

Early Growth in a Perturbed Universe: Dark Matter Halo Properties in 2LPT and  
ZA Simulations

By

Daniel J. Sissom

Dissertation

Submitted to the Faculty of the  
Graduate School of Vanderbilt University  
in partial fulfillment of the requirements  
for the degree of

DOCTOR OF PHILOSOPHY

in

PHYSICS

August, 2014

Nashville, TN

Approved:

Date:

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Jocelyn K. Holley-Bockelmann, Ph.D.

---

Andreas A. Berlind, Ph.D.

---

David A. Weintraub, Ph.D.

---

Shane M. Hutson, Ph.D.

---

Robert J. Scherrer, Ph.D.

## **ACKNOWLEDGMENTS**

This is where you thank the people that made your work possible: grant awarding agencies, advisers, your committee, mom and dad, whatever.

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# CHAPTER I

## Introduction

Text goes here. This is where we'll talk about the purpose of the project and the layout of this document.

The structure of this document is as follows: The remainder of this chapter, Chapter I, provides an introduction to the early universe and the processes that lead to galaxy-hosting dark matter halos, as well as the fundamentals of the computational theory for the numerical methods relevant to this discussion. Chapter II examines in more detail the specific numerical methods used for this work, with emphasis on the methodologies of the codes themselves, how they are implemented in the context of the overall simulation and analysis pipeline, and the results obtained at each step. Chapter III is a direct representation of the published paper which (more succinctly) presents an overview of the numerical methods and the main results in this work. Chapter IV is primarily the same material as previously submitted to fulfill the requirements of the Qualifying Exam, and is slightly edited to better suit the tone of this document. Chapter V concludes with a discussion of the results in this work and the greater implications to the overall field.

### I.1 Dark Matter Halos in the Early Universe

Text goes here.

#### I.1.1 Halo Formation and Growth

The attractive nature of gravity implies that regions of over-density become denser and regions of under-density become even more under-dense. A perfectly smooth dark matter field would continue to stay smooth, as net forces would balance to zero. However, small density perturbation generated during inflation in the otherwise

smooth primordial DM field trigger the inexorable collapse of dark matter into over-dense regions known as dark matter halos (Press & Schechter 1974; Bardeen et al. 1986).

The non-linear evolution of the collapse of density fluctuations may be approximated to first order by the spherical “top-hat” perturbation (Silk 1968; Peebles & Yu 1970; Peebles 1970; Gunn & Gott 1972). In this model, the perturbation is represented as an isolated, uniform sphere of dark matter. The region outside the sphere is unperturbed and does not influence the evolution of the sphere. This model affords an exact solution (Peebles 1980, 1993; Padmanabhan 1993, and references therein), but results in a collapse to infinite density. However, growth of initially small density inhomogeneities may interrupt the collapse by a rapid relaxation to a finite density virial equilibrium (Shapiro et al. 1999; Monaco 1998, and references therein).

The definition of a halo arises from the contrast in density between a virialized over-dense region and the density of the rest of the universe. For example, halos defined according to the spherical overdensity (SO) method are regions above a certain density threshold (Bryan & Norman 1998), either with respect to the critical density  $\rho_c = 3H^2/8\pi G$  or the background matter density  $\rho_b = \Omega_m \rho_c$ , where  $\Omega_m$  is the matter density of the Universe. The halo is then the region enclosed within a sphere with mean density  $\Delta \rho_c$  or  $\Delta \rho_b$ , where  $\Delta$  commonly ranges from  $\sim 100$  to  $\sim 500$  and is typically taken to be  $\sim 200$ . The radius of the sphere is typically called the virial radius  $R_{\text{vir}}$ , but may alternatively be denoted  $R_\Delta$ , where the specific choice of  $\Delta$  is listed (e.g.  $R_{200}$ ).

Dark matter halos form hierarchically (e.g., Cole et al. 2000; Conselice et al. 2003, and references therein). Small halos form first from gravitational collapse and successively merge to form larger structures over time, which is often referred to as the “bottom-up” paradigm. This leads to a characteristic mass of assembling halos at each redshift, which, at  $z = 0$ , are clusters with mass  $\gtrsim 10^{14} M_\odot$ . A typical halo

undergoes a number of mergers throughout its evolution (e.g. Conselice et al. 2003; Genel et al. 2009; Fakhouri et al. 2010). Defining a major merger to have a mass ratio of 3 : 1 or less and a minor merger to have a mass ratio of 10 : 1 or less, a massive halo typically undergoes  $\sim 4 - 5$  major mergers after  $z \sim 3$ , with minor mergers occurring even more frequently. These mergers play a critical role in the mass assembly of a halo, and greatly influence the evolution of the hosted baryonic galaxy.

### I.1.2 The Mass Function

The number density of dark matter halos as a function of halo mass and redshift, often referred to simply as the mass function, is a key probe of cosmology. The original formulation of Press & Schechter (1974) is explored in more detail by Mo & White (2002) and Warren et al. (2006), and here we follow the notation of Mo & White (2002).

$$\sigma^2(R) = \frac{1}{2\pi^2} \int_0^\infty k^3 P(k) \tilde{W}^2(kR) \frac{dk}{k} \quad (\text{I.1})$$

$$\tilde{W}(x) = 3[\sin(kR) - kR \cos(kR)]/(kR)^3$$

$$n(M, z) \, dM = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}_0}{M} \frac{d\nu}{dM} \exp\left(-\frac{\nu^2}{2}\right) \, dM \quad (\text{I.2})$$

$$\nu \equiv \delta_c/[D(z)\sigma(M)]$$

$$\delta_c \approx 1.69$$

$$D(z) = g(z)/[g(0)(1+z)]$$

$$g(z) \approx \frac{5}{2} \Omega_m \left[ \Omega_m^{4/7} - \Omega_\Lambda + (1 + \Omega_m/2)(1 + \Omega_\Lambda/70) \right]^{-1} \quad (\text{I.3})$$

There are a number of ways to define a halo's mass. This becomes significant for mass-sensitive studies, such as the halo mass function (Press & Schechter 1974; Reed et al. 2007; Heitmann et al. 2006; Lukić et al. 2007), the number density of halos as a

function of mass and a key probe of cosmology. For a review, see, e.g., White (2001) and references therein. Additionally, see Voit (2005) and references therein for a more observation-focused discussion.

$$M_{\text{vir}} \equiv \frac{4\pi}{3} \Delta_{\text{vir}} \rho_m R_{\text{vir}}^3, \quad (\text{I.4})$$

where  $\rho_m = \Omega_M \rho_c$ .

$$V_{\text{circ}} = \sqrt{\frac{GM(< r)}{r}} \Big|_{\text{max}} \quad (\text{I.5})$$

### I.1.3 Density and Concentration

The halo density profile is a measure of the spherically-averaged dark matter density as a function of radius. For numerical halos in  $N$ -body simulations, the density profile is typically computed by dividing the member particles into logarithmically-spaced bins from the virial radius inward towards the center, summing the mass of the particles in each bin, and dividing by the volume of the shell to find the density.

DM halos almost universally display a characteristic shape in their density profiles. This shape is most often parameterized with the Navarro-Frenk-White (NFW) profile (Navarro et al. 1996):

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s} \left(1 + \frac{r}{R_s}\right)^2}, \quad (\text{I.6})$$

where  $\rho_0$  is the characteristic density and  $R_s$  is the scale radius where the inner  $\sim r^{-1}$  profile transitions to the outer  $\sim r^{-3}$  profile.

Halo concentration  $c$  provides a single-parameter quantization of the density profile. For the NFW profile, concentration is defined as  $c \equiv R_{\text{vir}}/R_s$ , where  $R_{\text{vir}}$  is the halo virial radius. Generally, at low redshift, low mass halos are more dense than high mass halos (Navarro et al. 1997a), and concentration decreases with redshift and increases in dense environments (Bullock et al. 2001b). Neto et al. (2007) addi-

tionally find that concentration decreases with halo mass. Various additional studies have explored concentration's dependence on characteristics of the power spectrum (Eke et al. 2001), cosmological model (Macciò et al. 2008), redshift (Gao et al. 2008; Muñoz-Cuartas et al. 2011), and halo merger and mass accretion histories (Wechsler et al. 2002; Zhao et al. 2003, 2009). For halos at high redshift, Klypin et al. (2011) find that concentration reverses and increases with mass for high mass halos, while Prada et al. (2012) find that concentration's dependence on mass and redshift is more complicated and is better described through  $\sigma(M, z)$ , the rms fluctuation amplitude of the linear density field.

Concentration may be estimated from a halo's virial mass  $M_{\text{vir}}$  and maximum circular velocity  $V_{\text{circ}}$ . Following Klypin et al. (2011), we outline this relationship for  $z = 0$  and as a function of redshift. The relation between the virial mass and maximum circular velocity may be given as (Klypin et al. 2001):

$$V_{\text{circ}} = \left[ G \frac{f(x_{\text{max}})}{f(c)} \frac{c}{x_{\text{max}}} \hat{\rho}^{1/3} \right]^{1/2} M_{\text{vir}}^{1/3}, \quad (\text{I.7})$$

$$\hat{\rho} = \frac{M_{\text{vir}}}{R_{\text{vir}}^3} = \frac{4\pi}{3} \Delta_{\text{vir}} \rho_c \Omega_M, \quad (\text{I.8})$$

$$f(x) = \ln(1 + x) - \frac{x}{1 + x}, \quad (\text{I.9})$$

where  $x = r/R_s$ ,  $x_{\text{max}} = 2.15$ ,  $\Delta_{\text{vir}}$  is the overdensity limit that defines the virial radius,  $\rho_c$  is the critical density, and  $\Omega_M$  is the matter contribution to the average density of the universe. At  $z = 0$ ,  $\Delta_{\text{vir}} = 360$  and  $\Omega_M = 0.27$ , which yields

$$V_{\text{circ}}(M_{\text{vir}}) = \frac{6.72 \times 10^{-3} M_{\text{vir}}^{1/3} \sqrt{c}}{\sqrt{\ln(1 + c) - c/(1 + c)}} \quad (\text{I.10})$$

for  $M_{\text{vir}}$  in units of  $h^{-1} M_\odot$  and  $V_{\text{circ}}$  in units of  $\text{km s}^{-1}$ . Klypin et al. (2011) find that

at  $z = 0$ , this yields the approximation

$$c(M_{\text{vir}}) = 9.60 \left( \frac{M_{\text{vir}}}{10^{12} h^{-1} M_{\odot}} \right)^{-0.075} \quad (\text{I.11})$$

for distinct halos and

$$c(M_{\text{sub}}) = 12 \left( \frac{M_{\text{sub}}}{10^{12} h^{-1} M_{\odot}} \right)^{-0.12} \quad (\text{I.12})$$

for subhalos. Figure I.1 plots concentration as a function of virial mass from  $z = 0$  to  $z = 5$ . The dotted lines are given by

$$c(M_{\text{vir}}, z) = c_0(z) \left( \frac{M_{\text{vir}}}{10^{12} h^{-1} M_{\odot}} \right)^{-0.075} \times \left[ 1 + \left( \frac{M_{\text{vir}}}{M_0(z)} \right)^{0.26} \right], \quad (\text{I.13})$$

where  $c_0(z)$  and  $M_0(z)$  are free parameters for each  $z$ . Concentration displays a decreasing trend with mass at low redshift. At higher redshift, however, concentration flattens out and reverses its trend, increasing with mass for the most massive halos.

Figure I.2 plots concentration as a function of redshift for two representative halo masses. For a given fixed halo mass, concentration decreases with redshift for low redshift, then increases again with redshift at high redshift. The black curves are given by

$$c(M_{\text{vir}}, z) = c(M_{\text{vir}}, 0) [\delta^{4/3}(z) + \kappa(\delta^{-1}(z) - 1)], \quad (\text{I.14})$$

where  $\delta(z)$  is the linear growth factor of fluctuations normalized to  $\delta(0) = 1$  and  $\kappa$  is a free parameter. For the masses shown in the figure,  $\kappa = 0.084$  for  $M = 3 \times 10^{11} h^{-1} M_{\odot}$  and  $\kappa = 0.135$  for  $M = 3 \times 10^{12} h^{-1} M_{\odot}$ .

Using the same method of determining concentration from halo virial mass and maximum circular velocity, Prada et al. (2012) find that the complex mass and redshift dependence of concentration found by Klypin et al. (2011) may be simplified to a universal U-shaped profile when viewed as a function of the linear rms fluctuation of the density field  $\sigma(M, z)$ . Figure I.3 plots  $c$  as a function of  $\log \sigma^{-1}$  for redshifts from

$z = 0$  to  $z = 6$  for halos from the Bolshoi (Klypin et al. 2011) and MultiDark (Prada et al. 2012) simulations. If we define

$$x \equiv \left( \frac{\Omega_{M,0}}{\Omega_{\Lambda,0}} \right)^{1/3} a, \quad (\text{I.15})$$

$$a \equiv (1+z)^{-1} \quad (\text{I.16})$$

where  $\Omega_{M,0}$  and  $\Omega_{\Lambda,0}$  are the matter and cosmological constant contributions to the density of the universe at  $z = 0$ , then the overplotted curve is given by

$$c(M, z) = B_0(x)C(\sigma'), \quad (\text{I.17})$$

$$\sigma' = B_1(x)\sigma(M, x), \quad (\text{I.18})$$

$$C(\sigma') = A \left[ \left( \frac{\sigma'}{b} \right)^c + 1 \right] \exp \left( \frac{d}{\sigma'^2} \right), \quad (\text{I.19})$$

where  $A = 2.881$ ,  $b = 1.257$ ,  $c = 1.022$ , and  $d = 0.060$ . The rms density fluctuation may be approximated as

$$\sigma(M, x) = D(x) \frac{16.9y^{0.41}}{1 + 1.102y^{0.20} + 6.22y^{0.333}}, \quad (\text{I.20})$$

where

$$y \equiv \left[ \frac{M}{10^{12} h^{-1} \text{ M}_\odot} \right]^{-1}, \quad (\text{I.21})$$

$$D(x) = \frac{5}{2} \left( \frac{\Omega_{M,0}}{\Omega_{\Lambda,0}} \right)^{1/3} \frac{\sqrt{1+x^3}}{x^{3/2}} \int_0^x \frac{x^{3/2} \, dx}{(1+x^3)^{3/2}}. \quad (\text{I.22})$$

The functions  $B_0(x)$  and  $B_1(x)$  are defined such that they equal unity at  $z = 0$  for WMAP5 parameters:

$$B_0(x) = \frac{c_{\min}(x)}{c_{\min}(1.393)}, \quad (\text{I.23})$$

$$B_1(x) = \frac{\sigma_{\min}^{-1}(x)}{\sigma_{\min}^{-1}(1.393)}, \quad (\text{I.24})$$

where

$$c_{\min}(x) = c_0 + (c_1 - c_0) \left[ \frac{1}{\pi} \arctan[\alpha(x - x_0)] + \frac{1}{2} \right], \quad (\text{I.25})$$

$$\sigma_{\min}^{-1}(x) = \sigma_0^{-1} + (\sigma_1^{-1} - \sigma_0^{-1}) \left[ \frac{1}{\pi} \arctan[\beta(x - x_1)] + \frac{1}{2} \right], \quad (\text{I.26})$$

$$c_0 = 3.618, \quad c_1 = 5.033, \quad \alpha = 6.948, \quad x_0 = 0.424, \quad (\text{I.27})$$

$$\sigma_0^{-1} = 1.047, \quad \sigma_1^{-1} = 1.646, \quad \beta = 7.386, \quad x_1 = 0.526. \quad (\text{I.28})$$

The resulting curve closely follows the data at all redshifts from  $z = 0$  to  $z = 6$ , with a minimum concentration of  $\sim 5$  at a well-defined scale of  $\sigma \sim 0.71$ . The relation may also be seen as a function of mass without rescaling to  $z = 0$  by plotting Equations I.17-I.19, as shown in Figure I.4.

#### I.1.4 Halos as Hosts to Baryonic Processes

Early-forming dark matter halos provide an incubator for the baryonic processes that transform the surrounding space and allow galaxies to form. Initial gas accretion can lead to the formation of the first Pop-III stars (Couchman & Rees 1986; Tegmark et al. 1997; Abel et al. 2000, 2002), which, upon their death, can collapse into the seeds for supermassive black holes (SMBHs) (Madau & Rees 2001; Islam et al. 2003; Alvarez et al. 2009; Jeon et al. 2012) or enrich the surrounding medium with metals through supernovae (Heger & Woosley 2002; Heger et al. 2003). The radiation from these early quasars (Shapiro & Giroux 1987; Madau et al. 1999; Fan et al. 2001), Pop-III stars (Gnedin & Ostriker 1997; Venkatesan et al. 2003; Alvarez et al. 2006), and proto-galaxy stellar populations (Bouwens et al. 2012; Kuhlen & Faucher-Giguère 2012) all play a key role in contributing to the re-ionizing the universe by around  $z = 6$  (Barkana & Loeb 2001). Additionally, halo mergers can drastically increase the temperature of halo gas through shock heating, increasing X-ray luminosity (Sinha & Holley-Bockelmann 2009), and contribute to the unbinding of gas to form the

warm-hot intergalactic medium (Bykov et al. 2008; Sinha & Holley-Bockelmann 2010; Tanaka et al. 2012).

## I.2 Computational Theory

In this section, we present a broad overview of the fundamental theory and driving equations of computational astrophysics that are relevant to this work. Specific code implementations, such as the  $N$ -body simulation code GADGET-2 and the halo finder ROCKSTAR, are discussed in Chapter II, so here we instead focus on the mathematical concepts that form the basis these codes rely on and have in common with varied other implementations. Specifically, in this section, we discuss collisionless dynamics in  $N$ -body simulations and simulation initialization with the Zel'dovich approximation (ZA) and second-order Lagrangian perturbation theory (2LPT). As the simulations used in our study are of collisionless dark matter only, we forgo a discussion of collisional hydrodynamics.

### I.2.1 Collisionless Dynamics and $N$ -body Simulations

Astrophysical simulations of stars or dark matter, in essence, track a collisionless fluid, which is described in the continuum limit by the collisionless Boltzmann equation (CBE)

$$\frac{df(\mathbf{x}, \mathbf{v}, t)}{dt} \equiv \frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0 \quad (\text{I.29})$$

coupled to the Poisson equation

$$\nabla^2 \Phi(\mathbf{x}, t) = 4\pi G \int f(\mathbf{x}, \mathbf{v}, t) d\mathbf{v} \quad (\text{I.30})$$

in an expanding background Universe, typically according to the Friedmann-Lemaître-Robertson-Walker metric. Here,  $\Phi$  is the self consistent potential, and the distribution function  $f(\mathbf{x}, \mathbf{v}, t)$  gives the mass density in phase space. The high-dimensionality of the problem, however, makes directly solving the coupled system of equations in-

tractable. Instead, the  $N$ -body method, in which the phase-space density is sampled with a finite number  $N$  of tracer particles, is used to evolve the system in time. For the following discussion, we primarily follow the notation in Springel (2005).

The system of particles is described by the Hamiltonian

$$H(\mathbf{x}_1, \dots, \mathbf{x}_N, \mathbf{p}_1, \dots, \mathbf{p}_N, t) = \sum_i \frac{\mathbf{p}_i^2}{2m_i a(t)^2} + \frac{1}{2} \sum_{ij} \frac{m_i m_j \varphi(\mathbf{x}_i - \mathbf{x}_j)}{a(t)}, \quad (\text{I.31})$$

where  $\mathbf{x}_i$  are comoving coordinate vectors with corresponding canonical momenta  $p_i = a^2 m_i \dot{\mathbf{x}}_i$  and  $a(t)$  is the time evolution of the scale factor that introduces explicit time dependence to the Hamiltonian. For simulations with periodic boundary conditions, the interaction potential  $\varphi(\mathbf{x})$  for a cube of size  $L^3$  is the solution of

$$\nabla^2 \varphi(x) = 4\pi G \left[ -\frac{1}{L^3} + \sum_{\mathbf{n}} \tilde{\delta}(\mathbf{x} - \mathbf{n}L) \right], \quad (\text{I.32})$$

where the sum over  $\mathbf{n} = (n_1, n_2, n_3)$  iterates over all integer triplets. Here, the mean density is subtracted, and the dynamics of the system follow

$$\nabla^2 \phi(\mathbf{x}) = 4\pi G[\rho(\mathbf{x}) - \bar{\rho}] \quad (\text{I.33})$$

with peculiar potential

$$\phi(x) = \sum_i m_i \varphi(\mathbf{x} - \mathbf{x}_i). \quad (\text{I.34})$$

For non-periodic (vacuum) boundary conditions, the interaction potential for point masses simplifies to

$$\varphi(\mathbf{x}) = -\frac{G}{|\mathbf{x}|} \quad (\text{I.35})$$

for large separations.

At small particle separations as  $|\mathbf{x}_i - \mathbf{x}_j| \rightarrow 0$ , particle accelerations computed via

the standard force law

$$\mathbf{a}_i = - \sum_{j \neq i} \frac{Gm_j |\mathbf{x}_i - \mathbf{x}_j|}{|\mathbf{x}_i - \mathbf{x}_j|^3} \quad (\text{I.36})$$

approach a numerical singularity that can introduce unphysical results for finite time-steps. To avoid this scenario, numerical simulations employ a softening parameter  $\epsilon > 0$  in the force law so that it does not diverge for small particle separations. As a simple example, the softening parameter may be added to the particle displacement in the denominator of the Newtonian force law:

$$\mathbf{F}_i = - \sum_{j \neq i} \frac{Gm_i m_j |\mathbf{x}_i - \mathbf{x}_j|}{(|\mathbf{x}_i - \mathbf{x}_j|^2 + \epsilon^2)^{3/2}}. \quad (\text{I.37})$$

More generally, the single particle density distribution function  $\tilde{\delta}(\mathbf{x})$  of Equation I.32 is the Dirac  $\delta$ -function convolved with a gravitational softening kernel of comoving scale  $\epsilon$ . The specific choice of softening is dependent on the type of simulation and the system of study. The softening parameter is typically on the order of the mean inter-particle separation.

Directly calculating forces for every particle from every other particle inherently requires a double sum, implying a computational cost of  $\mathcal{O}(N^2)$  algorithm complexity scaling. For large  $N$ , this quickly becomes computationally expensive. While the accuracy afforded by direct summation is sometimes necessary, such as for collisional systems like high-density star clusters, most studies can tolerate random force errors up to  $\sim 1\%$  (Hernquist et al. 1993), introducing the possibility of approximation methods. There are a number of implementations for force approximations, but a typical result is a reduction of algorithmic complexity from  $\mathcal{O}(N^2)$  to  $\mathcal{O}(N \log N)$ . The specific implementation employed by GADGET-2 is discussed in Section II.2.1.

### I.2.2 Perturbation Theory and Particle Displacement

In order to retrieve reliable results from  $N$ -body simulations, generation of accurate initial conditions for a given cosmology is imperative. For cosmological simulations, the goal in creating initial conditions is to assign particle positions and velocities that are appropriate for a given simulation starting redshift  $z_{\text{start}}$  and consistent with the evolution of the Universe up to that point.

The subtle  $\mathcal{O}(10^{-5})$  density perturbations in place at the CMB epoch are vulnerable to numerical noise and intractable to simulate directly. Instead, a displacement field is applied to the particles to evolve them semi-analytically, nudging them from their initial positions to an approximation of where they should be at a more reasonable starting redshift for the numerical simulation. Starting at a later redshift aids in avoiding interpolation systematics and round-off errors (Lukić et al. 2007).

For this discussion, we will assume a  $\Lambda$ CDM Universe, where the initial density distribution is described by a Gaussian random field defined by the power spectrum. We wish to transform the information encoded in the power spectrum into a distribution of discrete particles at  $z_{\text{start}}$  that may then be evolved numerically. The first step is to create a representation of the density field in Fourier space. As the choice of power spectrum constrains the statistics of the density field and not its specific distribution, the specific realization of the field is generated from a random seed. The typical procedure is to create a set of uniform random phases and assign amplitudes drawn from the Rayleigh distribution (Efstathiou et al. 1985). The density field may then be used as a basis for creating a particle distribution.

Beginning from a uniform lattice of Lagrangian positions, particles are displaced to new Eulerian positions and assigned velocities according to a displacement field  $\Psi$  that is derived from the density field. The two most common methods for obtaining this displacement field are the Zel'dovich approximation (ZA, Zel'dovich 1970) and second-order Lagrangian perturbation theory (2LPT, Buchert 1994; Buchert et al.

1994; Bouchet et al. 1995; Scoccimarro 1998). Initial conditions created with ZA displace initial particle positions and velocities via a linear field (Klypin & Shandarin 1983; Efstathiou et al. 1985), while 2LPT initial conditions add a second-order correction term to the expansion of the displacement field (Scoccimarro 1998; Sirko 2005; Jenkins 2010).

### I.2.2.1 Particle Displacement with za and 2lpt

In this section, we give an overview of the equations necessary to generate initial conditions for  $N$ -body simulations using ZA and 2LPT. These results are fully described in Appendix D1 of Scoccimarro (1998), and are largely reproduced here following that notation.

As mentioned above, our goal is to displace particles from their initial positions  $\mathbf{q}$  to final Eulerian particle positions  $\mathbf{x}$  via a displacement field  $\Psi(\mathbf{q})$ :

$$\mathbf{x} = \mathbf{q} + \Psi(\mathbf{q}). \quad (\text{I.38})$$

If we define the conformal time  $\tau = \int dt/a(t)$ , where  $a(t)$  is the scale factor, and the conformal expansion rate  $\mathcal{H} \equiv d \ln a/d\tau = Ha$ , where  $H$  is the Hubble constant, then the equation of motion for particle trajectories  $\mathbf{x}(\tau)$  is given by

$$\frac{d^2\mathbf{x}}{d\tau^2} + \mathcal{H}(\tau) \frac{d\mathbf{x}}{d\tau} = -\nabla\Phi, \quad (\text{I.39})$$

where  $\Phi$  is the gravitational potential and  $\nabla$  is the gradient operator in Eulerian coordinates  $\mathbf{x}$ . Using  $1 + \delta(\mathbf{x}) = J^{-1}$ , where  $\delta(\mathbf{x}) \equiv [\rho(\mathbf{x}, t) - \bar{\rho}]/\bar{\rho}$  is the density contrast and the Jacobian determinant is  $J(\mathbf{q}, \tau) \equiv \det(\delta_{ij} + \Psi_{i,j})$ , where  $\Psi_{i,j} \equiv \partial\Psi_i/\partial\mathbf{q}_j$ , we may take the divergence of I.39 to obtain

$$J(\mathbf{q}, \tau) \nabla \cdot \left[ \frac{d^2\mathbf{x}}{d\tau^2} + \mathcal{H}(\tau) \frac{d\mathbf{x}}{d\tau} \right] = \frac{3}{2}\Omega\mathcal{H}^2(J-1). \quad (\text{I.40})$$

Using  $\nabla_i = (\delta_{ij} + \Psi_{i,j})^{-1} \nabla_{\mathbf{q}_j}$ , where the gradient operator in Lagrangian coordinates  $\nabla_{\mathbf{q}} \equiv \partial/\partial \mathbf{q}$ , this equation may be rewritten in terms of Lagrangian coordinates.

The solution to this transformed equation is given to first order by the Zel'dovich approximation:

$$\nabla_{\mathbf{q}} \cdot \Psi^{(1)} = -D_1(\tau) \delta(\mathbf{q}), \quad (\text{I.41})$$

where  $\delta(\mathbf{q})$  is the Gaussian density field determined by the initial conditions and  $D_1(\tau)$  is the linear growth factor, which obeys

$$\frac{d^2 D_1}{d\tau^2} + \mathcal{H}(\tau) \frac{dD_1}{d\tau} = \frac{3}{2} \Omega \mathcal{H}^2(\tau) D_1. \quad (\text{I.42})$$

The Zel'dovich approximation solution for the particle displacement field is then given by

$$\mathbf{x}(\mathbf{q}, \tau) = \mathbf{q} + \Psi(\mathbf{q}, \tau) \approx \mathbf{q} - D_1(\tau) \nabla \phi^{(1)}(\mathbf{q}), \quad (\text{I.43})$$

where  $\phi^{(1)}(\mathbf{q})$  is a Lagrangian potential given by the initial conditions. The velocities of particles initially at  $\mathbf{q}$  are given by

$$\mathbf{v} \approx -D_1(\tau) \mathcal{H}(\tau) f \nabla \phi^{(1)}(\mathbf{q}), \quad (\text{I.44})$$

where  $f(\Omega, \Lambda)$  is defined as

$$f_i(\Omega, \Lambda) \equiv \frac{d \ln D_i}{d \ln a} = \frac{1}{\mathcal{H}} \frac{d \ln D_i}{d\tau}. \quad (\text{I.45})$$

The second-order (2LPT) correction is found by a perturbative solution to the non-linear equation for  $\Psi(\mathbf{q})$  (Equation I.40 transformed to Lagrangian coordinates), expanding about the linear (ZA) solution (Equation I.41) to yield (e.g. Bouchet et al. 1995)

$$\nabla_{\mathbf{q}} \cdot \Psi^{(2)} = \frac{1}{2} D_2(\tau) \sum_{i \neq j} \left[ \Psi_{i,i}^{(1)} \Psi_{j,j}^{(1)} - \Psi_{i,j}^{(1)} \Psi_{j,i}^{(1)} \right], \quad (\text{I.46})$$

where  $D_2(\tau)$  is the second-order growth factor, which may be approximated as  $D_2(\tau) \approx -3D_1^2(\tau)/7$  (Bouchet et al. 1995). The displacement field may then be written in terms of two Lagrangian potentials  $\phi^{(1)}$  and  $\phi^{(2)}$ :

$$\mathbf{x}(\mathbf{q}) = \mathbf{q} - D_1 \nabla_q \phi^{(1)} + D_2 \nabla_q \phi^{(2)}. \quad (\text{I.47})$$

Likewise, the comoving velocities are then given to second order by

$$\mathbf{v} \equiv \frac{d\mathbf{x}}{dt} = -D_1 f_1 H \nabla_q \phi^{(1)} + D_2 f_2 H \nabla_q \phi^{(2)}. \quad (\text{I.48})$$

The logarithmic derivatives of the growth factors  $f_i$  may be approximated as  $f_1 \approx \Omega^{5/9}$  and  $f_2 \approx 2\Omega^{6/11}$  (Bouchet et al. 1995). The potentials  $\phi^{(1)}$  and  $\phi^{(2)}$  are derived by solving a pair of Poisson equations (Buchert et al. 1994):

$$\nabla_q^2 \phi^{(1)}(\mathbf{q}) = \delta^{(1)}(\mathbf{q}), \quad (\text{I.49})$$

$$\nabla_q^2 \phi^{(2)}(\mathbf{q}) = \delta^{(2)}(\mathbf{q}), \quad (\text{I.50})$$

where  $\delta^{(1)}(\mathbf{q})$  is the linear overdensity, and  $\delta^{(2)}(\mathbf{q})$  is the second-order overdensity given by

$$\delta^{(2)}(\mathbf{q}) = \sum_{i>j} \left\{ \phi_{,ii}^{(1)}(\mathbf{q}) \phi_{,jj}^{(1)}(\mathbf{q}) - \left[ \phi_{,ij}^{(1)}(\mathbf{q}) \right]^2 \right\}, \quad (\text{I.51})$$

where  $\phi_{,ij}^{(n)} \equiv \partial^2 \phi^{(n)} / \partial \mathbf{q}_i \partial \mathbf{q}_j$  (Jenkins 2010).

### I.2.2.2 Transients and the Advantages of 2lpt

A primary concern when generating cosmological initial conditions is the effects of non-linear decaying modes, or transients, which introduce deviations from the growing modes of the exact dynamics. Linear growing modes of density and velocity perturbations are correctly reproduced by ZA. However, ZA has shown to be inaccu-

rate in regards to higher-order growing modes and non-linear correlations (Grinstein & Wise 1987; Juszkiewicz et al. 1993; Bernardeau 1994; Catelan & Moscardini 1994; Juszkiewicz et al. 1995), and fails to accurately represent statistical quantities that probe phase correlations of density and velocity fields (Scoccimarro 1998).

We cannot expect accurate simulation results until enough time has passed for transients to have sufficiently decayed away. Transients are damped proportional to  $1/a$  in ZA. In 2LPT, however, transients are damped more quickly as  $1/a^2$ . Therefore, structure in 2LPT should be accurate after fewer  $e$ -folding times than in ZA (Scoccimarro 1998; Crocce et al. 2006; Jenkins 2010). Reed et al. (2013) suggest that for 2LPT-initialized simulations, between 10 and 50 expansion factors are needed before the relevant epoch of halo formation if percent level accuracy is to be achieved.

The practical result is that high- $\sigma$  DM density peaks at high redshift are suppressed in ZA compared with 2LPT for a given starting redshift (Crocce et al. 2006). While differences in ensemble halo properties, such as the halo mass function, between simulation initialization methods are mostly washed away by  $z = 0$  (Scoccimarro 1998), discrepancies between ZA and 2LPT remain at earlier redshifts (Reed et al. 2013), though these trends are relatively less studied (Lukić et al. 2007).

### I.2.2.3 Initial Redshift

When setting up an  $N$ -body simulation, it is critical to choose an appropriate starting redshift, determined by box size and resolution (Lukić et al. 2007). As 2LPT more accurately displaces initial particle positions and velocities, initialization with 2LPT allows for a later starting redshift compared with an equivalent ZA-initialized simulation. However, many ZA simulations do not take this into account, starting from too late an initial redshift and not allowing enough  $e$ -foldings to adequately dampen away numerical transients (Crocce et al. 2006; Jenkins 2010). In order to characterize an appropriate starting redshift, the relation between the initial rms particle displace-

ment and mean particle separation must be considered. The initial rms displacement  $\Delta_{\text{rms}}$  is given by

$$\Delta_{\text{rms}}^2 = \frac{4\pi}{3} \int_{k_f}^{k_{\text{Ny}}} P(k, z_{\text{start}}) dk, \quad (\text{I.52})$$

where  $k_f = 2\pi/L_{\text{box}}$  is the fundamental mode,  $L_{\text{box}}$  is the simulation box size,  $k_{\text{Ny}} = \frac{1}{2}Nk_f$  is the Nyquist frequency of an  $N^3$  simulation, and  $P(k, z_{\text{start}})$  is the power spectrum at starting redshift  $z_{\text{start}}$ . In order to avoid the “orbit crossings” that reduce the accuracy of the initial conditions,  $\Delta_{\text{rms}}$  must be some factor smaller than the mean particle separation  $\Delta_p = L_{\text{box}}/N$  (Holley-Bockelmann et al. 2012). For example, making orbit crossing a  $\sim 10\sigma$  event imposes  $\Delta_{\text{rms}}/\Delta_p = 0.1$ . However, for small-volume, high-resolution simulations, this quickly leads to impractical starting redshifts, placing such a simulation well into the regime of introducing errors from numerical noise caused by roundoff errors dominating the smooth potential. A more relaxed requirement of  $\Delta_{\text{rms}}/\Delta_p = 0.25$ , which makes orbit crossing a  $\sim 4\sigma$  event, often proves a more practical choice.

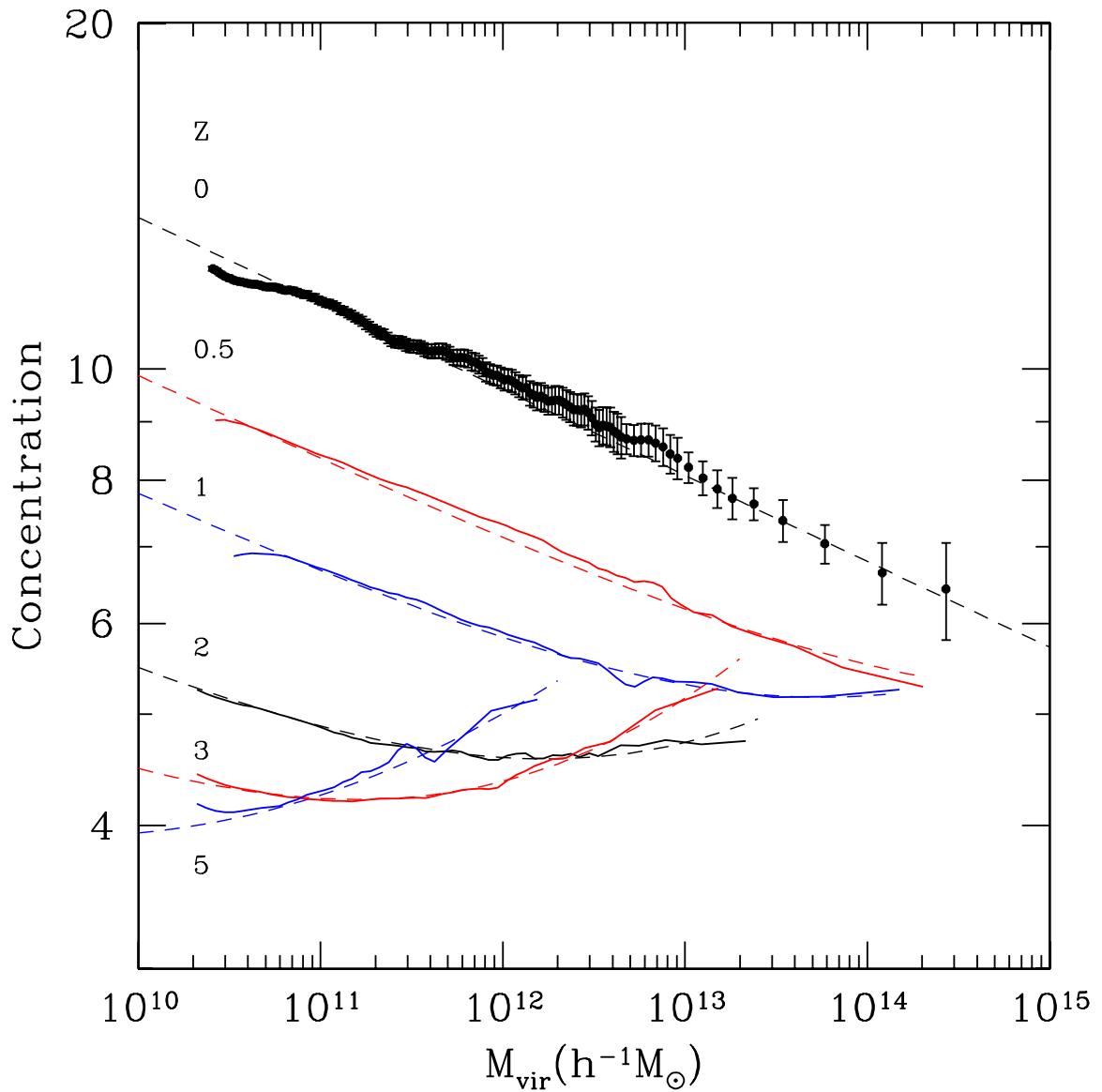


Figure I.1: Concentration as a function of virial mass for distinct halos from  $z = 0$  to  $z = 5$ . Symbols and solid curves are numerical results, while the dashed curves are analytical fits (Equation I.13). Concentration decreases with increasing mass except for high-mass halos at high redshift, for which the concentration flattens and increases with mass. (Klypin et al. 2011)

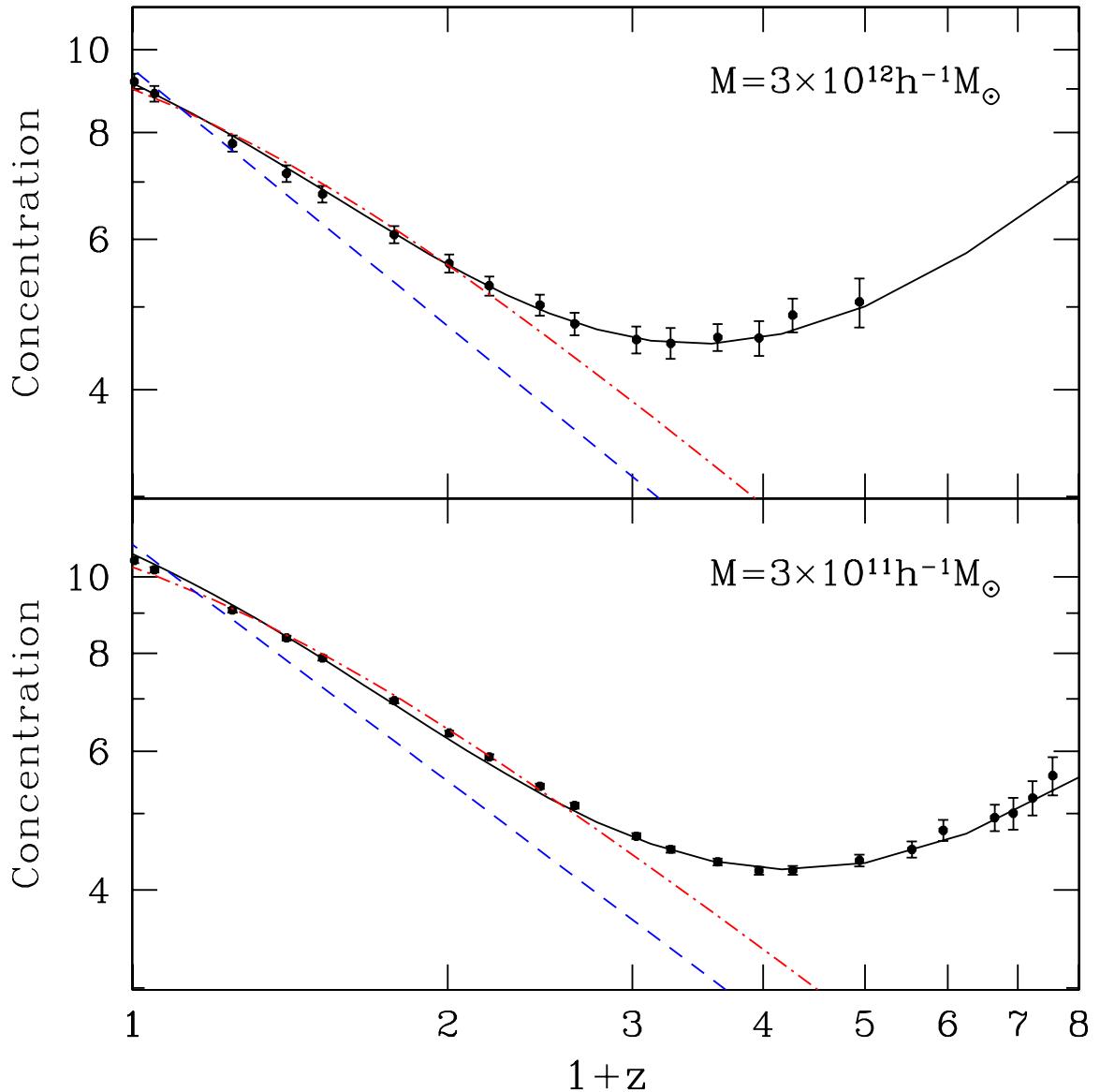


Figure I.2: Concentration as a function of redshift for two representative halo masses. Black dots are simulation results. The dashed blue curves show the power law  $c \propto (1+z)^{-1}$  and the dot-dashed red curves are  $c \propto \delta$ . The solid black curves are given by Equation I.14. Concentration initially decreases with redshift, but reverses and increases with redshift for high redshift. Concentration for both masses reaches a minimum of  $c_{\min} \approx 4 - 4.5$ . (Klypin et al. 2011)

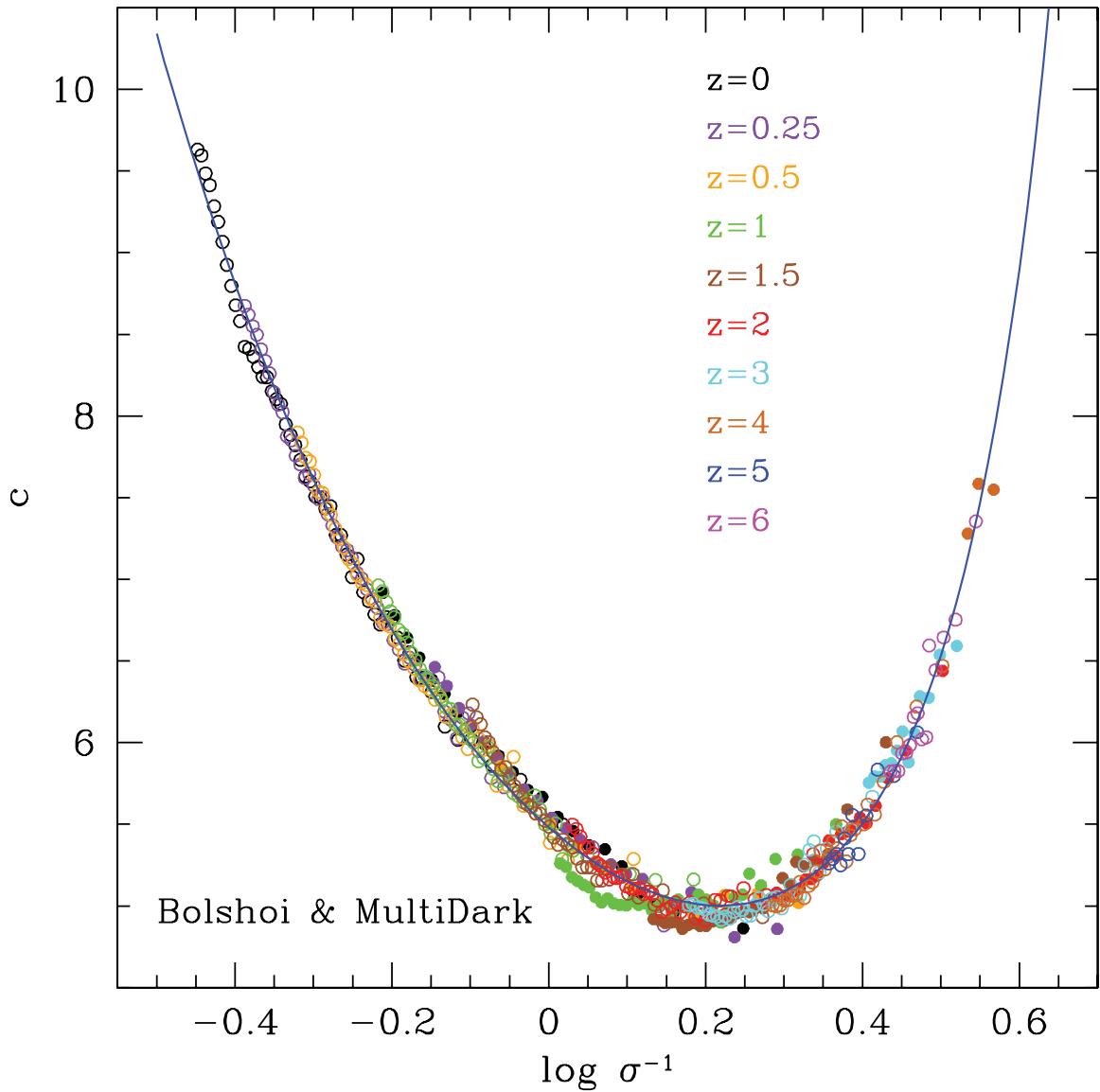


Figure I.3: Halo concentration  $c$  as a function of  $\log \sigma^{-1}$  for halos in the Bolshoi and MultiDark simulations. The results are rescaled to  $z = 0$ . The solid curve  $C(\sigma')$  is given by Equation I.19. A universal minimum concentration of  $\sim 5$  is seen at  $\sigma \sim 0.71$ . (Prada et al. 2012)

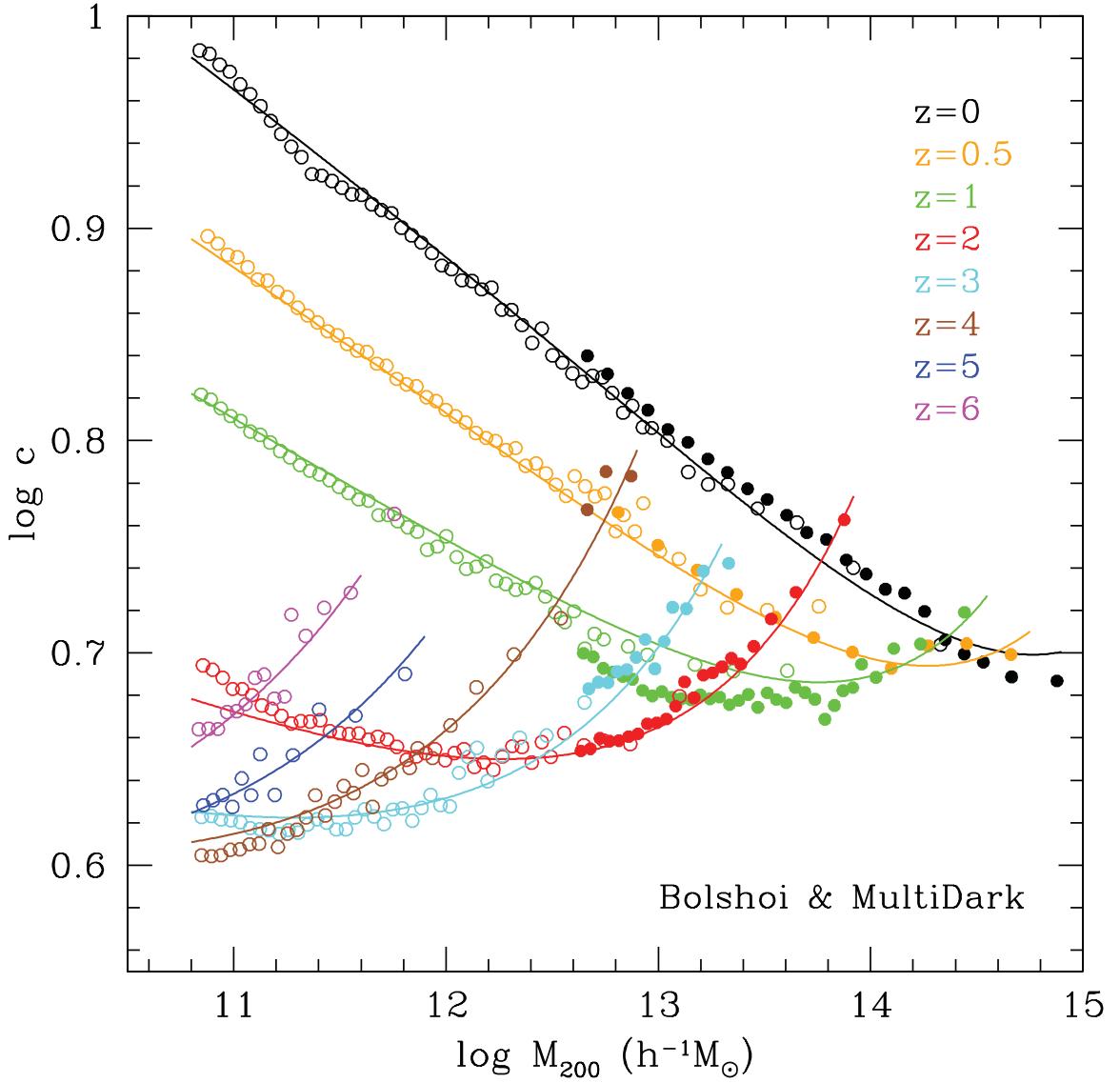


Figure I.4: Halo concentration  $c$  as a function of halo mass at various redshifts for halos in the Bolshoi (open circles) and MultiDark (filled circles) simulations. The overplotted curves are given by Equations I.17-I.19. The analytical approximations fit the data within a few percent. (Prada et al. 2012)

## CHAPTER II

### Numerical Methods

Text goes here. Here, we will discuss the computational tools used, their inner workings, and how they are implemented to accomplish their purpose in the pipeline.

