K_A^2}{K_{\parallel}} \right)^{1/2} \right]^2 \quad (\text{G.22})$$

Since $\eta_{CD} \propto 1/n_{\parallel}^2$ we see that current drive efficiency is maximized when $n_{\parallel}^2(\rho, \theta)$ is minimized – the inequality in Eq. (G.22) must be set to equality.

At this point there is an important subtlety that must be taken into account. The issue is that the wavelength spectrum of the applied klystron source is not a delta function – it has a finite half width, $\Delta n_{\parallel} \approx 0.2$ and a negative lobe. For simplicity, we have modeled the spectrum as rectangular and ignore the negative lobe. The negative lobe is accounted for through the value of η_{LH} , since this power obviously does not drive current in the desired direction. Now, Eq. (G.22) is an inequality and we want to minimize $n_{\parallel}^2(\rho, \theta)$ over all ρ for the given θ where the power is absorbed. Therefore, we must use the equality sign in Eq. (G.22) for the strictest case – that $n_{\parallel}(\hat{\rho}_J, \theta) = n_{\parallel}(\rho_J, \theta) - \Delta n_{\parallel}$ – where $\hat{\rho}_J$ and ρ_J (both as yet undetermined) are the corresponding strictest and average radii where power is absorbed.

With this in mind, after substituting the simplified expressions for the elements of the dielectric tensor we obtain

$$\begin{aligned} n_{\parallel}^2(\hat{\rho}_J, \theta) &= \left[\left(1 - \frac{1-\hat{\omega}^2}{\hat{\omega}^2} X \right)^{1/2} + X^{1/2} \right]^2 \\ X(\hat{\rho}_J, \theta) &= \omega_{pe}^2(\hat{\rho}_J) / \Omega_e^2(\hat{\rho}_J, \theta) \\ \hat{\omega}^2(\hat{\rho}_J, \theta) &= \omega^2 / \Omega_e(\hat{\rho}_J, \theta) \Omega_i(\hat{\rho}_J, \theta) \end{aligned} \quad (\text{G.23})$$

The question now is how do we choose the frequency: $\hat{\omega}$? There are actually three constraints on the frequency and we must choose the strictest one to determine $\hat{\omega}^2$.

2554 The constraints are as follows:

$$\begin{aligned}
\omega^2 &> \omega_{LH}^2(\hat{\rho}_J, \theta) && \text{Avoid mode conversion before reaching } \hat{\rho}_J, \theta \\
\omega^2 &> 4\omega_{LH}^2(\hat{\rho}_J, \theta) && \text{Avoid the PDI before reaching } \hat{\rho}_J, \theta \\
\omega^2 &> k_{\perp}^2(\hat{\rho}_J, \theta) v_{\alpha}^2 && \text{Avoid coupling to } \alpha \text{ particles before reaching } \hat{\rho}_J, \theta
\end{aligned} \tag{G.24}$$

2555 Here, $\omega_{LH}^2(\hat{\rho}_J, \theta) = \omega_{pi}^2 / (1 + \omega_{pe}^2 / \Omega_e^2)$ is the square of the lower hybrid frequency and
2556 $v_{\alpha} = (2E_{\alpha} / m_{\alpha})^{1/2}$ is the alpha particle speed. Also, PDI denotes parametric decay
2557 instability. The second and third constraints are approximate values, used here for
2558 simplicity.

2559 Each of these constraints is substituted into the expression for n_{\parallel}^2 . We find that in
2560 the regime of interest the α particle coupling requirement is the strictest. We thus
2561 choose the frequency to satisfy $\omega / k_{\perp} = v_{\alpha}$, or in normalized units:

$$n_{\perp}^2(\hat{\rho}_J, \theta) = \frac{c^2}{v_{\alpha}^2} \tag{G.25}$$

2562 This expression is simplified by evaluating n_{\perp}^2 using Eq. (G.19) coupled with n_{\parallel}^2 given
2563 by Eq. (G.22)

$$n_{\perp}^2(\hat{\rho}_J, \theta) = -\frac{K_{\parallel}}{K_{\perp}^{1/2}} \left(-\frac{K_A^2}{K_{\parallel}} \right)^{1/2} = \frac{m_i}{m_e} \frac{X^{3/2}}{\hat{\omega} [\hat{\omega}^2(1+X) - X]} \tag{G.26}$$

2564 Eq. (G.26) is a quadratic equation for $\hat{\omega}^2$, which can be easily solved, yielding:

$$\begin{aligned}
\hat{\omega}^2(\hat{\rho}_J, \theta) &= \frac{1}{2} \frac{X}{1+X} + \frac{1}{2} \left[\frac{X^2}{(1+X)^2} + 4\gamma^2 \frac{X^3}{1+X} \right]^{1/2} \\
\gamma &= \frac{m_i}{m_e} \frac{1}{n_{\perp}^2} = \frac{2m_i E_{\alpha}}{m_e m_{\alpha} c^2} = 8.562
\end{aligned} \tag{G.27}$$

2565 This value of $\hat{\omega}^2$ is substituted into Eq. (G.23) to obtain the desired expression for
2566 $n_{\parallel}^2 = n_{\parallel}^2(X)$.