#### II.1 Simulation Initialization

We have already discussed the fundamentals of particle displacement with ZA and 2LPT in Section I.2.2.1, so this section will instead provide an overview of the steps performed in the numerical implementation of simulation initialization. The code used to generate ZA and 2LPT initial conditions for the simulations used in this study follows the prescription detailed in Appendix D2 of Scoccimarro (1998), so we will simply summarize what is presented there. For this section, a tilde will denote Fourier-space quantities.

Beginning with a linear power spectrum, a Gaussian density field  $\tilde{\delta}(\mathbf{k})$ , with wave number  $\mathbf{k}$ , is generated in Fourier space. Equation I.49 is then used to find the Fourier space first-order potential  $\tilde{\phi}^{(1)}(\mathbf{k})$ , after which an inverse fast Fourier transform (FFT) is applied to produce  $\phi^{(1)}(\mathbf{q})$ . The first-order particle displacements and velocities are then found from Equations I.43 and I.44 by differencing  $\phi^{(1)}(\mathbf{q})$  along the three coordinate vectors to obtain  $\nabla_{\mathbf{q}}\phi^{(1)}$ , providing the solution according to ZA.

The 2LPT displacements and velocities are derived from the ZA solution by using  $\nabla_{\mathbf{q}}\phi^{(1)}$  to find the terms of the sum in Equation I.51. The diagonal terms  $\nabla_{11}^2\phi^{(1)}$ ,  $\nabla_{22}^2\phi^{(1)}$ ,  $\nabla_{33}^2\phi^{(1)}$  are obtained by diagonally differencing the components of the  $\nabla_{\mathbf{q}}\phi^{(1)}$  array. These are multiplied together to obtain the first term of Equation I.51. The non-diagonal terms  $\phi_{,ij}^{(1)}(\mathbf{q})$  are found by differencing  $\nabla_{\mathbf{q}}\phi^{(1)}$ , and the results are accumulated to form the second term of Equation I.51. An FFT is applied to  $\delta^{(2)}(\mathbf{q})$ , Equation I.50 is solved in Fourier space, and an inverse FFT is applied to the resulting

$\tilde{\phi}^{(2)}(\mathbf{k})$  to yield  $\phi^{(2)}(\mathbf{q})$ . The second-order potential  $\phi^{(2)}(\mathbf{q})$  is then differenced in each direction to yield  $\nabla_{\mathbf{q}}\phi^{(2)}$ . With both  $\nabla_{\mathbf{q}}\phi^{(1)}$  and  $\nabla_{\mathbf{q}}\phi^{(2)}$ , Equations I.47 and I.48 are used to find particle displacements and velocities, providing the solution for 2LPT.

## II.2 Simulations with Gadget-2

We use the massively parallel TreeSPH (Hernquist & Katz 1989) cosmological  $N$ -body simulation code GADGET-2 (Springel et al. 2001; Springel 2005) for the dark matter simulations presented in this work. In this section, we give an overview of the fundamentals of the GADGET-2 code, followed by details of our particular simulations.

### II.2.1 Gadget-2

GADGET-2 is a massively parallel cosmological  $N$ -body simulation code which calculates gravitational forces via a hierarchical multipole expansion and ideal gas parameters via smoothed particle hydrodynamics (SPH; Gingold & Monaghan 1977). This section will discuss the gravitational algorithms used to compute forces and the time integration method used to advance the simulation. As our simulations are collisionless only, we do not discuss the details of the implementation of gas dynamics in GADGET-2.

#### II.2.1.1 Gravitational Algorithms

Force computation suffers from a numerical singularity as the separation between two particles approaches zero, as discussed in Section I.2.1. A modification of the force law is therefore required at small separation scales. Force softening is accomplished in GADGET-2 using a spline kernel (Monaghan & Lattanzio 1985)  $W(|x|, h = 2.8\epsilon)$ , where

$$W(r, h) = \frac{8}{\pi h^3} \begin{cases} 1 - 6\left(\frac{r}{h}\right)^2 + 6\left(\frac{r}{h}\right)^3, & 0 \leq \frac{r}{h} \leq \frac{1}{2} \\ 2\left(1 - \frac{r}{h}\right)^3, & \frac{1}{2} < \frac{r}{h} \leq 1, \\ 0, & \frac{r}{h} > 1. \end{cases} \quad (\text{II.1})$$

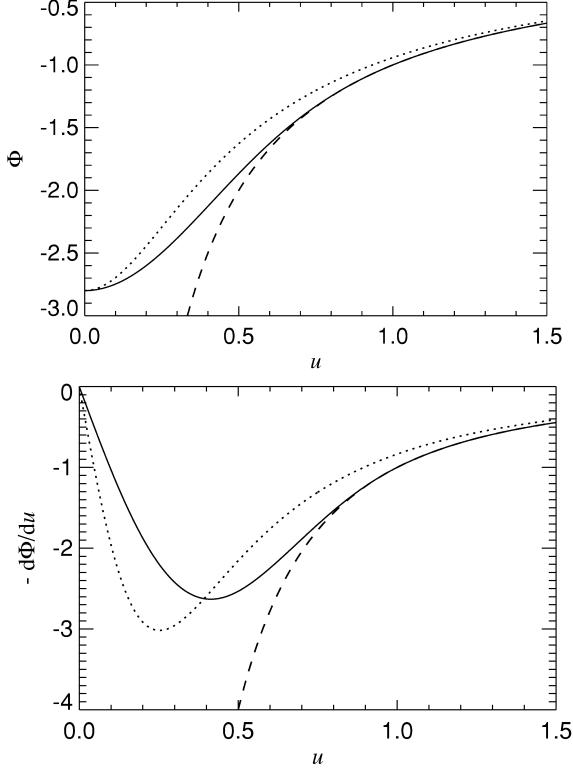


Figure II.1: Potential (*left*) and force (*right*) softening. The solid curves are the spline softening of Equation II.1. Curves for Plummer softening (dotted) and Newton’s law (dashed) are provided for comparison. Here,  $h = 1.0$  and  $\epsilon = h/2.8$ . Figure from Springel et al. (2001).

An example of this softening is shown in Figure II.1 for the potential and force.

As discussed in Section I.2.1, direct summation  $N$ -body techniques are prohibitively slow for modern simulations. GADGET-2 therefore makes use of a hierarchical multipole expansion technique, often called a “tree” algorithm, using the Barnes-Hut octal tree (Barnes & Hut 1986) algorithm. This method recursively divides the simulation volume into eight cells at each level of refinement, continuing the division until each cell contains only one particle. A visual description of this process in two dimensions is given in Figure II.2. Distant particles can then be grouped together for the force calculation, reducing the algorithm complexity to  $\mathcal{O}(N \log N)$ .

The Barnes-Hut octal tree algorithm begins with a cubic cell encompassing the entire simulation volume. The cell is then divided into eight daughter cells. If the cell

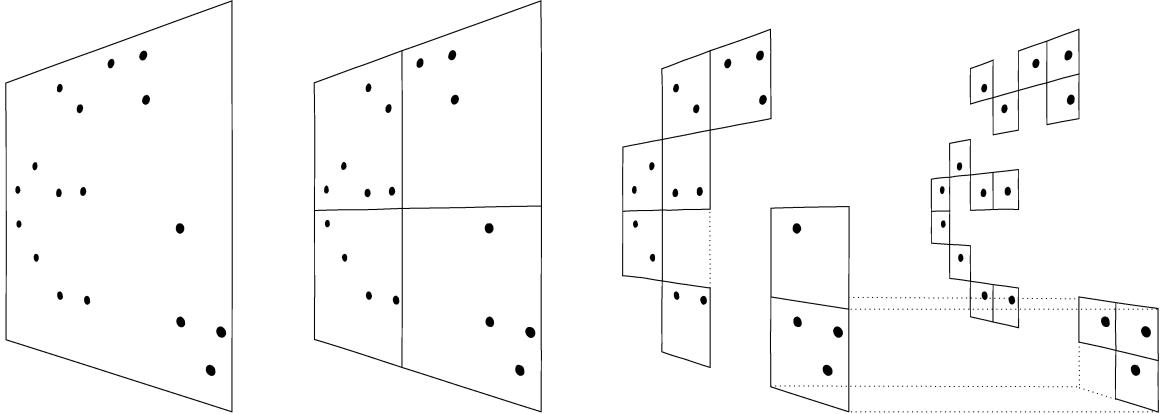


Figure II.2: Barnes-Hut oct-tree in two dimensions. The simulation volume is recursively partitioned into cells until each contains only one particle each. Empty cells may be ignored. Figure from Springel et al. (2001).

contains no particles, it is ignored. If it contains one particle, the dividing process for that cell ends there. If it contains more than one particle, the process continues recursively, dividing daughter cells into eight octants each, until each cell contains either one or no particles. A multipole expansion of all daughter cells is then found for each node, or “leaf.”

The accuracy of the force computation can be set by choosing how far to “walk” the tree. For each particle, the goal is to calculate the gravitational accelerations from all other particles accurately and quickly. There is a trade off, however, as increasing the accuracy of the tree code toward that of a direct summation approach also increases the runtime complexity toward that of an  $\mathcal{O}(N^2)$  algorithm. The balance between runtime and accuracy is controlled by the opening angle parameter  $\alpha$ . A node of mass  $M$  and extension  $l$  will be considered for usage if

$$\frac{GM}{r^2} \left( \frac{l}{r} \right)^2 \leq \alpha |a|, \quad (\text{II.2})$$

where  $r$  is the distance from the particle to the node and  $|a|$  is the total acceleration from the previous time-step. Nodes that are massive, large, and near enough to fall

outside this criterion are opened so that the daughter cells are recursively considered.

GADGET-2 can optionally make use of a hybrid approach for calculating forces, called the TreePM method (Xu 1995; Bode et al. 2000; Bagla 2002), where long range forces are computed using a particle-mesh algorithm instead of the Barnes-Hut octal tree. The GADGET-2 implementation of TreePM follows that of Bagla & Ray (2003).

### II.2.1.2 Time Integration

The  $N$ -body Hamiltonian is separable such that  $H = H_{\text{kin}} + H_{\text{pot}}$ . Time evolution operators for each of  $H_{\text{kin}}$  and  $H_{\text{pot}}$  may be computed exactly, leading to “drift” and “kick” operators (Quinn et al. 1997):

$$D_t(\Delta t) : \begin{cases} p_i &\mapsto p_i, \\ x_i &\mapsto x_i + \frac{p_i}{m_i} \int_t^{t+\Delta t} \frac{dt}{a^2}, \end{cases} \quad (\text{II.3})$$

$$K_t(\Delta t) : \begin{cases} x_i &\mapsto x_i, \\ p_i &\mapsto p_i + f_i \int_t^{t+\Delta t} \frac{dt}{a}, \end{cases} \quad (\text{II.4})$$

where

$$f_i = - \sum_j m_i m_j \frac{\partial \phi(x_{ij})}{\partial x_i} \quad (\text{II.5})$$

is the force in particle  $i$ .

A time evolution operator  $U(\Delta t)$  for an interval  $\Delta t$  may be approximated by combining the above two operators, where each fall a half time-step after the previous operation:

$$\tilde{U}(\Delta t) = D\left(\frac{\Delta t}{2}\right) K(\Delta t) D\left(\frac{\Delta t}{2}\right), \quad (\text{II.6})$$

or

$$\tilde{U}(\Delta t) = K\left(\frac{\Delta t}{2}\right) D(\Delta t) K\left(\frac{\Delta t}{2}\right), \quad (\text{II.7})$$

which gives us a leapfrog integrator constructed as a drift-kick-drift (DKD) or kick-drift-kick (KDK) operator. DKD and KDK are symplectic and time reversible, as both  $D_i$  and  $K_i$  are symplectic.

Cosmological simulations inherently contain a large dynamic range in time scales. Maintaining a constant time-step would be computationally prohibitive and wasteful, as high-density regions like the centers of galaxies require orders of magnitude smaller time-steps than low-density regions like the intergalactic medium. GADGET-2 therefore uses adaptive individual time-steps which are much more computationally efficient. The time-step criterion for collisionless particles is

$$\Delta t_{\text{grav}} = \min \left[ \Delta t_{\text{max}}, \left( \frac{2\eta\epsilon}{|a|} \right)^{1/2} \right], \quad (\text{II.8})$$

where  $\eta$  is an accuracy parameter,  $\epsilon$  is the gravitational softening, and  $a$  is the particle's acceleration. The maximum allowed timestep is  $\Delta t_{\text{max}}$ , which is usually chosen to be a small fraction of the dynamical time of the system.

GADGET-2 allows particles to take on time-steps as a power of two subdivision of a global time-step. A particle is allowed to move to a smaller time-step at any time. However, moving to a larger time-step is only allowed on every second iteration and when this would lead to synchronization with the higher time-step hierarchy.

### II.2.2 Simulations

We use GADGET-2 to evolve six dark matter-only cosmological volumes from  $z_{\text{start}} = 300$  to  $z = 6$  in a  $\Lambda$ CDM universe. Each simulation is initialized using WMAP-5 (Komatsu et al. 2009) parameters. For each of the three simulation pairs, we directly compare 2LPT and zA by identically sampling the CMB transfer function and displacing the initial particle positions to the same starting redshift using 2LPT and zA. The three sets of simulations differ only by the initial phase sampling random seed. Each volume contains  $512^3$  particles in a  $10 h^{-1}$  Mpc box.

Following Heitmann et al. (2010), we choose conservative simulation parameters in order to ensure high accuracy in integrating the particle positions and velocities. We have force accuracy of 0.002, integration accuracy of 0.00125, and softening of  $0.5 h^{-1}$  kpc, or 1/40 of the initial mean particle separation. We use a uniform particle mass of  $5.3 \times 10^5 h^{-1} M_{\odot}$ . We select PMGRID, which defines the Fourier grid, to be 1024, SMTH, which defines the split between short- and long-range forces, to be 1.5 times the mesh cell size, and RCUT, which controls the maximum radius for short-range forces, to be 6.0 times the mesh cell size.

### II.3 Halo Finding with Rockstar

ROCKSTAR (Robust Overdensity Calculation using K-Space Topoloogically Adaptive Refinement; Behroozi et al. 2013) is a halo finder based on the hierarchical refinement of friends-of-friends (FOF) groups in six phase space dimensions and, optionally, one time dimensions. It has been shown (Knebe et al. 2011) to be robust in recovering halo properties, determining substructure, and providing accurate particle member lists, even for notoriously difficult scenarios such as for low particle count halos and halos undergoing major merger events.

#### II.3.1 Halo Finding

Halo finding in ROCKSTAR is broken down into a number of steps, leading from the particle distribution of a simulation snapshot to the recovery of individual halo properties. FOF overdensity groups are distributed among the analysis processors which build hierarchies of FOF subgroups in phase space, determine particle membership for halos, compute host halo/subhalo relationships, remove unbounded particles, and compute halo properties. A summary of each of these steps is provided below.

### II.3.1.1 FOF Groups

The 3D friends-of-friends algorithm groups particles together if they fall within a set linking length of each other. The linking length is often chosen as a fraction  $b$  of the mean interparticle distance, with typical values ranging from  $b = 0.15$  to  $b = 0.2$  (More et al. 2011). As ROCKSTAR only uses FOF groups for breaking up the simulation volume to be distributed to individual processors, it is able to use a modified algorithm for calculating FOF groups that is an order of magnitude faster than the typical procedure of finding all particles within the linking length for every particle. For particles with more than 16 neighbor particles, the neighbor finding process is skipped for the neighboring particles. Instead, particles are linked to the same group if they are within two linking lengths of the original particle. This method runs much faster than the standard FOF algorithm, and links together at minimum the same particles. With this approach, run time decreases instead of increases with increasing linking length. ROCKSTAR therefore uses a large linking length of  $b = 0.28$ . The FOF groups are distributed among the available processors according to individual processor load.

### II.3.1.2 Phase-Space FOF Hierarchy

Within each FOF group, FOF subgroups are found hierarchically in phase-space. A phase-space linking length is adaptively chosen so that a constant fraction  $f$  of particles are linked together with at least one other particle. For two particles  $p_1$  and  $p_2$ , the phase-space distance metric is defined as (Gottloeber 1998)

$$d(p_1, p_2) = \left( \frac{|\vec{x}_1 - \vec{x}_2|^2}{\sigma_x^2} + \frac{|\vec{v}_1 - \vec{v}_2|^2}{\sigma_v^2} \right)^{1/2}, \quad (\text{II.9})$$

where  $\sigma_x$  and  $\sigma_v$  are the particle position and velocity dispersions for the FOF group. The phase-space distance to the nearest neighbor is computed for each particle, the linking length is chosen such that  $f = 0.7$ , and a new FOF subgroup is determined.

This process is repeated recursively on the new FOF subgroups until a minimum threshold of 10 particles is reached at the deepest level of the hierarchy.

### II.3.1.3 Converting FOF Subgroups to Halos

Seed halos are created for each of the deepest level subgroups in the FOF hierarchy. Particles from successively higher levels of the hierarchy are then assigned to the seed halos until all particles in the original FOF group are accounted for. To suppress extraneous seed halo generation due to noise, seed halos are merged if their positions and velocities are within  $10\sigma$  of Poisson uncertainties of each. Specifically, the halos are merged if

$$\sqrt{(x_1 - x_2)^2 \mu_x^{-2} + (v_1 - v_2)^2 \mu_v^{-2}} < 10\sqrt{2}, \quad (\text{II.10})$$

with

$$\mu_x = \sigma_x / \sqrt{n}, \quad (\text{II.11})$$

$$\mu_v = \sigma_v / \sqrt{n}, \quad (\text{II.12})$$

where  $\sigma_x$  and  $\sigma_v$  are the position and velocity dispersions of the smaller seed halo, and  $n$  is the number of particles of the smaller seed halo.

If a parent FOF group contains multiple seed halos, particles are assigned to the closest seed halo in phase space. The distance between a halo  $h$  and a particle  $p$  is given by

$$d(h, p) = \left( \frac{|\vec{x}_h - \vec{x}_p|^2}{r_{\text{dyn,vir}}^2} + \frac{|\vec{v}_h - \vec{v}_p|^2}{\sigma_v^2} \right)^{1/2}, \quad (\text{II.13})$$

$$r_{\text{dyn,vir}} = v_{\text{max}} t_{\text{dyn,vir}} = \frac{v_{\text{max}}}{\sqrt{\frac{4}{3}\pi G \rho_{\text{vir}}}}, \quad (\text{II.14})$$

where the seed halo currently has velocity dispersion  $\sigma_v$  and maximum circular velocity  $v_{\text{max}}$ . Here, “vir” refers to the virial overdensity as defined by Bryan & Norman (1998) for  $\rho_{\text{vir}}$ , which is 360 times the background density at  $z = 0$ . ROCKSTAR does, however, allow other choices for density definitions.

#### II.3.1.4 Substructure

Satellite membership is assigned based on phase-space distances before calculating halo masses. Equation II.13 is used to find the distance to all other halos with a greater number of particles, treating each halo center as a particle. The halo is then assigned to be a subhalo of the closest larger halo within the same FOF group, if one exists. If data from an earlier time-step is available, then halo cores at the current time-step are linked to halos from the previous time-step based on the largest contribution to the current halo core’s particle membership.

Halo masses are then determined so that particles assigned to the host are not counted in the mass of the subhalo, but particles in the subhalo are included in the mass of the host. Subhalo membership is then recalculated such that subhalos are those that fall within  $r_\Delta$  of more massive host halos.

### II.3.2 Halo Properties

Halo positions based on maximum density peaks are more accurate than those found by averaging all FOF halo particles (Knebe et al. 2011). As ROCKSTAR has already determined the halo density distribution when calculating the FOF subgroup hierarchy, halo positions are readily calculated by taking the average position of the particles in the inner subgroup which best minimizes the Poisson error.

The velocity of the halo core can be substantial offset from that of the halo bulk (Behroozi et al. 2013). The velocity for the halo is calculated as the average velocity of the particles within the innermost 10% of the halo radius, as the galaxy hosted by the halo should be most associated with the halo core.

Halo masses are calculated using the spherical overdensity (SO) out to various density thresholds, including the virial threshold of Bryan & Norman (1998) and density thresholds relative to the background density and the critical density. Mass calculations include all particles from the substructure contained in the halo, and

can optionally remove unbound particles. As subhalo particles can be isolated from those of the host halo, mass calculations for substructure can also be obtained with spherical overdensities using only the particles belonging to the subhalo.

The scale radius  $R_s$  is determined by dividing halo particles up into up to 50 radial equal-mass bins, with a minimum of 15 particles per bin, and fitting an NFW profile to the bins to find the maximum-likelihood fit. The Klypin scale radius (Klypin et al. 2011), which uses  $v_{\max}$  and  $M_{\text{vir}}$  to calculate  $R_s$ , is also determined.

A number of other parameters are calculated, including the angular momentum, halo spin parameter (Peebles 1969), Bullock spin parameter (Bullock et al. 2001a), central position offset (defined as the distance between the halo density peak and the halo center of mass), central velocity offset (defined as the difference between the halo core velocity and the bulk velocity), ratio of kinetic to potential energy, and ellipsoidal shape parameters (Zemp et al. 2011).

#### II.4 CrossMatch

Having pairs of corresponding 2LPT and ZA simulations necessitates a method for reliably matching halos between the two if we wish to compare properties of companion halos. To accomplish this, we use the CROSSMATCH code initially developed by Manodeep Sinha. CROSSMATCH uses particle IDs to find matching halos based on the percentage of common constituent particles. The code was modified for this study to import and process the BGC2 files output by the ROCKSTAR halo finder.

As dynamical variations between 2LPT and ZA simulations can cause companion halos to diverge in their evolutionary history, we cannot rely on bulk halo properties such as mass or central position as a primary means of matching. CROSSMATCH therefore relies on ID-based particle matching to pair halos. Companion simulations are initialized with identical particle ID schemes, and CROSSMATCH can then use these particle IDs to find pairs that are most likely to be the “same” halo for a

given simulation snapshot. At the most basic level, CROSSMATCH reads in halo and particle lists from a halo finder such as ROCKSTAR, iterates through the lists from one simulation, and finds the halo with the largest number of shared particles from the other simulation.

As CROSSMATCH needs to run on data from simulations with large numbers of particles, total runtime becomes a concern. A naive approach would be to iterate through the first particle list, and for every particle, linearly search through the entire second particle list to find which halo a particle belongs to. This would result in an  $\mathcal{O}(N^2)$  runtime complexity. To decrease runtime to an acceptable level, the second particle list is first sorted by particle ID using a standard QuickSort algorithm, which then enables the use of a more efficient binary search. This reduces runtime complexity to an  $\mathcal{O}(N \log N)$  algorithm. Halos from the second simulation are then ranked by the percentage of particles in common with the halo from the first simulation, and the best match is selected.

## II.5 Analysis

In this section, we discuss the details of the pipeline used for this work, including the analysis and plotting codes, databases, and automation scripts. We also present an overview of the results obtained at each step. A more in depth discussion of the observed trends and interpretations of results are presented in Sections III.3 and III.4.

As a high-level overview, we gather snapshots from previously run 2LPT and ZA simulations, find halos in each snapshot with ROCKSTAR, match halos between simulations with CROSSMATCH, and compare the differences in various properties between corresponding 2LPT and ZA halos, primarily as functions of redshift and halo mass. The specific codes developed for and used in our analysis are provided in the Appendices, and are referenced with the relevant discussions below.

## II.5.1 Halo Properties with Rockstar

Halos are identified and measured with the ROCKSTAR halo finder, which is discussed in detail in Section II.3. Here, we discuss the setup necessary to run ROCKSTAR, as well as its output files, post-processing steps, and particle list extraction.

### II.5.1.1 Simulation Snapshots and Rockstar Setup

We run ROCKSTAR on snapshots from each of our six simulation boxes. Each box has 62 snapshots, with  $512^3$  dark matter particles each. Due to the large size of the snapshot data and the per-user disk space quota of the ACCRE cluster, only one box is able to be processed at a time.

For each snapshot, a ROCKSTAR run directory is set up with a number of configuration files and scripts, including the ROCKSTAR configuration file (Appendix A.1), PBS submission script (Appendix A.2), a script to clean files from previous runs and begin a new run (Appendix M.3), and a script for post-processing generated output files (Appendix A.3). A directory for particle data contains a link to the actual simulation snapshot and a file containing a list of snapshot files, which in this case contains one item. A directory is also created for output halo data files. We discuss automation of run directory setup and simultaneous launching of multiple ROCKSTAR instances in Section II.6.

The parameter file controls various configuration options including simulation type, physical units, cosmological parameters, I/O options, halo definitions, and process setup. ROCKSTAR has native support for GADGET’s snapshot format and can automatically import cosmological parameters and box size. Length and mass scales must be input to convert from simulation units. ROCKSTAR uses periodic boundary conditions based on the number of analysis processes. Periodic boundary conditions are assumed if using a multiple of eight analysis processes and are not assumed if using one analysis process. For ROCKSTAR to output BGC2 files (discusses below in

Section II.5.1.2), the path of a file containing a list of snapshot filenames must be set as the BGC2 snapnames option. Halo virial radius and mass definitions may be set to either virial or a multiple of either the critical or background density. We select halos to be defined by the virial radius and mass. We are interested in defining halos as spherical overdensity halos rather than friends-of-friends halos, so we also choose to define halo properties based on all particles within the virial radius, whether or not they are energetically bound to the halo.

ROCKSTAR is run as a server-client setup. This is designed so that one processor acts as a director and output manager, one or more processors read in the input snapshots, and the remaining processors or compute nodes do the actual processing on different segments of the simulation box. ROCKSTAR uses sockets for communication between the server process and the worker processes if running on multiple nodes. However, we were unable to configure ROCKSTAR in a way that it would run across multiple compute nodes, so we run each instance of ROCKSTAR on one node, with ten processor cores for the necessary functions. One processor acts as the server, one as the snapshot reader, and the remaining eight as halo finders.

### II.5.1.2 Rockstar Output and Post-processing

ROCKSTAR outputs halo information in ASCII plaintext, binary, and BGC2 binary formats. As mentioned above, we run ROCKSTAR with eight worker processes per snapshot. Each worker process outputs its own set of data files, with each file covering a separate octant of the simulation box plus a small overlap region. Halos with particles in the overlap region are saved based on the location of their centers. In addition to the per-processor output, a composite list of halos (and only halos) from all worker processors are created.

Through its various output files, ROCKSTAR provides a large number of measured halo properties. Whether or not full friends-of-friends particle lists are saved

is controlled via the configuration file. Spherical overdensity particle lists are saved when utilizing BGC2 output. Particle data include particle ID, position, and velocity. Particle mass is not included as our simulations have uniform particle mass. Halo information consists of a large number of parameters, including halo ID, number of constituent particles, masses to various radii, position, velocity, angular momentum, spin, virial radius, scale radius, shape parameters, energy parameters, position and velocity offsets between the center of mass and the peak density, and parent halo ID.

As previously mentioned, we want halos defined based on spherical overdensity particle lists. These are only available from ROCKSTAR’s BGC2 binary output format, with all other available particle lists consisting of friends-of-friends particles. The BGC2 files consist of a 1024 byte header, halo data of 72 bytes per halo, and particle data with 32 bytes per particle. The header consists of an unsigned 8-byte integer, 16 8-byte signed integers, 19 8-byte double-precision floating point numbers, and extra padding out to 1024 bytes. The halo data consists of 2 8-byte signed integers, 2 8-byte unsigned integers, and 10 4-byte floating point numbers per halo. The particle data consists of 1 8-byte signed integer and 6 4-byte floating point numbers per particle. There is a 4-byte offset before the header, and 8-byte offsets between the header and halo data and between the halo data and particle data. The reader is referred to the `bgc2.h` header of the ROCKSTAR source code for further information on the contents of each structure. Our python code for reading in BGC2 files is presented in Appendix C. C code for reading in BGC2 files is bundled with the ROCKSTAR source code.

After ROCKSTAR is run, some post-processing of the output is needed. By default, ROCKSTAR does not provide information on membership information for substructure. Two scripts—one for the composite halo list and one for the BGC2 files—are provided with ROCKSTAR to cycle back through the halo lists and find the “parents,” or the halo in which a given subhalo is contained. A script is also provided to convert

halo information in the BGC2 files to ASCII plaintext. Our script for running these post-processing steps is presented in Appendix A.3.

### II.5.2 Density Profile Fitting

While ROCKSTAR’s output includes measurements for halo virial and scale radii, and thus concentration, we independently fit NFW density profiles to halos and measure concentration as a verification of ROCKSTAR’s fitting. The full density profile python code is presented in Appendix D. This section is included for completeness only, as we find that only a small fraction of halos are well fit by our method, and we instead rely on concentration measurements directly from ROCKSTAR for subsequent analysis.

#### II.5.2.1 Density Profiles

For each halo, a list of constituent spherical overdensity particles is obtained from the post-processed BGC2 catalog from ROCKSTAR’s output. For our purposes here, the relevant parameters are particle mass and position. We also use the values for each halo’s center position and virial radius as found by ROCKSTAR.

Density profiles are then constructed by binning the particle positions in logarithmic radial bins from the resolution limit of the simulation to the halo virial radius and multiplying by particle mass. Before being passed to the fitting routine, density profiles are normalized to unity for both virial radius and maximum density.

#### II.5.2.2 Fitting

Halos are fit using the CurveFit routine from the SciPy Optimize library. It uses the Levenberg-Marquardt algorithm (Marquardt 1963) for non-linear least squares fitting.

CurveFit is called by providing a model function, independent variable, measured dependent variable, and optionally weights for the dependent variable and initial guesses for fit coefficients. Here, our fit function is the NFW dark matter density

profile (see Equation III.1). The free parameters to be fit are the scale radius  $R_s$  and the characteristic density  $\rho_0$ .

As the least squares algorithm is sensitive to local minima, care must be taken in choosing initial guesses for the fit coefficients. Additionally, large dynamic range in the fit parameters tended to produce poor results. We explored a number of solutions to improve solution stability, including fitting in logarithmic space and randomizing the initial guesses and picking the best solution. We found the best results were achieved by normalizing the data to unity for both radius and density, and choosing initial guesses within an order of magnitude for a typical halo, namely, normalized  $R_s = 0.1$  and normalized  $\rho_0 = 1.0$ .

Some halos with irregular profiles presented the problem of the fitting algorithm choosing an unphysical scale radius larger than the virial radius of the halo. In order to heavily penalize this option from being chosen by the fitting algorithm, the dependent variable returned by the model function must differ from the input measured dependent variable as much as possible. However, we discovered that the transition must also be smooth, as a disjointed jump such as, say, returning a very large number for every value if  $R_s > R_{\text{vir}}$  would cause the algorithm to fail. We achieve this smooth transition penalty by adding the term  $(R_s - 1)e^r$  to the density returned by the model function if the fitting algorithm tries to guess a value of  $R_s$  larger than  $R_{\text{vir}}$ . However, while this did force halos to have definable concentrations, these halos often ended up with best fit scale radii equal to or just slightly less than the virial radii.

As we fit halos over a large range in redshift, we found low particle count halos to have noisy density profiles that were inherently more difficult to properly fit. Throughout our analysis, we use a lower bound of 100 particles to define a halo. At high redshift, even the largest halos are just beginning to cross this threshold. With such few particles spread across the number of bins necessary to properly define a

density profile, we are left with only a handful of particles per bin. In Figure II.3, we compare one of the largest halos at  $z = 14$  with one of the largest halos at the end of the simulation at  $z = 6$ .

### II.5.2.3 Characterization of Uncertainty

An initial motivation for finding our own concentration parameters independent from ROCKSTAR is that ROCKSTAR does not provide information about the quality of its density profile fits. We assign Poisson errors to the density in each bin such that  $\sigma_\rho = \rho\sqrt{N}/N$ , where  $\rho$  is the density and  $N$  is the number of particles in each bin. These uncertainties are then provided as weights to the CurveFit routine. Upon finding a best fit, the routine provides the fit parameters and an estimation of the uncertainty in those parameters via a covariance matrix, which we use to uncertainty in the concentration. Additionally, we find the  $\chi^2$  for the overall fit, which we use as an indicator of whether to accept or reject the fit for a given halo.

### II.5.2.4 Concentration Comparison to Rockstar

Overall, we do not find good agreement with ROCKSTAR. Using a script (see Appendix H) to compare the concentrations derived from our fits with those from ROCKSTAR. At  $z = 6$ , we find that only 26% of halos fit by our method have concentrations within 20% of concentrations as measured by ROCKSTAR. We have slightly more agreement with high mass halos, with 37% agreement if we only consider the most massive 10% of halos. Additionally, we do not find good fits for every halo. If the distribution of particles would produce too few bins or the fitting routine exceeded a maximum number of iterations to find a stable solution, the halo is not fit. We also exclude halos with fits returned with very large  $\chi^2$  values. Because of the discrepancies in our results and the fact that we do not find acceptable fits for every halo, we use the more complete ROCKSTAR data for the final concentration measurements used in the remainder of our analysis.

### II.5.3 Cross-matched Halo Catalog

With halo catalogs generated by ROCKSTAR for both 2LPT and ZA simulations, we need to be able to directly compare corresponding halos from the two suites of simulations. We match halos between simulations based on constituent particles with the `CROSSMATCH` code modified to import ROCKSTAR’s BGC2 binary output files. Properties of the matched halos are then compiled into one large database per box for further filtering and analysis.

#### II.5.3.1 Cross-matching

Our simulations are initialized with identical particle ID schemes, and we are thus able to uniquely identify and track matching particles between simulations and match halos based on the largest number of shared particles. As the full implementation of the `CROSSMATCH` code is previously discussed in Section II.4, we only briefly summarize its place in our analysis pipeline here. The script in Appendix M.2 sets up the directory structure for the `CROSSMATCH` analysis and copies the `CROSSMATCH` parameter files (Appendices B.1 and B.2) to the appropriate run directories. `CROSSMATCH` is then run for each snapshot via the submission script in Appendix M.7, which is run for each simulation box.

One caveat of the `CROSSMATCH` code is that matches are not necessarily unique. For each halo in the first simulation, only one best match halo will be selected from the second simulation. However, there may be other halos from the first simulation that also have the same halo from the second simulation selected as a best match. To counter this, we run `CROSSMATCH` in both directions—once matching ZA halos to 2LPT halos and once matching 2LPT halos to ZA halos—and choose best match halos as those that are matched in both directions. This assures a unique one-to-one matching between 2LPT and ZA halos. The code and submission script that select the best matches from the 2LPT-first and ZA-first cross-matched halo lists are presented

in Appendix E.1.

### II.5.3.2 Database Aggregation and Filtering

We now have raw halo data we need for further study, but are also left with a large number of disparate files that contain this information. For every snapshot, we have cross-simulation halo matching information from `CROSSMATCH` and the best match selection script, independent density profile and concentration measurement information from the density profile program, and original halo properties and host halo membership information from `ROCKSTAR` spread across plaintext and `BGC2` binary files for each processor on which `ROCKSTAR` was run, all for three simulation boxes each for both `2LPT` and `ZA`.

We combine the information from all of these file into one centralized database per snapshot with the database generation program and submission script in Appendix F. The program reads in all of the source data files, finds companion halos from the output of `CROSSMATCH`, and outputs all available data for each halo pair aggregated together. The program is run for each of our 62 snapshots per simulation box, giving 186 total database files.

With the first version of our database generation code, total runtime became a significant factor. The halo matching code was initially implemented in a naive double loop search through all the data files to find collect halo pair properties. Pure python loop structures are exceedingly slow for larger data sets, and an initial estimate gave a runtime on the order of weeks or months. This was unacceptable, as there are many snapshots, and the aggregation may need to be performed multiple times if any of the previous steps in the analysis pipeline were to be modified. The code was therefore rewritten to take full advantage of the vectorization of the NumPy library, achieving a massive speedup to a runtime of order a few seconds.

In order to retain a centralized database of all available information for matched

halos, we do not filter out halos at this step. Subsequent analysis, however, does remove halo pairs from consideration in certain circumstances. For early analysis involving our independent density profile fitting, we remove halos based on evidence of a poor fit, including halos that have measured concentrations greater than 100 or less than 1,  $\rho_0$  less than zero, or  $\chi^2$  greater than 10. For all analysis, we remove halos with fewer than 100 particles and halos that exist as substructure in a larger host halo.

#### II.5.4 Halo Comparison

With a catalog of DM halos cross-matched between 2LPT and ZA simulations, we are able to directly compare properties on a halo-by-halo basis. At this stage, we are mostly concerned with a qualitative comparison between individual halos in order to judge the overall success of halo matching and the broad differences in halo evolution arising from differences in simulation initialization.

##### II.5.4.1 Match Verification

In order to compare halo evolution between 2LPT and ZA simulations, we first need to ensure that the halos being compared do actually represent the same halo in each simulation. The `CROSSMATCH` code as well as its implementation in our analysis pipeline are discussed above, so here we instead focus on the plots used as a visual sanity check on the resulting matches. The python code used to generate these plots is listed in Appendix G.1.

As we wish to compare halos that may have followed different evolutionary paths in their respective 2LPT or ZA simulations, we are unable to do a hard cut on a single parameter such as mass, radius, position or particle distribution. However, large variances in any of these properties can hint at a problem in the matching algorithm. We therefore perform a quick visual check on a number of halo pairs by plotting their relative positions, radii, and constituent particle distributions in order to verify that

the CrossMatch code performed as expected.

An example of this comparison is shown in Figure II.4, where we plot two large matching halos at  $z = 6$ . Particles belonging to the halos are plotted as points, with 2LPT halo particles in blue and zA halo particles in red. The virial radii of the two halos are represented by the black circles. The virial radii and particle distributions are very similar, and there is only a small offset in position. We consider this a successful match.

#### II.5.4.2 Morphology

The morphology of a dark matter halo can provide insight into its structural evolution and merger history. Features such as tidal features, irregular shapes, and offset nuclei hint at recent merger activity, while more symmetrical distributions suggest a quieter recent history. We compare DM particle distributions of matched halos by observing the projected density map along three axis vectors. The python code for plotting these, as well as the density profiles discussed below, is listed in Appendix G.2.

By comparing the projected density morphologies of companion 2LPT and zA halos, we get a qualitative impression of the differences in their current evolutionary state. We found the inner nuclear region to often display the most discernible difference in structure between the two halos. For halo pairs where this difference is most apparent, such as one halo having a single central core with the other halo having two distinct density peaks, we believe the most likely cause to be an offset in merger epochs between the two simulations. In this case, the snapshot from one simulation would catch the merger in progress, with multiple unsettled density peaks still visible, while the other simulation snapshot would catch the halo after it has settled into a more virialized state.

As an example of this, we plot comparisons of two  $z = 6$  halo pairs in Figures II.5 and II.6. The top two rows of panels of each show XY, XZ, and YZ projections of the

dark matter density for the 2LPT and zA halo on the first and second row, respectively. The density map is shown with a logarithmic color scale, and equal density contours are marked with white curves. Figure II.5 shows a pair of large halos that display similar central structure. These halos are unlikely to have largely differed in their evolution shortly prior to the snapshot. Figure II.6, however, shows a halo pair with differing nuclear structure. The zA halo displays two distinct central density peaks, while the 2LPT halo shows only a single more relaxed core.

#### II.5.4.3 Density Profiles

The code listed in Appendix G.2, which produces the density projections discussed above, also plots comparisons of the halos' density profiles. We have addressed the creation of density profiles in Section II.5.2, and here the same method is used for each profile. In this case, we wish to directly compare the profiles of the companion 2LPT and zA halos, so they are plotted together, alongside the 2-D density projections discussed in the previous section.

We again consider the halo pairs compared in Figures II.5 and ??, where the bottom two panels of each display the density profiles of the 2LPT and zA halos, respectively. Halo particles are binned in logarithmically-spaced radial bins from the virial radius inward to the simulation resolution limit. The profiles are fit with the NFW profile model with free parameters for scale radius and characteristic density. The resulting fit is overplotted as red curves, and the scale radius is marked with the vertical purple dot-dash lines.

The halos in Figure II.5 display very similar central morphology and are both well-fit by the NFW profile. The more relaxed and spherically symmetrical halos such as these tend to be easier to fit well than more irregular halos. The measured scale radii for these halos are also very similar, and combined with the similar virial radii, produce similar concentration values. The halos in Figure II.6 display a more

differing structure. While the 2LPT halo is relatively symmetrical, allowing it to be relatively well-fit by the NFW model, the ZA halo has two distinct central density peaks, causing the NFW fit to be a bit less accurate. Here, there is a marked difference in the resulting scale radii, with the 2LPT halo displaying a larger concentration than its ZA companion.

### II.5.5 Difference Distributions

We now turn our focus to the ensemble halo population as a whole. Comparing individual companion halos can realistically only give a qualitative picture of differences arising between 2LPT and ZA simulations, as the large number of halos necessitates consideration of only a small percentage of the sample. We therefore need a consistent way of measuring the behavior of the entire population. In this section, we discuss how we measure these differences in halo populations using the codes listed in Appendix I. In particular, the analysis code itself is listed in Appendix I.1, the script to run the analysis on the combined halo population from all three simulation boxes is listed in Appendix I.2, the script to run the analysis on the simulation boxes independently is listed in Appendix I.3, and the script to collect the resulting statistics from all the individual snapshots into one database is listed in Appendix I.4.

#### II.5.5.1 Histograms

We wish to explore differences in a number of halo properties, so we construct a generic distribution so that any measured halo quantity  $q$  can be considered. The distribution should highlight the differences between 2LPT and ZA halo populations while remaining unbiased to the choice of simulation initialization. This leaves us with a distribution of the differences between 2LPT and ZA quantities, normalized by the average of the two:

$$\Delta q = \frac{q_{\text{2LPT}} - q_{\text{ZA}}}{q_{\text{avg}}}, \quad (\text{II.15})$$

where  $q_{\text{avg}} = \frac{1}{2}(q_{2\text{LPT}} + q_{\text{ZA}})$ . Defined in this way, difference distributions of, e.g., virial mass  $\Delta M_{\text{vir}}$ , concentration  $\Delta c$ , or the offset distance between the central density peak and the center of mass  $\Delta X_{\text{off}}$  can all be considered on equal footing. We create distribution histograms of  $\Delta q$  for various halo quantities both for the combined halo catalog from the stacked simulation boxes and for the individual simulation boxes separately.

### II.5.5.2 Fitting

In order to extract a number of statistical quantities and to get a better high-level feel for the leading behavior of the distributions, we wish to fit a statistical model to the data histograms. While the data would seem to distributed according to a Gaussian distribution at first glance, we found the deviations from Gaussianity to be more significant than could be ignored. After significant trial and error, we found the  $\Delta q$  distributions to be best described by a generalized normal distribution (Nadarajah 2005) with the probability density function

$$f(x) = \frac{\beta}{2\alpha\Gamma(1/\beta)} e^{(|x-\mu|/\alpha)^\beta}, \quad (\text{II.16})$$

where  $\mu$  is the mean,  $\alpha$  is the scale parameter,  $\beta$  is the shape parameter, and  $\Gamma$  is the gamma function

$$\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx. \quad (\text{II.17})$$

The shape parameter  $\beta$  is restricted to  $\beta \geq 1$ . This allows the distribution to potentially vary from a Laplace distribution ( $\beta = 1$ ) to a uniform distribution ( $\beta = \infty$ ) and includes the normal distribution ( $\beta = 2$ ). The distribution has variance

$$\sigma^2 = \frac{\alpha^2\Gamma(3/\beta)}{\Gamma(1/\beta)} \quad (\text{II.18})$$

and excess kurtosis

$$\gamma_2 = \frac{\Gamma(5/\beta)\Gamma(1/\beta)}{\Gamma(3/\beta)^2} - 3. \quad (\text{II.19})$$

The distribution is symmetric, and thus has no skewness by definition. As such, the values obtained for the skew of the distribution are measured directly from the data.

We use the CurveFit module from the SciPy library for all of our functional fitting. CurveFit is a non-linear least squares fitting routine that can fit an arbitrary input function to data with optional uncertainties. It can return estimates of the free parameters of the model, as well as a covariance matrix used to determine the uncertainties in the fit coefficients.

We found our fitting routine to be fairly sensitive to differences in initial guess of fit coefficients. CurveFit is not guaranteed to find global minima, and can become stuck in local extrema. This ends up being most probable when trying to find multiple fit coefficients with large dynamic range. We found the best way to address this was to scale the data to unity in each dimension whenever possible. In the case of our difference histograms, the standard deviations of the distributions are typically around order unity, so it was only necessary to normalized the counts. We also found that we achieved better results when fitting in logarithmic space.

We explored a number of halo parameters, but found the most interesting distributions to be those for virial mass and concentration. In Figure II.7, we plot histograms of  $\Delta M_{\text{vir}}$  and  $\Delta c$  in the left and right columns, respectively, for three representative simulation snapshots at  $z = 14.7$ ,  $z = 10.3$ , and  $z = 6.0$ . Data from the entire sample are plotted as blue histograms, data for the top 25% of halo pairs, sorted by 2LPT halo mass, are plotted as grey-filled green histograms, and the generalized normal distribution fits are overplotted as red dashed curves.

## II.5.6 Redshift Trends

Up to this point, we have only considered one snapshot at a time. While we have observed variations with redshift, this has not been explicitly quantified. In this section, we consider the statistical quantities derived from the generalized normal distribution fits from the previous section as functions of redshift. The code used for this analysis is listed in Appendix J.

### II.5.6.1 Mean and Standard Deviation

Representing the mean and standard deviation of the distributions is relatively straightforward. For the fit generalized normal distributions, we record values for the mean, uncertainty in the mean, standard deviation, and uncertainty in the standard deviation. We also record the mean and standard deviation of the underlying distribution as directly measured from the data.

In Figure II.8, we plot the mean and standard deviation of the distributions for mass and concentration, as well as the rms value derived from the data, all as functions of redshift. The mean is plotted as blue points with error bars, the standard deviation is plotted as two black dashed lines that represent  $\mu \pm \sigma$ , and the rms is plotted as a dotted green line.

In this case, we wish to be conservative with the error bars on the mean. Since we have a measurement for the mean both from the fitting distribution and the underlying data, we can incorporate both of these into our result. The points plotted in Figure II.8 are the mean measured from the fit distribution, and the error bars are the uncertainty in the mean estimated from the least squares routine. However, if the mean measured directly from the data falls outside the error bars, the error bars are expanded to encompass that measurement. This is most often not a concern, as the means for most snapshots are very close together. However, when there is a slight discrepancy between the fit and data values, the error bars will reflect this.

### II.5.6.2 Skew

The generalized normal distributions we use to fit our  $\Delta q$  histograms are symmetrical by definition and therefore have no inherent skew. This was a simplifying assumption necessary to use a well-defined distribution as well as reduce the number of free parameters during fitting. We do note, however, that the skew of our underlying data is often large enough to not be ignored.

Therefore, we need an alternate way to measure skew and its uncertainty. We use the skew routine from the SciPy statistics library, which defines skew as

$$\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}}, \quad (\text{II.20})$$

where  $\mu_m$  are central moments given by

$$\mu_m = E[(X - \mu)^m] = \sum_k (x_k - \mu)^m p(x_k) \quad (\text{II.21})$$

$$= \sum_{k=0}^m (-1)^{m-k} \binom{m}{k} \mu^{m-k} \mu'_k, \quad (\text{II.22})$$

with non-central moments  $\mu'_m$  given by

$$\mu'_m = E[X^m] = \sum_k x_k^m p(x_k), \quad (\text{II.23})$$

where  $p(x_k)$  is the probability density function. The skew is then measured from the entire halo sample for the three combined simulation boxes. Uncertainty in skew is evaluated by taking the skew of the three boxes as independent measurements. The results for skew as a function of redshift are plotted as blue curves for the  $\Delta M_{\text{vir}}$  and  $\Delta c$  distributions in Figure II.9.

### II.5.6.3 Kurtosis

Variable kurtosis is a fundamental part of the generalized normal distribution, so we may therefore derive the kurtosis directly from the fit distribution parameters. The generalized normal distribution is defined in terms of a shape parameter  $\beta$ , which does introduce some complexity in the conversion to kurtosis. The shape parameter is converted to excess kurtosis by way of Equation III.13. As this definition includes the Gamma function, a number of steps are required to convert the uncertainty in shape parameter to the uncertainty in kurtosis, which we outline below.

The standard deviation of a function  $f(x_1, x_2, \dots, x_n)$  is, in general, given by

$$s_f = \sqrt{\sum_x \left( \frac{\partial f}{\partial x} \right)^2 s_x^2} \quad (\text{II.24})$$

with summation over all independent variables  $x$ . The generalized normal distribution

$$f(x) = \frac{\beta}{2\alpha\Gamma(1/\beta)} e^{-(|x-\mu|/\alpha)^\beta} \quad (\text{II.25})$$

with mean  $\mu$ , scale parameter  $\alpha$ , and shape parameter  $\beta$ , has excess kurtosis

$$\gamma_2 = \frac{\Gamma(5/\beta)\Gamma(1/\beta)}{\Gamma(3/\beta)^2} - 3. \quad (\text{II.26})$$

The gamma function

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \quad (\text{II.27})$$

has the first derivative

$$\Gamma'(x) = \Gamma(x)\psi_0(x) \quad (\text{II.28})$$

where the digamma function  $\psi_0$  is the derivative of the logarithm of the gamma

function and is given by

$$\psi_0(x) = \int_0^\infty \left( \frac{e^{-t}}{t} - \frac{e^{-xt}}{1-e^{-t}} \right) dt \quad (\text{II.29})$$

if the real part of  $x$  is positive.

We now apply (II.24) to (III.13) to find the standard deviation of the excess kurtosis:

$$s_{\gamma_2} = \sqrt{\left( \frac{d\gamma_2}{d\beta} \right)^2 s_\beta^2} \quad (\text{II.30})$$

$$= s_\beta \frac{d\gamma_2}{d\beta} \quad (\text{II.31})$$

$$= s_\beta \frac{d}{d\beta} \left[ \frac{\Gamma(5/\beta)\Gamma(1/\beta)}{\Gamma(3/\beta)^2} - 3 \right]. \quad (\text{II.32})$$

Making the substitution  $x = 1/\beta$  and  $dx = -1/\beta^2 d\beta$ , taking the derivative, and doing a bit of algebra, we have:

$$s_{\gamma_2} = s_\beta \frac{d\gamma_2}{dx} \frac{dx}{d\beta} \quad (\text{II.33})$$

$$= s_\beta \left( -\frac{1}{\beta^2} \right) \frac{d}{dx} \left[ \frac{\Gamma(5x)\Gamma(x)}{\Gamma(3x)} - 3 \right] \quad (\text{II.34})$$

$$= -s_\beta x^2 \left\{ \frac{\Gamma(3x)^2 \frac{d}{dx} [\Gamma(5x)\Gamma(x)] - \Gamma(5x)\Gamma(x) \frac{d}{dx} [\Gamma(3x)^2]}{\Gamma(3x)^4} \right\} \quad (\text{II.35})$$

$$= -s_\beta \frac{x^2}{\Gamma(3x)^4} \{ \Gamma(3x)^2 [5\Gamma(5x)\psi_0(5x)\Gamma(x) + \Gamma(5x)\Gamma(x)\psi_0(x)] - \Gamma(5x)\Gamma(x)[6\Gamma(3x)^2\psi_0(3x)] \} \quad (\text{II.36})$$

$$= s_\beta \frac{x^2}{\Gamma(3x)^4} \{ 6\Gamma(5x)\Gamma(3x)^2\Gamma(x)\psi_0(3x) - \Gamma(5x)\Gamma(3x)^2\Gamma(x)[5\psi_0(5x) + \psi_0(x)] \} \quad (\text{II.37})$$

$$= s_\beta \frac{x^2}{\Gamma(3x)^4} \{ \Gamma(5x)\Gamma(3x)^2\Gamma(x)[6\psi_0(3x) - 5\psi_0(5x) - \psi_0(x)] \} \quad (\text{II.38})$$

$$= s_\beta x^2 \frac{\Gamma(5x)\Gamma(x)}{\Gamma(3x)^2} [6\psi_0(3x) - 5\psi_0(5x) - \psi_0(x)]. \quad (\text{II.39})$$

Substituting back in for  $x$  and recognizing an occurrence of  $\gamma_2$ , we have the result

$$s_{\gamma_2} = s_\beta \frac{1}{\beta^2} (\gamma_2 + 3) [6\psi_0(3/\beta) - 5\psi_0(5/\beta) - \psi_0(1/\beta)] \quad (\text{II.40})$$

with which we can find the uncertainty in the kurtosis given the value and uncertainty of the shape parameter  $\beta$ .

With a method of determining the uncertainty in kurtosis established, we may now examine the results. In Figure II.9, we plot the kurtosis and associated uncertainties as a function of redshift as red curves for distributions of  $\Delta M_{\text{vir}}$  and  $\Delta c$ .

## II.5.7 Mass Trends

So far, our analysis has mostly focused on the behavior of the entire halo sample as a single unit. However, there is also a wealth of information available when the statistics for our sample are viewed as functions of halo mass. In this section, we explore our halo ensemble more deeply by dividing into bins of mass and viewing the behavior of the resulting subsamples. In this way, we are able to explore differences in low- and high-mass halos, as well as quantify the explicit mass dependencies. The codes used for this analysis are listed in Appendix K.

### II.5.7.1 Binning and Fitting

When representing the mass dependence of our various halo properties, we wished to do so in a way that was both straightforward to quantify and visually descriptive of the overall distribution of the data. We found the best way to accomplish this was to provide a dual representation, with the data both binned in mass for least-squares fitting and binned two dimensionally in mass and  $\Delta q$ , with a color scale representing bin density, for a human reader to more easily see the relative population of the parameter space.

First, the data is binned on a 2-D grid. We found this to be the most natural way

to visually represent the distribution of the data, as some features like population sparseness at high redshift, asymmetry, and large differences in number between low- and high-mass halos would be more difficult to convey with only average mass bin means and standard deviations. The binned data are plotted with a logarithmic color scale and smoothed with a Gaussian kernel.

As a technical aside, we note that plotting bins with zero members with a logarithmic color scale naturally leads to poor results. We counter this by artificially counting one half halo for bins that are otherwise empty, and rescale the color representation to make anything less than one unit per bin display the minimum color value.

As an alternate representation, and mainly for the benefit of a more quantitative analysis, we bin the data along the average halo mass axis. For each bin, we measure the mean and standard deviation of the data. The uncertainty in the mean is then calculated as the standard deviation divided by the square root of the number of particles in the bin. We find a linear fit to the bin means using our standard least-squares approach, weighted by the mean uncertainties.

Example plots are provided in Figures II.10 and II.11 to demonstrate this approach. The 2-D binned data is plotted using a logarithmic color scale to represent the number density of halos in a given cell. The bin means and associated uncertainties are plotted as the black points with error bars. The standard deviation to either side of the mean is plotted as black dotted lines. The least-squares fit to the bin means is plotted as a solid magenta line.

#### II.5.7.2 Trends with Redshift

To better analyze the time evolution of the mass dependence, we need a more compact representation than simply looking at successive individual redshift snapshots. The most informative individual parameter from these plots is the slope of the linear fit

line for  $\Delta q$  as a function of average halo mass. We therefore plot the slopes and associated uncertainties for each snapshot as a function of redshift, with the results for  $\Delta M_{\text{vir}}$  and  $\Delta c$  displayed in Figure II.12. The data are then fit with our linear least-squares routine, and the fit is overplotted as a red dashed line.

### II.5.8 Alternate Difference Distributions

The distributions of  $\Delta q$  that have been discussed up to this point are an excellent measure of the overall behavior of the halo population differences between 2LPT and zA simulations. However, as these distributions rely on the average quantity  $q_{\text{avg}} = (q_{\text{2LPT}} + q_{\text{zA}})/2$  for normalization, quantities like the fraction of halo pairs differing by a given amount between simulations are more difficult to extract. We therefore redefine our distribution quantity to instead use a normalization factor of  $q_{\text{zA}}$ :

$$\delta q = \frac{q_{\text{2LPT}} - q_{\text{zA}}}{q_{\text{zA}}}, \quad (\text{II.41})$$

which allows for a more direct count of halo pairs. Statistics for these distributions are saved alongside the output for  $\Delta q$  distributions with the codes in Appendix I.

#### II.5.8.1 Equivalent Displacement

The question may be asked why these distributions have not been used all along, as they more readily offer more quantitative values for our halo populations. Our previous distributions of  $\Delta q$  are symmetrical between 2LPT and zA quantities, which allows us to be completely unbiased as to which simulation initialization is correct. The distributions of  $\delta q$  lose this symmetry, and are only defined for  $\delta q \geq -1$  for positive quantities like mass and concentration.

For this analysis, we therefore need a way to consider halo pairs that differ by a certain amount in either direction (e.g. pairs that differ in quantity  $q$  by 10%, whether

$q$  is larger in 2LPT or ZA). Rearranging Equation II.41 yields

$$q_{\text{2LPT}} = (\delta q + 1)q_{\text{ZA}}, \quad (\text{II.42})$$

and making the substitution  $x = \delta q + 1$  gives us

$$q_{\text{2LPT}} = xq_{\text{ZA}}. \quad (\text{II.43})$$

For a given  $x$ , we want to find  $x_{\text{eq}}$  such that  $x_{\text{eq}} = 1/x$ . Substituting now for  $x$  and  $x_{\text{eq}}$  and rearranging gives us

$$\delta q_{\text{eq}} = \frac{1}{\delta q + 1} - 1, \quad (\text{II.44})$$

the value for which a halo pair with a larger  $q$  in ZA would differ by the same factor as a halo pair with a larger  $q$  in 2LPT.

### II.5.8.2 Redshift Trends

In Figures II.13 and II.14, we plot statistics for our  $\delta q$  distributions as functions of redshift. In Figure II.13, we plot the  $\delta q$  of the peak of the distribution, as well as the  $\delta q$  values where 50%, 10%, and 1% of halo pairs fall at or above  $\delta q$ . In Figure II.14, we plot the fraction of halo pairs  $f_h$  that fall outside various  $\delta q$  values. The solid curves represent the fraction of halo pairs that have a 2LPT mass or concentration at least 1.1, 1.5, 2.0, or 5.0 times that of the corresponding ZA halo. Dashed curves represent the same values, regardless of whether the 2LPT or ZA mass or concentration is higher. This is the same as counting halos that fall above a given  $\delta q$  as well as below the corresponding  $\delta q_{\text{eq}}$ . The code for creating these plots is listed in Appendix L.

## II.6 Automation

Dealing with the large number of data files, programs, and pipeline steps used in our analysis quickly becomes prohibitive in terms of time and complexity when each

must be dealt with completely “by hand.” In order to shorten the time needed for a full analysis of the data down to a reasonably human-scale level, a certain level of automation is required. A combination of shell scripting and basic parallelization was used to this effect. This has the added benefit of providing a self-documenting reproducibility to the analysis that was invaluable for the inevitable times when an error was discovered and the entire pipeline had to be re-run from the beginning. In this section, we will give a very brief summary of the automation steps taken and the scripts written for these tasks. Scripts run locally or launched manually are written in Bash, while job scripts that are submitted to the ACCRE compute cluster use the PBS syntax for communication with the scheduler and Bash for the remaining logic.

The creation of the directory structure for analysis with ROCKSTAR and subsequent halo catalog generation steps was done using the script listed in Appendix M.1. The creation of the directory structure for CROSSMATCH was done using the script listed in M.2. Individual instances of ROCKSTAR may be run on individual snapshots with the script in Appendix M.3, while all snapshots may be run as a batch job using the scripts in Appendices M.4 and M.5 for 2LPT and ZA snapshots, respectively. The output from ROCKSTAR is run through a post-processing step that is automated using the script in Appendix M.6. The CROSSMATCH program is run with the script in Appendix M.7, and the python code to generate density profiles is launched with the script in Appendix M.8. A number of other Bash scripts, PBS submission scripts, and Python programs, which we have already discussed in the above sections, were used for automation of the remainder of the analysis pipeline.

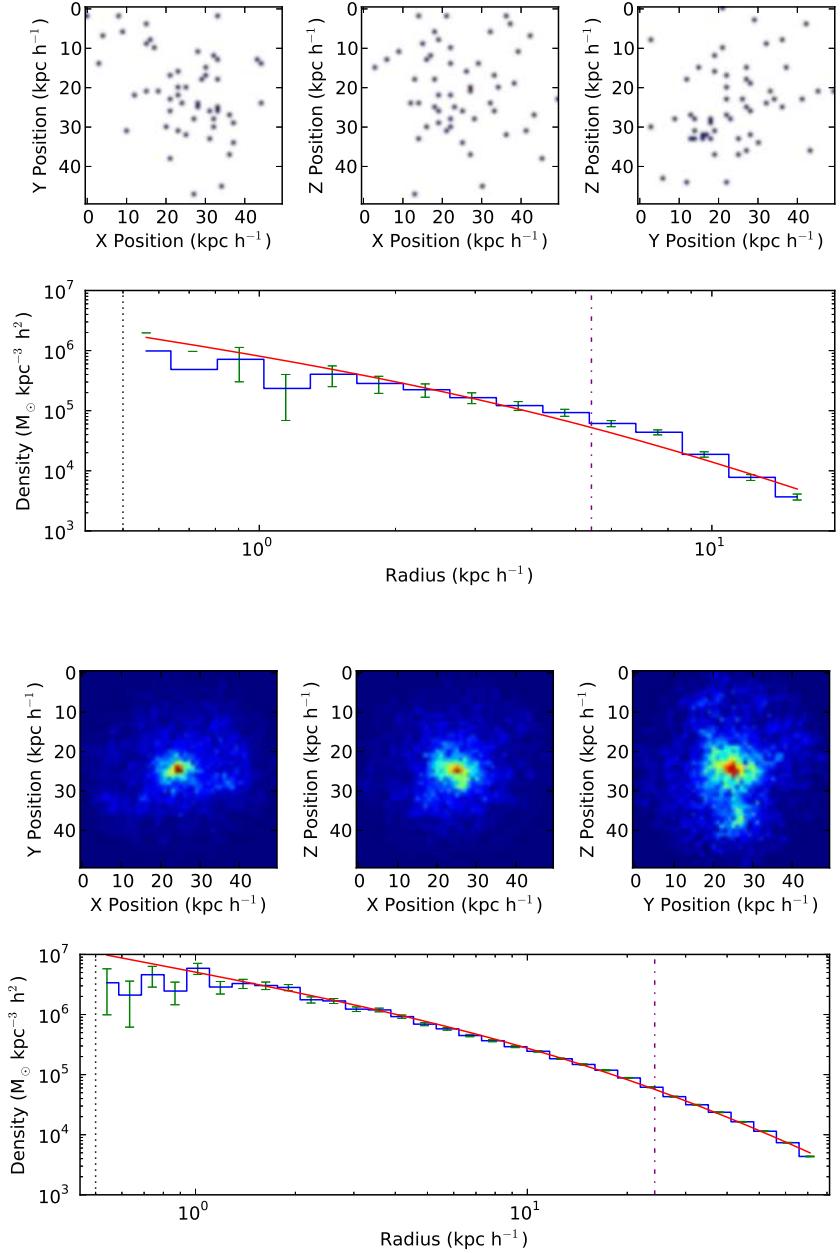


Figure II.3: Spacial projections and density profiles for two large halos at  $z = 14$  (top) and  $z = 6$  (bottom). Both halos are from the Box 1 2LPT simulation, and are the largest halos at their respective redshifts. The density profiles are fit with an NFW profile, and the resulting scale radius is plotted as a vertical dot-dash purple line.

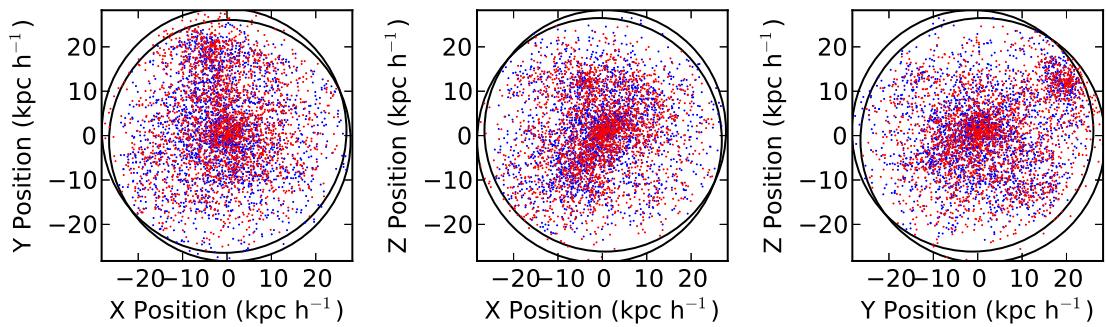


Figure II.4: Example of halo particle matching at  $z = 6$ . Blue dots are 2LPT halo particles, and red dots are ZA halo particles. Black circles are the virial radii of the halos. Good matches are achieved for halos, with only slight drift between simulations.

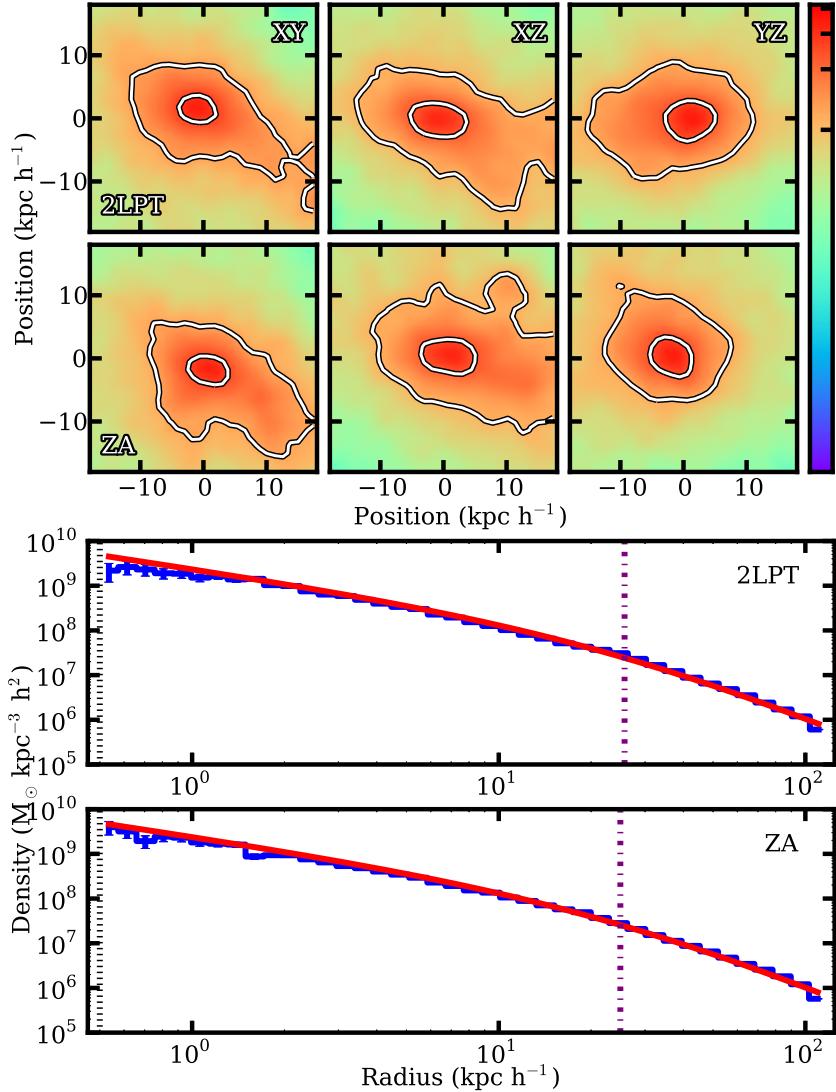


Figure II.5: Two large matched halos at  $z = 6$  with similar nuclear structure. *Top two rows:* Projected density maps, with XY, XZ, and YZ views of the central nuclear region of the halos. Density is represented by a logarithmic color scale, and equal density contours are plotted as white curves. The first and second rows depict the 2LPT and ZA halo, respectively. *Bottom two rows:* Radially-binned halo density profiles fit with the NFW density profile model. The blue stepped profiles are the binned data, red curves are the fit NFW models, black dashed lines are the resolution limit of the simulation, and purple dot-dash lines are the measured scale radius.

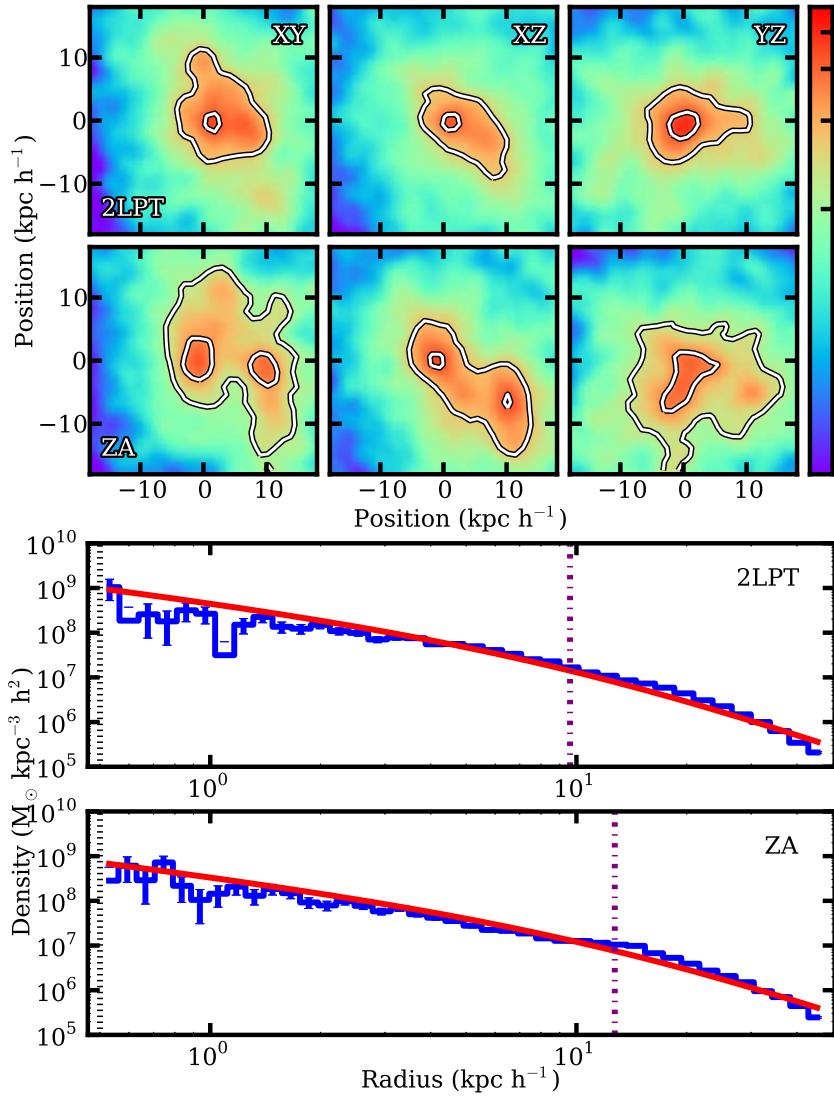
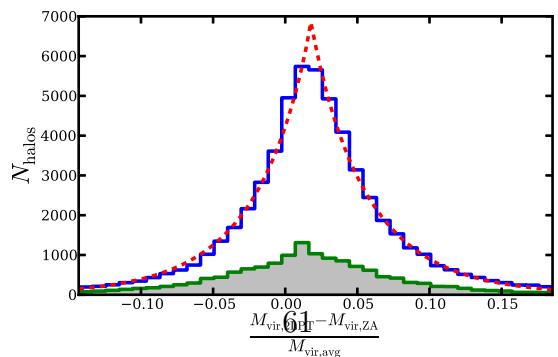
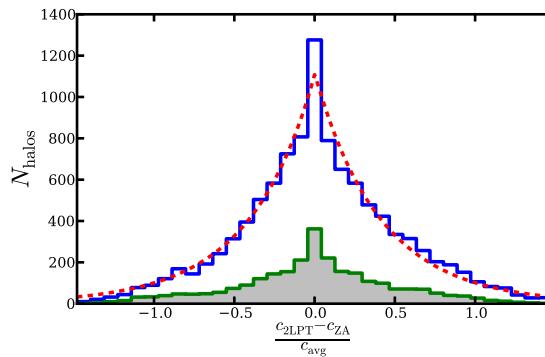
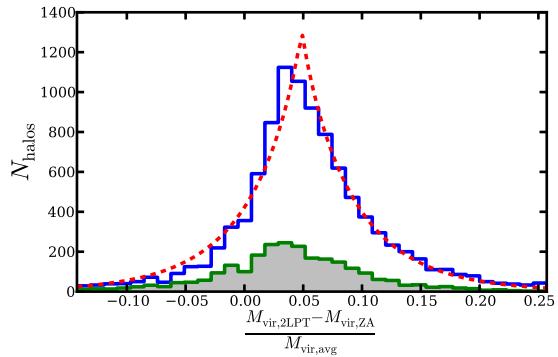
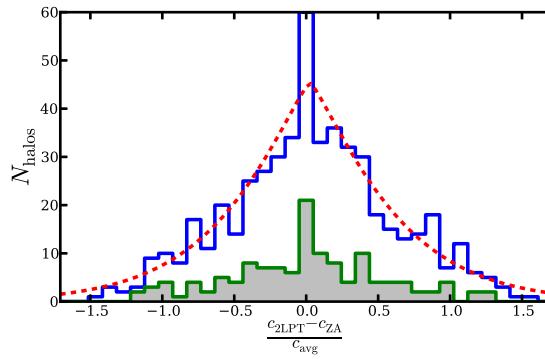
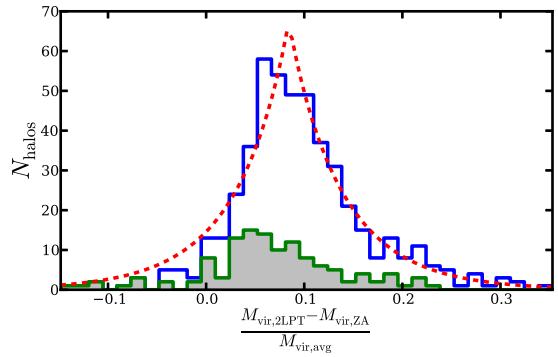


Figure II.6: Like Figure II.5, but for two large matched halos at  $z = 6$  with differing nuclear structure.



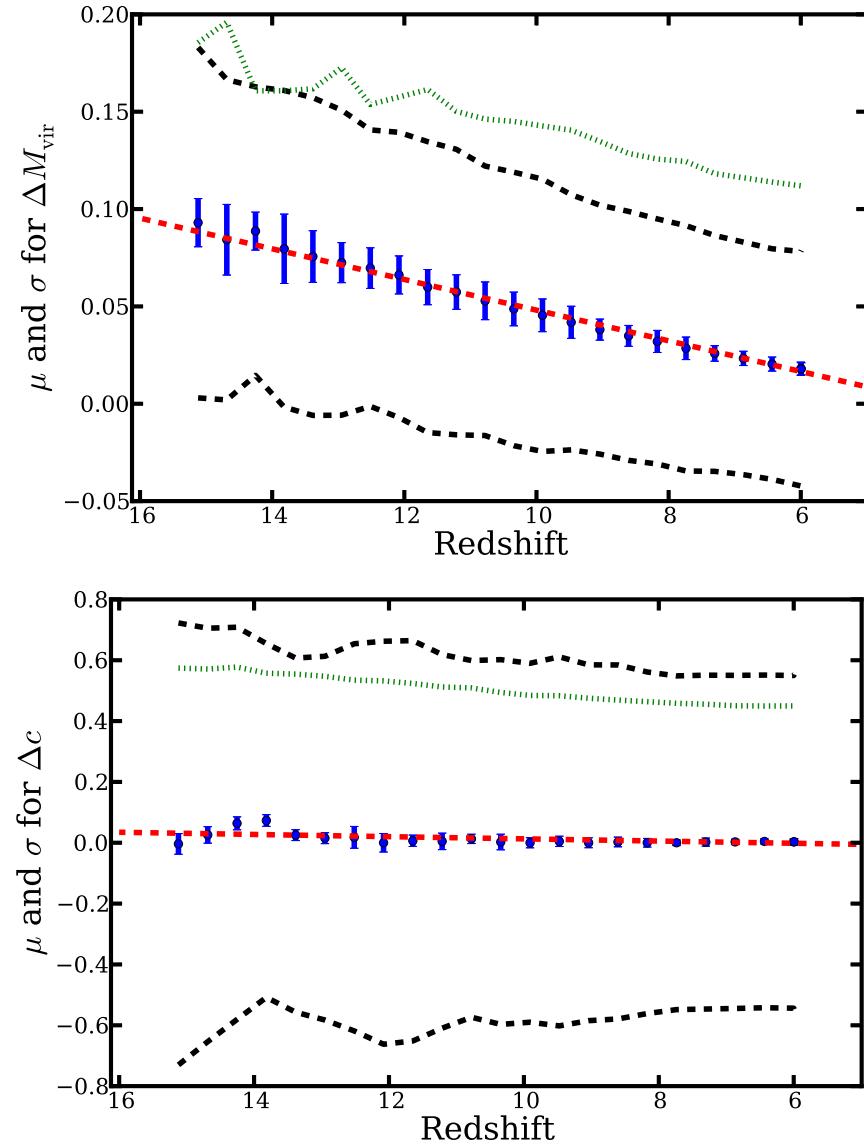


Figure II.8: Mean, standard deviation, and rms as functions of redshift for  $\Delta M_{\text{vir}}$  (top) and  $\Delta c$  (bottom). The mean is plotted as blue points,  $\mu \pm \sigma$  is plotted as the black dashed curves, and rms values are plotted as a green dotted curve. The red dashed line is a linear fit to the mean.

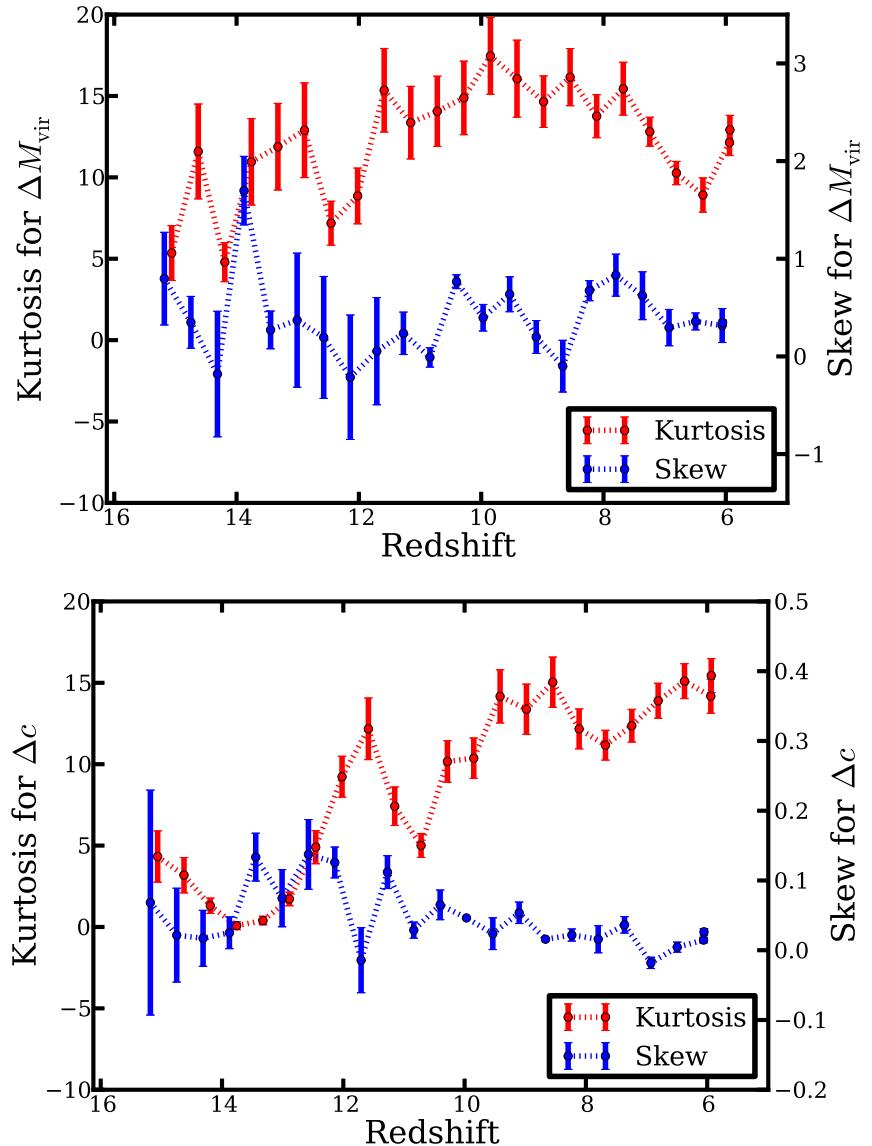


Figure II.9: Skew (blue curve) and excess kurtosis (red curve) from generalized normal distribution fits as functions of redshift for  $\Delta M_{\text{vir}}$  (top) and  $\Delta c$  (bottom). For both plots, the left axis is the scale for kurtosis and the right axis is the scale for skew.

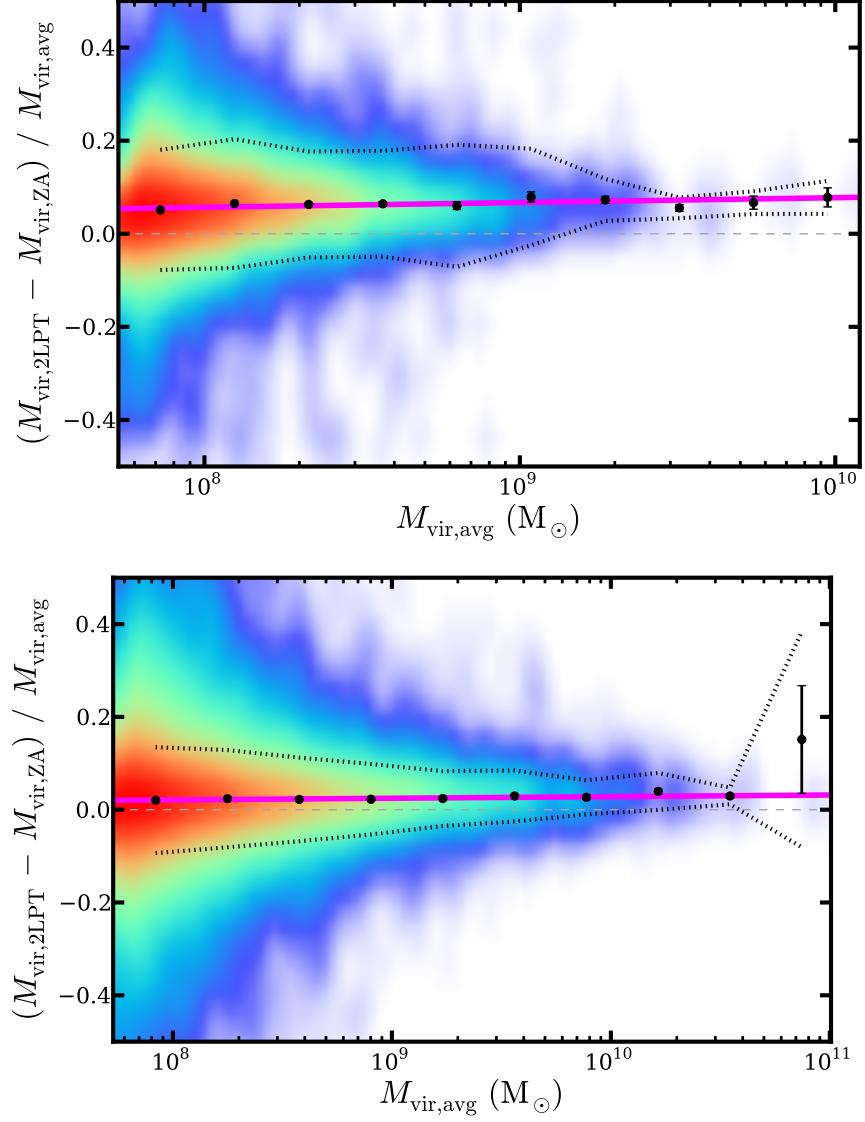


Figure II.10:  $\Delta M_{\text{vir}}$  as a function of  $M_{\text{vir,avg}}$ . For the 2-D color histogram, halos are counted in rectangular bins and smoothed with a Gaussian kernel with a logarithmic color scale. The halos are also divided into logarithmically-spaced bins in average virial mass, and the mean for each bin is plotted as a black point. The black dotted curves are the standard deviation around the mean. The magenta line is the linear least-squares best fit to the bin means. The light grey dashed line at  $\Delta q = 0$  is provided to guide the eye. The two panels correspond to snapshots at  $z = 10.3$  and  $z = 6.0$ . These plots are provided as examples of the output at this stage of the analysis and are further discussed in Chapter III.

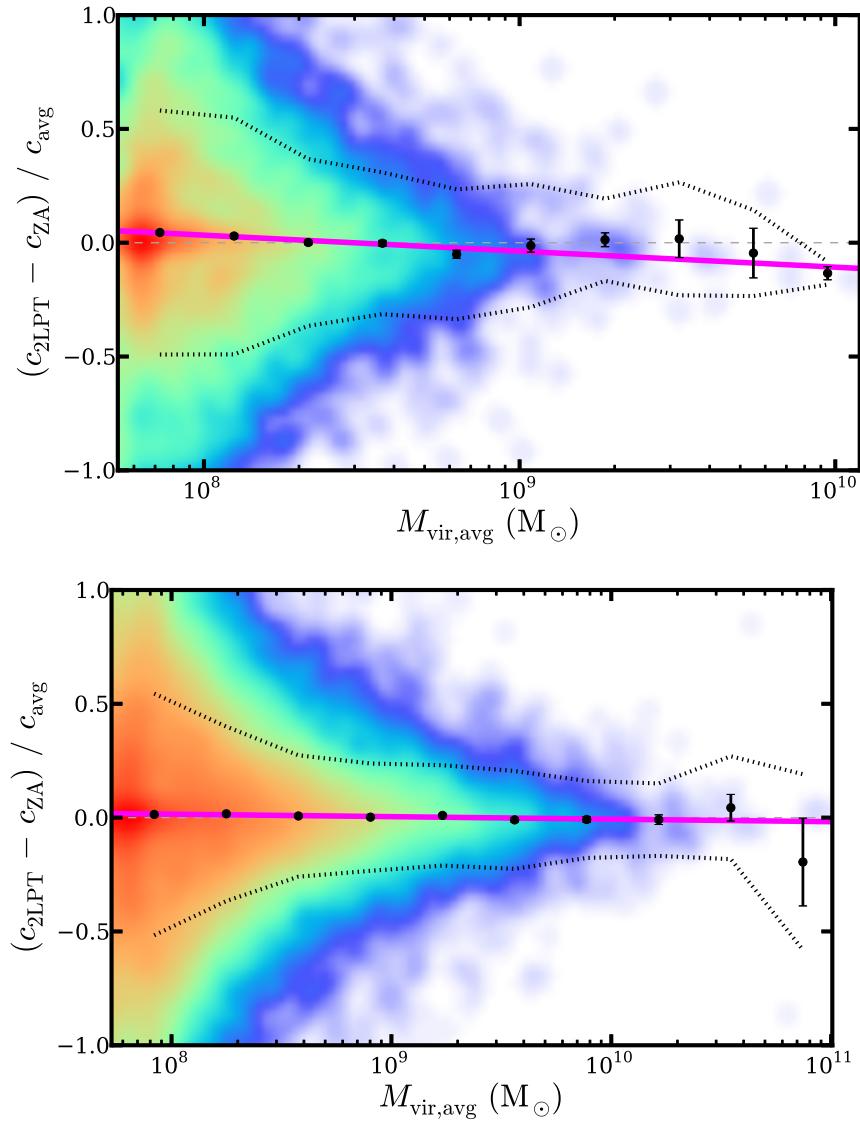


Figure II.11: Like Figure II.10, but for  $\Delta c$  instead of  $\Delta M_{\text{vir}}$  as a function of average halo mass.