2567 Calculation of $\hat{\rho}_j$

2568 The calculation of $\hat{\rho}_j$ requires a very lengthy analysis of Landau damping. We can
 2569 bypass this complication by making use of a simple rule of thumb that is reasonably
 2570 accurate. This rule states that lower hybrid power is absorbed and driven current
 2571 produced in a somewhat narrow layer of the plasma profile whose location is deter-
 2572 mined by the requirement that the parallel phase velocity be approximately equal to
 2573 three times the electron thermal speed,

$$\frac{\omega}{k_{\parallel}} \approx 3v_T \quad (\text{G.28})$$

2574 The equation can be rewritten in terms of \hat{n}_{\parallel} leading to a transcendental algebraic
 2575 equation for $\hat{\rho}_j$,

$$(1 + \nu_T) (1 - \hat{\rho}_J^2)^{\nu_T} n_{\parallel}^2(\hat{\rho}_J, \theta) = \frac{m_e c^2}{18\bar{T}} = \frac{28.39}{\bar{T}_k} \quad (\text{G.29})$$

2576 This is a simple equation to solve numerically.

2577 Calculation of ρ_j

2578 The last step in the analysis is to map the results at the strictest absorption location –
 2579 (ρ, θ) – to the center of the absorption layer (ρ_J, θ) where the current drive efficiency
 2580 is defined. This is easily done by noting that power is always absorbed in at the local
 2581 radius where $\omega/k_{\parallel} = 3v_{Te}$. Consequently, the relations at ρ_J are related to those at
 2582 $\hat{\rho}_J$ by:

$$\begin{aligned} (1 + \nu_T) (1 - \hat{\rho}_J^2)^{\nu_T} n_{\parallel}^2(\hat{\rho}_J, \theta) &= \frac{28.39}{\bar{T}_k} \\ (1 + \nu_T) (1 - \rho_J^2)^{\nu_T} n_{\parallel}^2(\rho_J, \theta) &= \frac{28.39}{\bar{T}_k} \end{aligned} \quad (\text{G.30})$$

2583 Since $n_{\parallel}(\hat{\rho}_J, \theta) = n_{\parallel}(\rho_J, \theta) - \Delta n_{\parallel}$, it follows that $\hat{\rho}_J$ and ρ_J are related by:

$$\frac{(1 - \rho_J^2)^{\nu_T}}{(1 - \hat{\rho}_J^2)^{\nu_T}} = \left[1 - \frac{\Delta n_{\parallel}}{n_{\parallel}(\rho_J, \theta)} \right]^2 \rightarrow \rho_J^2 = 1 - (1 - \hat{\rho}_J^2) \left[1 - \frac{\Delta n_{\parallel}}{n_{\parallel}(\rho_J, \theta)} \right]^{2/\nu_T} \quad (\text{G.31})$$

2584 Note that in general: $\rho_J > \hat{\rho}_J$. The strictest location determining $n_{\parallel}(\hat{\rho}_J, \theta)$ is the
 2585 innermost radial point on the temperature profile where power is absorbed.

2586 Abridged Modus Operandi

2587 Assume the following quantities are given as inputs: $B_0, \theta, \bar{n}_{20}, \bar{T}_k, \varepsilon, \Delta n_{\parallel}, \eta_{LH}$. Carry
 2588 out the following steps:

2589 1. Solve the equations below simultaneously to determine $n_{\parallel}^2(\hat{\rho}_J, \theta), \hat{\omega}^2(\hat{\rho}_J, \theta)$, and $\hat{\rho}_J$

2590

$$\begin{aligned} n_{\parallel}^2(\hat{\rho}_J, \theta) &= \left[\left(1 - \frac{1 - \hat{\omega}^2}{\hat{\omega}^2} X \right)^{1/2} + X^{1/2} \right]^2 \\ \hat{\omega}^2(\hat{\rho}_J, \theta) &= \frac{1}{2} \frac{X}{1+X} + \frac{1}{2} \left[\frac{X^2}{(1+X)^2} + 4\gamma^2 \frac{X^3}{1+X} \right]^{1/2} \\ (1 + \nu_T)(1 - \hat{\rho}_J^2)^{\nu_T} n_{\parallel}^2(\hat{\rho}_J, \theta) &= \frac{m_e c^2}{2\bar{T}} = \frac{28.39}{\bar{T}_k} \end{aligned} \quad (\text{G.32})$$

2591 2. Solve for $\tilde{\eta}(\hat{\rho}_J, \theta)$

$$\tilde{\eta}(\hat{\rho}_J, \theta) = CM R \eta_0 \quad (\text{G.33})$$

2592 3. Solve for $n_{\parallel}(\rho_J, \theta)$

$$n_{\parallel}(\rho_J, \theta) = n_{\parallel}(\hat{\rho}_J, \theta) + \Delta n_{\parallel} \quad (\text{G.34})$$

2593 4. Solve for ρ_J

$$\rho_J^2 = 1 - (1 - \hat{\rho}_J^2) \left[1 - \frac{\Delta n_{\parallel}}{n_{\parallel}(\rho_J, \theta)} \right]^{2/\nu_T} \quad (\text{G.35})$$

2594 5. Re-evaluate $\tilde{\eta}(\rho_J, \theta)$ by substituting the values of $\rho_J, n_{\parallel}(\rho_J, \theta)$ into Eq. (G.33).

2595 6. Solve for η_{CD}

$$\eta_{CD} = \frac{1}{2\pi} \left(\frac{en_{20}}{m_e \nu_0 v_{Te}} \right) \tilde{\eta} = 0.06108 \frac{\eta_{LH}}{\ln \Lambda} (1 + \nu_T) \bar{T}_k (1 - \rho_J^2)^{\nu_r} \tilde{\eta}(\rho_J, \theta) \quad (\text{G.36})$$

2596 In the end there will have to be some iteration with the rest of the analysis to make
 2597 sure the values of \bar{n}_{20} and \bar{T}_k are self-consistent.

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