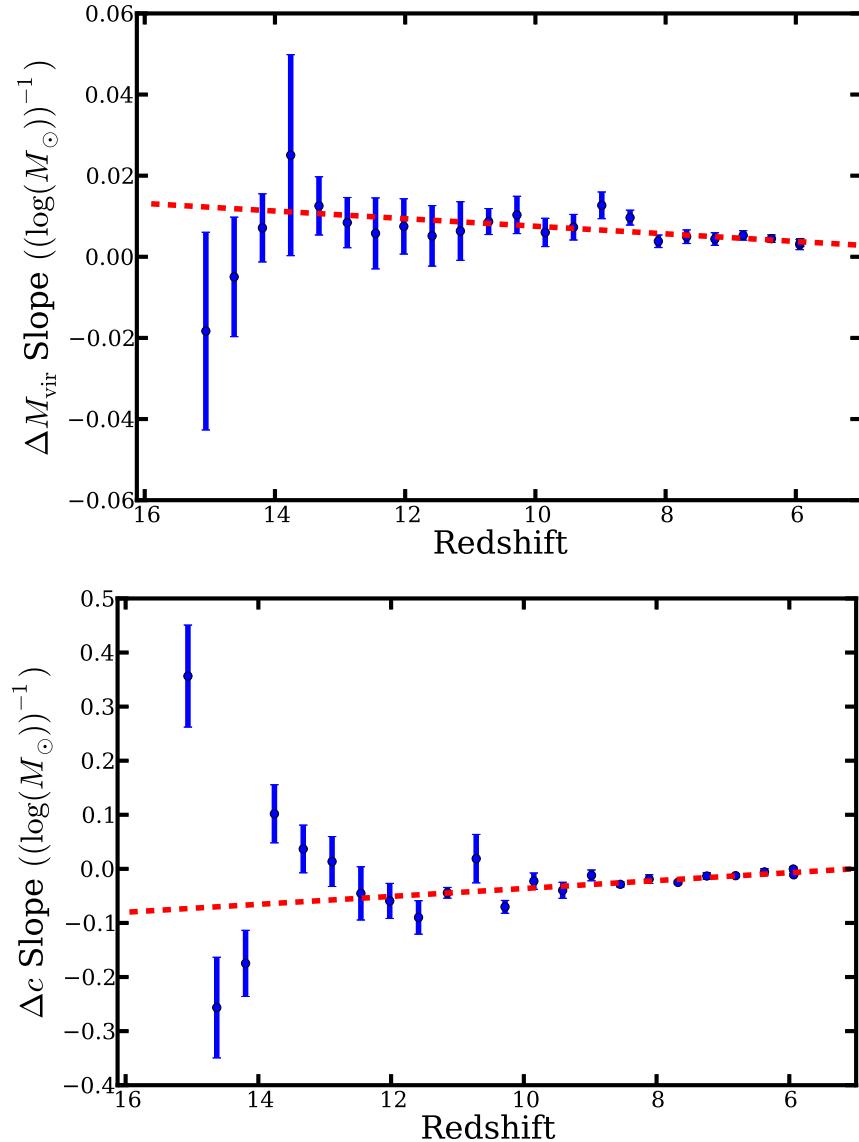


Figure II.12: Slopes of the  $\Delta q$  vs.  $M_{\text{vir,avg}}$  fit functions. The top and bottom panels correspond to the  $\Delta M_{\text{vir}}$  and  $\Delta c$  plots of Figures II.10 and II.11. Linear least-squares fits to the data are overplotted as red dashed lines. These plots are provided as examples of the output at this stage of the analysis and are further discussed in Chapter III.

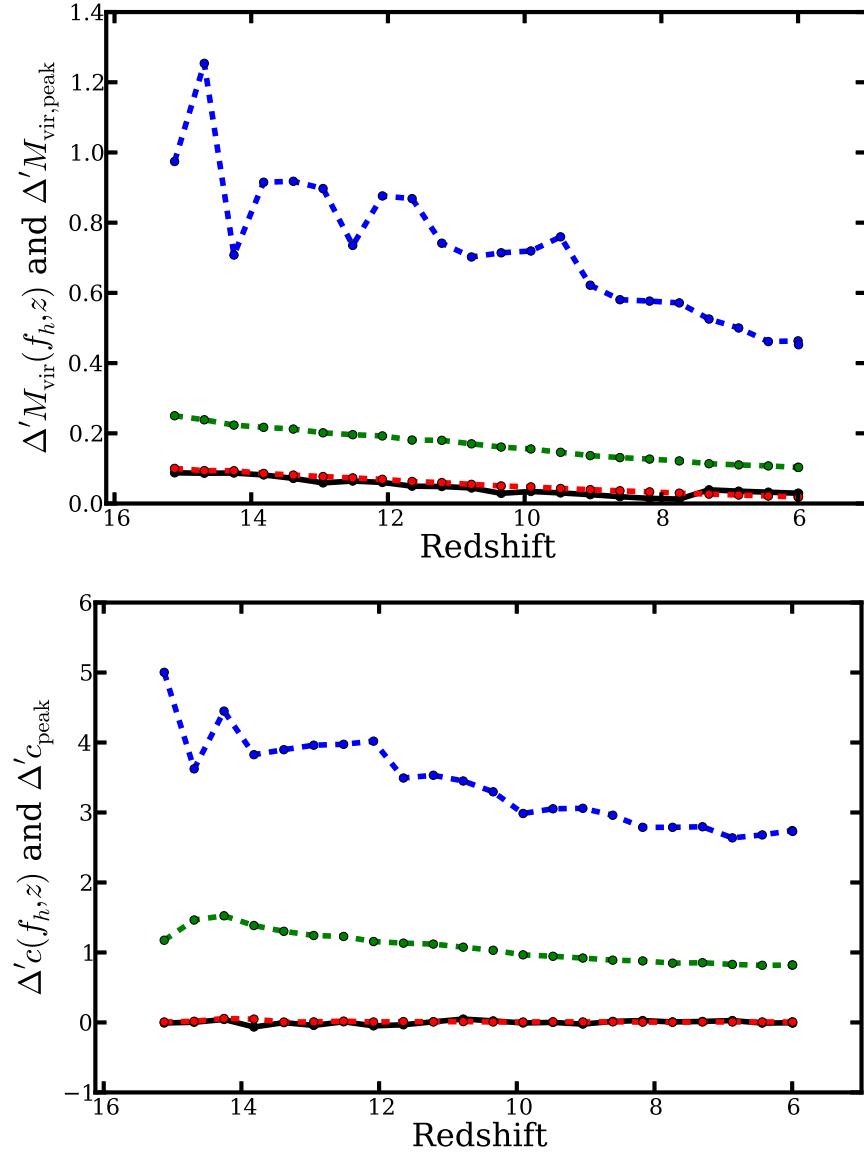


Figure II.13: Statistics for distributions of  $\delta M_{\text{vir}}$  (*top*) and  $\delta c$  (*bottom*) as functions of redshift. The  $\delta q$  of the peak of the distribution (black curve), and the  $\delta q$  where 50% (red dashed curve), 10% (green dashed curve), and 1% (blue dashed curve) of the halos fall at or above  $\delta q$ . These plots are provided as examples of the output at this stage of the analysis and are further discussed in Chapter III.

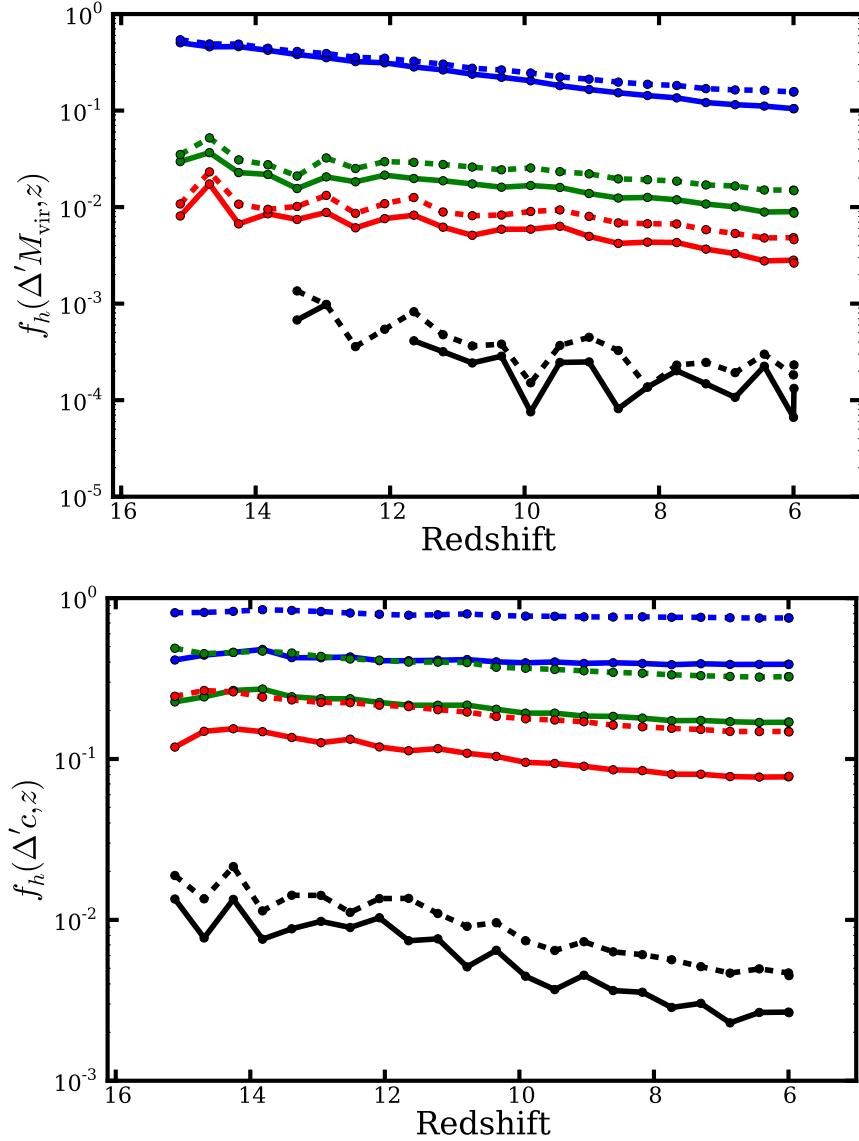


Figure II.14: Statistics for distributions of  $\delta M_{\text{vir}}$  (top) and  $\delta c$  (bottom) as functions of redshift. The fraction of halos with  $\delta q$  greater than 0.10 (solid blue curve), 0.50 (solid green curve), 1.00 (solid red curve), and 4.00 (solid black curve). The dashed curves additionally count halo pairs with  $\delta q$  lower than the corresponding equivalent displacements of -0.09, -0.33, -0.50, and -0.80, respectively (see Equation II.44). These plots are provided as examples of the output at this stage of the analysis and are further discussed in Chapter III.

## CHAPTER III

### Exploring Dark Matter Halo Populations in 2lpt and za Simulations

We study the structure and evolution of dark matter halos from  $z = 300$  to  $z = 6$  for two cosmological N-body simulation initialization techniques. While the second order Lagrangian perturbation theory (2LPT) and the Zel'dovich approximation (ZA) both produce accurate present day halo mass functions, earlier collapse of dense regions in 2LPT can result in larger mass halos at high redshift. We explore the differences in dark matter halo mass and concentration due to initialization method through three 2LPT and three ZA initialized cosmological simulations. We find that 2LPT induces more rapid halo growth, resulting in more massive halos compared to ZA. This effect is most pronounced for high mass halos and at high redshift, with a fit to the mean normalized difference between 2LPT and ZA halos as a function of redshift of  $\mu_{\Delta M_{\text{vir}}} = (7.88 \pm 0.17) \times 10^3 z - (3.07 \pm 0.14) \times 10^{-2}$ . Halo concentration is, on average, largely similar between 2LPT and ZA, but retains differences when viewed as a function of halo mass. For both mass and concentration, the difference between typical individual halos can be very large, even for symmetrically distributed quantities, highlighting the shortcomings of ZA-initialized simulations for high- $z$  halo population studies.

#### III.1 Introduction

The pre-reionization epoch is a time of significant evolution of early structure in the universe. Rare density peaks in the otherwise smooth dark matter (DM) sea lead to the collapse and formation of the first dark matter halos. For example, at  $z = 20$ ,  $10^7 M_\odot$  halos are  $\sim 4\sigma$  peaks, and  $10^8 M_\odot$  halos, candidates for hosting the first supermassive black hole seeds, are  $\sim 5\sigma$  peaks.

These early-forming dark matter halos provide an incubator for the baryonic pro-

cesses that transform the surrounding space and allow galaxies to form. Initial gas accretion can lead to the formation of the first Pop-III stars (Couchman & Rees 1986; Tegmark et al. 1997; Abel et al. 2000, 2002), which, upon their death, can collapse into the seeds for supermassive black holes (SMBHs) (Madau & Rees 2001; Islam et al. 2003; Alvarez et al. 2009; Jeon et al. 2012) or enrich the surrounding medium with metals through supernovae (Heger & Woosley 2002; Heger et al. 2003). The radiation from these early quasars (Shapiro & Giroux 1987; Madau et al. 1999; Fan et al. 2001), Pop-III stars (Gnedin & Ostriker 1997; Venkatesan et al. 2003; Alvarez et al. 2006), and proto-galaxy stellar populations (Bouwens et al. 2012; Kuhlen & Faucher-Giguère 2012) all play a key role in contributing to the re-ionizing of the universe by around  $z = 6$  (Barkana & Loeb 2001). Additionally, halo mergers can drastically increase the temperature of halo gas through shock heating, increasing X-ray luminosity (Sinha & Holley-Bockelmann 2009), and contribute to the unbinding of gas to form the warm-hot intergalactic medium (Bykov et al. 2008; Sinha & Holley-Bockelmann 2010; Tanaka et al. 2012).

While a number of parameters are required to fully characterize a DM halo, a first-order description can be obtained from its mass and density profile. There are a number of ways to define a halo's mass, the subtleties of which becomes significant for mass-sensitive studies, such as the halo mass function (Press & Schechter 1974; Reed et al. 2007; Heitmann et al. 2006; Lukić et al. 2007). For a review, see, e.g., White (2001) and references therein. Additionally, see Voit (2005) and references therein for a more observation-focused discussion.

From a simulation standpoint, the two most common ways to obtain halo mass are to define either spherical overdensity halos or friends-of-friends (FOF) halos. The spherical overdensity method identifies regions above a certain density threshold, either with respect to the critical density  $\rho_c = 3H^2/8\pi G$  or the background density  $\rho_b = \Omega_m \rho_c$ , where  $\Omega_m$  is the matter density of the universe. The mass is then the

mass enclosed in a sphere of some radius with mean density  $\Delta\rho_c$ , where  $\Delta$  commonly ranges from  $\sim 100$  to  $\sim 500$ . Alternatively, the FOF method finds particle neighbors and neighbors of neighbors defined to be within some separation distance (Einasto et al. 1984; Davis et al. 1985). Halo mass, then, is simply the sum of the masses of the constituent particles.

The density profile of a DM halo is determined by radially binning the constituent particles into spherical shells, and determining the average density per shell, giving a characteristic  $\rho(r)$ . The most widely used model for the DM halo density profile is the NFW (Navarro et al. 1996) profile

$$\rho(r) = \frac{\rho_0}{\frac{r}{R_s} \left(1 + \frac{r}{R_s}\right)^2}, \quad (\text{III.1})$$

where  $\rho_0$  is the characteristic density, and the scale radius  $R_s$  is the break radius between the inner  $\sim r^{-1}$  and outer  $\sim r^{-3}$  density profiles.

The halo density profile is quantified by the halo concentration  $c \equiv R_{\text{vir}}/R_s$ .  $R_{\text{vir}}$  is the halo virial radius, which is often defined as the radius at which the average interior density is some factor  $\Delta_c$  times the critical density of the universe  $\rho_c$ , where  $\Delta_c$  is typically  $\sim 200$ . Generally, at low redshift, low mass halos are more dense than high mass halos (Navarro et al. 1997a), and concentration decreases with redshift and increases in dense environments (Bullock et al. 2001b). Neto et al. (2007) additionally find that concentration decreases with halo mass. Various additional studies have explored concentration's dependence on characteristics of the power spectrum (Eke et al. 2001), cosmological model (Macciò et al. 2008), redshift (Gao et al. 2008; Muñoz-Cuartas et al. 2011), and halo merger and mass accretion histories (Wechsler et al. 2002; Zhao et al. 2003, 2009). For halos at high redshift, Klypin et al. (2011) find that concentration reverses and increases with mass for high mass halos, while Prada et al. (2012) find that concentration's dependence on mass and redshift is more

complicated and is better described through  $\sigma(M, z)$ , the RMS fluctuation amplitude of the linear density field.

The subtle  $\mathcal{O}(10^{-5})$  density perturbations in place at the CMB epoch are vulnerable to numerical noise and intractable to simulate directly. Instead, a displacement field is applied to the particles to evolve them semi-analytically, nudging them from their initial positions to an approximation of where they should be at a more reasonable starting redshift for the numerical simulation. Starting at a later redshift saves computation time as well as avoiding interpolation systematics and round-off errors (Lukić et al. 2007).

The Zel'dovich approximation (Zel'dovich 1970) and 2nd-order Lagrangian Perturbation Theory (Buchert 1994; Buchert et al. 1994; Bouchet et al. 1995; Scoccimarro 1998) are the two canonical frameworks for the initial particle displacement involved in generating simulation initial conditions. Zel'dovich approximation (ZA, hereafter) initial conditions (Klypin & Shandarin 1983; Efstathiou et al. 1985) displace initial particle positions and velocities via a linear field, while 2nd-order Lagrangian Perturbation Theory (2LPT, hereafter) initial conditions (Scoccimarro 1998; Sirk 2005; Jenkins 2010) add a second-order correction term to the expansion of the displacement field.

Following Jenkins (2010), we briefly outline the second-order Lagrangian perturbation theory and compare it to the Zel'dovich approximation. In 2LPT, a displacement field  $\Psi(\mathbf{q})$  is applied to the initial positions  $\mathbf{q}$  to yield the Eulerian final comoving positions

$$\mathbf{x} = \mathbf{q} + \Psi. \quad (\text{III.2})$$

The displacement field is given in terms of two potentials  $\phi^{(1)}$  and  $\phi^{(2)}$  by

$$\mathbf{x} = \mathbf{q} - D_1 \nabla_q \phi^{(1)} + D_2 \nabla_q \phi^{(2)}, \quad (\text{III.3})$$

with linear growth factor  $D_1$  and second-order growth factor  $D_2 \approx -3D_1^2/7$ . The subscripts  $\mathbf{q}$  refer to partial derivatives with respect to the Lagrangian coordinates  $\mathbf{q}$ . Likewise, the comoving velocities are given, to second order, by

$$\mathbf{v} = -D_1 f_1 H \nabla_{\mathbf{q}} \phi^{(1)} + D_2 f_2 H \nabla_{\mathbf{q}} \phi^{(2)}, \quad (\text{III.4})$$

with Hubble constant  $H$  and  $f_i = d \ln D_i / d \ln a$ , with expansion factor  $a$ . The relations  $f_1 \approx \Omega^{5/9}$  and  $f_2 \approx 2\Omega^{6/11}$ , with matter density  $\Omega$ , apply for flat models with a non-zero cosmological constant (Bouchet et al. 1995). The  $f_1$ ,  $f_2$ , and  $D_2$  approximations here are very accurate for most actual  $\Lambda$ CDM initial conditions, as  $\Omega$  is close to unity at high starting redshift (Jenkins 2010). We may derive  $\phi^{(1)}$  and  $\phi^{(2)}$  by solving a pair of Poisson equations

$$\nabla_{\mathbf{q}}^{(1)}(\mathbf{q}) = \delta^{(1)}(\mathbf{q}), \quad (\text{III.5})$$

with linear overdensity  $\delta^{(1)}(\mathbf{q})$ , and

$$\nabla_{\mathbf{q}}^{(2)}(\mathbf{q}) = \delta^{(2)}(\mathbf{q}). \quad (\text{III.6})$$

The second order overdensity  $\delta^{(2)}(\mathbf{q})$  is related to the linear overdensity field by

$$\delta^{(2)}(\mathbf{q}) = \sum_{i>j} \left\{ \phi_{,ii}^{(1)}(\mathbf{q}) \phi_{,jj}^{(1)}(\mathbf{q}) - \left[ \phi_{,ij}^{(1)}(\mathbf{q}) \right]^2 \right\}, \quad (\text{III.7})$$

where  $\phi_{,ij} \equiv \partial^2 \phi / \partial q_i \partial q_j$ . For initial conditions from the Zel'dovich approximation, or first-order Lagrangian initial conditions, the  $\phi^{(2)}$  terms of Equations III.3 and III.4 are ignored.

Cosmological simulations that follow the initial collapse of dark matter density peaks into virialized halos often neglect to consider the nuances of initialization method. Non-linear decaying modes, or transients, will be damped as  $1/a$  in ZA. In 2LPT, however, transients are damped more quickly as  $1/a^2$ . It should be ex-

pected, then, that structure in 2LPT will be accurate after fewer  $e$ -folding times than in ZA (Scoccimarro 1998; Crocce et al. 2006; Jenkins 2010). The practical result is that high- $\sigma$  DM density peaks at high redshift are suppressed in ZA compared with 2LPT for a given starting redshift (Crocce et al. 2006).

While differences in ensemble halo properties, such as the halo mass function, between simulation initialization methods are mostly washed away by  $z = 0$  (Scoccimarro 1998), trends at earlier redshifts are less studied (Lukić et al. 2007). In this paper, we explore the effects of ZA and 2LPT on the evolution of halo populations at high redshift. It is thought that 2LPT allows initial DM overdensities to get a “head start” compared with ZA, allowing earlier structure formation, more rapid evolution, and larger possible high-mass halos for a given redshift. We explore this possibility by evolving a suite of simulations from  $z = 300$  to  $z = 6$  and comparing the resulting differences in halo properties arising from initialization with ZA and 2LPT in these otherwise identical simulations.

We discuss the simulations, halo finding, and analysis methods in Section III.2, results in Section III.3, implications, caveats, and future work in Section III.4, and a summary of our results and conclusions in Section III.5.

### III.2 Numerical Methods

We use the Nbody tree/SPH code GADGET-2 (Springel et al. 2001; Springel 2005) to evolve six dark matter-only cosmological volumes from  $z_{start} = 300$  to  $z = 6$  in a  $\Lambda$ CDM universe. Each simulation is initialized using WMAP-5 (Komatsu et al. 2009) parameters. For each of the three simulation pairs, we directly compare 2LPT and ZA by identically sampling the CMB transfer function and displacing the initial particle positions to the same starting redshift using 2LPT and ZA. The three sets of simulations differ only by the initial phase sampling random seed. Each volume contains  $512^3$  particles in a  $10 h^{-1}$  Mpc box. Full simulation details are discussed in

Holley-Bockelmann et al. (2012).

One facet often overlooked when setting up an  $N$ -body simulation is an appropriate starting redshift, determined by box size and resolution (Lukić et al. 2007). As 2LPT more accurately displaces initial particle positions and velocities, initialization with 2LPT allows for a later starting redshift compared with an equivalent ZA-initialized simulation. However, many ZA simulations do not take this into account, starting from too late an initial redshift and not allowing enough e-foldings to adequately dampen away numerical transients. (Crocce et al. 2006; Jenkins 2010). In order to characterize an appropriate starting redshift, the relation between the initial RMS particle displacement and mean particle separation must be considered. The initial RMS displacement  $\Delta_{\text{rms}}$  is given by

$$\Delta_{\text{rms}}^2 = \frac{4\pi}{3} \int_{k_f}^{k_{\text{Ny}}} P(k, z_{\text{start}}) dk, \quad (\text{III.8})$$

where  $k_f = 2\pi/L_{\text{box}}$  is the fundamental mode,  $L_{\text{box}}$  is the simulation box size,  $k_{\text{Ny}} = \frac{1}{2}Nk_f$  is the Nyquist frequency of an  $N^3$  simulation, and  $P(k, z_{\text{start}})$  is the power spectrum at starting redshift  $z_{\text{start}}$ . In order to avoid the “orbit crossings” that reduce the accuracy of the initial conditions,  $\Delta_{\text{rms}}$  must be some factor smaller than the mean particle separation  $\Delta_p = L_{\text{box}}/N$  (Holley-Bockelmann et al. 2012). For example, making orbit crossing a  $\sim 10\sigma$  event imposes  $\Delta_{\text{rms}}/\Delta_p = 0.1$ . However, for small-volume, high-resolution simulations, this quickly leads to impractical starting redshifts. Continuing our example, satisfying  $\Delta_{\text{rms}}/\Delta_p \sim 0.1$  for a  $10h^{-1}$  Mpc,  $512^3$  simulation suggests  $z_{\text{start}} \approx 799$ . Starting at such a high redshift places such a simulation well into the regime of introducing errors from numerical noise caused by roundoff errors dominating the smooth potential. A more relaxed requirement of  $\Delta_{\text{rms}}/\Delta_p = 0.25$  yields  $z_{\text{start}} = 300$ , which we adopt for this work.

For each of our six simulations, we use the 6-D phase space halo finder code

ROCKSTAR (Behroozi et al. 2013) to identify spherical overdensity halos at each timestep. ROCKSTAR follows an adaptive hierarchical refinement of friends-of-friends halos in 6-D phase space, allowing determination of halo properties such as halo mass, position, virial radius, internal energy, and number of subhalos. ROCKSTAR tracks halos down to a threshold of around 20 particles, but we use a more conservative 100 particle threshold for our analysis. We use all particles found within the virial radius to define our halos and their properties.

We identify matching halos based on the highest fraction of matching particles contained in each at any given timestep. We remove halo pairs where either one or both halos are considered subhalos (i.e. a halo must not be contained within another halo) and pairs with fewer than 100 particles in either 2LPT or ZA. We are left with approximately 60,000 total halo pairs for our three boxes at  $z = 6$ . With halo catalogs matched between simulations, we can compare properties of individual corresponding halos. To mitigate the effects of cosmic variance on our small volumes, we “stack” the three simulation boxes for each initialization method, and combine the halos from each into one larger sample in our analysis.

Halo concentration is derived from ROCKSTAR’s output for  $R_s$  and  $R_{\text{vir}}$ . Here,  $R_{\text{vir}}$  is the virial radius as defined by Bryan & Norman (1998). Figure III.1 makes evident the difficulty in fitting density profiles and obtaining concentration measurements for typical realistic halos. Large substructure, as displayed by the ZA halo, can disrupt the radial symmetry of the halo and cause significant deviations in the density profile. Centering can also be an issue in these cases. Due to these complications, there are a number of approaches for finding halo concentrations (Prada et al. 2012), but for consistency, we use the values derived from ROCKSTAR’s fitting for our concentration measurements.

At each simulation snapshot, we measure and compare a number of parameters for halos in both 2LPT and ZA simulations. For each quantity  $q$ , we create histograms

of  $\Delta q$ , the normalized difference in  $q$  between halos in the 2LPT and ZA simulations, defined as

$$\Delta q = \frac{q_{\text{2LPT}} - q_{\text{ZA}}}{q_{\text{avg}}}, \quad (\text{III.9})$$

where  $q_{\text{avg}} = \frac{1}{2}(q_{\text{2LPT}} + q_{\text{ZA}})$ . For each of these, we fit the  $\Delta q$  histograms with a generalized normal distribution (Nadarajah 2005) with the probability density function

$$f(x) = \frac{\beta}{2\alpha\Gamma(1/\beta)} e^{(|x-\mu|/\alpha)^\beta}, \quad (\text{III.10})$$

where  $\mu$  is the mean,  $\alpha$  is the scale parameter,  $\beta$  is the shape parameter, and  $\Gamma$  is the gamma function

$$\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx. \quad (\text{III.11})$$

The shape parameter  $\beta$  is restricted to  $\beta \geq 1$ . This allows the distribution to potentially vary from a Laplace distribution ( $\beta = 1$ ) to a uniform distribution ( $\beta = \infty$ ) and includes the normal distribution ( $\beta = 2$ ). The distribution has variance

$$\sigma^2 = \frac{\alpha^2\Gamma(3/\beta)}{\Gamma(1/\beta)} \quad (\text{III.12})$$

and excess kurtosis

$$\gamma_2 = \frac{\Gamma(5/\beta)\Gamma(1/\beta)}{\Gamma(3/\beta)^2} - 3. \quad (\text{III.13})$$

The distribution is symmetric, and thus has no skewness by definition. As such, the values for skew presented below are measured directly from the data.

As our fitting distributions are symmetrical and skew must therefore be measured directly from the data, in order to derive uncertainties for skew, we measure the skew of the distributions for each of our three simulation boxes individually as well as for the single stacked data set. Uncertainty in skew is then simply the standard deviation of the mean of the skew of the three individual boxes.

Determining the uncertainty in the kurtosis is slightly more involved, as kurtosis is determined by a transformation of the generalized normal distribution's shape parameter  $\beta$  according to Equation III.13. Following the standard procedure for propagation of uncertainty, we calculate the standard deviation of the kurtosis as

$$s_{\gamma_2} = \sqrt{\left(\frac{d\gamma_2}{d\beta}\right)^2 s_\beta^2} \quad (\text{III.14})$$

$$= s_\beta \frac{d}{d\beta} \left[ \frac{\Gamma(5/\beta)\Gamma(1/\beta)}{\Gamma(3/\beta)^2} - 3 \right]. \quad (\text{III.15})$$

The derivative of the gamma function is

$$\Gamma'(x) = \Gamma(x)\psi_0(x), \quad (\text{III.16})$$

where the digamma function  $\psi_0$  is the derivative of the logarithm of the gamma function and is given by

$$\psi_0(x) = \int_0^\infty \left( \frac{e^{-t}}{t} - \frac{e^{-xt}}{1-e^{-t}} \right) dt \quad (\text{III.17})$$

if the real part of  $x$  is positive. Now, taking the derivative of  $\gamma_2$  and doing a bit of algebra yields

$$s_{\gamma_2} = s_\beta \frac{1}{\beta^2} (\gamma_2 + 3) [6\psi_0(3/\beta) - 5\psi_0(5/\beta) - \psi_0(1/\beta)], \quad (\text{III.18})$$

with which we can find the uncertainty in the kurtosis given the value and uncertainty of the shape parameter  $\beta$  estimated from the least squares fit routine.

In addition to distributions of  $\Delta q$ , we also consider distributions of

$$\Delta'q = \frac{q_{\text{2LPT}} - q_{\text{ZA}}}{q_{\text{ZA}}} \quad (\text{III.19})$$

in order to better quantify the fraction of halos differing by a given amount between 2LPT and ZA simulations. This distribution is inherently non-symmetrical, and is only defined for  $\Delta'q \geq -1$  for positive quantities like mass and concentration. In order to consider halo pairs that differ by a certain amount in either direction (e.g. pairs that differ by 10%, whether larger in 2LPT or ZA), a relation for equivalent displacement is required. Rearranging Equation III.19 yields

$$q_{\text{2LPT}} = (\Delta'q + 1)q_{\text{ZA}}, \quad (\text{III.20})$$

and making the substitution  $x = \Delta'q + 1$  gives us

$$q_{\text{2LPT}} = xq_{\text{ZA}}. \quad (\text{III.21})$$

For a given  $x_1$ , we want an  $x_2$  such that  $x_2 = 1/x_1$ . Substituting now for  $x_1$  and  $x_2$  and rearranging gives us

$$\Delta'q_2 = \frac{1}{\Delta'q_1 + 1} - 1, \quad (\text{III.22})$$

the value for which a halo pair with a larger  $q$  in ZA would differ by the same factor as a halo pair with a larger  $q$  in 2LPT where  $\Delta'q = \Delta'q_1$ .

### III.3 Results

With our catalog of matched dark matter halos, we directly compare differences in halo properties arising from initialization with 2LPT vs ZA. We consider halos on a pair-by-pair basis as well as the entire sample as a whole. Overall, we find 2LPT halos undergo more growth at a given redshift than their ZA counterparts.

#### III.3.1 Individual halo pairs

We compare large scale morphologies, density profiles, and various other halo properties for halo pairs on an individual halo-by-halo basis for several of the most massive

halos. Morphologies appear similar for most halos, indicating good halo matches between simulations. However, many pairs display differences in central morphology, such as the number and separation of central density peaks. We interpret these cases to be examples of differences in merger epochs, in which case one halo may still be undergoing a major merger, while its companion is in a more relaxed post-merger state. We give an example of one such pair at  $z = 6$  in Figure III.1. The top two rows show density projections of the nuclear regions for a large 2LPT and matching ZA halo (first and second rows, respectively). We find the ZA halo to contain two distinct density peaks with a separation of  $\sim 10$  kpc, while the 2LPT halo displays only a single core. On the third and fourth rows, we plot the density profiles of the same two halos (2LPT and ZA, respectively). Here, with nearly identical virial radii, it is readily seen that the 2LPT halo is more concentrated than the ZA halo.

### III.3.2 Differences in ensemble halo properties

For the halo population as a whole, we consider distributions of halo virial mass  $M_{\text{vir}}$  and concentration  $c$ . We plot histograms of  $\Delta M_{\text{vir}}$  and  $\Delta c$  in the left and right columns, respectively, of Figure III.2 for three representative timesteps at redshifts of  $z = 14.7$ ,  $z = 10.3$ , and  $z = 6.0$ . For each panel, the blue histogram features the entire halo sample, and the smaller gray-filled green histogram displays only the top 25% most massive halos, ordered by 2LPT mass. Fits to the primary histograms are overplotted as red dashed curves.

Throughout the simulation, we find a tendency for 2LPT halos to be more massive. At  $z = 15$ , the mean of the  $\Delta M_{\text{vir}}$  distribution is  $(9.3 \pm 1.2) \times 10^{-2}$ . The mean is consistently positive (heavier 2LPT halos) and is most displaced from zero at high redshift. The peak of the distribution gradually moves closer to zero as we progress in redshift. We find the least difference between paired halos for the final snapshot at  $z = 6$ , with  $\mu_{\Delta M_{\text{vir}}} = (1.79 \pm 0.31) \times 10^{-2}$ .

The higher-order moments of the  $\Delta M_{\text{vir}}$  distribution are of interest as well, as we find significant deviation from a Gaussian distribution. As we use the symmetrical generalized normal distribution as our fit function, the skewness of the data is unable to be measured from the fit itself. However, a qualitative deviation from symmetry can be readily observed. By  $z = 6$ , we end up with a rather symmetrical distribution, with both sides of the histogram equally well described by our fit. However, at higher redshift, we note a marked increase in skewness and deviation from this symmetry. As redshift increases, we observe an increasing difference between the fit curve and the bins to the left of the histogram peak.

We find the distributions to be much closer to a Laplace distribution than a Gaussian, with shape parameter consistently sitting at or very close to  $\beta = 1$ . Compared to a Gaussian distribution, the larger excess kurtosis implies a narrower central peak and heavier outlying tails. Our fit constrains  $\beta \geq 1$ , so the kurtosis of the data itself could potentially be higher than the fit implies.

We find no overall preference for more concentrated 2LPT or ZA halos. In contrast to the  $\Delta M_{\text{vir}}$  histograms,  $\Delta c$  shows very little deviation from symmetry about zero. Throughout the simulation, we find the distributions to have a mean close to zero and negligible skew. The widths of the distributions are much larger than those for  $\Delta M_{\text{vir}}$ , with an order of magnitude difference by  $z = 6$ . As with mass, concentration histograms are sharply peaked with heavy tails, implying a tendency for halo pairs to move towards the extremes of either very similar or very discrepant concentrations.

### III.3.3 Time evolution of mass and concentration

In Figure III.3, we more quantitatively assess the evolution of our various trends hinted at in Figure III.2. Here, we plot the mean, root mean square (RMS), standard deviation, skew, and kurtosis for  $\Delta M_{\text{vir}}$  and  $\Delta c$  as functions of redshift. Uncertainty in the mean is estimated directly by the least squares fitting routine.

Table III.1: Coefficients for linear least squares fits from Figure III.3.

	<i>A</i>	<i>B</i>
$\Delta M_{\text{vir}}$	$(7.88 \pm 0.17) \times 10^{-3}$	$(-3.07 \pm 0.14) \times 10^{-2}$
$\Delta c$	$(3.62 \pm 0.95) \times 10^{-3}$	$(-2.34 \pm 0.84) \times 10^{-2}$

The mean for  $\Delta M_{\text{vir}}$  is positive and highest at high redshift, trending toward zero by the end of the simulation. Distributions for  $\Delta c$  retain means close to and consistent with zero. Standard deviation decreases slightly for both  $\Delta M_{\text{vir}}$  and  $\Delta c$ . From  $z = 15$  to  $z = 6$ , standard deviation falls from  $(9.0 \pm 1.5) \times 10^{-2}$  to  $(6.08 \pm 0.31) \times 10^{-2}$  for  $\Delta M_{\text{vir}}$  and from  $0.73 \pm 0.11$  to  $0.551 \pm 0.026$  for  $\Delta c$ .

We find least square linear fits for both mean  $\Delta M_{\text{vir}}$  vs  $z$  and mean  $\Delta c$  vs  $z$ . Coefficients for slope  $A$  and y-intercept  $B$  for the fit equation  $\mu = Az + B$  are given in Table III.1 for both cases. We find a significant trend for  $\Delta M_{\text{vir}}$ , with a slope  $\sim 46\sigma$  from zero. Conversely, the slope for  $\Delta c$  is much smaller and, considering the larger spread of the underlying distributions, can be considered negligible. For  $\Delta M_{\text{vir}}$ , the y-intercept coefficient  $B$  likely has little meaning in terms of the actual behavior at  $z = 0$ , as we expect the trend to level out at later redshift.

We do note, however, that the mean can be deceiving as an indicator of total difference between halo populations, especially when it is close to zero as with concentration. It should be noted that while the mean can indicate a lack of average difference between the whole sample of 2LPT and ZA halos, there can still be very large discrepancies between many individually paired halos. We visualize this by plotting the RMS of  $\Delta M_{\text{vir}}$  and  $\Delta c$ , which is plotted as a green dotted line. Unlike the mean, standard deviation, and kurtosis, which are measured from fits to the histograms, RMS is measured directly from the data and is not dependent on fitting. The large RMS values are indicative of how much overall difference can arise between 2LPT and ZA halos, even though the differences may average to zero when considering the entire population. The RMS for both  $\Delta M_{\text{vir}}$  and  $\Delta c$  starts highest at high redshift—0.19 for

$\Delta M_{\text{vir}}$  and 0.57 for  $\Delta c$  at  $z = 15$ —and steadily decreases throughout the simulation, reaching minimums of 0.11 for  $\Delta M_{\text{vir}}$  and 0.45 for  $\Delta c$  by  $z = 6$ .

Additionally, it is of interest to consider the percentage of halo pairs that are “wrong” at some given time, regardless of whether the quantity is higher in 2LPT or ZA. For example, if we count halos outside a slit of  $\epsilon = 10\%$  around  $\Delta q = 0$ , we find that by  $z = 6$ , 14.6% of halo pairs still have substantially mismatched masses, and 74.3% have mismatched concentrations. It is evident that a substantial percentage of halo pairs can have markedly different growth histories, even when there is little or no offset in the ensemble halo population average.

Kurtosis is consistently large for both mass and concentration, with a slight increasing trend throughout the simulation for concentration. It reaches maximum values of  $17.5 \pm 2.4$  at redshift 10 for  $\Delta M_{\text{vir}}$  and  $15.4 \pm 1.0$  at the end of the simulation at redshift 6 for  $\Delta c$ . Skew is positive for much of the simulation for mass, but is much smaller for concentration. We find average skews of  $0.39 \pm 0.29$  for  $\Delta M_{\text{vir}}$  and  $0.045 \pm 0.028$  for  $\Delta c$ . These higher moment deviations from Gaussianity hint at the non-linear dynamics at play in halo formation.

The narrow peak and heavy tails of the distribution may indicate a fair amount of sensitivity to initial differences in halo properties, in that halo pairs that start out within a certain range of the mean are more likely to move closer to the mean, while pairs that are initially discrepant will diverge even further in their characteristics. This is indicative of the non-linear gravitational influence present during halo evolution, and is further supported by a kurtosis that increases with time.

The skew at high redshift for  $\Delta M_{\text{vir}}$  may give another hint at the non-linear halo formation process. Runaway halo growth causes more massive halos to even favor faster mass accretion and growth. The positively skewed distributions show a picture of 2LPT halo growth in which initial differences in mass are amplified most readily in the earliest forming and most massive halos, again indicating the extra kick-start to

halo growth provided by 2LPT initialization. While the slight decrease in skew with redshift may be counter-intuitive to this notion, it is likely that the large number of newly formed halos begin to mask the signal from the smaller number of large halos displaying this effect.

### III.3.4 Dependence of mass and concentration differences on halo mass

We consider  $\Delta M_{\text{vir}}$  and  $\Delta c$  as a function of average halo mass  $M_{\text{vir,avg}} = (M_{\text{vir,2LPT}} + M_{\text{vir,ZA}})/2$  for three representative timesteps in Figure III.4. The data are binned in average virial mass, for which means and standard deviations are provided as the black points and black dotted lines, respectively. The error bars on the black points represent the uncertainty in the mean and are the standard deviation divided by the number of halos in that bin. We additionally bin the data in rectangular bins on a 2-D grid with a logarithmic color map in order to provide a more natural representation of the distribution of the data. Linear fits to the bin means are overplotted in magenta.

We find that  $\Delta M_{\text{vir}}$  tends to increase with increasing  $M_{\text{vir,avg}}$  for most snapshots. 2LPT halos are consistently more massive than their ZA counterparts, and, aside from the highest redshift snapshots, this difference increases with average halo mass. While less massive halo pairs have a larger spread in the difference in 2LPT and ZA mass, more massive halo pairs are consistently heavier in 2LPT than in ZA. At redshift 15, the transition snapshot between negative and positive slopes, the least squares fit to the data produces the fit equation  $\Delta M_{\text{vir}} = -(0.5 \pm 1.5) \times 10^{-2} \log(M_{\text{vir,avg}}) + (0.15 \pm 0.12)$ . The slope of the fit lines then become positive and trends back towards zero as we progress in redshift, with a fit of  $\Delta M_{\text{vir}} = (3.49 \pm 0.99) \times 10^{-3} \log(M_{\text{vir,avg}}) - (6.8 \pm 8.3) \times 10^{-3}$  by  $z = 6$ .

We additionally find a trend for more massive halo pairs to be more concentrated in ZA. This trend is somewhat stronger than for  $\Delta M_{\text{vir}}$ , but again, high  $z$  snapshots differ from the trend. The fit equations for  $z = 15$  and  $z = 6$  are  $\Delta c = -(0.256 \pm$

Table III.2: Coefficients for linear least squares fits from Figure III.5.

	<i>A</i>	<i>B</i>
$\Delta M_{\text{vir}}$	$(9.4 \pm 2.4) \times 10^{-4}$	$(-1.8 \pm 1.8) \times 10^{-3}$
$\Delta c$	$(-7.3 \pm 1.9) \times 10^{-3}$	$(3.7 \pm 1.4) \times 10^{-2}$

$0.093) \log(M_{\text{vir,avg}}) + (2.07 \pm 0.76)$  and  $\Delta c = -(1.10 \pm 0.31) \times 10^{-2} \log(M_{\text{vir,avg}}) + (0.103 \pm 0.026)$ , respectively. The negative slope for most of the redshift range might be expected, as halo concentration is expected to decrease with increasing mass, at least at later redshift (Neto et al. 2007), and we find high mass halos to be more massive in 2LPT than in ZA for all but the highest redshift snapshots. However, the dependence of concentration on mass and redshift at high redshift is more complicated (Klypin et al. 2011; Prada et al. 2012). The data have a larger variance than  $\Delta M_{\text{vir}}$  by a factor of  $\sim 2$ . Again, mass dependence is smallest by  $z = 6$ . To reconcile these trends with the symmetrical concentration distributions of Figure III.2, we note that the trends in mass may be hidden by integration across the entire mass range and still result in overall  $\Delta c$  distributions symmetric about zero.

The slopes of the fits to the  $\Delta q$  vs.  $M_{\text{vir,avg}}$  data are plotted in Figure III.5. Linear least-squares fits are overplotted as red dashed lines. We find a trend for there to be more  $\Delta q$  dependence on  $M_{\text{vir,avg}}$  with increasing redshift, except for the highest  $z$  snapshots, which seem to reverse the trend. Coefficients  $A$  and  $B$  for the fit equation Slope =  $Az + B$  are listed in Table III.2. The data are well-fit by the best fit line for most of the redshift range, except for  $z \gtrsim 14$  for  $\Delta M_{\text{vir}}$  and  $z \gtrsim 13$  for  $\Delta c$ , which begin to deviate from the trend. This may simply be due to the fluctuations inherent when dealing with the low number of matched halos available in our sample at these very high redshifts.

### III.3.5 Fractional differences in halo populations

As our distributions of  $\Delta q$  rely on the average quantity  $q_{\text{avg}} = (q_{\text{2LPT}} + q_{\text{ZA}})/2$  for normalization, it can be difficult to extract certain statistics, such as the fraction of halo pairs differing by a certain amount between 2LPT and ZA simulations. To address this, for this section, we redefine our difference distributions to instead use  $q_{\text{ZA}}$  as the normalization factor (see Equation III.19). In Figure III.6, we plot, as functions of redshift, statistics derived from these alternate fractional difference distributions  $\Delta' M_{\text{vir}}$  and  $\Delta' c$ . In the left column, we plot the  $\Delta' q$  of the peak of the distribution along with the  $\Delta' q$  where various percentages of the halo pairs fall at or above  $\Delta' q$ .

As the  $\Delta' q$  value of peak of the distribution is the location of the mode, it represents the most typical halo pair. While concentration differences remain close to zero throughout the simulation, mass difference peak moves from a  $\Delta' M_{\text{vir}}$  of  $9 \times 10^{-2}$  at  $z = 15$  to  $3 \times 10^{-2}$  at  $z = 6$ . The 1% of halo pairs with the largest excess 2LPT mass have 2LPT mass at least twice ZA mass at  $z = 15$  and 1.5 times ZA mass at  $z = 6$ . For concentration, the 1% most 2LPT concentrated halo pairs differ by at least a factor of 6 at  $z = 15$  and 4 at  $z = 6$ .

In the right column of Figure III.6, we plot the fraction of halos  $f_h$  that fall outside various  $\Delta' q$  values. The solid lines represent halo pairs that have  $\Delta' q$  greater than or equal to the listed values, i.e., the fraction of halo pairs where the 2LPT halo has a virial mass or concentration that is at least 1.1, 1.5, 2.0, or 5.0 times that of its corresponding ZA halo. The dashed lines additionally count halos with  $\Delta' q$  at or below the corresponding equivalent displacement (see Equation III.22) and represent the fraction of halo pairs where one halo has a virial mass or concentration at least 1.1, 1.5, 2.0, or 5.0 times that of its companion, regardless of whether the 2LPT or ZA value is higher.

We find that half of halo pairs are at least 10% more massive in 2LPT at  $z = 15$ . By  $z = 6$ , this has fallen to 10%. Furthermore, 1% are at least twice as massive in

2LPT at  $z = 15$ , and by  $z = 6$ , this has only reduced to 0.3%. Halos in 2LPT are at least twice as concentrated as their ZA counterparts for at least 12% of the halo population at  $z = 15$  and at least 8% by  $z = 6$ . Halo pairs that are at least 5 times as concentrated in 2LPT make up 1.3% of the sample at  $z = 15$  and 0.3% at  $z = 6$ .

If we consider only the difference in properties between paired halos, regardless of whether the 2LPT or ZA halo has the higher mass or concentration, we include an even larger percentage of the population. This equates to additionally considering halo pairs that are less than or equal to the equivalent displacement as defined in Equation III.22. We find 54% of the halo pairs differ in mass by at least 10% at  $z = 15$ , with 16% differing by  $z = 6$ . Halos that are at least twice as massive in either 2LPT or ZA account for 1.1% at  $z = 15$  and 0.5% at  $z = 6$ . Halos that are at least twice as concentrated in either 2LPT or ZA account for 25% at  $z = 15$  and 15% at  $z = 6$ .

### III.4 Discussion

As we evolve our DM halo population from our initial redshift to  $z = 6$ , we find that simulation initialization with 2LPT can have a significant effect on halo population compared to initialization with ZA. The second order displacement boost of 2LPT provides a head start on the initial collapse and formation of DM halos. This head start manifests itself further along in a halo's evolution as more rapid growth and earlier mergers. 2LPT halos are, on average, more massive than their ZA counterparts at a given redshift, with a maximum mean  $\Delta M_{\text{vir}}$  of  $(9.3 \pm 1.2) \times 10^{-2}$  at  $z = 15$ . The larger mass for 2LPT halos is more pronounced for higher mass pairs, while 2LPT halo concentration is larger on the small mass end. Both mass and concentration differences trend towards symmetry about zero as halos evolve in time, with the smallest difference observed at the end of the simulations at  $z = 6$ , with a mean  $\Delta M_{\text{vir}}$  of  $(1.79 \pm 0.31) \times 10^{-2}$ . Casual extrapolation of our observed trends with

redshift to today would indicate that, barring structure like massive clusters that form at high redshift, 2LPT and ZA would produce very similar halo populations by  $z = 0$ . However, the larger differences at high redshift should not be ignored.

The earlier formation times and larger masses of halos seen in 2LPT-initialized simulations could have significant implications with respect to early halo life during the Dark Ages. Earlier forming, larger halos affect the formation of Pop-III stars, and cause SMBHs to grow more rapidly during their infancy (Holley-Bockelmann et al. 2012) and produce more powerful early AGN. The epoch of peak star formation may also be shifted earlier. This could additionally increase the contribution of SMBHs and early star populations to the re-ionization of the universe. Larger early halos may also increase clustering, speed up large scale structure formation, and influence studies of the high- $z$  halo mass function, abundance matching, gas dynamics, and galaxy formation.

In these discussions, it is important to note that it is wrong to assume that the ZA halo properties are the “correct” halo properties, even in a statistical sense. While halo mass suggests the most obvious shortcoming of ZA simulations, even properties such as concentration—that show little difference on average between 2LPT and ZA—can have large discrepancies on an individual halo basis. Failure to consider uncertainties in halo properties for high  $z$  halos in ZA simulations can lead to catastrophic errors.

We note a few caveats with our simulations and analysis. We did not exclude substructure when determining the properties of a halo, and although this would not change the broad conclusions herein, care must be taken when comparing to works which remove subhalo particles in determining halo mass and concentration. Halo matching is not perfect, as it is based on one snapshot at a time, and may miss-count halos due to merger activity and differences in merger epochs. However, we believe this effect to be minor. While we compared ROCKSTAR’s output with our own fitting

routines and found them to broadly agree, ROCKSTAR does not provide goodness of fit parameters for its NFW profile fitting and  $R_s$  measurements. It also may be debated whether it makes sense to even consider concentration of halos at high redshift which are not necessarily fully virialized.

As ROCKSTAR does not provide goodness-of-fit parameters for its internal density profile measurements used to derive concentration, error estimates for concentration values of individual halos are unknown. Additionally, proper density profile fitting is non-trivial, as the non-linear interactions of numerical simulations rarely result in simple spherical halos that can be well described using spherical bins. Halo centering issues may also come into play, although ROCKSTAR does claim to perform well in this regard.

We use a simulation box size of only  $(10 \text{ Mpc})^3$ . This is too small to effectively capture very large outlier density peaks. We would, however, expect these large uncaptured peaks to be most affected by 2LPT initialization, so the effects presented here may even be dramatically underestimated. Additionally, a larger particle number would allow us to consider smaller mass halos than we were able to here, and to better resolve all existing structure. A higher starting redshift could probe the regime where 2LPT initialization contributes the most. It would also be of interest to evolve our halo population all the way to  $z = 0$ . The addition of baryons in a fully hydrodynamical simulation could also affect halo properties. These points may be addressed in future studies.

### III.5 Conclusion

We analyzed three 2LPT and zA simulation pairs and tracked the spherical overdensity dark matter halos therein with the 6-D phase space halo finder code ROCKSTAR to compare the effect of initialization technique on properties of particle-matched dark matter halos from  $z = 300$  to  $z = 6$ . This approach allowed us to directly compare

matching halos between simulations and isolate the effect of using 2LPT over ZA. In summary, we found the following:

- 2LPT halos get a head start in the formation process and grow faster than their ZA counterparts. Companion halos in 2LPT and ZA simulations may have offset merger epochs and differing nuclear morphologies.
- 2LPT halos are, on average, more massive than ZA halos. At  $z = 15$ , the mean of the  $\Delta M_{\text{vir}}$  distribution is  $(9.3 \pm 1.2) \times 10^{-2}$ , and 50% of 2LPT halos are at least 10% more massive than their ZA companions. By  $z = 6$ , the mean  $\Delta M_{\text{vir}}$  is  $(1.79 \pm 0.31) \times 10^{-2}$ , and 10% of 2LPT halos are at least 10% more massive.
- This preference for more massive 2LPT halos is dependent on redshift, with the effect most pronounced at high  $z$ . This trend is best fit by  $\Delta M_{\text{vir}} = (7.88 \pm 0.17) \times 10^{-3}z - (3.07 \pm 0.14) \times 10^{-2}$ .
- Earlier collapse of the largest initial density peaks causes the tendency for more massive 2LPT halos to be most pronounced for the most massive halos, a trend that increases with redshift. We find a trend of  $\Delta M_{\text{vir}} = (1.03 \pm 0.46) \times 10^{-2} \log(M_{\text{vir,avg}}) - (2.6 \pm 3.8) \times 10^{-2}$  for  $z = 10$ . By  $z = 6$ , this has flattened to  $\Delta M_{\text{vir}} = (3.49 \pm 0.99) \times 10^{-3} \log(M_{\text{vir,avg}}) - (6.8 \pm 8.3) \times 10^{-3}$ . As a function of redshift, the slopes of these equations are fit by Slope =  $(9.4 \pm 2.4) \times 10^{-4}z - (1.8 \pm 1.8) \times 10^{-3}$ .
- Halo concentration, on average, is similar for 2LPT and ZA halos. However, even by the end of the dark ages, the width of the  $\Delta c$  distribution— $\sigma_{\Delta c} = 0.551 \pm 0.026$  at  $z = 6$ —is large and indicative of a significant percentage of halos with drastically mismatched concentrations, despite the symmetrical distribution of  $\Delta c$ . At  $z = 15$ , 25% of halo pairs have at least a factor of 2 concentration difference, with this falling to 15% by  $z = 6$ .

- There is a trend for ZA halos to be more concentrated than 2LPT halos at high mass. We find  $\Delta c = -(0.256 \pm 0.093) \log(M_{\text{vir,avg}}) + (2.07 \pm 0.76)$  at  $z = 15$  and  $\Delta c = -(1.10 \pm 0.31) \times 10^{-2} \log(M_{\text{vir,avg}}) - (0.103 \pm 0.026)$  at  $z = 6$ . The slopes of these equations, as a function of redshift, are fit by  $\text{Slope} = -(7.3 \pm 1.9) \times 10^{-3}z + (3.7 \pm 1.4) \times 10^{-2}$ . This is not visible in the symmetrical  $\Delta c$  distributions, as the trends are roughly centered about zero and are washed away when integrated across the entire mass range.

We have found that choice of initialization technique can play a significant role in the properties of halo populations during the pre-reionization dark ages. The early halo growth displayed 2LPT simulations, or conversely the delayed halo growth arising from the approximations made in ZA-initialized simulations, makes careful attention to simulation initialization imperative, especially for studies of halos at high redshift. It is recommended that future  $N$ -body simulations be initialized with 2LPT, and that previous high- $z$  or high-mass halo studies involving ZA-initialized simulations be viewed with the potential offsets in halo mass and concentration in mind.

This work was conducted using the resources of the Advanced Computing Center for Research and Education (ACCRE) at Vanderbilt University, Nashville, TN. We also acknowledge the support of the NSF CAREER award AST-0847696. We would like to thank the referee for helpful comments, as well as the first author's graduate committee, who provided guidance throughout this work.

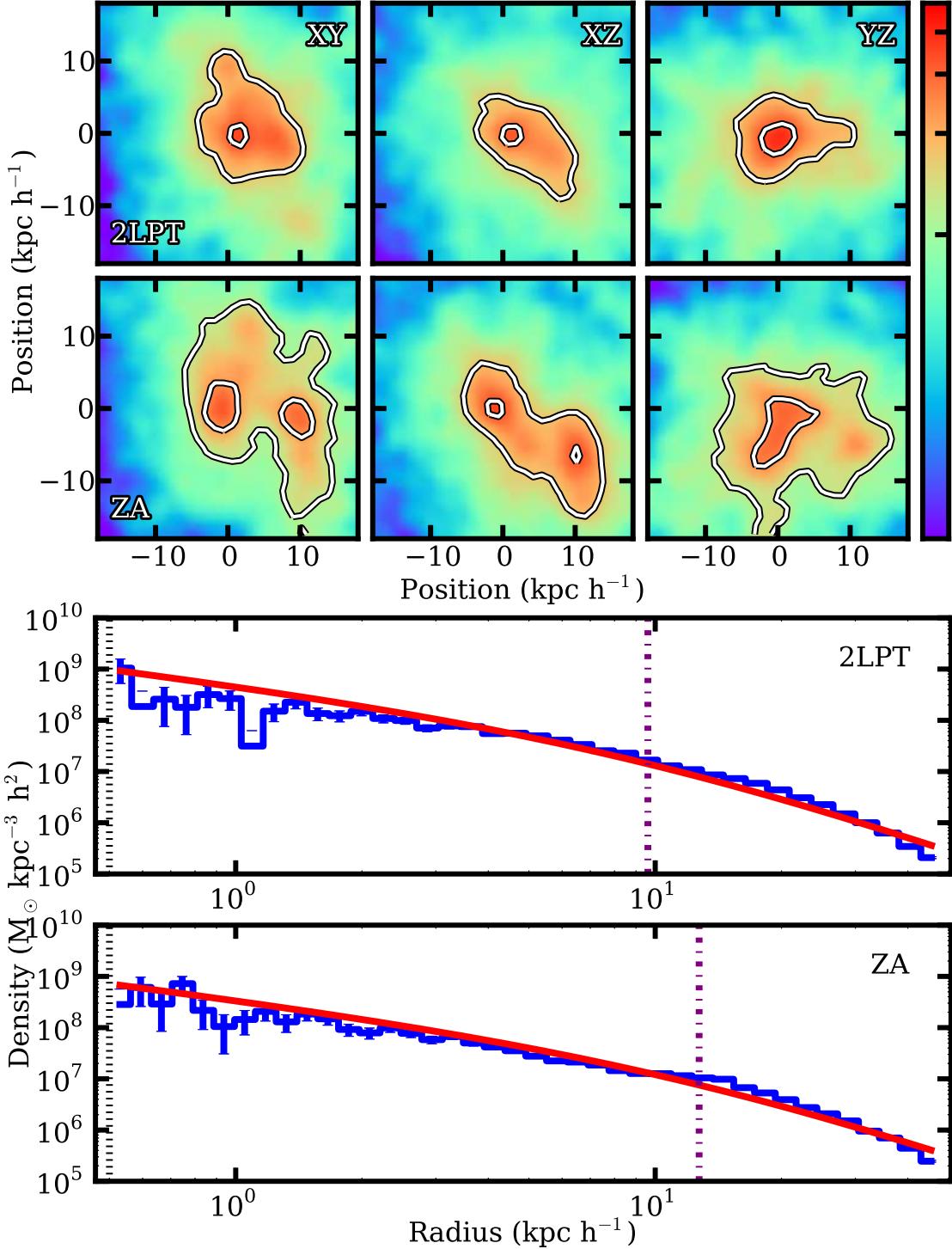
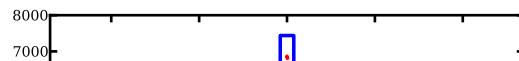
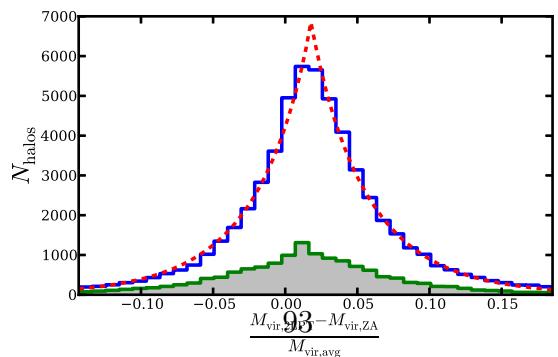
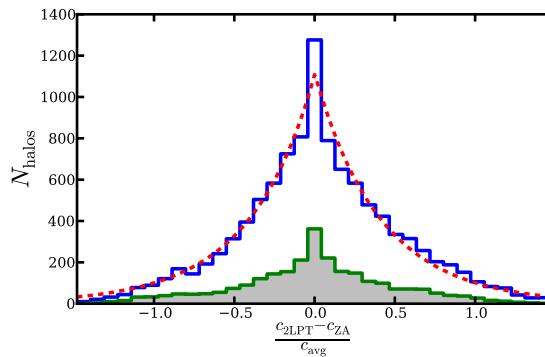
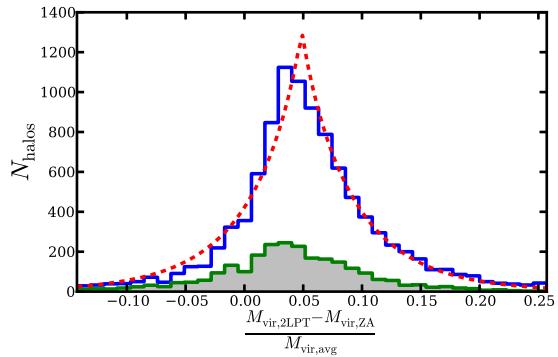
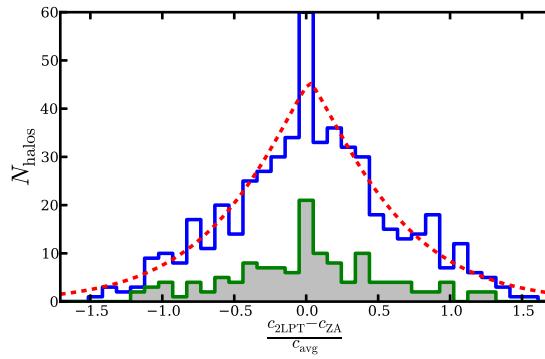
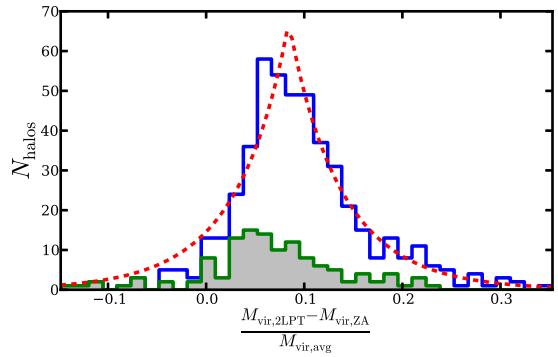


Figure III.1: *Top two rows:* Density projections for two matching halos at  $z = 6$ . The first and second row are 2LPT and ZA, respectively. The halos appear to be either undergoing or have recently undergone a major merger. The 2LPT halo appears to be more relaxed and further along in the merger process, while the ZA halo lags behind, still displaying two distinct cores. The halos have masses of  $5.95 \times 10^9 M_\odot$  for 2LPT and  $5.85 \times 10^9 M_\odot$  for ZA. *Bottom two rows:* Density profiles for the same two halos as above. NFW profiles are fit to logarithmic radial bins of particle position and are overplotted as red curves. The purple dot-dash lines mark the scale radii. The black dotted lines mark the resolution limit of the simulations.



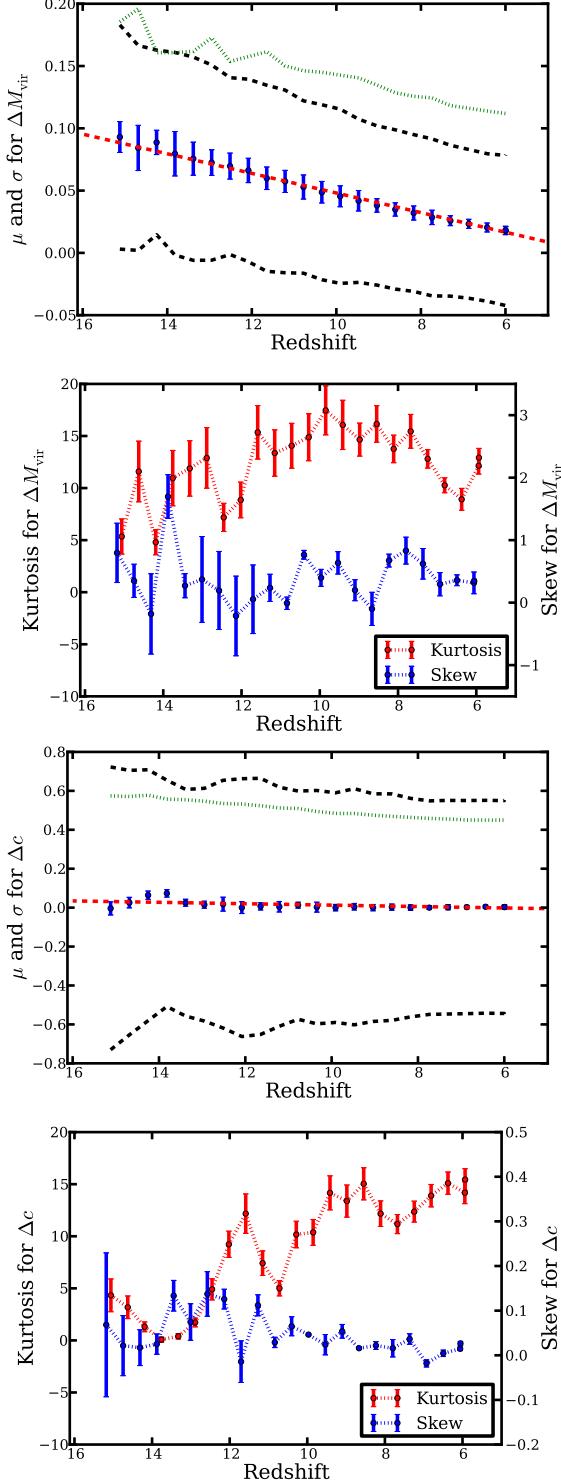
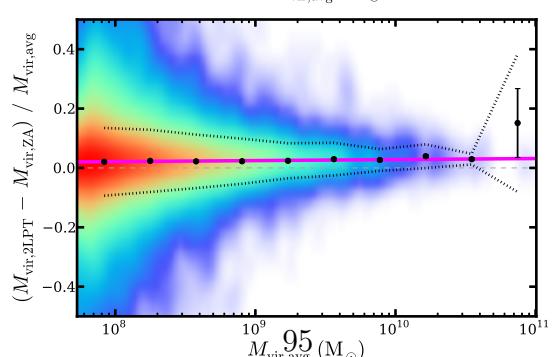
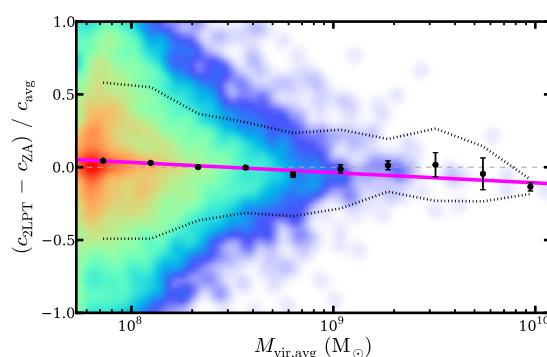
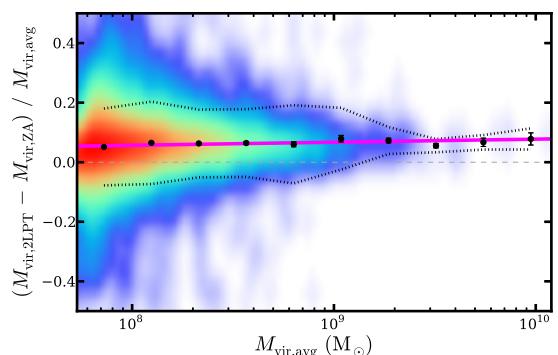
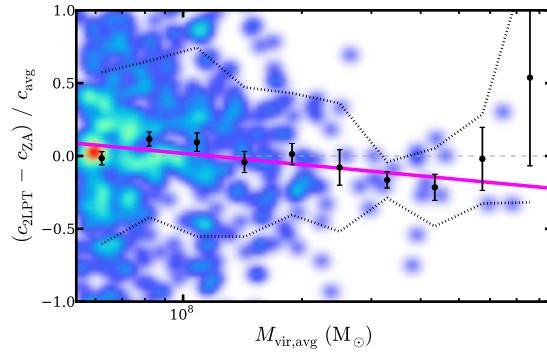
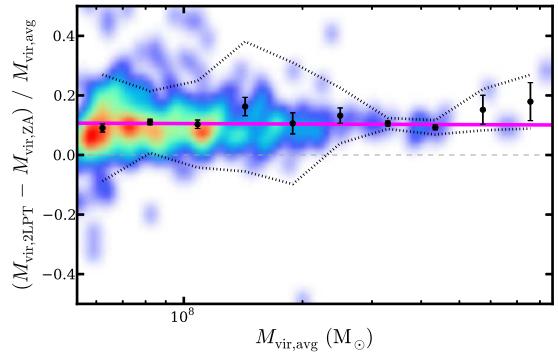


Figure III.3: Mean, standard deviation, and RMS (left column) and skew and excess kurtosis (right column) as functions of redshift for  $\Delta M_{\text{vir}}$  (top row) and  $\Delta c$  (bottom row). In the left column,  $\mu$  is plotted as blue points, and  $\mu \pm \sigma$  is plotted as the black dashed line, and RMS values are plotted as a green dotted line. The red dashed line is a linear fit to the mean. We find a significant trend for  $\mu$  for  $\Delta M_{\text{vir}}$  to be more positive at higher redshift and gradually shift toward zero as the simulation progresses, with a fit function of  $\mu_{\Delta M_{\text{vir}}} = (7.88 \pm 0.17) \times 10^{-3}z - (3.07 \pm 0.14) \times 10^{-2}$ . The mean for  $\Delta c$ , however, remains at or very near zero for most of the simulation and is fit by  $\mu_{\Delta c} = (3.62 \pm 0.95) \times 10^{-3}z - (2.34 \pm 0.84) \times 10^{-2}$ . The  $\Delta M_{\text{vir}}$  and  $\Delta c$  distributions narrow over time, with a slight decrease in  $\sigma$ . In the right column, we plot skew (blue line) and excess kurtosis (red line). Skew is positive for much of the simulation for  $\Delta M_{\text{vir}}$ , but is much smaller for  $\Delta c$ . Kurtosis is large (much more peaked than Gaussian) for both  $\Delta M_{\text{vir}}$  and  $\Delta c$  throughout much of the simulation, and especially at later redshift.



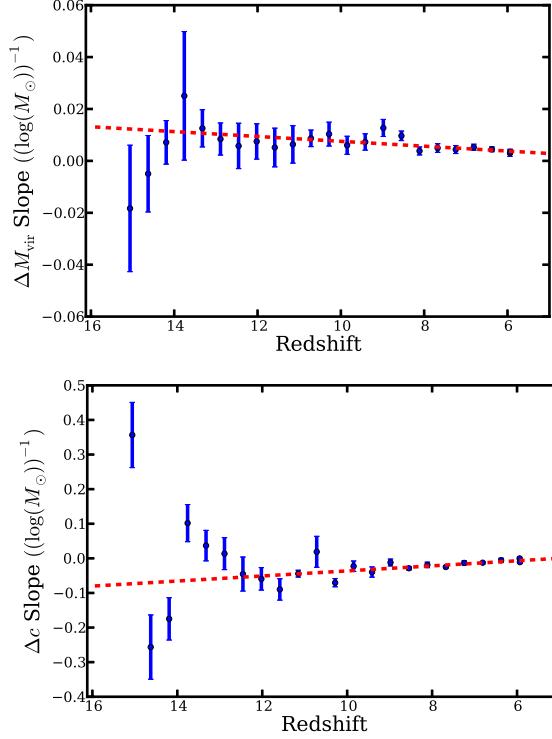


Figure III.5: Slopes of the  $\Delta q$  vs.  $M_{\text{vir,avg}}$  fit functions. The left and right panels correspond to the  $\Delta M_{\text{vir}}$  and  $\Delta c$  plots in the left and right columns, respectively, of Figure III.4. Linear least-squares fits to the data are overplotted as red dashed lines. Overall, we find a trend of positive and increasing slope with redshift for  $\Delta M_{\text{vir}}$  and negative and decreasing slope with redshift for  $\Delta c$ . We find fit equations of  $\text{Slope} = (9.4 \pm 2.4) \times 10^{-4}z - (1.8 \pm 1.8) \times 10^{-3}$  for  $\Delta M_{\text{vir}}$  and  $\text{Slope} = -(7.3 \pm 1.9) \times 10^{-3}z + (3.7 \pm 1.4) \times 10^{-2}$  for  $\Delta c$ . Snapshots at very high redshift,  $z \gtrsim 14$  for  $\Delta M_{\text{vir}}$  and  $z \gtrsim 13$  for  $\Delta c$ , begin to deviate from these trends. However, it is uncertain if this deviation is significant due to the low number statistics of our sample at such high  $z$ .

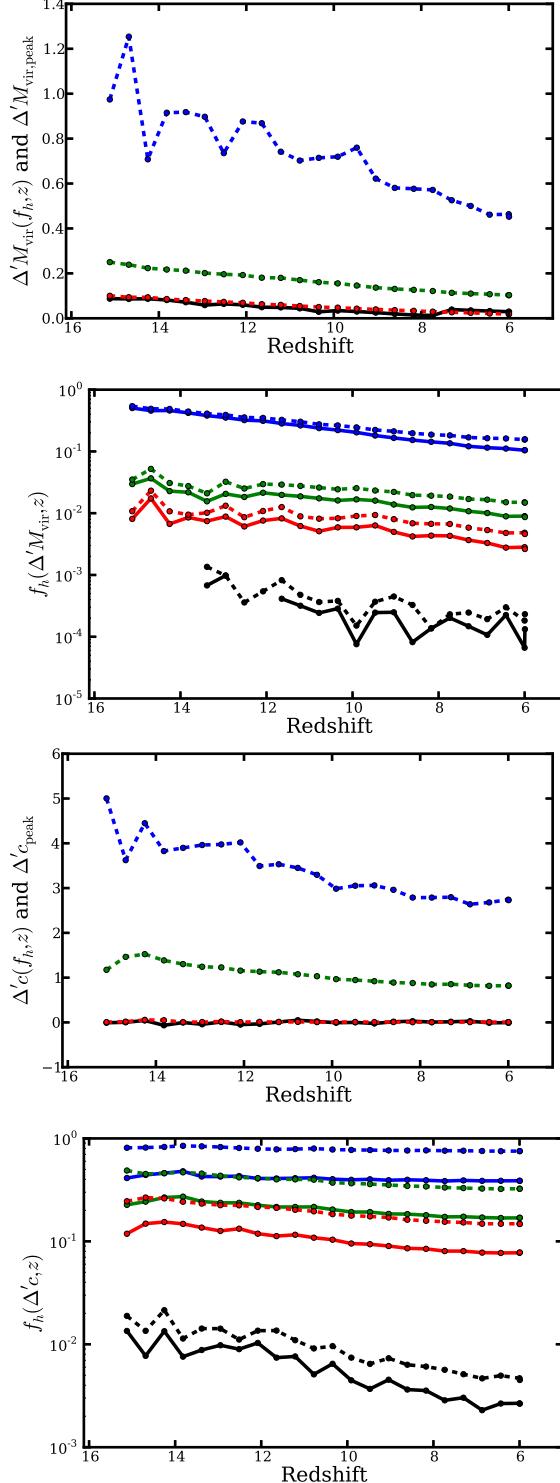


Figure III.6: Fractional error distributions statistics for  $\Delta'M_{\text{vir}}$  (top row) and  $\Delta'c$  (bottom row) as functions of redshift. *Left column:* The  $\Delta'q$  of the peak of the distribution (black line), and the  $\Delta'q$  where 50% (red dashed line), 10% (green dashed line), and 1% (blue dashed line) of the halos fall at or above  $\Delta'q$ . As with distributions of  $\Delta M_{\text{vir}}$ ,  $\Delta'M_{\text{vir}}$  has the largest positive displacement at high redshift and steadily decreases throughout the simulation. Additionally,  $\Delta'c$  maintains a peak near zero and has a spread much larger than that of  $\Delta'M_{\text{vir}}$ . *Right column:* The fraction of halos with  $\Delta'q$  greater than 0.10 (solid blue line), 0.50 (solid green line), 1.00 (solid red line), and 4.00 (solid black line). The dashed lines additionally count halo pairs with  $\Delta'q$  lower than the corresponding equivalent displacements of -0.09, -0.33, -0.50, and -0.80, respectively (see Equation III.22). We find that 50% of 2LPT halos are at least 10% more massive than their ZA companions at  $z = 15$ , reducing to 10% by  $z = 6$ . Halos in 2LPT are at least twice as concentrated for 12% of halos at  $z = 15$  and 7.8% of halos at  $z = 6$ .

## CHAPTER IV

### Supermassive Black Holes and Their Hosts

#### IV.1 Introduction

The study of the evolution of galaxies and the growth of the supermassive black holes at their cores go hand in hand. Although the typical length scales for the two can vary by many orders of magnitude, they seem inexorably linked. Observational correlations between galaxy and supermassive black hole properties hint at an underlying co-evolution driven by shared mechanisms.

##### IV.1.1 Galaxy Properties

How do we describe a galaxy? Being extended, resolvable objects, galaxies provide a unique wealth of observable characteristics not obtainable from point sources such as stars. While many characteristics can be deduced about point sources, the actual observations themselves come down to measuring position on the sky and measuring flux as a function of frequency and time. From this information, all that we know about stars and other point sources, such as temperature, age, size, and composition, can be inferred. However, for extended objects like galaxies, we are given more to work with.

###### IV.1.1.1 Color

A galaxy's color is determined by its stellar component. While a galaxy in itself may be resolvable, for all but the most nearby of galaxies, individual stars are not. What we see when looking at a particular small section of a galaxy is the averaged-together light from stars in that section.

Broadly, bluer late-type spirals have a  $u - r$  color of around 1.3 – 2.0, while redder early-type galaxies have a  $u - r$  color of around 2.3 – 2.7. The color of a galaxy can be a

good indicator for its age and evolutionary stage. Star formation processes generally tend to produce many smaller, cooler, redder stars and fewer larger, hotter, bluer stars. These small, cool stars are much longer-lived than their massive counterparts, while the large, warm stars are much brighter. After star formation turns off, the short-lived blue stars begin to die off, and the galaxy becomes redder, as more of the fraction of total light comes from the red end of the population.

#### IV.1.1.2 Morphology

The extended nature of galaxies allows us to observe their morphology. The classification scheme originally devised by Hubble (1926) places galaxies into the four broad categories: elliptical, spiral, lenticular, and irregular. Elliptical galaxies tend to be larger, redder, have less gas, and dominated by more radial orbits. Spiral galaxies tend to be smaller, bluer, have more gas, and have more of a disk component. Spirals can have a number of arms, a central bulge, and a central bar. Lenticular galaxies are middle-of-the-road galaxies, with both a strong central bulge like an elliptical, and an extended disk like a spiral, however without spiral arms. Irregular galaxies tend to defy this simple classification scheme, and can be found in any number of configurations.

Figure IV.1 is a cartoon of the classification scheme. To the left of the diagram are elliptical galaxies. The subcategories are an indication of the shape of the galaxy, with the most spherical on the left and progressing to more flattened shapes to the right. On the right of the diagram are spiral galaxies. These are broken into two branches, based on whether or not the galaxy contains a central bar. Moving from right to left, the spiral arms of the galaxies become more tightly wound, and the central bulges become more dominant. At the center of the diagram where the spiral fork meets the elliptical line, lie lenticular galaxies. Irregular galaxies are, as the name would imply, irregular and do not fall on the diagram.

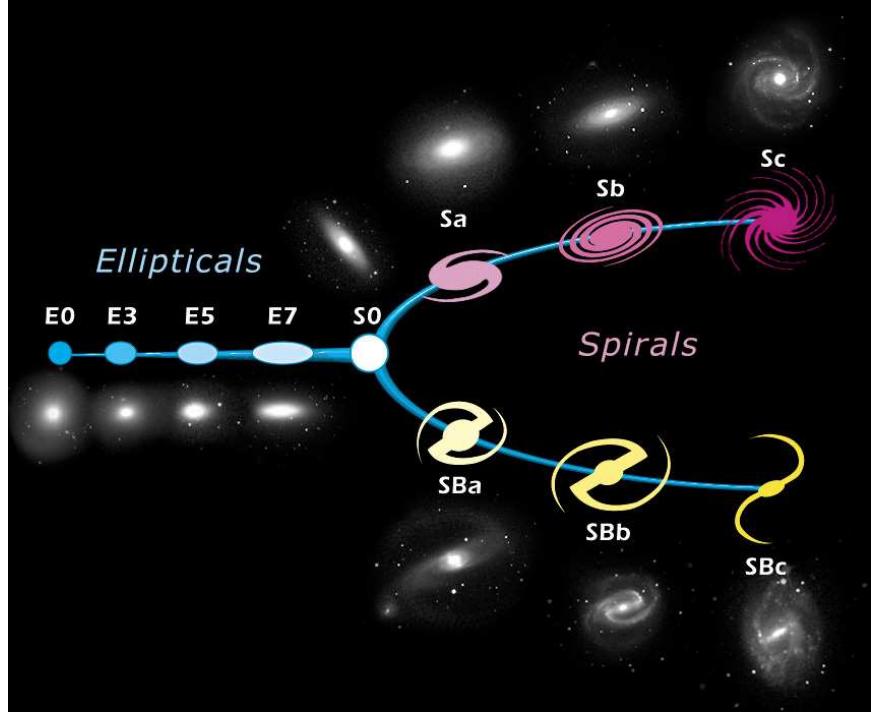


Figure IV.1: The Hubble tuning fork. On the left of the diagram are elliptical galaxies. E0 galaxies are the most spherical, while E7 are the most flattened or elongated. S0 are lenticular galaxies. The top branch on the right are spiral galaxies with no bar, while the bottom right branch are spiral galaxies with a bar. Both progress from tightly wound spiral arms and large bulges to loosely wound spiral arms and small to no bulges, going from Sa to Sc or SBa to SBc.

#### IV.1.2 Supermassive Black Hole Properties

A non-merging black hole, much like an elementary particle, can be described simply by its mass, charge, and spin. Its effect on its local spacetime, infalling matter, and surrounding environment all come back to these three parameters. However, determination of these parameters and the study of how black holes interact with their surroundings can be quite involved.

Black holes are, by their very nature, black, and difficult to observe. We cannot see light emitted directly from a black hole as we would a star, since a black hole is defined as an object massive and compact enough to not allow light within its event horizon to escape. We are forced, therefore, to employ other methods of measuring

black holes.

Thus far, the majority of progress in the measurement of black hole properties has been in measuring mass. There are a number of ways to measure the mass of a black hole. Here, we will briefly discuss masers, stellar dynamics, gas dynamics, and reverberation mapping as methods of measuring a supermassive black hole's mass.

Astrophysical masers are sources of stimulated spectral line emission in the microwave band formed in regions of high-density gas comprised of molecules such as hydroxyl, formaldehyde, and water (Lo 2005). Since the emission frequencies of these sources are very well constrained, high-accuracy Doppler shifts can be determined. These Doppler shifts can then be used to determine velocities for the masers, and thus how much mass is enclosed by their orbits. If these masers lie very close to the supermassive black hole (SMBH) in the center of their galaxy, the enclosed mass can be constrained to be primarily that of the SMBH.

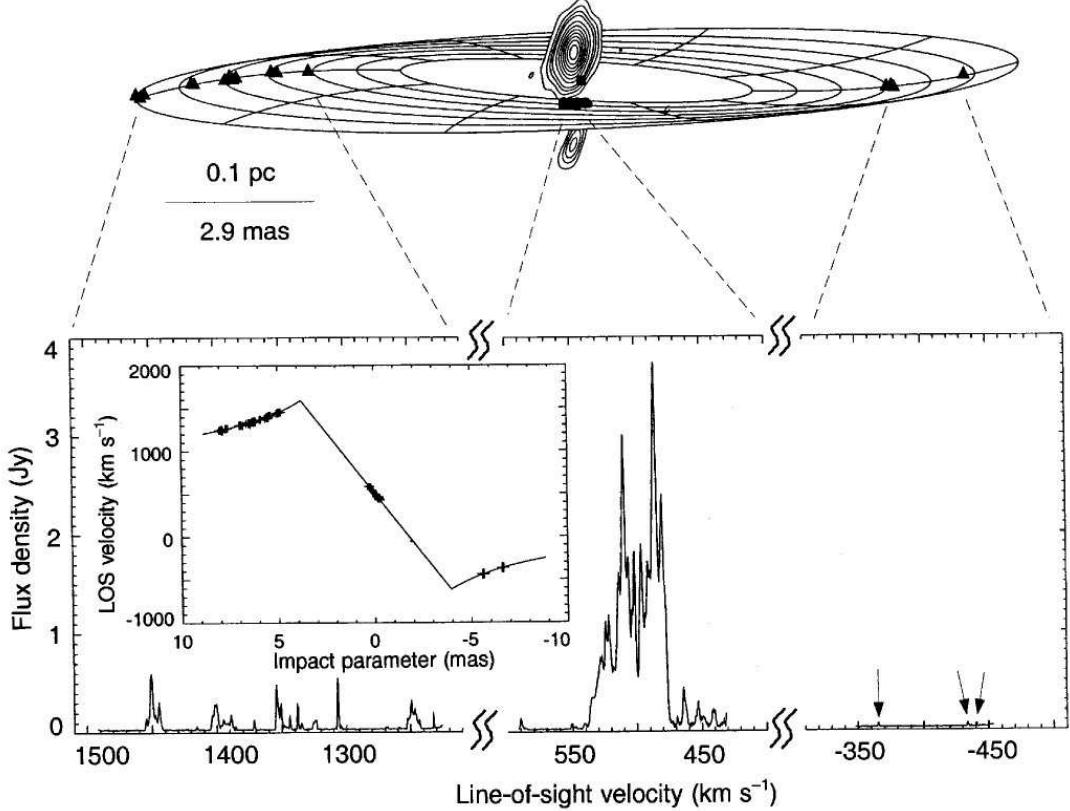


Figure IV.2: Maser orbits fit to a warped disk for NGC4258. Masers can also be useful for distance determinations. Here, the positions and velocities of water masers are able to be fit to a warped disk model surrounding a supermassive black hole. This allows the interpolation of physical radii away from the black hole, giving us both the black hole mass and a standard ruler to allow precise determination of the distance to NGC4258. (Herrnstein et al. 1999)

Stellar dynamics and gas dynamics both probe light coming from matter near the black hole. The width of broadened spectral lines from either the stars or gas can be used to determine a velocity dispersion for the matter local to the SMBH. This velocity dispersion, therefore, can then be used to determine the potential through which the matter is traveling, and thus the mass of the black hole.

A special case of stellar dynamics for which the orbits of the constituent stars can be resolved—namely, for the case of our own Milky Way—adds another dimension to our knowledge of the stellar orbits. Over time, we can observe the proper motion on the sky for these orbits. Combining these measurements with Doppler measurements

for radial velocity yields full orbital solutions. Then, it simply requires Kepler’s laws to determine the mass of the SMBH.

Reverberation mapping can be thought of as “echo-mapping” the gas disk around a SMBH. Continuum emission very near the black hole travels outward and stimulates broad line emission in surrounding gas. Any changes in the continuum emission will take time to propagate to the broad line region, since the speed of light is finite. By measuring the timing difference in the change in continuum emission and change in stimulated broad line emission, the physical distance from the SMBH to the broad line region can be inferred. With this radius, and the velocity of the gas in the broad line region measured by the width of the broadened lines, a black hole mass can be determined (Blandford & McKee 1982).

### IV.1.3 Correlations

Correlations between varying properties of galaxies and black holes can provide much deeper insight into the dynamics that shape the evolution of both. Of particular interest here are the fundamental plane of elliptical galaxies, the  $M - \sigma$  relation, and the green valley-AGN relation.

#### IV.1.3.1 The M-Sigma Relation

If we consider the all the observable properties of a galaxy and compare them to the mass of its SMBH, the tightest correlation can be found with the velocity dispersion  $\sigma$  of the galaxy’s bulge. Such a tight correlation is surprising, as the sphere of influence of a typical SMBH does not extend much past order a few pc, while bulges exist on scales of a kpc or greater. In essence, the supermassive black hole and the outer edges of the bulge shouldn’t “feel” each other. Nevertheless, the correlation is indeed there, suggesting some mechanism that influences—or is influenced by—both of them. Göltekin et al. (2009) use a sample of 49  $M_{BH}$  measurements and 19 upper limits to measure this correlation, and find  $\log(M_{BH}/M_\odot) = \alpha + \beta \log(\sigma/200 \text{ km s}^{-1})$

with  $(\alpha, \beta, \epsilon_0) = (8.12 \pm 0.08 M_\odot, 4.24 \pm 0.41 M_\odot, 0.44 \pm 0.06 M_\odot)$  for all galaxies and  $(\alpha, \beta, \epsilon_0) = (8.23 \pm 0.08 M_\odot, 3.96 \pm 0.42 M_\odot, 0.31 \pm 0.06 M_\odot)$  for ellipticals, where  $\epsilon_0$  is the intrinsic scatter in the relation.

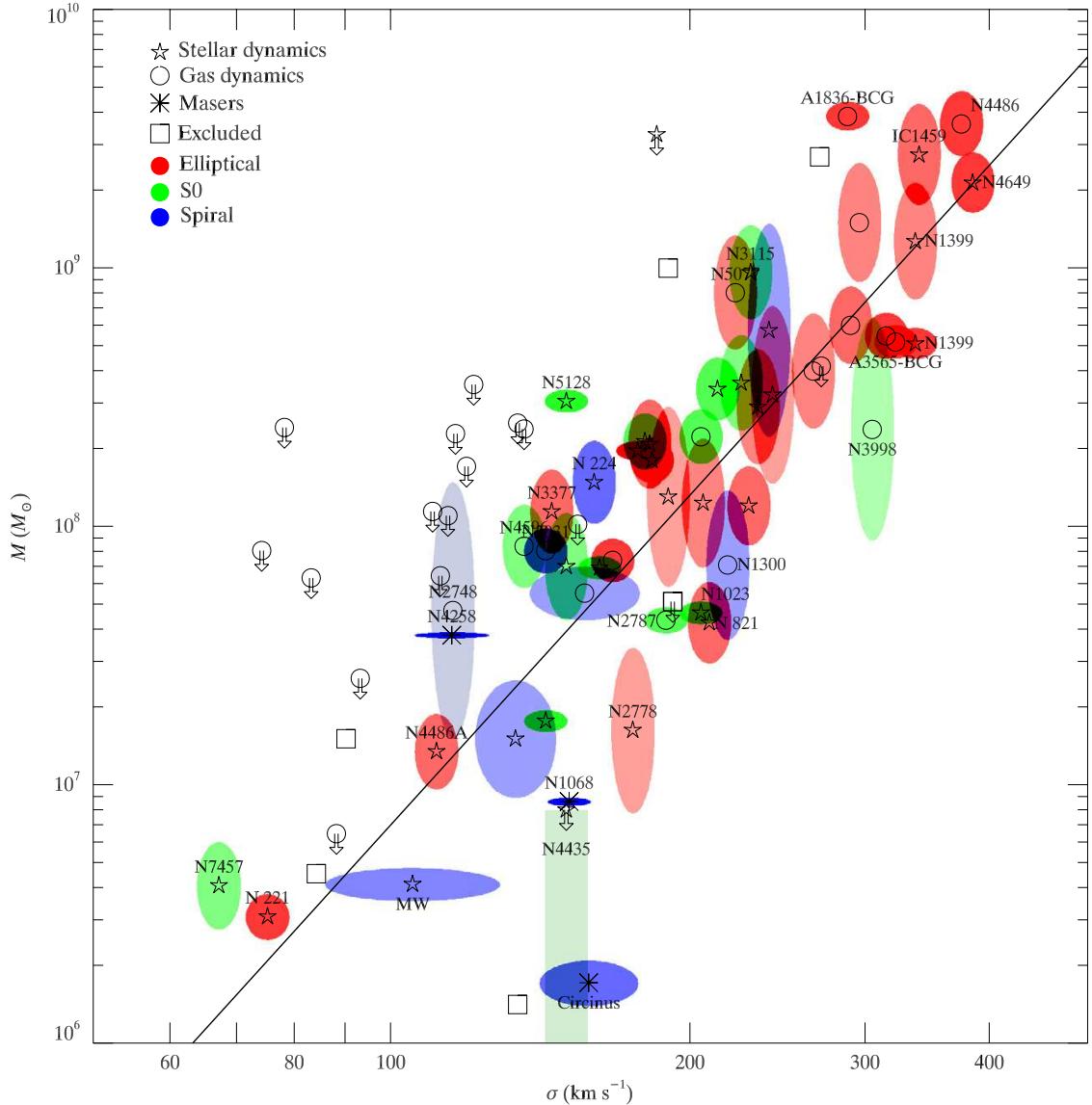


Figure IV.3: The M- $\sigma$  relation for galaxies with dynamical measurements. Black hole mass is plotted vs velocity dispersion of its host spheroid. The symbols represent the method by which the black hole mass was measured: pentagrams for stellar dynamics, circles for gas dynamics, and asterisks for masers. Upper limits are given by arrows. Error ellipses are colored by galaxy type, with red for elliptical galaxies, green for lenticular galaxies, and blue for spiral galaxies. The saturation of the color is inversely proportional to the area of the ellipse. For this sample, the best fit relation is  $M_{BH} = 10^{8.12} M_{\odot} (\sigma / 200 \text{ km s}^{-1})^{4.24}$ . Galaxies not included in this fit are labeled as squares. (Gültekin et al. 2009)

#### IV.1.3.2 The Fundamental Plane

While not a direct correlation with the properties of supermassive black holes, the fundamental plane of elliptical galaxies offers insight into the characteristics of their hosts. The fundamental plane is a three-parameter correlation between properties of elliptical galaxies: velocity dispersion, effective radius, and surface brightness. This correlation (Figure IV.4) between these three parameters is tighter than the combination of any two alone (Djorgovski & Davis 1987). The fit for this correlation can be given as  $\log R_e = 0.36(\langle I \rangle_e / \mu_B) + 1.4 \log \sigma_0$ , where  $R_e$  is the effective radius in kpc,  $\langle I \rangle_e$  is the mean surface brightness interior to  $R_e$  in units of  $\mu_B$ , and  $\sigma_0$  is the velocity dispersion in  $\text{km s}^{-1}$  (Binney & Merrifield 1998).

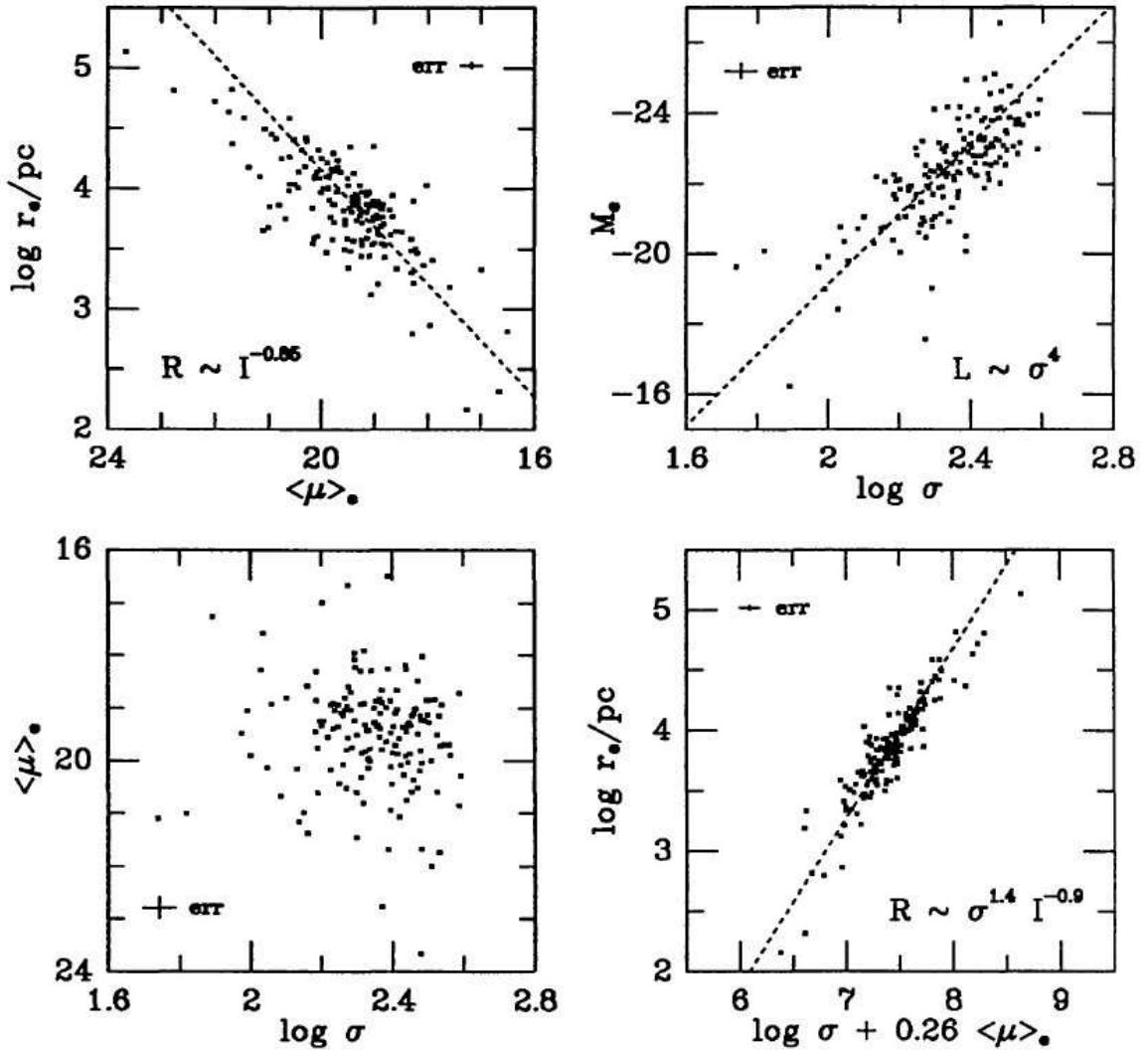


Figure IV.4: The fundamental plane for elliptical galaxies. *Top panels:* The top panels show the one-parameter scaling relations, with the relation between radius and mean surface brightness on the left and the relation between luminosity and velocity dispersion (the Faber-Jackson relation) on the right. *Bottom left:* The relation between the surface brightness and velocity dispersion. This is an almost face-on view of the fundamental plane. *Bottom right:* The relation between the effective radius and the combination of surface brightness and velocity dispersion. This is the edge-on view of the fundamental plane. (Kormendy & Djorgovski 1989)

#### IV.1.3.3 The Green Valley

When considering both the color and stellar mass of a galaxies, a correlation emerges where many galaxies lie in either the “blue cloud” of bluer, lower mass galaxies, or

the “red sequence” of redder, generally higher mass galaxies. The area between these two is known as the “green valley” and, while not as populated as the blue cloud or red sequence, holds special interest when active galactic nuclei (AGN) are considered. AGN are very luminous regions at the centers of some galaxies. Schawinski et al. (2010) show that galaxies falling on the green valley are much more likely to host AGN than galaxies on the blue cloud or red sequence, hinting at an underlying link between the evolution of galaxies, and the activity at their centers.

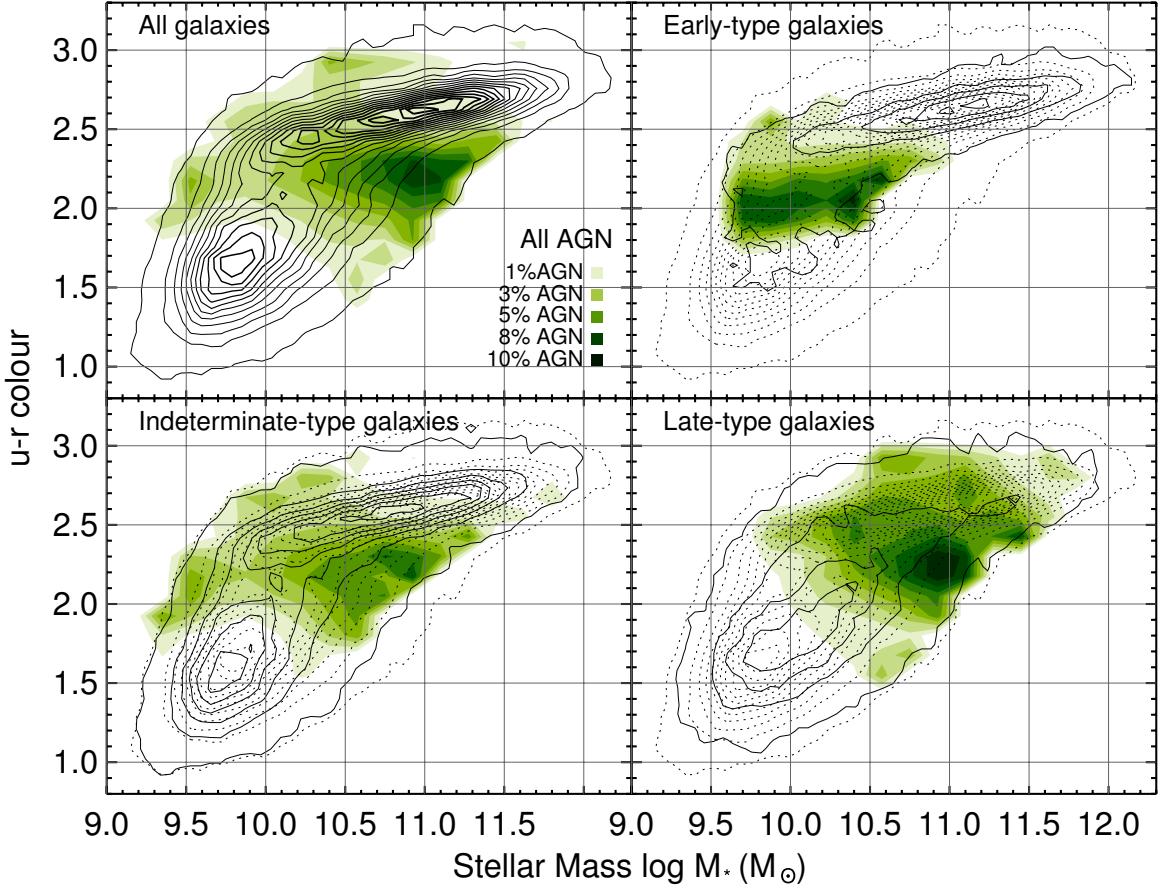


Figure IV.5: Distribution of the fraction of galaxies containing AGN. Galaxy color in  $u-r$  is plotted vs stellar mass. The contours are the galaxy population for all galaxies (top-left), early-type galaxies (top right), intermediate-type galaxies (bottom left), and late type galaxies (bottom right). For the three sub-samples, dotted contours represent the full sample for comparison. The green shaded contours represent the fraction of galaxies in that subsample that contain active galactic nuclei. It can be clearly seen that the AGN fraction is highest for galaxies falling within the green valley. (Schawinski et al. 2010)

## IV.2 Galaxy Evolution

### IV.2.1 Dark Matter Halos

Every galaxy resides inside a dark matter halo. Often about an order of magnitude larger in both radius and mass than the baryonic component, dark matter halos dominate the large-scale behavior of galaxies. Dark matter is matter that is thought to interact very weakly or not at all with light and ordinary matter, except gravita-

tionally. Evidence for dark matter comes from a number of sources, including the relatively flat rotational velocity curve of galaxies, the velocity dispersion of galaxies, gravitational lensing measurements, galaxy clustering, and the offset between the gas and dominant mass measured in the Bullet cluster. Here we will briefly discuss the evidence from flat rotation curves.

If there were no dark matter component and only the baryonic components (i.e. stars and gas) contributed to the galactic potential, we would expect the rotational velocity of galaxies to fall off with radius. However, observations show that the rotation curve remains relatively flat (Rubin et al. 1980). Figure IV.6 shows several observed rotation curves.

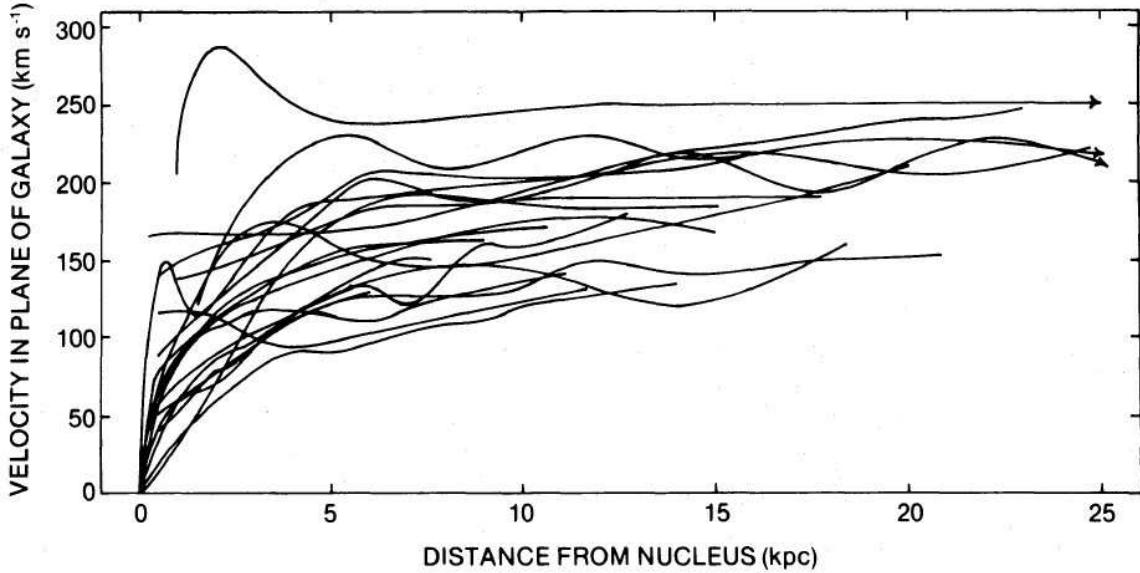


Figure IV.6: Rotation curves for 21 Sc galaxies. It is readily identifiable that the rotation curves do not fall off as would be expected for galaxies without a dark matter component. (Rubin et al. 1980)

Navarro et al. (1997b) found that dark matter halos generally follow the same density profile, regardless of mass. This universal dark matter density profile can be given as

$$\rho(r) \propto \frac{1}{(r/a)(1+r/a)^2}, \quad (\text{IV.1})$$

where  $a$  is the radius where the profile transitions from an  $r^{-1}$  power law to an  $r^{-3}$  power law.

#### IV.2.2 Galaxy Mergers

Galaxy mergers are the fundamental mechanism by which galaxies grow and evolve. Collisions between galaxies trigger processes that can alter nearly all the properties of the galaxies. Naturally, mergers increase the mass of galaxies. Starting from small perturbations in the early universe, gravity slowly pulls matter together to form larger and larger clumps. These clumps of gas and dark matter eventually form stars, beginning what we think of as typical galaxies, and over time, these galaxies merge together into larger and larger galaxies.

Mergers affect many other properties of galaxies as well. Mergers distort the shapes of galaxies, causing long tidal tails to form and the entire morphology to appear irregular. The disk structures of spiral galaxies that form from the settling of the rotational component are distorted and “puffed up” into components with ever increasing bulge-like properties.

Mergers can trigger wide-scale starburst events, where a large portion of gas goes into the formation of stars. Much of the gas component of the galaxy can subsequently be blown out by the winds from the supernovae of short-lived O and B stars. This shuts off star formation, and as the stellar population is no longer replenished with new high-mass stars, the galaxy becomes progressively redder as large stars die.

The general trend is for mergers to move galaxies from the right side of the Hubble tuning fork towards the left, turning blue, gas rich spirals into red, gas poor ellipticals. This process is aided by the AGN feedback also triggered during galaxy mergers, as we discuss in the following section.

## IV.3 Supermassive Black Hole Growth

Supermassive black holes grow by two primary mechanisms, binary mergers and gas accretion. Through a combination of these, black holes can grow to as large as  $\sim 10^9 - 10^{10} M_\odot$  by  $z = 0$ .

### IV.3.1 Binary Mergers

When two galaxies merge, the supermassive black holes at their hearts begin a process that will eventually lead to their coalescence. There are generally thought to be three stages to this journey. First, the black holes sink towards the center of the merged galaxy through mass segregation and dynamical friction until they form a bound orbit with each other. Then, the black holes tighten their orbit through three-body scattering of nearby stars. Finally, as the black holes become close enough together for general relativistic effects to come into play, gravitational waves are emitted and radiate away the remaining orbital energy until the binary coalesces.

#### IV.3.1.1 Dynamical Friction and Inspiral

During the majority of the inspiral process, the black holes do not “feel” each other’s gravitational pull. Instead, interactions with the galaxy itself push the holes together.

As it travels through a galaxy, a black hole—or any massive body—is slowed by the surrounding field of matter. Gravitational attraction pulls surrounding matter toward the black hole. However, as the black hole is moving with respect to the local medium, the attracted particles will tend to fall behind the black hole. This creates a wake of overdensity that gravitationally attracts the black hole from behind and slows its velocity. Chandrasekhar (1943) develops this notion of dynamical friction for the motion of a star through a sea of other stars. If the distribution of velocities of the surrounding particles is Maxwellian, the acceleration on the black hole can be

written as

$$\frac{d\mathbf{v}_M}{dt} = -\frac{4\pi G^2 M \rho \ln \Lambda}{v_M^3} \left[ \text{erf}(X) - \frac{2X}{\sqrt{\pi}} e^{-X^2} \right] \mathbf{v}_M, \quad (\text{IV.2})$$

where  $\mathbf{v}_M$  is the velocity of the black hole,  $M$  is its mass,  $\rho$  is the density of surrounding matter,  $\text{erf}$  is the error function,  $\ln \Lambda$  is the Coulomb logarithm, and  $X \equiv v_M/(\sqrt{2}\sigma)$  where  $\sigma$  is the velocity dispersion of the surrounding medium (Binney & Tremaine 1988). As the black hole is slowed by dynamical friction, it loses angular momentum and sinks towards the center of the galaxy's potential well.

#### IV.3.1.2 The Final Parsec Problem

Dynamical friction and mass segregation can only take us so far. Once the black holes are close enough together, they form a bound binary orbit. This generally occurs for separations of around a few to tens of parsecs. This presents a problem, however, since the orbit needs to shrink to around  $10^{-2}$ – $10^{-3}$  pc in order for gravitational wave emission to remove energy from the orbit in a significant amount. The orbit can be tightened with three-body scattering of stars that wander through the orbit of the binary, however, in the spherical galaxies where mergers often take place, there is a depletion of stars with orbits that intersect the binary. Khan et al. (2011), however, show that the non-spherical, triaxial potential typical of post-merger galaxy remnants can efficiently funnel stars through the orbit of the black hole binary with sufficient intensity to tighten the binary orbit to the gravitational wave regime.

#### IV.3.1.3 Gravitational Waves and Recoil Kicks

Once the black hole binary separation reaches the point where strong field general relativistic effects come into play, we no longer require external influences to nudge the black holes together. In the final plunge toward coalescence, the black hole binary sheds energy through emission of gravitational radiation. As energy is radiated away, the binary tightens its orbit until the two black holes merge into one. Following this

coalescence, the resultant black hole undergoes a “ringdown” phase, in which the distorted space time settles back down into a black hole that can again be simply described by mass, charge, and spin.

The emission of gravitational waves has two interesting consequences. First, the radiation from two merging supermassive black holes is extremely loud, and can potentially provide an observational signature of the process for gravitational wave observatories. Second, the gravitational waves carry linear momentum, leading to a recoil “kick” imparted to the black hole merger remnant.

Recent advances in numerical relativity simulations have provided a much deeper insight into the black hole binary merger process than has been previously available. Waveforms produced from these simulations (Figure IV.7) can be used to predict what gravitational wave observatories such as LIGO and LISA would expect to observe for signals originating from merging supermassive black hole binaries. Having these waveforms as templates for comparison to data can greatly increase the signal to noise ratio for these detectors, potentially allowing the gravitational wave events to be seen among the sea of noise. These waveforms produced from simulations of the last few orbits of inspiral through the merger and ringdown can be combined with waveforms suggested from post-Newtonian approximations for the longer duration inspiral to provide a complete extended signal to match against.

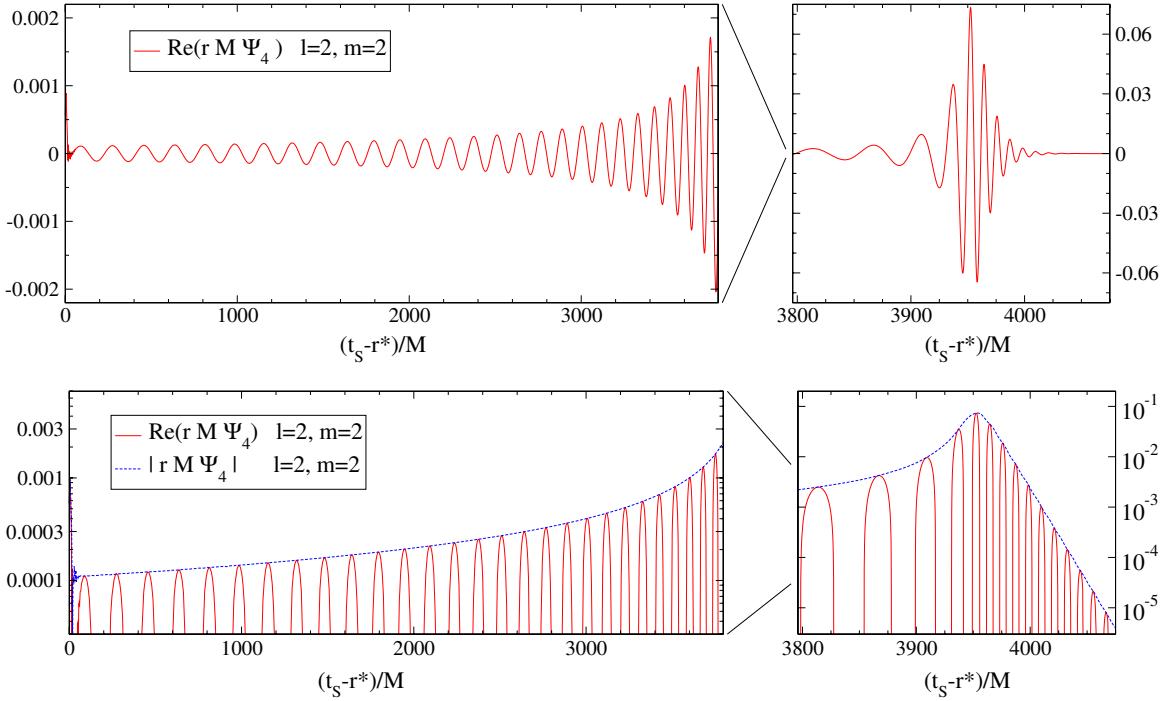


Figure IV.7: Gravitational waveform for an equal-mass, non-spinning black hole binary merger. This is the final waveform, extrapolated to infinity, from the numerical relativity simulation of Scheel et al. (2009). The waveform is shown on the top panel with a linear y-axis and on the bottom panel with a logarithmic y-axis. The left panels are the earlier stages of inspiral, and the right panels show the merger and ringdown stages.

For asymmetric mergers, gravitational radiation is emitted anisotropically. This causes a recoil kick, in which the gravitational waves impart a net velocity to the final black hole with respect to the original center of mass. The magnitude and direction of this kick are dependent on the mass ratio of the binary and the spins of the two black holes—in all, a 7-dimensional parameter space. This large parameter space has been largely explored with numerical relativistic simulations, and analytic equations can be fit to the data to predict the recoil from a given merger configuration. Holley-Bockelmann et al. (2008), give these equations as

$$\mathbf{v}_{kick} = (1 + e) [\hat{\mathbf{x}}(v_m + v_{\perp} \cos \xi) + \hat{\mathbf{y}}v_{\perp} \sin \xi + \hat{\mathbf{z}}v_{\parallel}], \quad (\text{IV.3})$$

where

$$v_m = A \frac{q^2(1-q)}{(1+q)^5} \left[ 1 + B \frac{q}{(1+q)^2} \right], \quad (\text{IV.4})$$

$$v_{\perp} = H \frac{q^2}{(1+q)^5} \left( \alpha_2^{\parallel} - q \alpha_1^{\parallel} \right), \quad (\text{IV.5})$$

$$v_{\parallel} = K \cos(\Theta - \Theta_0) \frac{q^2}{(1+q)^5} \left( \alpha_2^{\perp} - q \alpha_1^{\perp} \right). \quad (\text{IV.6})$$

Here, the fitting constants are  $A = 1.2 \times 10^4 \text{ km s}^{-1}$ ,  $B = -0.93$ ,  $H = (7.3 \pm 0.3) \times 10^3 \text{ km s}^{-1}$ , and  $K = (6.0 \pm 0.1) \times 10^4 \text{ km s}^{-1}$ . The  $\hat{z}$  unit vector is in the direction of the orbital angular momentum, and  $\perp$  and  $\parallel$  refer to components perpendicular and parallel to  $\hat{z}$ , respectively. The fitting parameters are the eccentricity  $e$ , the mass ratio  $q \equiv M_2/M_1$ , and the reduced spin parameters  $\alpha_i \equiv S_i/M_i^2$  where  $S$  is the spin angular momentum. The orientation of the merger is given by the angles  $\Theta$ ,  $\Theta_0$ , and  $\xi$  (Holley-Bockelmann et al. 2008).

Slices through this parameter space are shown in Figure IV.8. For certain configurations of the merger, the recoil velocity can be very high. Very asymmetric mergers can produce recoils as high as  $\sim 4000 \text{ km s}^{-1}$ . These large recoils can be enough for the black hole to escape the potential well of its host galaxy and be ejected. Even less extreme recoil kicks can affect the evolution of black holes, as the kicked black hole can oscillate about its host's center, potentially changing its local gas environment and accretion rate.

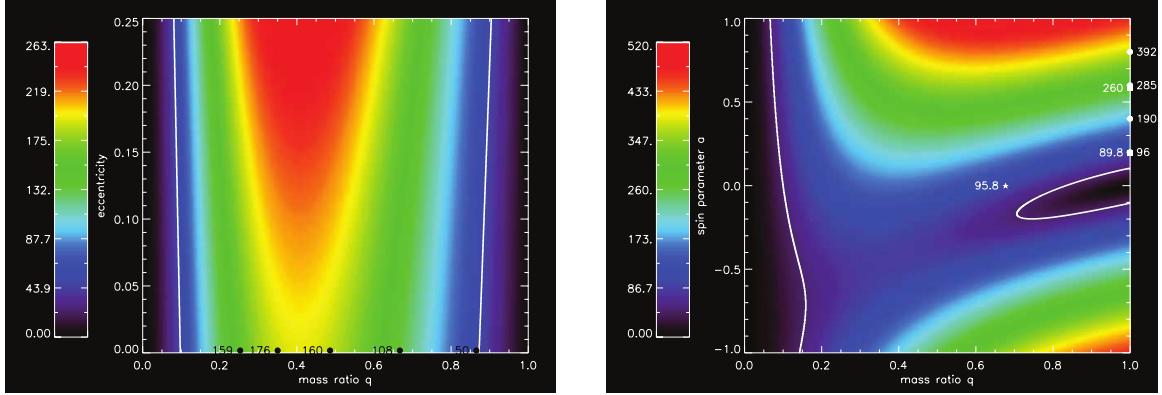


Figure IV.8: *Left:* Gravitaional wave recoil velocity from a merger of nonspinning black holes as a function of eccentricity and mass ratio. Data from numerical relativity simulations (González et al. 2007) are overlaid along the zero eccentricity line. The overlaid white contours are the escape velocity of a typical globular cluster,  $50 \text{ km s}^{-1}$ . *Right:* Gravitational wave recoil kick velocity as a function of spin parameter and mass ratio for a merger of spinning black holes on a circular orbit with spins perpendicular to the orbital plane of the binary and anti-aligned with each other. Again, the  $50 \text{ km s}^{-1}$  escape velocity of a globular cluster is overlaid as white contours. Results from numerical relativity simulations are over-plotted: squares for Koppitz et al. (2007), cirlces for Herrmann et al. (2007), and star for Brügmann et al. (2004). (Holley-Bockelmann et al. 2008)

### IV.3.2 Accretion

Although mergers play an important role in the evolution of supermassive black holes, gas accretion can often dominate in terms of mass growth. Gas can fall into a black hole in a number of ways. Here, we will discuss accretion onto a moving black hole, spherical accretion onto a stationary black hole, and disk accretion onto a stationary black hole.

#### IV.3.2.1 Bondi-Hoyle-Lyttleton Accretion

Let us first consider a massive object, in this case our black hole, moving through a uniform density gas medium. Just as in the case of dynamical friction, particles close enough to the black hole will feel a gravitational attraction, causing them to move toward the black hole. As they move closer, the black hole is also moving through

the medium, causing the gas particles to focus behind the black hole. As the particle stream reaches the wake directly behind the black hole, it collides with opposing streams, causing the angular momentum to go to zero. If these particles are bound, they will proceed to fall onto the black hole. Hoyle & Lyttleton (1939) derive an impact parameter for which particles will be accreted,

$$\sigma < \sigma_{HL} = \frac{2GM}{v_\infty^2}, \quad (\text{IV.7})$$

and a mass accretion from the wake column at a rate of

$$\dot{M}_{HL} = \pi \sigma_{HL}^2 v_\infty \rho_\infty = \frac{4\pi G^2 M^2 \rho_\infty}{v_\infty^3}, \quad (\text{IV.8})$$

where  $v_\infty$  and  $\rho_\infty$  are the velocity and density far away from the black hole, respectively. Expanding upon this analysis, Bondi & Hoyle (1944) suggest that the accretion rate should rather be

$$\dot{M}_{BH} = \frac{2\alpha\pi G^2 M^2 \rho_\infty}{v_\infty^3}, \quad (\text{IV.9})$$

where  $\alpha$  is a constant between 1 and 2, with a typical value of around 1.25.

For an accretor at rest in an isotropic gas medium, one would expect accretion to be a spherical process. Bondi (1952) considers this configuration, and finds the accretion rate for this “temperature-limited” case to be

$$\dot{M}_{Bondi} = \frac{2\pi G^2 M^2 \rho_\infty}{c_{s,\infty}^3}, \quad (\text{IV.10})$$

where  $c_{s,\infty}$  is the speed of sound far away from the black hole.

Extrapolating between this result and the “velocity-limited” case of Equation IV.9 suggests (Bondi 1952)

$$\dot{M}_{BH} = \frac{2\pi G^2 M^2 \rho_\infty}{(c_{s,\infty}^2 + v_\infty^2)^{3/2}} \quad (\text{IV.11})$$

as an order of magnitude estimate of the more general case of accretion. Numerical simulations (Shima et al. 1985) suggest an additional factor of 2 is needed for better agreement with simulation results, giving us a generally applicable formula for the accretion rate,

$$\dot{M}_{BH} = \frac{4\pi G^2 M^2 \rho_\infty}{(c_{s,\infty}^2 + v_\infty^2)^{3/2}}. \quad (\text{IV.12})$$

#### IV.3.2.2 Disk Accretion and Active Galactic Nuclei

Active galactic nuclei play a fundamental role in the evolution of both supermassive black holes and their host galaxies. As gas falls in to a black hole in the center of a galaxy, its angular momentum forces it into an accretion disk. As matter moves towards the SMBH, it transfers its gravitational potential energy to thermal energy. For accretion disks around supermassive black holes, this can cause the disk to emit large amounts of electromagnetic radiation (Lin & Papaloizou 1996).

This emitted radiation is important in a number of ways. Most critical to the SMBH itself is the radiation pressure exerted on infalling matter. This radiation pressure sets an upper limit on the rate of accretion, as there is a point where the force from emitted radiation balances the force of gravity for infalling gas (Rybicki & Lightman 1979). This limit, known as the Eddington limit, is given by

$$L_{Edd} = 4\pi GMcm_H/\sigma_T = 1.25 \times 10^{38} \text{erg s}^{-1}(M/M_\odot), \quad (\text{IV.13})$$

where  $c$  is the speed of light,  $m_H$  is the mass of hydrogen, and  $\sigma_T$  is the Thompson cross section.

The radiation given off by the accretion disk affects galactic properties as well. Powerful AGN can strip away gas from the center of the galaxy, halting star formation. This can quickly change a galaxy from a blue, gaseous, star forming galaxy into one that is red, dry, and dead.

## IV.4 Conclusion

We have seen that galaxies and the supermassive black holes at their centers both have their most dramatic periods of evolution around the same time. Galaxy mergers grow both the galaxy and the SMBH. Galaxies grow and become more elliptical as mergers bring in additional mass on orbits that can disrupt their gaseous disks. These mergers also bring in counterpart supermassive black holes that fall toward the center of the galaxy and merge with the central SMBH, while also triggering accretion events and AGN feedback that pump energy back into the galaxy, shutting off star formation.

### IV.4.1 Correlations

In light of these shared growth mechanisms, the correlations mentioned in Section IV.1 begin to move from a purely observational coincidence to a natural result of co-evolution. The  $M-\sigma$  relation is a natural byproduct of the simultaneous growth of supermassive black holes and their galaxies during merger events. The mass of the SMBH increases due to the merging of binary companions and increased levels of accretion, while the host mass, and thus velocity dispersion, increases due to the infalling galaxy itself. Likewise, the overabundance of AGN in galaxies lying in the green valley is the consequence of simultaneous change. Mergers both trigger highly luminous AGN feedback and cause an inexorable shift from the blue cloud, through the green valley, to the red sequence. Even the increase in scatter of the  $M-\sigma$  relation at low masses can be explained by the galaxies having lower mass, and therefore being more likely to allow a gravitational wave recoil kicked black hole of a given velocity to escape.

### IV.4.2 Open Questions

In the end, there remain a number of open questions. How can very large supermassive black holes form so early? What is dark matter actually made of? How do galaxies retain their black holes if merger recoils can kick them with velocities greater than the

escape velocity of the galaxy? Over what range are our correlations truly valid? These are just some of the questions that are currently being investigated, and promise to provide a rich field of study for years to come.

## **CHAPTER V**

### **Conclusion**

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# Appendices

## Appendix A

### Rockstar Configuration and Execution

#### A.1 Single Node Configuration File (Text)

```
1 #Rockstar Halo Finder
2 #Parallel config file for multi-cpu, multi-snapshot halo finding
3 #Note that periodic boundary conditions are assumed for NUM_WRITERS > 1.
4 #See README for details.
5
6 #Once compiled ("make"), run Rockstar server as
7 # ./rockstar -c parallel.cfg
8 #Then launch the reading/analysis tasks with:
9 # ./rockstar -c auto-rockstar.cfg
10 #You will have to launch at least NUM_BLOCKS+NUM_WRITERS processes.
11
12 FILE_FORMAT = "GADGET2" # or "ART" or "ASCII"
13 PARTICLE_MASS = 0 # must specify (in Msun/h) for ART or ASCII
14
15 # You should specify cosmology parameters only for ASCII formats
16 # For GADGET2 and ART, these parameters will be replaced with values from the
17 # particle data file
18 SCALE_NOW = 1
19 h0 = 0.7
20 O1 = 0.73
21 Om = 0.27
22
23 # For GADGET2, you may need to specify conversion parameters.
24 # Rockstar's internal units are Mpc/h (lengths) and Msun/h (masses)
25 GADGET_LENGTH_CONVERSION = 1e-3
26 GADGET_MASS_CONVERSION = 1e+10
27
28 # This specifies the use of multiple processors:
29 PARALLEL_IO = 1
30
31 # Output full particle information as well as halos for N number of procs
32 FULL_PARTICLE_CHUNKS = 0
33
34 # This should be less than 1/5 of BOXSIZE
35 OVERLAP_LENGTH = 1.5
36
37 # This specifies how many CPUs you want to analyze the particles:
38 NUM_WRITERS = 8
39
40 # Calculate radii and other halo properties using unbound (0) or only bound (1) particles (default 1)
41 BOUND_PROPS = 0
42
43 # This sets the virial radius/mass definition ("vir", "XXXc", or "XXXb")
44 MASS_DEFINITION = "vir"
45
46 # This specifies the I/O filenames:
47 OUTBASE = "halos"
48 INBASE = "particles"
49 NUM_SNAPS = 1
50 NUM_BLOCKS = 1
51 #BGC2_SNAPNAMES = "snapnames.lst"
52 #FILENAME = "particles_<snap>.<block>.dat"
```

#### A.2 PBS Submission Script (Bash)

```
1#!/bin/sh
2#PBS -M djsissom@gmail.com
3#PBS -m bae
4#PBS -l nodes=1:ppn=10
5#PBS -l pmem=3000mb
6#PBS -l mem=30000mb
7#PBS -l walltime=0:30:00
8#PBS -o out.log
9#PBS -j oe
10
11# Change to working directory
12echo $PBS_NODEFILE
13cd $PBS_O_WORKDIR
14
15# Start the server
16rockstar -c onenode.cfg &> server.out &
17
18# Wait for auto-rockstar.cfg to be created
19perl -e 'sleep 1 while (!(-e "halos/auto-rockstar.cfg"))'
20mv halos/auto-rockstar.cfg .
21
22# Execute the reader processes
23mpicexec -verbose -n 1 rockstar -c auto-rockstar.cfg >> clients.out 2>&1 &
24sleep 20
```

```

25
26 # Execute the analysis processes
27 mpixexec -verbose -n 8 rockstar -c auto-rockstar.cfg >> clients.out 2>&1
28
29 # - end of script

```

### A.3 Post-Processing Script (Bash)

```

1 #!/bin/bash
2
3 echo 'running finish_bgc2...'
4 ~/projects/programs/nbody/rockstar/Rockstar-0.99.9/util/finish_bgc2 -c onenode.cfg -s 0
5
6 echo 'running bgc2_to_ascii...'
7 ~/projects/programs/nbody/rockstar/Rockstar-0.99.9/util/bgc2_to_ascii -c onenode.cfg -s 0 > halos/all_halos.bgc2.
     ascii
8
9 echo 'running find_parents...'
10 ~/projects/programs/nbody/rockstar/Rockstar-0.99.9/util/find_parents halos/out_0.list 10.0 > halos/out_0.list.
     parents
11
12 echo 'finished'

```

## Appendix B

### CrossMatch Modifications and Configuration

#### B.1 2lpt First Configuration File (Text)

```
1 MIN_SNAPSHOT_NUM 0
2 MAX_SNAPSHOT_NUM 0
3
4 MAX_RANK_LOC      0
5
6 OUTBASE           crossmatch_2lpt_first
```

#### B.2 za First Configuration File (Text)

```
1 MIN_SNAPSHOT_NUM 0
2 MAX_SNAPSHOT_NUM 0
3
4 MAX_RANK_LOC      0
5
6 OUTBASE           crossmatch_za_first
```

## Appendix C

### BGC2 Import Code (Python)

```
1 #!/usr/bin/env python
2
3 import sys
4 import struct
5
6 def read_bgc2(filename):
7     offset = 4
8     groupoffset = 8
9     particleoffset = 8
10
11    headersize = 1024
12    groupsize = 4*8 + 10*4
13    particlesize = 1*8 + 6*4
14
15    headerformat = '=Q\u16q\u19d'
16    groupformat = '=2qu2Q\u10f'
17    particleformat = '=q\u6f'
18
19    print "Reading"+filename+"..."
20    fd = open(filename, 'rb')
21    bin_string = fd.read()
22    fd.close()
23    print "Finished reading file."
24    bin_string = bin_string[offset:]
25
26    # Header stuff
27    header_bin = bin_string[:headersize]
28    header_pad = headersize - 36*8
29    header = list(struct.unpack(headerformat, header_bin[:-header_pad]))
30
31    # Group stuff
32    ngroups = header[8]
33    print 'ngroups=%u', ngroups
34    groupstart = headersize + groupoffset
35    groupend = groupstart + ngroups*groupsize
36    group_bin = bin_string[groupstart:groupend]
37    group = []
38    for i in range(ngroups):
39        group.append(list(struct.unpack(groupformat, group_bin[i*groupsize:(i+1)*groupsize])))
40
41    # Particle stuff
42    particlestart = headersize + groupoffset + ngroups*groupsize + particleoffset
43    particle_bin = bin_string[particlestart:]
44    particle = []
45    p_start = 0
46    for i in range(ngroups):
47        npart = group[i][2]
48        particle.append([])
49        for j in range(npart):
50            particle[i].append(list(struct.unpack(particleformat, particle_bin[p_start:p_start+particlesize])))
51            p_start += particlesize
52        p_start += particleoffset
53
54    print "Finished parsing bgc2 file"
55    return header, group, particle
56
57
58 def main():
59     header, group, particle = read_bgc2(sys.argv[1])
60
61     print 'Header contents:'
62     for value in header:
63         print value
64     print
65
66     print 'Group[0] contents:'
67     for value in group[0]:
68         print value
69     print
70
71     print 'Particles in group[0]:'
72     for part in particle[0]:
73         print part
74     print
75
76     print 'Group[1] contents:'
77     for value in group[1]:
78         print value
79     print
80
81     print 'Particles in group[1]:'
82     for part in particle[1]:
```

```
83     print part
84
85
86
87
88
89 if __name__ == '__main__':
90     main()
```

## Appendix D

### Density Profile Code (Python)

```
1 #!/usr/bin/env python
2
3 import sys
4 import bgc2
5 import numpy as np
6 import matplotlib.pyplot as plt
7 from matplotlib.ticker import MultipleLocator
8 from scipy.optimize import curve_fit
9 from scipy.stats import chisquare
10
11 #read_mode = 'ascii2'
12 read_mode = 'bgc2'
13
14 if read_mode == 'bgc2':
15     use_bgc2 = True
16     use_all = False
17     individual_masses = False
18     halo_id = 146289
19     nbins = 50
20     nfit = 500
21     ooms = 3.0
22     mass_scale = 1.0
23     common_mass = 5.33423e5
24     dist_scale = 1.0e3
25     #res_limit = 0.488
26     #res_limit = 4.0
27     res_limit = 0.5
28     #res_limit = 10.0
29     draw_frac = 0.1
30     tick_base_major = 100.0
31     tick_base_minor = 10.0
32     find_com = False
33 elif read_mode == 'ascii':
34     use_bgc2 = False
35     use_all = True
36     individual_masses = True
37     halo_id = 0
38     nbins = 100
39     nfit = 500
40     ooms = 5.0
41     mass_scale = 1.0e12
42     dist_scale = 200.0
43     res_limit = 1.0e-2
44     draw_frac = 2.0e-4
45     tick_base_major = 80.0
46     tick_base_minor = 20.0
47     find_com = True
48 elif read_mode == 'ascii2':
49     use_bgc2 = False
50     use_all = True
51     individual_masses = True
52     halo_id = 0
53     nbins = 100
54     nfit = 500
55     ooms = 3.5
56     mass_scale = 1.0e10
57     dist_scale = 1.0
58     #res_limit = 3.0e-1
59     res_limit = 1.0
60     draw_frac = 1.0e-2
61     tick_base_major = 200.0
62     tick_base_minor = 40.0
63     find_com = True
64 else:
65     sys.exit(98712)
66
67 #outfile = 'asciitest_halo_properties.txt'
68 outfile = 'density_profile_halos.dat'
69 comfile = 'center_of_mass.txt'
70
71 make_plot = False
72 #make_plot = True
73 draw_density = True
74 #plot_base = 'asciitest_density_profile.fig.'
75 plot_base = 'figure_'
76 plot_ext = '.eps'
77 dist_units = 'kpc'
78 xlabel_proj = r'X\u20d7Position\u20d7(%s\u20d7h\u20d7{-1})\u20d7' % (dist_units), r'X\u20d7Position\u20d7(%s\u20d7h\u20d7{-1})\u20d7' % (dist_units), r'Y\u20d7
    Position\u20d7(%s\u20d7h\u20d7{-1})\u20d7' % (dist_units)]
79 ylabel_proj = [r'Y\u20d7Position\u20d7(%s\u20d7h\u20d7{-1})\u20d7' % (dist_units), r'Z\u20d7Position\u20d7(%s\u20d7h\u20d7{-1})\u20d7' % (dist_units), r'Z\u20d7
    Position\u20d7(%s\u20d7h\u20d7{-1})\u20d7' % (dist_units)]
80 xlabel_prof = r'Radius\u20d7(%s\u20d7h\u20d7{-1})\u20d7' % (dist_units)
```

```

81 ylabel_prof = r'Density_(M_{\odot}\%s^{-3}h^2)', % (dist_units)
82 npixels = 50
83
84 #common_mass = 1.0e-7
85 #common_mass = 1.0e5
86 mass_col = 0
87 pos_cols = (1,2,3)
88 vel_cols = (4,5,6)
89 halo_id_col = 0
90
91 grav_const = 4.3e-6 # kpc M_sun^-1 (km/s)^2
92
93
94 def read_files(files):
95     data = 0
96     for file in files:
97         print 'Reading file %s...' % (file)
98         if data == 0:
99             data = np.genfromtxt(file, comments='#')
100        else:
101            data = np.append(data, np.genfromtxt(file, comments='#'), axis=0)
102    print 'Finished reading files.'
103    return data
104
105
106 def my_chisq(ydata, ymod, deg=2, sd=None):
107     """
108     Returns the reduced chi-square error statistic for an arbitrary model,
109     chisq/nu, where nu is the number of degrees of freedom. If individual
110     standard deviations (array_usd) are supplied, then the chi-square error
111     statistic is computed as the sum of squared errors divided by the standard
112     deviations. See http://en.wikipedia.org/wiki/Goodness_of_fit for reference.
113
114     ydata, ymod, sd assumed to be Numpy arrays. deg integer.
115
116     Usage:
117     >>> chisq=redchisq(ydata, ymod, n, sd)
118     where
119     ydata: data
120     ymod: model evaluated at the same x points as ydata
121     n: number of free parameters in the model
122     sd: uncertainties in ydata
123
124     Rodrigo Nemmen
125     http://goo.gl/8S10o
126     """
127     # Chi-square statistic
128     if sd==None:
129         chisq=np.sum((ydata-ymod)**2)
130     else:
131         chisq=np.sum( ((ydata-ymod)/sd)**2 )
132
133     # Number of degrees of freedom assuming 2 free parameters
134     nu=ydata.size-1-deg
135     return chisq/nu
136
137
138 def calc_m_enclosed(mass, pos):
139     r = np.sqrt(pos[:,0]**2 + pos[:,1]**2 + pos[:,2]**2)
140     r = np.sort(r)
141     first_good_bin = 0
142     for i in range(len(r)):
143         if r[i] > res_limit:
144             first_good_bin = i
145             break
146     print 'r1=%', r[first_good_bin-1]
147     print 'r2=%', r[first_good_bin]
148     print 'r3=%', r[first_good_bin+1]
149     m_extra = mass[0] * first_good_bin
150     r = r[first_good_bin:]
151     #m_enclosed = np.zeros(len(r))
152     #for i in range(len(r)):
153     #    m_enclosed[i] = mass[0] * (i + 1.0)
154     m_enclosed = (np.arange(len(r)) + 1.0) * mass[0] + m_extra
155     return r, m_enclosed
156
157
158 def calc_density_profile(mass, pos):
159     r = np.sqrt(pos[:,0]**2 + pos[:,1]**2 + pos[:,2]**2)
160     max_r = r.max()
161     #min_r = max_r / 10**oms
162     min_r = res_limit
163     log_range = np.log10(max_r) - np.log10(min_r)
164
165     #global nbins
166     local_nbins = float(nbins + 1)
167     #nbins = len(r) / 1000
168     while True:
169         bins = np.arange(local_nbins)
170         bins = max_r * 10.0**((log_range * bins / (local_nbins-1.0) - log_range))
171         bin_mass, r_bins = np.histogram(r, bins, weights=mass)
172         if (bin_mass == 0.0).any():

```

```

173     local_nbins -= 1
174     continue
175   else:
176     break
177
178 #print 'Binning particles using bin edges of \n', r_bins
179
180 rho = bin_mass / (sphere_vol(r_bins[1:]) - sphere_vol(r_bins[:-1]))
181
182 N_bin, blah = np.histogram(r, bins)
183 rho_err = poisson_error(N_bin) * rho
184
185 return r_bins, rho, rho_err
186
187
188 def logbin(pos):
189   r = np.sqrt(pos[:,0]**2 + pos[:,1]**2 + pos[:,2]**2)
190   max_r = r.max()
191   min_r = max_r / 10**ooms
192   log_range = np.log10(max_r) - np.log10(min_r)
193
194   global nbins
195   nbins = float(nbins + 1)
196   bins = np.arange(nbins)
197   bins = max_r * 10.0**(log_range * bins / (nbins-1.0) - log_range)
198
199   hist, bin_edges = np.histogram(r, bins)
200 #print 'Binning particles using bin edges of \n', bin_edges
201 return hist, bin_edges
202
203
204 def poisson_error(N):
205   err = np.sqrt(N) / N
206   return err
207
208
209 def sphere_vol(r):
210   volume = (4.0 / 3.0) * np.pi * r**3
211   return volume
212
213
214 def get_rho_0(R_s, R_vir):
215   H = 70.0e-3 # km s^-1 kpc^-1
216   G = 4.3e-6 # kpc M_sun^-1 (km/s)^2
217   rho_crit = 3.0 * H**2 / (8.0 * np.pi * G)
218
219   v = 178
220   c = R_vir / R_s
221   g = 1.0 / (np.log(1.0+c) - c/(1.0+c))
222   delta_char = v * c**3 * g / 3.0
223
224   return rho_crit * delta_char
225
226
227 def nfw_fit_rho0(r, R_s, rho_0):
228   if R_s >= 1.0:
229     return (R_s - 1.0) * np.exp(r) + rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
230   return rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
231
232
233 def nfw_fit_rho0_log(r, R_s, rho_0):
234   r = 10.0**r
235   R_s = 10.0**R_s
236   rho_0 = 10.0**rho_0
237   profile = rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
238   return np.log10(profile)
239
240
241 def nfw_def_rho0(R_vir):
242   def _nfw_def_rho0(r, R_s):
243     rho_0 = get_rho_0(R_s, R_vir)
244     return rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
245   return _nfw_def_rho0
246
247
248 def nfw_databin_rho0(rho_0):
249   def _nfw_databin_rho0(r, R_s):
250     return rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
251   return _nfw_databin_rho0
252
253
254 def dm_profile_fit_rho0_log(r, R_s, rho_0, alpha):
255   r = 10.0**r
256   R_s = 10.0**R_s
257   rho_0 = 10.0**rho_0
258   alpha = 10.0**alpha
259   profile = rho_0 / ((r / R_s) * (1.0 + r / R_s)**alpha)
260   return np.log10(profile)
261
262
263 def dm_profile_fit_rho0(r, R_s, rho_0, alpha):
264   return rho_0 / ((r / R_s) * (1.0 + r / R_s)**alpha)

```

```

265
266
267 def dm_profile_def_rho0(R_vir):
268     def _dm_profile_def_rho0(r, R_s, alpha):
269         rho_0 = get_rho_0(R_s, R_vir)
270         return rho_0 / ((r / R_s) * (1.0 + r / R_s)**alpha)
271     return _dm_profile_def_rho0
272
273
274 def dm_profile_databin_rho0(rho_0):
275     def _dm_profile_databin_rho0(r, R_s, alpha):
276         return rho_0 / ((r / R_s) * (1.0 + r / R_s)**alpha)
277     return _dm_profile_databin_rho0
278
279
280 def nfw_cdf(r, R_s, rho_0):
281     r = 10.0**r
282     R_s = 10.0**R_s
283     rho_0 = 10.0**rho_0
284     profile = rho_0 * R_s * (np.log(1.0 + r / R_s) - 1.0 / (1.0 + r / R_s))
285     return np.log10(profile)
286
287
288 def nfw_cdf_nolog(r, R_s, rho_0):
289     profile = rho_0 * R_s * (np.log(1.0 + r / R_s) - 1.0 / (1.0 + r / R_s))
290     return profile
291
292
293 def mass_profile(s, c):
294     g = 1.0 / (np.log(1.0 + c) - c / (1.0 + c))
295     return g * (np.log(1.0 + c * s) - c * s / (1.0 + c * s))
296
297
298 def fit_mass_profile(s, m_enclosed, err=None, R_vir=None):
299     #for i in range(len(s)):
300     #    if s[i] > res_limit:
301     #        first_good_bin = i
302     #        break
303     first_good_bin = 0
304
305     #popt, pcov = curve_fit(nfw_cdf, np.log10(r), np.log10(m_outside), sigma=np.log10(err))
306     #popt, pcov = curve_fit(nfw_cdf, np.log10(r), np.log10(m_outside))
307     #popt = 10.0**popt
308     #pcov = 10.0**pcov
309     popt, pcov = curve_fit(mass_profile, s, m_enclosed)
310
311     print 'fit_params:', popt
312     print 'covariance:', pcov
313     nfw_r = np.linspace(s[0], s[-1], nfit)
314     nfw_fit = mass_profile(nfw_r, popt[0])
315     chi2_fit = mass_profile(s, popt[0])
316
317     chi2 = chisquare(np.log10(m_enclosed[first_good_bin:]), np.log10(chi2_fit[first_good_bin:]))
318     chi2_nolog = chisquare(m_enclosed[first_good_bin:], chi2_fit[first_good_bin:])
319     print 'chi_square:', chi2
320     print 'chi_square_nolog:', chi2_nolog
321     return nfw_r, nfw_fit, popt, pcov, chi2[0]
322
323
324 def fit_profile(r, rho, err=None, R_vir=None):
325     first_good_bin = 0
326     # for i in range(len(r)):
327     #     if r[i] > res_limit:
328     #         rho_0_databin = rho[i]
329     #         first_good_bin = i
330     #         break
331     # print 'first_good_bin = ', first_good_bin
332
333     ##### choose one fitting type #####
334     #popt, pcov = curve_fit(nfw_fit_rho0, r, rho, sigma=err)
335     #popt, pcov = curve_fit(nfw_def_rho0(R_vir), r, rho, p0=[10.0], sigma=err)
336     #popt, pcov = curve_fit(nfw_databin_rho0(rho_0_databin), r, rho, sigma=err)
337     blah = 3
338     if blah == 0:
339         for i in range(100):
340             a = 2.0 * np.random.random() * 0.1 * r.max()
341             b = 2.0 * np.random.random() * 10.0
342             c = 2.0 * np.random.random() * 2.0
343             try:
344                 popt, pcov = curve_fit(dm_profile_fit_rho0, r, rho, p0=[a,b,c], sigma=err)
345             except RuntimeError:
346                 continue
347             if (popt[0] < r.max()) and (popt[2] >= 0.0):
348                 break
349             elif i >= 99:
350                 print 'no good fit found for this halo...'
351     #     return None, None, None, None
352     elif blah == 1:
353         #a = r.max() / 100.0
354         a = 0.001
355         b = rho[first_good_bin]
356         c = 0.001

```

```

357     #popt, pcov = curve_fit(dm_profile_fit_rho0, r, rho, sigma=err)
358     print '-----'
359     print 'rho_0before', b
360     #try:
361     #    popt, pcov = curve_fit(dm_profile_fit_rho0, r, rho, p0=[a,b,c], sigma=err, maxfev=1, xtol=100.0)
362     #    popt, pcov = curve_fit(dm_profile_fit_rho0, r, rho, p0=[a,b,c], sigma=err, xtol=1.0e-1)
363     #except RuntimeError:
364     #    print 'just checking for now...'
365     #    print 'rho_0after', popt[1]
366     #    sys.exit()
367 elif blah == 2:
368     #popt, pcov = curve_fit(dm_profile_fit_rho0_log, np.log10(r), np.log10(rho), sigma=np.log10(err))
369     popt, pcov = curve_fit(nfw_fit_rho0_log, np.log10(r), np.log10(rho), sigma=np.log10(err))
370     popt = 10.0*popt
371     pcov = 10.0*pcov
372 elif blah == 3:
373     popt, pcov = curve_fit(nfw_fit_rho0, r, rho, sigma=err, p0 = [0.1, 1.0])
374
375 #popt, pcov = curve_fit(dm_profile_def_rho0(R_vir), r, rho, sigma=err)
376 #popt, pcov = curve_fit(dm_profile_databin_rho0(rho_0_databin), r, rho, sigma=err)
377 #-----
378
379 print 'fit_params', popt
380 print 'covariance', pcov
381
382 nfw_r = np.linspace(r[0], r[-1], nfit)
383 ##### choose one fitting type #####
384 nfw_fit = nfw_fit_rho0(nfw_r, popt[0], popt[1])
385 #nfw_fit = nfw_def_rho0(R_vir)(nfw_r, popt[0])
386 #nfw_fit = nfw_databin_rho0(rho_0_databin)(nfw_r, popt[0])
387 #nfw_fit = dm_profile_fit_rho0(nfw_r, popt[0], popt[1], popt[2])
388 #nfw_fit = dm_profile_def_rho0(R_vir)(nfw_r, popt[0], popt[1])
389 #nfw_fit = dm_profile_databin_rho0(rho_0_databin)(nfw_r, popt[0], popt[1])
390 #####
391 ##### choose one fitting type #####
392 chi2_fit = nfw_fit_rho0(r, popt[0], popt[1])
393 #chi2_fit = nfw_def_rho0(R_vir)(r, popt[0])
394 #chi2_fit = nfw_databin_rho0(rho_0_databin)(r, popt[0])
395 #chi2_fit = dm_profile_fit_rho0(r, popt[0], popt[1], popt[2])
396 #chi2_fit = dm_profile_def_rho0(R_vir)(r, popt[0], popt[1])
397 #chi2_fit = dm_profile_databin_rho0(rho_0_databin)(r, popt[0], popt[1])
398 #####
399
400 #chi2 = my_chisq(rho, chi2_fit, 2, err)
401 chi2 = chisquare(rho, chi2_fit)
402 print 'chi_square', chi2
403 chi2 = chi2[0]
404
405 return nfw_r, nfw_fit, popt, pcov, chi2
406
407
408 def draw_projection(fig, place, plot_lim, x, y):
409     ax = plt.subplot(2,3,place+1, aspect='equal')
410     im = ax.plot(x, y, linestyle='.', marker='.', markersize=1, markeredgecolor='blue')
411     ax.set_xlabel(xlabel_proj[place])
412     ax.set_ylabel(ylabel_proj[place])
413     ax.set_xlim(-plot_lim, plot_lim)
414     ax.set_ylim(-plot_lim, plot_lim)
415     ax.xaxis.set_major_locator(MultipleLocator(tick_base_major))
416     ax.xaxis.set_minor_locator(MultipleLocator(tick_base_minor))
417     ax.yaxis.set_major_locator(MultipleLocator(tick_base_major))
418     ax.yaxis.set_minor_locator(MultipleLocator(tick_base_minor))
419     return fig
420
421
422 def draw_density_projection(fig, place, plot_lim, x, y):
423     limits = [[-plot_lim, plot_lim], [-plot_lim, plot_lim]]
424     ax = plt.subplot(2,3,place+1, aspect='equal')
425     #ax.set_xlim(-plot_lim, plot_lim)
426     #ax.set_ylim(-plot_lim, plot_lim)
427     #im = ax.plot(x, y, linestyle='.', marker='.', markersize=1, markeredgecolor='blue')
428     z, xedges, yedges = np.histogram2d(x, y, bins = npixels, range = limits)
429     #z = np.log10(z)
430     im = ax.imshow(z.T, extent=(-plot_lim, plot_lim, -plot_lim, plot_lim), interpolation='gaussian', origin='lower')
431     ax.locator_params(nbins=6)
432     ax.set_xlabel(xlabel_proj[place])
433     ax.set_ylabel(ylabel_proj[place])
434     # ax.xaxis.set_major_locator(MultipleLocator(tick_base_major))
435     # ax.xaxis.set_minor_locator(MultipleLocator(tick_base_minor))
436     # ax.yaxis.set_major_locator(MultipleLocator(tick_base_major))
437     # ax.yaxis.set_minor_locator(MultipleLocator(tick_base_minor))
438     return fig
439
440
441 def draw_density_profile(fig, r, rho, err=None):
442     ax = plt.subplot(2,1,2)
443     im = ax.loglog(r, rho, linestyle='steps-mid')
444     line1 = ax.axvline(res_limit, color='black', linestyle=':')
445     #ax.set_xlim(r_bins[0], r_bins[-1])
446     ax.set_xlim(r[0] - (r[1]-r[0]), r[-1] + (r[-1]-r[-2]))
447     ax.set_xlabel(xlabel_prof)

```

```

448     ax.set_ylabel(ylabel_prof)
449     if err != None:
450         err_bars = ax.errorbar(r, rho, yerr=err, linestyle='None')
451     return fig, ax
452
453
454 def draw_nfw_profile(fig, ax, r, rho, R_s=None):
455     ax.loglog(r, rho, linestyle='--', color='red')
456     if R_s != None:
457         line = ax.axvline(R_s, color='purple', linestyle='-.')
458     return fig
459
460
461 def calc_kinetic_energy(mass, vel):
462     vsq = vel[:,0]**2 + vel[:,1]**2 + vel[:,2]**2
463     energy = 0.5 * np.sum(mass*vsq)
464     return energy
465
466
467 def calc_potential_energy(mass, pos):
468     local_sqrt = np.sqrt
469     partial_sum = 0.0
470     for i in range(len(mass)):
471         for j in range(len(mass)):
472             if j != i:
473                 r_diff = local_sqrt((pos[i,0] - pos[j,0])**2 + (pos[i,1] - pos[j,1])**2 + (pos[i,2] - pos[j,2])**2)
474                 partial_sum = partial_sum - mass[i]*mass[j]/r_diff
475     energy = partial_sum * grav_const / 2.0
476     return energy
477
478
479 def calc_angular_momentum(mass, pos, vel):
480     ang_mom_x = np.sum(mass * (pos[:,1] * vel[:,2] - pos[:,2] * vel[:,1]))
481     ang_mom_y = np.sum(mass * (pos[:,2] * vel[:,0] - pos[:,0] * vel[:,2]))
482     ang_mom_z = np.sum(mass * (pos[:,0] * vel[:,1] - pos[:,1] * vel[:,0]))
483     ang_mom = np.sqrt(ang_mom_x**2 + ang_mom_y**2 + ang_mom_z**2)
484     return ang_mom
485
486
487 def main():
488     with open(outfile, 'w') as fd:
489         fd.write('#halo_mass concentration R_vir R_s +- err rho_0 +- err alpha +- err chi_square\n')
490         fd.write('#halo_id halo_mass pos_x_pos_y_pos_z_pos_nbins_N_part\n')
491         fd.write('err_rho_0 R_vir R_s +- err_rho_0 +- err_chi_square\n')
492         fd.write('nbins N_part\n')
493
494     # with open(comfile, 'w') as fd:
495     #     fd.write('#id mass dx dy dz\n')
496
497     # if use_bgc2 == True:
498     #     header, halos, particles = bgc2.read_bgc2(sys.argv[1])
499     #     for i in range(len(halos)):
500     #         if halos[i][halo_id_col] == halo_id:
501     #             index = i
502     #             halo_particles = np.asarray(particles[index])
503     #             pos = halo_particles[:,pos_cols[0]:pos_cols[0]+3] * dist_scale
504     #             r_vir = halos[index][4] * dist_scale
505     #         else:
506     #             # Read in particle files
507     #             data = read_files(sys.argv[1:])
508     #             # Select particles with a given halo ID and convert positions from Mpc to kpc
509     #             if use_all == False:
510     #                 halo_particles = data[np.where(data[:,halo_id_col] == halo_id)]
511     #             if use_all == True:
512     #                 halo_particles = data
513     #             del data
514     #             pos = halo_particles[:,pos_cols[0]:pos_cols[0]+3] * dist_scale
515     #             r_vir = 241.48
516     #             #r_vir = pos.max()
517
518     for input_file in sys.argv[1:]:
519         if use_bgc2 == True:
520             header, halos, particles = bgc2.read_bgc2(sys.argv[1])
521             header, halos, particles = bgc2.read_bgc2(input_file)
522             halos = np.asarray(halos)
523             indices = np.argsort(halos[:,2])           # sort by number of particles
524             indices = indices[::-1]                   # start with the biggest
525         else:
526             data = read_files([input_file])
527             # Select particles with a given halo ID and convert positions from Mpc to kpc
528             if use_all == False:
529                 particles = [data[np.where(data[:,halo_id_col] == halo_id)]]
530             if use_all == True:
531                 particles = [data]
532             del data
533
534             itteration = 0
535             #for index in range(len(halos)):
536             #for index in range(1):
537             #for index in indices[:10]:
538             for index in indices:
539                 if ((len(particles[index]) >= 100) and (halos[index][1] == -1)):
```

```

538     print '-----',
539
540     halo_particles = np.asarray(particles[index])
541     pos = halo_particles[:,pos_cols[0]:pos_cols[0]+3] * dist_scale
542     vel = halo_particles[:,vel_cols[0]:vel_cols[0]+3]
543
544     if use_bgc2 == True:
545         halo_id = halos[index][0]
546         r_vir = halos[index][4] * dist_scale
547         halo_mass = halos[index][5]
548         halo_pos = np.array([halos[index][6] * dist_scale, halos[index][7] * dist_scale, halos[index][8] *
549         dist_scale])
550         halo_vel = np.array([halos[index][9], halos[index][10], halos[index][11]])
551     else:
552         r_vir = 241.48
553         halo_id = 0
554         #halo_mass = mass[0] * len(halo_particles)
555         halo_pos = np.array([0.0, 0.0, 0.0])
556         halo_vel = np.array([0.0, 0.0, 0.0])
557
558     if individual_masses == True:
559         mass = halo_particles[:,mass_col] * mass_scale
560     else:
561         mass = np.ones(halo_particles.shape[0]) * common_mass * mass_scale
562
563     if use_bgc2 == False:
564         halo_mass = mass[0] * len(halo_particles) #fix placement of this for ascii test
565
566     print 'Using %d particles in halo %d.' % (halo_particles.shape[0], halo_id)
567
568     # Find center of mass
569     if find_com == True:
570         mass_tot = mass.sum()
571         m_pos = mass.reshape(mass.shape[0],1) * pos
572         com = m_pos.sum(axis=0) / mass_tot
573         pos = pos - com
574         print 'Center of mass = (%g, %g, %g)' % (com[0], com[1], com[2])
575     else:
576         pos = pos - halo_pos
577         vel = vel - halo_vel
578
579     #with open(comfile, 'a') as fd:
580     #    fd.write("%d %g %g %g\n" % (halo_id, halo_mass, halo_pos[0] - com[0], halo_pos[1] - com[1],
581     halo_pos[2] - com[2]))
582
583     # Bin halo particles into logarithmic shells and compute density
584     r_bins, rho, rho_err = calc_density_profile(mass, pos)
585
586     if len(r_bins) < 5:
587         print 'Too few bins. Skipping this halo.'
588         with open(outfile, 'a') as fd:
589             fd.write("%8d%8d\n" % (halo_id, halo_mass, halo_pos[0], halo_pos[1], halo_pos[2], -9999, -9999, -9999,
590             -9999, -9999, -9999, len(halo_particles)))
591         continue
592
593     # hist, r_bins = logbin(pos)
594     # err = poisson_error(hist)
595     # rho = mass * hist / (sphere_vol(r_bins[1:]) - sphere_vol(r_bins[:-1]))
596     # rho_err = err * rho
597     mid_bins = 10.0**((0.5 * (np.log10(r_bins[1:]) + np.log10(r_bins[:-1]))))
598     print 'nbins =', len(mid_bins)
599
600     # Don't pass NaN's to fitting routine
601     rho_err_nonan = np.copy(rho_err)
602     nan_check = np.isnan(rho_err_nonan)
603     for i in range(len(rho_err_nonan)):
604         if (nan_check[i] == True):
605             # rho[i] = 1.0e-10
606             if (mid_bins[i] < res_limit) or (nan_check[i] == True):
607                 rho_err_nonan[i] = 1.0e10
608
609     # m_enclosed = calc_m_enclosed(mass, pos)
610
611     # Fit an NFW profile to the data
612     try:
613         nfw_r, nfw_fit, popt, pcov, chisq = fit_profile(mid_bins / r_vir, rho / rho.max(), err = rho_err_nonan /
614         rho.max(), R_vir = 1.0)
615         #nfw_r, nfw_fit, popt, pcov, chisq = fit_mass_profile(r / r_vir, m_enclosed / halo_mass)
616         nfw_r = nfw_r * r_vir
617         nfw_fit = nfw_fit * rho.max()
618         scale_radius = popt[0] * r_vir
619         scale_radius_err = pcov[0,0] * r_vir
620         rho_0 = popt[1] * rho.max()
621         rho_0_err = pcov[1,1] * rho.max()
622         concentration = r_vir / scale_radius
623         concentration_err = concentration * scale_radius_err / scale_radius
624
625     # Print parameters

```

```

625     print 'r_vir=', r_vir
626     print "rho_0=%g+/-%g" % (rho_0, rho_0_err)
627     print "scale_radius=%g+/-%g" % (scale_radius, scale_radius_err)
628     print "concentration=%g+/-%g" % (concentration, concentration_err)
629
630 #put these back sometime#####
631 #     kin_energy = calc_kinetic_energy(mass, vel)
632 #     pot_energy = calc_potential_energy(mass, pos)
633 #     ang_mom = calc_angular_momentum(mass, pos, vel)
634 #
635 #     ttow = 2.0 * abs(kin_energy / pot_energy)
636 #     lambda_spin = ang_mom * np.sqrt(abs(kin_energy + pot_energy)) / (grav_const * (np.sum(mass))**2.5)
637     kin_energy = 0.0
638     pot_energy = 0.0
639     ang_mom = 0.0
640
641     ttow = 0.0
642     lambda_spin = 0.0
643 ##########
644
645     if isinstance(pcov, float):
646         print "info:covariance returned, skipping this halo..."
647         with open(outfile, 'a') as fd:
648             fd.write("%8d%16.12g%14.10g%14.10g%14d+-%14d%14d%14d+-%14d%14d%14d\n" % (halo_id, halo_mass, halo_pos[0], halo_pos[1], halo_pos[2], -9999, -9999, -9999, -9999,
649             -9999, -9999, -9999, len(halo_particles)))
650         continue
651
652     #Write parameters to file
653     with open(outfile, 'a') as fd:
654         #fd.write("%g %g %g %g +- %g %g +- %g %g\n" % (halo_mass, concentration, r_vir,
655         scale_radius, pcov[0,0], rho_0, pcov[1,1], alpha, pcov[2,2], chisq))
656         fd.write("%8d%16.12g%14.10g%14.10g%14.10g%14.6g%14.10g%14.10g%14.6g%14.10
657         g%14.6g%14.10g%8d%8d\n" % (halo_id, halo_mass, halo_pos[0], halo_pos[1], halo_pos[2],
658         concentration, concentration_err, r_vir, scale_radius, scale_radius_err, rho_0, rho_0_err, chisq, len(r_bins),
659         len(halo_particles)))
660
661
662     ##########
663     # Plot density profile histogram
664     if (make_plot == True) and (iteration < 10):
665         # Find the maximum of x, y, or z to be limit of projection plots
666         plot_lim = pos.max()
667         # Pick only a certain percentage of particles for projection plots
668         if (draw_frac < 1.0):
669             np.random.shuffle(pos)
670             pos = pos[:int(draw_frac*pos.shape[0])]

671         fig = plt.figure()
672         if draw_density == True:
673             fig = draw_density_projection(fig, 0, plot_lim, pos[:,0], pos[:,1])
674             fig = draw_density_projection(fig, 1, plot_lim, pos[:,0], pos[:,2])
675             fig = draw_density_projection(fig, 2, plot_lim, pos[:,1], pos[:,2])
676         else:
677             fig = draw_projection(fig, 0, plot_lim, pos[:,0], pos[:,1])
678             fig = draw_projection(fig, 1, plot_lim, pos[:,0], pos[:,2])
679             fig = draw_projection(fig, 2, plot_lim, pos[:,1], pos[:,2])
680             fig, ax = draw_density_profile(fig, mid_bins, rho, err=rho_err) #put this back for binning
681             #fig, ax = draw_density_profile(fig, r, m_enclosed) #take this out for binning
682             fig = draw_nfw_profile(fig, ax, nfw_r, nfw_fit, R_s=scale_radius)
683             #fig = draw_nfw_profile(fig, ax, nfw_r, blah_fit, R_s=20.0)
684             fig.tight_layout()
685             plt.savefig(plot_base+str(iteration)+plot_ext)
686
687         #sys.exit()
688         iteration += 1
689
690
691 if __name__ == '__main__':
692     main()

```

## Appendix E

### CrossMatch Best Match Code

#### E.1 Best Match (Python)

```
1 #!/usr/bin/env python
2
3 import sys
4 import getopt
5 import numpy as np
6
7
8 def main():
9     # read in files
10    print 'reading files...'
11    with open(sys.argv[1]) as f:
12        matches1 = f.readlines()
13    with open(sys.argv[2]) as f:
14        matches2 = f.readlines()
15    print 'done reading files'
16
17    header = matches1[2:6]
18    header.insert(0, '#Best matches for bi-directional crossmatch\n')
19    header.insert(1, '\n')
20
21    matches1 = matches1[7:]
22    matches2 = matches2[7:]
23
24    # convert to numpy arrays
25    print 'converting to numpy arrays...'
26    match_array1 = np.asarray([line.split() for line in matches1], dtype=int)
27    match_array2 = np.asarray([line.split() for line in matches2], dtype=int)
28    print 'done converting'
29
30    # find matches that exist in both lists
31    print 'finding matches...'
32    mask = np.zeros(len(match_array1), dtype=bool)
33    for i, line in enumerate(match_array1):
34        id1 = line[id1_col]
35        id2 = line[id2_col]
36        tmp = (match_array2[:,id1_col] == id2)
37        tmp = (match_array2[tmp,id2_col] == id1)
38        mask[i] = tmp.any()
39        if i % 1000 == 0:
40            print "Finished line", i
41
42    print 'done matching'
43
44    out_array = match_array1[mask]
45
46    # write results
47    print 'writing results...'
48    with open(sys.argv[3], 'w') as f:
49        f.writelines(( "%s" % line for line in header))
50        np.savetxt(f, out_array, fmt='%10d')
51
52    print 'Finished.'
53
54
55 id1_col      = 4
56 npart1_col   = 5
57 id2_col      = 1
58 npart2_col   = 2
59 ncommon_col  = 6
60 hnum1_col    = 3
61 hnum2_col    = 0
62
63
64 if __name__ == '__main__':
65     main()
```

#### E.2 PBS Submission Script (Bash)

```
1 #!/usr/bin/env bash
2 #PBS -M djsissoom@gmail.com
3 #PBS -m bae
4 #PBS -l nodes=27:ppn=1
5 #PBS -l pmem=20000mb
6 #PBS -l mem=54000mb
7 #PBS -l walltime=0:30:00
8 #PBS -o out.log
9 #PBS -j oe
10
11 #nodes=186:ppn=1
```

```

12 #pmem=20000mb
13 #mem=372000mb
14
15 minsnap=53
16 maxsnap=61
17
18 minbox=1
19 maxbox=3
20
21 # Change to working directory
22 echo $PBS_NODEFILE
23 cd $PBS_O_WORKDIR
24
25 for ((i=$minbox; i<=$maxbox; i++)); do
26
27   for ((snap=$minsnap; snap<=$maxsnap; snap++)); do
28
29     if [ $snap -lt 10 ]; then
30       j=0$snap
31     elif [ $snap -lt 100 ]; then
32       j=0$snap
33     fi
34
35     base_dir=~/projects/simulations/rockstar/box${i}
36     crossmatch_dir=${base_dir}/crossmatch/snap${j}
37     first_file=${crossmatch_dir}/crossmatch_2lpt_first_000.txt
38     second_file=${crossmatch_dir}/crossmatch_za_first_000.txt
39     outfile=${crossmatch_dir}/crossmatch_000.txt
40     logfile=${crossmatch_dir}/best_crossmatch.log
41
42     echo "Starting box${i} snap${j}..."
43
44   {
45     mpiexec -verbose -n 1 ./best_crossmatch.py ${first_file} ${second_file} ${outfile} > ${logfile} 2>&1
46     echo "Finished box${i} snap${j}"
47   } &
48
49 done
50
51 done
52
53 wait
54 # - end of script

```

## Appendix F

### Database Generation Code

#### F.1 Halo Match (Python)

```
1 #!/usr/bin/env python
2
3 import sys
4 import getopt
5 import numpy as np
6
7
8 def main():
9     # read and parse command line arguments
10    opts, args = getopt.getopt(sys.argv[1:])
11    output_file, match_file, densprof_files, parents_files, ascii_files = parse_args(opts, args)
12
13    # read in headers as lists and data as numpy arrays
14    match_header, match_data = read_files(match_file, header_line = 3)
15    densprof_header1, densprof_data1 = read_files(densprof_files[0], header_line = 0)
16    densprof_header2, densprof_data2 = read_files(densprof_files[1], header_line = 0)
17    parents_header1, parents_data1 = read_files(parents_files[0], header_line = 0)
18    parents_header2, parents_data2 = read_files(parents_files[1], header_line = 0)
19    ascii_header1, ascii_data1 = read_files(ascii_files[:len(ascii_files)/2]), header_line = 0)
20    ascii_header2, ascii_data2 = read_files(ascii_files[(len(ascii_files)/2):]), header_line = 0)
21    print 'Finished reading files.'
22
23    # filter matches, remove duplicate halo matches, and reorder match columns
24    print 'Filtering match data...'
25    match_data = filter_matches(match_data)
26    if filter_duplicate_matches:
27        match_data = filter_dups(match_data, unique_col = match_id1_col)
28        match_data = filter_dups(match_data, unique_col = match_id2_col)
29    if reorder_match_columns:
30        match_header, match_data = reorder_match_cols(match_header, match_data)
31
32    # calculate number of subhalos and add column to parents data and headers
33    print 'Finding number of subhalos...'
34    parents_header1.append('N_subs')
35    parents_header2.append('N_subs')
36    parents_data1 = count_subs(parents_data1)
37    parents_data2 = count_subs(parents_data2)
38
39    # create header
40    print 'Making header...'
41    header = make_header(match_header, densprof_header1, densprof_header2, \
42                          parents_header1, parents_header2, ascii_header1, ascii_header2)
43
44    # match halos
45    print 'Matching halos...'
46    halos = match_halos(match_data, [densprof_data1, densprof_data2, \
47                                      parents_data1, parents_data2, ascii_data1, ascii_data2])
48
49    # filter based on given criteria and sort
50    print 'Filtering halo data...'
51    if filter_halo_properties:
52        halos = filter_halos(halos)
53        if sort_col != None:
54            sort_mask = halos[:,sort_col].argsort()
55            sort_mask = sort_mask[::-1]
56            halos = halos[sort_mask]
57
58    # output matched table
59    print 'Writing results...'
60    write_results(output_file, header, halos)
61
62    print 'Finished.'
63
64
65 def get_args(arglist):
66     try:
67         opts, args = getopt.gnu_getopt(arglist, shortopts, longopts)
68     except getopt.GetoptError:
69         print "Invalid option(s)."
70         print help_string
71         sys.exit(2)
72     if opts == []:
73         print 'No options given.'
74         print help_string
75         sys.exit(2)
76     return opts, args
77
78
79 def parse_args(opts, args):
80     densprof_files = None
```

```

81 parentsfiles = None
82 asciiifiles = None
83 use_ascii = False
84 for opt in opts:
85     if (opt[0] == '-h') or (opt[0] == '--help') or (opts == None):
86         print help_string
87         sys.exit(0)
88     if (opt[0] == '-o') or (opt[0] == '--outfile'):
89         outfile = opt[1]
90     if (opt[0] == '-m') or (opt[0] == '--match'):
91         matchfile = opt[1]
92     if (opt[0] == '-d') or (opt[0] == '--density'):
93         densproffiles = create_append(densproffiles, opt[1])
94     if (opt[0] == '-p') or (opt[0] == '--parents'):
95         parentsfiles = create_append(parentsfiles, opt[1])
96     if (opt[0] == '-a'):
97         use_ascii = True
98 if use_ascii:
99     if len(args) % 2 != 0:
100        print 'Must have an even number of ascii files!'
101        sys.exit(3)
102    for arg in args:
103        asciiifiles = create_append(asciiifiles, arg)
104    return outfile, matchfile, densproffiles, parentsfiles, asciiifiles
105
106
107 def create_append(lst, value):
108     if lst == None:
109         lst = [value]
110     else:
111         lst.append(value)
112     return lst
113
114
115 def read_files(files, header_line = None, comment_char = '#'):
116     header = None
117     data = None
118     if type(files) == str:
119         files = [files]
120
121     if header_line != None:
122         with open(files[0], 'r') as fd:
123             for line in range(header_line):
124                 fd.readline()
125             header = fd.readline()
126             if header[0] != comment_char:
127                 print "Header must start with a %s" % comment_char
128                 sys.exit(4)
129             header = header[1:]
130             header = header.split()
131
132     for file in files:
133         print 'Reading file %s...' % (file)
134         if data == None:
135             data = np.genfromtxt(file, comments='#')
136         else:
137             data = np.append(data, np.genfromtxt(file, comments='#'), axis=0)
138
139     if header_line == None:
140         return data
141     else:
142         return header, data
143
144
145 def filter_matches(halos):
146     if filter_bad_matches:
147         halos = halos[:,match_id1_col] != -1]
148         halos = halos[:,match_id2_col] != -1]
149     if (min_npart != 0) and (min_npart != None):
150         halos = halos[:, match_npart1_col] >= min_npart]
151         halos = halos[:, match_npart2_col] >= min_npart]
152     if (minperc_ncommon != 0) and (minperc_ncommon != None):
153         halos = halos[:, match_ncommon_col] / halos[:, match_npart1_col] >= minperc_ncommon]
154         halos = halos[:, match_ncommon_col] / halos[:, match_npart2_col] >= minperc_ncommon]
155     return halos
156
157
158 def filter_dups(halos, unique_col = 0):
159     ncommon = halos[:, match_ncommon_col]
160     n1 = halos[:, match_npart1_col]
161     n2 = halos[:, match_npart2_col]
162     rank = ncommon**2 / (n1 * n2) - np.abs(n1 - n2) / (n1 + n2)
163
164     sort_mask = np.argsort(rank)
165     halos = halos[sort_mask]
166
167     unique, mask = np.unique(halos[:, unique_col], return_index=True)
168     halos = halos[mask]
169     return halos
170
171
172 def reorder_match_cols(match_header, match_data):

```

```

173     global match_id1_col
174     global match_id2_col
175     global match_hnum1_col
176     global match_hnum2_col
177     global match_npart1_col
178     global match_npart2_col
179     global match_ncommon_col
180
181     order = [match_id1_col, match_id2_col, \
182             match_hnum1_col, match_hnum2_col, \
183             match_npart1_col, match_npart2_col, \
184             match_ncommon_col]
185     match_header = [match_header[index] for index in order]
186     match_data = match_data[:, order]
187
188     match_id1_col = 0
189     match_id2_col = 1
190     match_hnum1_col = 2
191     match_hnum2_col = 3
192     match_npart1_col = 4
193     match_npart2_col = 5
194     match_ncommon_col = 6
195
196     return match_header, match_data
197
198
199 def count_subs(halos):
200     id = halos[:, id_col]
201     parents = halos[:, parents_col]
202     parents = parents[parents != -1]
203     nsubs = (id[:, np.newaxis] == parents).sum(axis = 1)
204     halos = np.column_stack((halos, nsubs))
205     return halos
206
207
208 def make_header(match, densprof1, densprof2, parents1, parents2, ascii1, ascii2):
209     # zeroeth line just lists column number
210     total_len = len(match + densprof1 + densprof2 + parents1 + parents2 + ascii1 + ascii2)
211     header_line0 = [str(i) for i in range(total_len)]
212     header_line0 = '\u001a'.join(header_line0)
213     header_line0 = '#' + header_line0
214
215     # first line denotes which file columns are from
216     match_repeat = len(match) - 4
217     densprof_repeat = len(densprof1 + densprof2) - 4
218     parents_repeat = len(parents1 + parents2) - 4
219     ascii_repeat = len(ascii1 + ascii2) - 4
220
221     match_part = '\u001a'.join(['|---', 'cross', 'match'] + ['|---'] * match_repeat + ['|---|'])
222     densprof_part = '\u001a'.join(['|---', 'density', 'profile'] + ['|---'] * densprof_repeat + ['|---|'])
223     parents_part = '\u001a'.join(['|---', 'rockstar', 'parents'] + ['|---'] * parents_repeat + ['|---|'])
224     ascii_part = '\u001a'.join(['|---', 'rockstar', 'ascii'] + ['|---'] * ascii_repeat + ['|---|'])
225
226     header_line1 = '\u001a'.join([match_part, densprof_part, parents_part, ascii_part])
227     header_line1 = '#' + header_line1
228
229     # second line labels 2lpt and za columns
230     tot_len = len(match + densprof1 + densprof2 + parents1 + parents2 + ascii1 + ascii2)
231     header_line2 = ['2lpt' if i % 2 == 0 else 'za' if i % 2 == 1 else 'blah' for i in range(tot_len - 1)]
232     header_line2.insert(len(match) - 1, 'matched')
233     header_line2 = '\u001a'.join(header_line2)
234     header_line2 = '#' + header_line2
235
236     # third line pulls labels from original file headers
237     match_part = match
238     densprof_part = interweave(densprof1, densprof2)
239     parents_part = interweave(parents1, parents2)
240     ascii_part = interweave(ascii1, ascii2)
241
242     header_line3 = match_part + densprof_part + parents_part + ascii_part
243     header_line3 = '\u001a'.join(header_line3)
244     header_line3 = '#' + header_line3
245
246     header = [header_line0, header_line1, header_line2, header_line3]
247     return header
248
249
250 def interweave(list1, list2):
251     newlist = list1 + list2
252     newlist[::2] = list1
253     newlist[1::2] = list2
254     return newlist
255
256
257 def interweave_np_2d(array1, array2):
258     newarray = np.empty((len(array1), len(array1[0]) + len(array2[0])))
259     newarray[:, ::2] = array1
260     newarray[:, 1::2] = array2
261     return newarray
262
263
264 def match_halos(matches, arrays):

```

```

265     halos = matches.copy()
266     for i, array in enumerate(arrays):
267         if array != None:
268             match_id_col = i % 2
269             halos = sort_stack(halos, array, match_id_col)
270
271     # interweave columns so that matching 2lpt/za columns are adjacent
272     tmp_halos = halos
273     halos = np.empty((len(tmp_halos), len(tmp_halos[0])))
274     halos[:,len(matches[0])] = matches
275     startcol = len(matches[0])
276     for i in range(0, len(arrays), 2):
277         colrange1 = len(arrays[i][0])
278         colrange2 = len(arrays[i+1][0])
279         endcol = startcol + colrange1 + colrange2
280
281         cols1 = tmp_halos[:,startcol:startcol+colrange1]
282         cols2 = tmp_halos[:,startcol+colrange1:startcol+colrange1+colrange2]
283
284         halos[:,startcol:endcol] = interweave_np_2d(cols1, cols2)
285         startcol = endcol
286
287     return halos
288
289 def sort_stack(halos, array, match_id_col):
290     # add empty columns to halos to later fill with halo data
291     rows = len(halos)
292     origcols = len(halos[0])
293     newcols = len(array[0])
294     empty = np.empty((rows, newcols))
295     empty[:] = np.nan
296     halos = np.column_stack((halos, empty))
297
298     # remove halos from array with no matches
299     match_id = halos[:, match_id_col]
300     array_id = array[:, id_col]
301     array_mask = np.in1d(array_id, match_id)
302     array = array[array_mask]
303
304     # create mask so we only add lines for halos in array
305     array_id = array[:, id_col]
306     halo_mask = np.in1d(match_id, array_id)
307     masked_halos = halos[halo_mask]
308
309     # create masks to sort by halo id
310     match_id_sort_mask = np.argsort(masked_halos[:, match_id_col])
311     sorted_masked_halos = masked_halos[match_id_sort_mask]
312
313     # sort array by halo id and copy to empty columns of view of halos
314     array_id_sort_mask = np.argsort(array[:,id_col])
315     sorted_masked_halos[:, origcols:] = array[array_id_sort_mask]
316
317     # 'unmask' - put data back in original halos
318     masked_halos[match_id_sort_mask] = sorted_masked_halos
319     halos[halo_mask] = masked_halos
320
321     return halos
322
323
324 def filter_halos(halos):
325     #todo
326     return halos
327
328
329 def write_results(output_file, header, halos):
330     format = get_format(halos[0])
331     with open(output_file, 'w') as fd:
332         for line in header:
333             fd.write(line + '\n')
334     np.savetxt(fd, halos, fmt=format)
335
336
337 def get_format(line):
338     format = ['%d' if col in int_cols else '%1.14g' for col in range(len(line))]
339     format = ''.join(format)
340     return format
341
342
343 help_string = '''
344 Available options are:
345     -h, --help
346     -v, --verbose
347     -o <outfile>, --outfile <outfile>
348     -m <matchlist>, --match <matchlist>
349     -d <densityprofile_file>, --density <densityprofile_file>
350     -p <parents_file>, --parents <parents_file>
351     -a <ascii_files>, --ascii <ascii_files> - must be last
352 '''
353 shortopts = "hvo:m:d:p:a"
354 longopts = ["help", "verbose", "outfile=", "matchfile=", "density=", "parents=", "ascii"]
355
356 int_cols = []

```

```

357 lt_vals = []
358
359 gt_cols = []
360 gt_vals = []
361
362 eq_cols = []
363 eq_vals = []
364
365 ne_cols = []
366 ne_vals = []
367
368 #int_cols = [0, 1, 2, 3, 4, 5, 6, 7, 8]
369 int_cols = []
370
371 match_id2_col      = 1
372 match_npart2_col   = 2
373 match_id1_col      = 4
374 match_npart1_col   = 5
375 match_ncommon_col  = 6
376 match_hnum1_col    = 3
377 match_hnum2_col    = 0
378
379 id_col             = 0      # col of each input file
380 sort_col            = 47     # col of final table - use None to turn off sorting
381 parents_col         = -1
382
383 filter_bad_matches = True
384 filter_duplicate_matches = False
385 reorder_match_columns = True
386 filter_halo_properties = False
387 min_npart          = 20    # use 0 or None to use all size halos
388 minperc_ncommon    = 0.05  # a fraction, use 0 or None to use any match percent
389
390
391 if __name__ == '__main__':
392     main()

```

## F.2 PBS Submission Script (Bash)

```

1#!/usr/bin/env bash
2#PBS -M djsissom@gmail.com
3#PBS -m bae
4#PBS -l nodes=1:ppn=1
5#PBS -l pmem=4000mb
6#PBS -l mem=4000mb
7#PBS -l walltime=1:00:00
8#PBS -o out.log
9#PBS -j oe
10
11 minsnap=0
12 maxsnap=61
13
14 minbox=1
15 maxbox=3
16
17 # Change to working directory
18 echo $PBS_NODEFILE
19 cd ${PBS_O_WORKDIR}
20
21 for ((i=$minbox; i<=$maxbox; i++)); do
22
23     for ((snap=$minsnap; snap<=$maxsnap; snap++)); do
24
25         if [ $snap -lt 10 ]; then
26             j=0${snap}
27         elif [ $snap -lt 100 ]; then
28             j=0${snap}
29         fi
30
31         base_dir=~/projects/simulations/rockstar/box${i}
32         crossmatch_dir=${base_dir}/crossmatch/snap${j}
33         snap_dir_2lpt=${base_dir}/2lpt/snap${j}
34         snap_dir_zap=${base_dir}/za/snap${j}
35         logfile=${crossmatch_dir}/match_halos.log
36
37         echo "Starting box${i} snap${j}..."
38
39         {
40             #mpixexec -verbose -n 1 \
41             ./match.py -o ${crossmatch_dir}/halos.dat \
42             -m ${crossmatch_dir}/crossmatch_000.txt \
43             -d ${snap_dir_2lpt}/halos/density_profile_halos.dat \
44             -d ${snap_dir_zap}/halos/density_profile_halos.dat \
45             -p ${snap_dir_2lpt}/halos/out_0.list.parents \
46             -p ${snap_dir_zap}/halos/out_0.list.parents \
47             -a \
48             ${snap_dir_2lpt}/halos/halos_0.*.ascii \
49             ${snap_dir_zap}/halos/halos_0.*.ascii \
50             > ${logfile} 2>&1
51
52         echo 'Aligning columns...' >> ${logfile} 2>&1

```

```
53     column -t ${crossmatch_dir}/halos.dat > ${crossmatch_dir}/tmp156546.dat 2>> ${logfile}
54     mv ${crossmatch_dir}/tmp156546.dat ${crossmatch_dir}/halos.dat 2>> ${logfile}
55     echo 'Finished.' >> ${logfile} 2>&1
56     echo "Finished_box${i}_snap${j}"
57 }
58 #} &
59
60 done
61
62 done
63
64 wait
65
66 # - end of script
```

## Appendix G

### Halo Comparison Code

#### G.1 Particle Comparison (Python)

```
1 #!/usr/bin/env python
2
3 import sys
4 import bgc2
5 import numpy as np
6 import matplotlib.pyplot as plt
7 from matplotlib.patches import Circle
8 from matplotlib.ticker import MultipleLocator
9 from scipy.optimize import curve_fit
10 from scipy.stats import chisquare
11
12 #id1, id2 = 727, 4420 # 2lpt first
13 #id1, id2 = 4416, 727 # za first
14
15 #id1, id2 = 4416, 4420 # both za
16 #id1, id2 = 4416, 4416 # both za
17
18 #id1, id2 = 653, 4355
19 #id1, id2 = 38, 3803
20 #id1, id2 = 155099, 80362
21 #id1, id2 = 98722, 14357
22 id1, id2 = 84289, 143514
23
24
25 #read_mode = 'ascii2'
26 read_mode = 'bgc2'
27
28 if read_mode == 'bgc2':
29     use_bgc2 = True
30     use_all = False
31     multiple_halos = True
32     individual_masses = False
33     halo_id = 146289
34     nbins = 50
35     nfit = 500
36     ooms = 3.0
37     mass_scale = 1.0
38     common_mass = 5.33423e5
39     dist_scale = 1.0e3
40     #res_limit = 0.488
41     res_limit = 4.0
42     #res_limit = 10.0
43     #draw_frac = 1.0e-2
44     draw_frac = 1.0
45     tick_base_major = 10.0
46     tick_base_minor = 1.0
47 elif read_mode == 'ascii':
48     use_bgc2 = False
49     use_all = True
50     individual_masses = True
51     halo_id = 0
52     nbins = 100
53     nfit = 500
54     ooms = 5.0
55     mass_scale = 1.0e12
56     dist_scale = 200.0
57     res_limit = 1.0e-2
58     draw_frac = 2.0e-4
59     tick_base_major = 80.0
60     tick_base_minor = 20.0
61 elif read_mode == 'ascii2':
62     use_bgc2 = False
63     use_all = True
64     individual_masses = True
65     halo_id = 0
66     nbins = 100
67     nfit = 500
68     ooms = 3.5
69     mass_scale = 1.0e10
70     dist_scale = 1.0
71     res_limit = 3.0e-1
72     draw_frac = 1.0e-2
73     tick_base_major = 200.0
74     tick_base_minor = 40.0
75 else:
76     sys.exit(98712)
77
78 outfile = 'halo_properties.txt'
79 comfile = 'center_of_mass.txt'
```

```

81 make_plot = True
82 plot_base = 'density_profile.fig.'
83 plot_ext = '.eps'
84 dist_units = 'kpc'
85 xlabel_proj = [r'X\u20d7Position\u20d7(%s\u20d7h\u20d7^{\{-1\}})', % (dist_units), r'X\u20d7Position\u20d7(%s\u20d7h\u20d7^{\{-1\}})', % (dist_units), r'Y\u20d7
Position\u20d7(%s\u20d7h\u20d7^{\{-1\}})', % (dist_units)]
86 ylabel_proj = [r'Y\u20d7Position\u20d7(%s\u20d7h\u20d7^{\{-1\}})', % (dist_units), r'Z\u20d7Position\u20d7(%s\u20d7h\u20d7^{\{-1\}})', % (dist_units), r'Z\u20d7
Position\u20d7(%s\u20d7h\u20d7^{\{-1\}})', % (dist_units)]
87 xlabel_prof = r'Radius\u20d7(%s\u20d7h\u20d7^{\{-1\}})', % (dist_units)
88 ylabel_prof = r'Density\u20d7(M_{\u20d7\odot}\u20d7%s\u20d7^{\{-3\}}\u20d7h\u20d7^{\{2\}})', % (dist_units)
89
90 #common_mass = 1.0e-7
91 #common_mass = 1.0e5
92 mass_col = 0
93 pos_cols = (1,2,3)
94 vel_cols = (4,5,6)
95 halo_id_col = 0
96
97 grav_const = 4.3e-6 # kpc M_sol^-1 (km/s)^2
98
99 profile_type = 0 # 0 -> nfw, fit rho_0
100          # 1 -> nfw, calculate rho_0
101          # 2 -> nfw, rho_0 middle of leftmost bin above resolution
102          # 3 -> fit outer slope, fit rho_0
103          # 4 -> fit outer slope, calculate rho_0
104          # 5 -> fit outer slope, rho_0 middle of leftmost bin above resolution
105
106 def read_files(files):
107     data = 0
108     for file in files:
109         print 'Reading\u20d7file\u20d7%s...', % (file)
110         if data == 0:
111             data = np.genfromtxt(file, comments='#')
112         else:
113             data = np.append(data, np.genfromtxt(file, comments='#'), axis=0)
114     print 'Finished\u20d7reading\u20d7files.'
115     return data
116
117
118 def calc_density_profile(mass, pos):
119     r = np.sqrt(pos[:,0]**2 + pos[:,1]**2 + pos[:,2]**2)
120     max_r = r.max()
121     #min_r = max_r / 10**ooms
122     min_r = res_limit
123     log_range = np.log10(max_r) - np.log10(min_r)
124
125     #global nbins
126     local_nbins = float(nbins + 1)
127     #nbins = len(r) / 1000
128     while True:
129         bins = np.arange(local_nbins)
130         bins = max_r * 10.0**(log_range * bins / (local_nbins-1.0) - log_range)
131         bin_mass, r_bins = np.histogram(r, bins, weights=mass)
132         if (bin_mass == 0.0).any():
133             local_nbins -= 1
134             continue
135         else:
136             break
137
138     #print 'Binning particles using bin edges of \n', r_bins
139
140     rho = bin_mass / (sphere_vol(r_bins[1:]) - sphere_vol(r_bins[:-1]))
141
142     N_bin, blah = np.histogram(r, bins)
143     rho_err = poisson_error(N_bin) * rho
144
145     return r_bins, rho, rho_err
146
147
148 def logbin(pos):
149     r = np.sqrt(pos[:,0]**2 + pos[:,1]**2 + pos[:,2]**2)
150     max_r = r.max()
151     min_r = max_r / 10**ooms
152     log_range = np.log10(max_r) - np.log10(min_r)
153
154     global nbins
155     nbins = float(nbins + 1)
156     bins = np.arange(nbins)
157     bins = max_r * 10.0**(log_range * bins / (nbins-1.0) - log_range)
158
159     hist, bin_edges = np.histogram(r, bins)
160     #print 'Binning particles using bin edges of \n', bin_edges
161     return hist, bin_edges
162
163
164 def poisson_error(N):
165     err = np.sqrt(N) / N
166     return err
167
168
169 def sphere_vol(r):
170     volume = (4.0 / 3.0) * np.pi * r**3

```

```

171     return volume
172
173
174 def get_rho_0(R_s, R_vir):
175     H = 70.0e-3 # km s^-1 kpc^-1
176     G = 4.3e-6 # kpc M_sun^-1 (km/s)^2
177     rho_crit = 3.0 * H**2 / (8.0 * np.pi * G)
178
179     v = 178
180     c = R_vir / R_s
181     g = 1.0 / (np.log(1.0+c) - c/(1.0+c))
182     delta_char = v * c**3 * g / 3.0
183
184     return rho_crit * delta_char
185
186
187 def nfw_fit_rho0(r, R_s, rho_0):
188     return rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
189
190
191 def nfw_fit_rho0_log(r, R_s, rho_0):
192     r = 10.0**r
193     R_s = 10.0**R_s
194     rho_0 = 10.0**rho_0
195     profile = rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
196     return np.log10(profile)
197
198
199 def nfw_def_rho0(R_vir):
200     def _nfw_def_rho0(r, R_s):
201         rho_0 = get_rho_0(R_s, R_vir)
202         return rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
203     return _nfw_def_rho0
204
205
206 def nfw_databin_rho0(rho_0):
207     def _nfw_databin_rho0(r, R_s):
208         return rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
209     return _nfw_databin_rho0
210
211
212 def dm_profile_fit_rho0_log(r, R_s, rho_0, alpha):
213     r = 10.0**r
214     R_s = 10.0**R_s
215     rho_0 = 10.0**rho_0
216     alpha = 10.0**alpha
217     profile = rho_0 / ((r / R_s) * (1.0 + r / R_s)**alpha)
218     return np.log10(profile)
219
220
221 def dm_profile_fit_rho0(r, R_s, rho_0, alpha):
222     return rho_0 / ((r / R_s) * (1.0 + r / R_s)**alpha)
223
224
225 def dm_profile_def_rho0(R_vir):
226     def _dm_profile_def_rho0(r, R_s, alpha):
227         rho_0 = get_rho_0(R_s, R_vir)
228         return rho_0 / ((r / R_s) * (1.0 + r / R_s)**alpha)
229     return _dm_profile_def_rho0
230
231
232 def dm_profile_databin_rho0(rho_0):
233     def _dm_profile_databin_rho0(r, R_s, alpha):
234         return rho_0 / ((r / R_s) * (1.0 + r / R_s)**alpha)
235     return _dm_profile_databin_rho0
236
237
238 def fit_profile(r, rho, err=None, R_vir=None):
239     for i in range(len(r)):
240         if r[i] > res.limit:
241             rho_0_databin = rho[i]
242             first_good_bin = i
243             break
244     ##### choose one fitting type #####
245     #popt, pcov = curve_fit(nfw_fit_rho0, r, rho, sigma=err)
246     #popt, pcov = curve_fit(nfw_def_rho0(R_vir), r, rho, p0=[10.0], sigma=err)
247     #popt, pcov = curve_fit(nfw_databin_rho0(rho_0_databin), r, rho, sigma=err)
248     blah = 2
249     if blah == 0:
250         for i in range(100):
251             a = 2.0 * np.random.random() * 0.1 * r.max()
252             b = 2.0 * np.random.random() * 10.0
253             c = 2.0 * np.random.random() * 2.0
254             try:
255                 popt, pcov = curve_fit(dm_profile_fit_rho0, r, rho, p0=[a,b,c], sigma=err)
256             except RuntimeError:
257                 continue
258             if (popt[0] < r.max()) and (popt[2] >= 0.0):
259                 break
260             elif i >= 99:
261                 print 'no good fit found for this halo...'
262     #     return None, None, None, None, None

```

```

263     elif blah == 1:
264         #a = r.max() / 100.0
265         a = 0.001
266         b = rho[first_good_bin]
267         c = 0.001
268         #popt, pcov = curve_fit(dm_profile_fit_rho0, r, rho, sigma=err)
269         print '-----',
270         print 'rho_0before=', b
271         #try:
272             #popt, pcov = curve_fit(dm_profile_fit_rho0, r, rho, p0=[a,b,c], sigma=err, maxfev=1, xtol=100.0)
273             popt, pcov = curve_fit(dm_profile_fit_rho0, r, rho, p0=[a,b,c], sigma=err, xtol=1.0e-1)
274         #except RuntimeError:
275             # print 'just checking for now...'
276             print 'rho_0after=', popt[1]
277             #sys.exit()
278     elif blah == 2:
279         #popt, pcov = curve_fit(dm_profile_fit_rho0_log, np.log10(r), np.log10(rho), sigma=np.log10(err))
280         popt, pcov = curve_fit(nfw_fit_rho0_log, np.log10(r), np.log10(rho), sigma=np.log10(err))
281         popt = 10.0*popt
282         pcov = 10.0*pcov
283     elif blah == 3:
284         popt, pcov = curve_fit(nfw_fit_rho0, r, rho, sigma=err)
285
286     #popt, pcov = curve_fit(dm_profile_def_rho0(R_vir), r, rho, sigma=err)
287     #popt, pcov = curve_fit(dm_profile_databin_rho0(rho_0_databin), r, rho, sigma=err)
288     #-----
289     print 'fit_params=', popt
290     print 'covariance=', pcov
291     nfw_r = np.linspace(r[0], r[-1], nfit)
292     #----- choose one fitting type -----#
293     nfw_fit = nfw_fit_rho0(nfw_r, popt[0], popt[1])
294     #nfw_fit = nfw_def_rho0(R_vir)(nfw_r, popt[0])
295     #nfw_fit = nfw_databin_rho0(rho_0_databin)(nfw_r, popt[0])
296     #nfw_fit = dm_profile_fit_rho0(nfw_r, popt[0], popt[1], popt[2])
297     #nfw_fit = dm_profile_def_rho0(R_vir)(nfw_r, popt[0], popt[1])
298     #nfw_fit = dm_profile_databin_rho0(rho_0_databin)(nfw_r, popt[0], popt[1])
299     #-----#
300     #----- choose one fitting type -----#
301     chi2_fit = nfw_fit_rho0(r, popt[0], popt[1])
302     #chi2_fit = nfw_def_rho0(R_vir)(r, popt[0])
303     #chi2_fit = nfw_databin_rho0(rho_0_databin)(r, popt[0])
304     #chi2_fit = dm_profile_fit_rho0(r, popt[0], popt[1], popt[2])
305     #chi2_fit = dm_profile_def_rho0(R_vir)(r, popt[0], popt[1])
306     #chi2_fit = dm_profile_databin_rho0(rho_0_databin)(r, popt[0], popt[1])
307     #-----#
308
309     chi2 = chisquare(np.log10(rho[first_good_bin:]), np.log10(chi2_fit[first_good_bin:]))
310     chi2_nolog = chisquare(rho[first_good_bin:], chi2_fit[first_good_bin:])
311     print 'chi_square=', chi2
312     print 'chi_square_nolog=', chi2_nolog
313     return nfw_r, nfw_fit, popt, pcov, chi2[0]
314
315
316 def draw_projection(fig, place, plot_lim, x, y):
317     ax = plt.subplot(1,3,place+1, aspect='equal')
318     im = ax.plot(x, y, linestyle=':', marker='.', markersize=1, markeredgecolor='blue')
319     ax.set_xlabel(xlabel_proj[place])
320     ax.set_ylabel(ylabel_proj[place])
321     ax.set_xlim(-plot_lim, plot_lim)
322     ax.set_ylim(-plot_lim, plot_lim)
323     # ax.xaxis.set_major_locator(MultipleLocator(tick_base_major))
324     # ax.xaxis.set_minor_locator(MultipleLocator(tick_base_minor))
325     # ax.yaxis.set_major_locator(MultipleLocator(tick_base_major))
326     # ax.yaxis.set_minor_locator(MultipleLocator(tick_base_minor))
327     return fig, ax
328
329
330 def draw_projection_again(fig, ax, x, y):
331     im = ax.plot(x, y, linestyle=':', marker='.', markersize=1, markeredgecolor='red')
332     return fig
333
334
335 def draw_density_profile(fig, r, rho, err=None):
336     ax = plt.subplot(2,1,2)
337     im = ax.loglog(r, rho, linestyle='steps-mid')
338     line1 = ax.axvline(res_limit, color='black', linestyle=':')
339     #ax.set_xlim(r_bins[0], r_bins[-1])
340     ax.set_xlim(r[0] - (r[1]-r[0]), r[-1] + (r[-1]-r[-2]))
341     ax.set_xlabel(xlabel_prof)
342     ax.set_ylabel(ylabel_prof)
343     if err != None:
344         err_bars = ax.errorbar(r, rho, yerr=err, linestyle='None')
345     return fig, ax
346
347
348 def draw_nfw_profile(fig, ax, r, rho, R_s=None):
349     ax.loglog(r, rho, linestyle='-', color='red')
350     if R_s != None:
351         line = ax.axvline(R_s, color='purple', linestyle='-.')
352     return fig
353
354
```

```

355 def calc_kinetic_energy(mass, vel):
356     vsq = vel[:,0]**2 + vel[:,1]**2 + vel[:,2]**2
357     energy = 0.5 * np.sum(mass*vsq)
358     return energy
359
360
361 def calc_potential_energy(mass, pos):
362     local_sqrt = np.sqrt
363     partial_sum = 0.0
364     for i in range(len(mass)):
365         for j in range(len(mass)):
366             if j != i:
367                 r_diff = local_sqrt((pos[i,0] - pos[j,0])**2 + (pos[i,1] - pos[j,1])**2 + (pos[i,2] - pos[j,2])**2)
368                 partial_sum = partial_sum - mass[i]*mass[j]/r_diff
369     energy = partial_sum * grav_const / 2.0
370     return energy
371
372
373 def calc_angular_momentum(mass, pos, vel):
374     ang_mom_x = np.sum(mass * (pos[:,1] * vel[:,2] - pos[:,2] * vel[:,1]))
375     ang_mom_y = np.sum(mass * (pos[:,2] * vel[:,0] - pos[:,0] * vel[:,2]))
376     ang_mom_z = np.sum(mass * (pos[:,0] * vel[:,1] - pos[:,1] * vel[:,0]))
377     ang_mom = np.sqrt(ang_mom_x**2 + ang_mom_y**2 + ang_mom_z**2)
378     return ang_mom
379
380
381 def main():
382     #for input_file in sys.argv[1:]:
383     #header1, halos1, particles1 = bgc2.read_bgc2(sys.argv[1])
384     #header2, halos2, particles2 = bgc2.read_bgc2(sys.argv[2])
385
386     nargs = len(sys.argv) - 1
387     if (float(nargs) % 2.0) != 0.0:
388         print 'number of arguments must be even'
389         sys.exit()
390
391     for i in range(nargs / 2):
392         i += 1
393         temp_header1, temp_halos1, temp_particles1 = bgc2.read_bgc2(sys.argv[i])
394         temp_header2, temp_halos2, temp_particles2 = bgc2.read_bgc2(sys.argv[(nargs / 2) + i])
395
396         if i == 1:
397             halos1, particles1 = temp_halos1, temp_particles1
398             halos2, particles2 = temp_halos2, temp_particles2
399         else:
400             halos1 = np.append(halos1, temp_halos1, axis=0)
401             halos2 = np.append(halos2, temp_halos2, axis=0)
402             particles1 = np.append(particles1, temp_particles1, axis=0)
403             particles2 = np.append(particles2, temp_particles2, axis=0)
404
405             halos1 = np.asarray(halos1)
406             halos2 = np.asarray(halos2)
407             #indices = np.argsort(halos[:,2])           # sort by number of particles
408             #indices = indices[::-1]                   # start with the biggest
409
410             iteration = 0
411             #for index in indices[:1000]:
412             #for index in indices:
413             for index in range(halos1.shape[0]):
414                 halo_id = halos1[index,0]
415                 if (halo_id == id1):
416                     print '-----',
417
418                     halo_particles1 = np.asarray(particles1[index])
419                     pos1 = halo_particles1[:,pos_cols[0]:pos_cols[0]+3] * dist_scale
420                     #vel1 = halo_particles1[:,vel_cols[0]:vel_cols[0]+3]
421
422                     r_vir1 = halos1[index][4] * dist_scale
423                     halo_mass1 = halos1[index][5]
424                     halo_pos1 = np.array([halos1[index][6] * dist_scale, halos1[index][7] * dist_scale, halos1[index][8] * dist_scale])
425                     #halo_vel1 = np.array([halos1[index][9], halos1[index][10], halos1[index][11]])
426
427                     print 'Using %d particles in halo %d.' % (halo_particles1.shape[0], halo_id)
428
429                     # Find center of mass
430                     #pos = pos - halo_pos
431                     #vel = vel - halo_vel
432
433                     # Pick only a certain percentage of particles for projection plots
434                     if (draw_frac < 1.0):
435                         np.random.shuffle(pos1)
436                         pos1 = pos1[:int(draw_frac*pos1.shape[0])]
437
438                     for index in range(halos2.shape[0]):
439                         halo_id = halos2[index,0]
440                         if (halo_id == id2):
441                             print '-----',
442
443                             halo_particles2 = np.asarray(particles2[index])
444                             pos2 = halo_particles2[:,pos_cols[0]:pos_cols[0]+3] * dist_scale
445                             #vel2 = halo_particles2[:,vel_cols[0]:vel_cols[0]+3]

```

```

446     r_vir2 = halos2[index][4] * dist_scale
447     halo_mass2 = halos2[index][5]
448     halo_pos2 = np.array([halos2[index][6] * dist_scale, halos2[index][7] * dist_scale, halos2[index][8] *
449     dist_scale])
450     #halo_vel2 = np.array([halos2[index][9], halos2[index][10], halos2[index][11]])
451
452     print 'Using %d particles in halo %d.' % (halo_particles2.shape[0], halo_id)
453
454     # Find center of mass
455     #pos = pos - halo_pos
456     #vel = vel - halo_vel
457
458     # Pick only a certain percentage of particles for projection plots
459     if (draw_frac < 1.0):
460         np.random.shuffle(pos2)
461         pos2 = pos2[:((draw_frac*pos2.shape[0]))]
462
463     # Find the maximum of x, y, or z to be limit of projection plots
464     center = (halo_pos1 + halo_pos2) / 2.0
465     pos1 = pos1 - center
466     pos2 = pos2 - center
467     halo_pos1 = halo_pos1 - center
468     halo_pos2 = halo_pos2 - center
469     plot_lim = np.append(pos1, pos2).max()
470
471     # Plot density profile histogram
472     if (make_plot == True):
473         fig = plt.figure()
474
475         fig, ax = draw_projection(fig, 0, plot_lim, pos1[:,0], pos1[:,1])
476         fig = draw_projection_again(fig, ax, pos2[:,0], pos2[:,1])
477         ax.add_patch(Circle((halo_pos1[0], halo_pos1[1]), r_vir1, fc="None", ec="black", lw=1))
478         ax.add_patch(Circle((halo_pos2[0], halo_pos2[1]), r_vir2, fc="None", ec="black", lw=1))
479
480         fig, ax = draw_projection(fig, 1, plot_lim, pos1[:,0], pos1[:,2])
481         fig = draw_projection_again(fig, ax, pos2[:,0], pos2[:,2])
482         ax.add_patch(Circle((halo_pos1[0], halo_pos1[2]), r_vir1, fc="None", ec="black", lw=1))
483         ax.add_patch(Circle((halo_pos2[0], halo_pos2[2]), r_vir2, fc="None", ec="black", lw=1))
484
485         fig, ax = draw_projection(fig, 2, plot_lim, pos1[:,1], pos1[:,2])
486         fig = draw_projection_again(fig, ax, pos2[:,1], pos2[:,2])
487         ax.add_patch(Circle((halo_pos1[1], halo_pos1[2]), r_vir1, fc="None", ec="black", lw=1))
488         ax.add_patch(Circle((halo_pos2[1], halo_pos2[2]), r_vir2, fc="None", ec="black", lw=1))
489
490         #fig, ax = draw_density_profile(fig, mid_bins, rho, err=rho_err)
491         #fig = draw_nfw_profile(fig, ax, nfw_r, nfw_fit, R_s=scale_radius)
492         #plt.tight_layout()
493         #plt.savefig(plot_base+str(itteration)+plot_ext)
494         plt.savefig('test.eps')
495
496     if __name__ == '__main__':
497         main()

```

## G.2 Density Comparison (Python)

```

1 #!/usr/bin/env python
2
3 import sys
4 import bgc2
5 import numpy as np
6 import matplotlib as mpl
7 mpl.use('Agg')
8 import matplotlib.pyplot as plt
9 from matplotlib.patches import Circle
10 from matplotlib import patheffects
11 from mpl_toolkits.axes_grid1 import ImageGrid
12 from scipy.stats import ks_2samp
13 from scipy.stats import chisquare
14 from scipy.optimize import curve_fit
15 from scipy.ndimage.filters import gaussian_filter
16 from ipdb import set_trace
17
18
19 ##### Note: only run one box pair at a time.
20 ##### ex: ./compare.py /crossmatch_dir/halos.dat /2lpt_dir/halos_0.*.bgc2 /za_dir/halos_0.*.bgc2
21
22 def main():
23     crossmatched_halo_file, bgc2_2lpt_files, bgc2_za_files = parse_args(sys.argv[1:])
24
25     header, halos = read_files(crossmatched_halo_file, header_line = 3)
26
27     bgc2_2lpt_header, bgc2_2lpt_halos, bgc2_2lpt_particles = get_bgc2_data(bgc2_2lpt_files)
28     bgc2_za_header, bgc2_za_halos, bgc2_za_particles = get_bgc2_data(bgc2_za_files)
29
30     header = np.asarray(header)
31     bgc2_2lpt_halos, bgc2_za_halos = map(np.asarray, (bgc2_2lpt_halos, bgc2_za_halos))
32
33     if sort_col != None:
34         halos = sort_by_column(halos, sort_col)
35     if remove_nonfit_halos:

```

```

36         halos = remove_nans(halos)
37     if global_filter_halos:
38         halos = filter_halos(halos)
39     if (nhalos != None) or (nhalos != 0):
40         #halos = halos[:nhalos]
41         halos = halos[0:700]           ##### hard coded for the moment
42         #halos = halos[10000:10050]
43
44     header, halos = add_c_columns(header, halos)
45     header = reduce_header(header)
46
47     for i, halo_pair in enumerate(halos):
48         make_plot(i, header, halo_pair, bgc2_2lpt_halos, bgc2_za_halos, \
49                    bgc2_2lpt_particles, bgc2_za_particles)
50
51     print 'Finished all plots.'
52
53
54 def parse_args(args):
55     crossmatched_halo_file = args[0]
56     if len(args[1:]) % 2 != 0:
57         print 'Must call with even number of bgc2 files... exiting.'
58         sys.exit(-1)
59     bgc2_files = args[1:]
60     bgc2_2lpt_files = bgc2_files[:len(bgc2_files)/2]
61     bgc2_za_files = bgc2_files[len(bgc2_files)/2:]
62     return crossmatched_halo_file, bgc2_2lpt_files, bgc2_za_files
63
64
65 def read_files(files, header_line = None, comment_char = '#'):
66     header = None
67     data = None
68     if type(files) == str:
69         files = [files]
70
71     if header_line != None:
72         with open(files[0], 'r') as fd:
73             for line in range(header_line):
74                 fd.readline()
75             header = fd.readline()
76             if header[0] != comment_char:
77                 print 'Header must start with a %s' % comment_char
78                 sys.exit(4)
79             header = header[1:]
80             header = header.split()
81
82     for file in files:
83         print 'Reading file %s...' % (file)
84         if data == None:
85             data = np.genfromtxt(file, comments=comment_char)
86         else:
87             data = np.append(data, np.genfromtxt(file, comments=comment_char), axis=0)
88
89     print 'Finished reading files.'
90     if header_line == None:
91         return data
92     else:
93         return header, data
94
95
96 def get_bgc2_data(bgc2_files):
97     header = None
98     halos = None
99     particles = None
100    for bgc2_file in bgc2_files:
101        print 'Reading file %s...' % (bgc2_file)
102        tmp_header, tmp_halos, tmp_particles = bgc2.read_bgc2(bgc2_file)
103        if header == None:
104            header = tmp_header
105            halos = tmp_halos
106            particles = tmp_particles
107        else:
108            halos = np.append(halos, tmp_halos, axis=0)
109            particles = np.append(particles, tmp_particles, axis=0)
110    print 'Finished reading bgc2 files.'
111    return header, halos, particles
112
113
114 def sort_by_column(halos, col):
115     print 'Sorting halos...'
116     mask = np.argsort(halos[:, col])
117     mask = mask[::-1]
118     halos = halos[mask]
119     return halos
120
121
122 def remove_nans(halos):
123     print 'Removing NaNs...'
124     halos = halos[halos[:, c_2lpt_col] != -9999]
125     halos = halos[np.isfinite(halos[:, c_2lpt_col])]
126     halos = halos[np.isfinite(halos[:, c_za_col])]
127     return halos

```

```

128
129
130 def filter_halos(halos):
131     print 'Filtering data...'
132     for col, val in zip(lt_cols, lt_vals):
133         halos = halos[halos[:, col] <= val]
134     for col, val in zip(gt_cols, gt_vals):
135         halos = halos[halos[:, col] >= val]
136     for col, val in zip(eq_cols, eq_vals):
137         halos = halos[halos[:, col] == val]
138     for col, val in zip(ne_cols, ne_vals):
139         halos = halos[halos[:, col] != val]
140
141
142
143 def add_c_columns(header, halos):
144     c1_rockstar = halos[:, Rv1_col] / halos[:, Rs1_col]
145     c2_rockstar = halos[:, Rv2_col] / halos[:, Rs2_col]
146     halos = np.column_stack((halos, c1_rockstar, c2_rockstar))
147     header = np.append(header, 'c_rockstar')
148     header = np.append(header, 'c_rockstar')
149
150     return header, halos
151
152
153 def reduce_header(header):
154     header_2lpt = header[print_cols_2lpt]
155     header_zap = header[print_cols_zap]
156     if (header_2lpt == header_zap).all():
157         header = header_2lpt
158     else:
159         print 'column mismatch... exiting'
160         set_trace()
161         sys.exit(123)
162
163
164 def make_plot(itteration, header, halo_pair, bgc2_halos_2lpt, bgc2_halos_zap, \
165               bgc2_particles_2lpt, bgc2_particles_zap):
166     id_2lpt = halo_pair[id_col_2lpt]
167     id_zap = halo_pair[id_col_zap]
168     properties_2lpt = halo_pair[print_cols_2lpt]
169     properties_zap = halo_pair[print_cols_zap]
170
171     # find 2lpt and zap halo from id
172     halo_index_2lpt = np.where(bgc2_halos_2lpt[:, halo_id_col] == id_2lpt)[0][0]
173     halo_index_zap = np.where(bgc2_halos_zap[:, halo_id_col] == id_zap)[0][0]
174
175     bgc2_halos_2lpt = bgc2_halos_2lpt[halo_index_2lpt]
176     bgc2_halos_zap = bgc2_halos_zap[halo_index_zap]
177
178     # convert particles to numpy arrays
179     bgc2_particles_2lpt = np.asarray(bgc2_particles_2lpt[halo_index_2lpt])
180     bgc2_particles_zap = np.asarray(bgc2_particles_zap[halo_index_zap])
181
182     # make density profiles
183     r_2lpt, rho_2lpt, rho_err_2lpt, r_vir_2lpt = density_profile(bgc2_halos_2lpt, bgc2_particles_2lpt)
184     r_zap, rho_zap, rho_err_zap, r_vir_zap = density_profile(bgc2_halos_zap, bgc2_particles_zap)
185
186     # fit density profiles
187     nfw_r_2lpt, nfw_rho_2lpt, r_s_2lpt = fit_profile(r_2lpt / r_vir_2lpt, rho_2lpt / rho_2lpt.max(), err =
188             rho_err_2lpt / rho_2lpt.max())
189     nfw_r_zap, nfw_rho_zap, r_s_zap = fit_profile(r_zap / r_vir_zap, rho_zap / rho_zap.max(), err =
190             rho_err_zap / rho_zap.max())
191
192     # de-normalize values
193     nfw_r_2lpt = nfw_r_2lpt * r_vir_2lpt
194     nfw_r_zap = nfw_r_zap * r_vir_zap
195     nfw_rho_2lpt = nfw_rho_2lpt * rho_2lpt.max()
196     nfw_rho_zap = nfw_rho_zap * rho_zap.max()
197     r_s_2lpt = r_s_2lpt * r_vir_2lpt
198     r_s_zap = r_s_zap * r_vir_zap
199
200     # find center of halos and plot limit
201     halo_pos_2lpt = bgc2_halos_2lpt[:, halo_pos_cols] * dist_scale
202     halo_pos_zap = bgc2_halos_zap[:, halo_pos_cols] * dist_scale
203     particle_pos_2lpt = bgc2_particles_2lpt[:, particle_pos_cols] * dist_scale
204     particle_pos_zap = bgc2_particles_zap[:, particle_pos_cols] * dist_scale
205
206     if wrap_box:
207         for i in range(3):
208             if abs(halo_pos_2lpt[i] - halo_pos_zap[i]) > box_size / 2.0:
209                 print "#####wrapping#####wrapping#####wrapping#####wrapping#####wrapping#####"
210                 if (halo_pos_2lpt[i] > halo_pos_zap[i]):
211                     halo_pos_zap[i] += box_size
212                     particle_pos_zap[:, i] += box_size
213                 if (halo_pos_2lpt[i] < halo_pos_zap[i]):
214                     halo_pos_2lpt[i] += box_size
215                     particle_pos_2lpt[:, i] += box_size
216             else:
217                 print "error in wrapping"
218                 sys.exit()

```

```

218     center = (halo_pos_2lpt + halo_pos_z) / 2.0
219     halo_pos_2lpt = halo_pos_2lpt - center
220     halo_pos_z = halo_pos_z - center
221     particle_pos_2lpt = particle_pos_2lpt - center
222     particle_pos_z = particle_pos_z - center
223
224     if zoom_projections:
225         plot_lim = zoom_scale
226     else:
227         plot_lim = np.append(particle_pos_2lpt, particle_pos_z).max()
228
229
230     r_vir_2lpt = bgc2_halos_2lpt[halo_r_col] * dist_scale
231     r_vir_z = bgc2_halos_z[halo_r_col] * dist_scale
232
233     if make_stats:
234         print 'generating plot...'
235         fig = plt.figure(figsize = (9.0, 6.0))
236         fig = make_projections(fig, 221, halo_pos_2lpt, halo_pos_z, particle_pos_2lpt, particle_pos_z, \
237                                r_vir_2lpt, r_vir_z, plot_lim)
238         ax = fig.add_subplot(223)
239         ax = draw_density_profile(ax, r_2lpt, rho_2lpt, err=rho_err_2lpt, color='blue', label='2lpt')
240         ax = draw_density_profile(ax, r_z, rho_z, err=rho_err_z, color='red', label='z')
241
242         ax = fig.add_subplot(122)
243         ax = draw_parameters(ax, header, properties_2lpt, properties_z)
244
245         fig.tight_layout()
246         plot_name = "%s%0.3d_(%d,%d)%s" % (plot_base, itteration, id_2lpt, id_z, plot_ext)
247         plt.savefig(plot_name, bbox_inches='tight')
248         print 'finished plot' + plot_name
249
250     if make_projection:
251         print 'generating density projection plot...'
252         fig = plt.figure(figsize = (9.0, 6.0))
253
254         if label_projection:
255             ax = fig.add_subplot(111, aspect=2.0/3.2)
256             ax = hide_axes(ax)
257             ax.set_xlabel(proj_xlabel)
258             ax.set_ylabel(proj_ylabel)
259
260             fig = make_projections(fig, 111, halo_pos_2lpt, halo_pos_z, particle_pos_2lpt, particle_pos_z, \
261                                    r_vir_2lpt, r_vir_z, plot_lim)
262             fig.tight_layout()
263             plot_name = "%s%0.3d_(%d,%d)%s" % (plot_base, itteration, id_2lpt, id_z, proj_name, plot_ext)
264             plt.savefig(plot_name, bbox_inches='tight')
265             print 'finished density projection plot' + plot_name
266
267     if make_density_profile:
268         print 'generating density profile plot...'
269         fig = plt.figure(figsize = (9.0, 12.0))
270
271         if label_projection:
272             ax = fig.add_subplot(211, aspect=2.0/3.2)
273             ax = hide_axes(ax)
274             ax.set_xlabel(proj_xlabel)
275             ax.set_ylabel(proj_ylabel)
276
277             fig = make_projections(fig, 211, halo_pos_2lpt, halo_pos_z, particle_pos_2lpt, particle_pos_z, \
278                                    r_vir_2lpt, r_vir_z, plot_lim)
279
280             ax = fig.add_subplot(212)
281             ax = hide_axes(ax)
282             ax.set_xlabel(prof_xlabel)
283             ax.set_ylabel(prof_ylabel)
284
285             #grid = ImageGrid(fig, 212, nrows_ncols=(2,1), axes_pad=0.24)
286
287             ax1 = fig.add_subplot(413)
288             ax1 = draw_density_profile(ax1, r_2lpt, rho_2lpt, err=rho_err_2lpt, color='blue')
289             ax1 = draw_nfw_profile(ax1, nfw_r_2lpt, nfw_rho_2lpt, R_s=r_s_2lpt, color='red')
290
291             ax2 = fig.add_subplot(414)
292             ax2 = draw_density_profile(ax2, r_z, rho_z, err=rho_err_z, color='blue')
293             ax2 = draw_nfw_profile(ax2, nfw_r_z, nfw_rho_z, R_s=r_s_z, color='red')
294
295         if equal_profile_axes:
296             ymin = min(ax1.get_ylim()[0], ax2.get_ylim()[0])
297             ymax = max(ax1.get_ylim()[1], ax2.get_ylim()[1])
298             ax1.set_ylim(ymin, ymax)
299             ax2.set_ylim(ymin, ymax)
300
301             xmin = min(ax1.get_xlim()[0], ax2.get_xlim()[0])
302             xmax = max(ax1.get_xlim()[1], ax2.get_xlim()[1])
303             ax1.set_xlim(xmin, xmax)
304             ax2.set_xlim(xmin, xmax)
305
306         if print_text:
307             ax1.text(0.95, 0.85, '2LPT', color='black', horizontalalignment='right', verticalalignment='center', transform=ax1.transAxes)

```

```

308         ax2.text(0.95, 0.85, 'ZA', color='black', horizontalalignment='right', verticalalignment='center',
309         transform=ax2.transAxes)
310
311     #fig.tight_layout()
312     plot_name = "%s%0.3d_(%d,%d)%s%s" % (plot_base, iteration, id_2lpt, id_za, dens_name, plot_ext)
313     plt.savefig(plot_name, bbox_inches='tight')
314     print 'finished density profile plot' + plot_name
315
316
317
318 def density_profile(halo, particles):
319     r_vir = halo[halo_r_col] * dist_scale
320     halo_pos = halo[halo_pos_cols] * dist_scale
321     #mass = np.ones(particles.shape[0]) * common_mass * mass_scale
322     mass = particles[:,particle_mass_col] * mass_scale
323     pos = particles[:,particle_pos_cols] * dist_scale
324     pos = pos - halo_pos
325
326     r_bins, rho, rho_err = calc_density_profile(mass, pos)
327     mid_bins = 10.0**((0.5 * (np.log10(r_bins[1:]) + np.log10(r_bins[:-1]))))
328
329     # Don't pass NaN's to fitting routine
330     rho_err_nonan = np.copy(rho_err)
331     nan_check = np.isnan(rho_err_nonan)
332     for i in range(len(rho_err_nonan)):
333         if (mid_bins[i] < res_limit) or (nan_check[i] == True):
334             rho_err_nonan[i] = 1.0e10
335
336     return mid_bins, rho, rho_err, r_vir
337
338
339 def calc_density_profile(mass, pos):
340     r = np.sqrt(pos[:,0]**2 + pos[:,1]**2 + pos[:,2]**2)
341     max_r = r.max()
342     min_r = res_limit
343     log_range = np.log10(max_r) - np.log10(min_r)
344     local_nbins = float(nbins + 1)
345     while True:
346         bins = np.arange(local_nbins)
347         bins = max_r * 10.0**log_range * bins / (local_nbins-1.0) - log_range
348         bin_mass, r_bins = np.histogram(r, bins, weights=mass)
349         if (bin_mass == 0.0).any():
350             local_nbins -= 1
351             continue
352         else:
353             break
354     rho = bin_mass / (sphere_vol(r_bins[1:]) - sphere_vol(r_bins[:-1]))
355     N_bin, blah = np.histogram(r, bins)
356     rho_err = poisson_error(N_bin) * rho
357     return r_bins, rho, rho_err
358
359
360 def sphere_vol(r):
361     volume = (4.0 / 3.0) * np.pi * r**3
362     return volume
363
364
365 def poisson_error(N):
366     err = np.sqrt(N) / N
367     return err
368
369
370 def fit_profile(r, rho, err=None, R_vir=None):
371     popt, pcov = curve_fit(nfw_profile, r, rho, sigma=err, p0=[0.1, 1.0])
372     R_s, rho_0 = popt[0], popt[1]
373     nfw_r = np.linspace(r[0], r[-1], nfit)
374     nfw_rho = nfw_profile(nfw_r, R_s, rho_0)
375     return nfw_r, nfw_rho, R_s
376
377
378 def nfw_profile(r, R_s, rho_0):
379     if R_s >= 1.0:
380         return (R_s - 1.0) * np.exp(r) + rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
381     return rho_0 / ((r / R_s) * (1.0 + r / R_s)**2)
382
383
384 def filter_column(x, x_col):
385     print 'Filtering data...'
386     x = x[x != -9999]
387     if x_col in lt_cols:
388         val = lt_vals[lt_cols.index(x_col)]
389         x = x[x <= val]
390     if x_col in gt_cols:
391         val = gt_vals[gt_cols.index(x_col)]
392         x = x[x >= val]
393     if x_col in eq_cols:
394         val = eq_vals[eq_cols.index(x_col)]
395         x = x[x == val]
396     if x_col in ne_cols:
397         val = ne_vals[ne_cols.index(x_col)]
398         x = x[x != val]

```

```

399     return x
400
401
402 def draw_hist(fig, ax, x, x_min=None, x_max=None, use_log=False, color=None, label=None):
403     if use_log:
404         xbins = np.logspace(np.log10(x_min), np.log10(x_max), num=nbins+1)
405         ax.set_xscale('log')
406     else:
407         xbins = np.linspace(x_min, x_max, num=nbins+1)
408
409     n, bins, patches = ax.hist(x, bins=xbins, histtype='step', log=ylog, color=color, label=label)
410
411     return fig, ax, n, bins, patches
412
413 def add_text(fig, ax, textstr):
414     props = dict(boxstyle='round', facecolor='white', alpha=0.7)
415     ax.text(0.02, 0.08, textstr, transform=ax.transAxes, fontsize=14,
416             verticalalignment='top', bbox=props)
417
418     return fig, ax
419
420 def make_projections(fig, position, halo_pos1, halo_pos2, pos1, pos2, r_vir1, r_vir2, plot_lim):
421     #grid = ImageGrid(fig, position, nrows_ncols=(2,3), axes_pad=0.05, cbar_mode='single')
422     grid = ImageGrid(fig, position, nrows_ncols=(2,3), axes_pad=0.12, cbar_mode='single')
423     for i, (x, y, hx, hy, r) in enumerate(zip(
424         (pos1[:,0], pos1[:,0], pos1[:,1], pos2[:,0], pos2[:,0], pos2[:,1]), \
425         (pos1[:,1], pos1[:,2], pos1[:,2], pos2[:,1], pos2[:,2], pos2[:,2]), \
426         (halo_pos1[0], halo_pos1[0], halo_pos1[1], halo_pos2[0], halo_pos2[0], halo_pos2[1]), \
427         (halo_pos1[1], halo_pos1[2], halo_pos1[2], halo_pos2[1], halo_pos2[2], halo_pos2[2]), \
428         (r_vir1, r_vir1, r_vir1, r_vir2, r_vir2, r_vir2))):
429         ax = grid[i]
430         draw_projection(ax, x, y, hx, hy, r, plot_lim)
431         if print_text:
432             if i == 0:
433                 ax.text(0.05, 0.12, '2LPT', color='white', horizontalalignment='left', verticalalignment='center',
434                         transform=ax.transAxes, path_effects=[patheffects.withStroke(linewidth=3, foreground='black')])
435             if i == 3:
436                 ax.text(0.05, 0.12, 'ZA', color='white', horizontalalignment='left', verticalalignment='center',
437                         transform=ax.transAxes, path_effects=[patheffects.withStroke(linewidth=3, foreground='black')])
438             if i == 0:
439                 ax.text(0.95, 0.88, 'XY', color='white', horizontalalignment='right', verticalalignment='center',
440                         transform=ax.transAxes, path_effects=[patheffects.withStroke(linewidth=3, foreground='black')])
441             if i == 1:
442                 ax.text(0.95, 0.88, 'XZ', color='white', horizontalalignment='right', verticalalignment='center',
443                         transform=ax.transAxes, path_effects=[patheffects.withStroke(linewidth=3, foreground='black')])
444             if i == 2:
445                 ax.text(0.95, 0.88, 'YZ', color='white', horizontalalignment='right', verticalalignment='center',
446                         transform=ax.transAxes, path_effects=[patheffects.withStroke(linewidth=3, foreground='black')])
447
448     return fig
449
450
451 def draw_projection(ax, x, y, hx, hy, r, plot_lim):
452     limits = [[-plot_lim, plot_lim], [-plot_lim, plot_lim]]
453     z, xedges, yedges = np.histogram2d(x, y, bins=npixels, range=limits)
454     if log_scale_projections:
455         z[z<1.0] = 0.5
456         #z = np.log10(z)
457         #z = np.log10(z)
458         #z[np.isinf(z)] = -0.1
459         plot_norm = mpl.colors.LogNorm(vmin = 1, vmax = z.max(), clip=True)
460         #plot_norm = None
461     else:
462         plot_norm = None
463     if extra_smoothing:
464         z = gaussian_filter(z, smoothing_radius)
465     im = ax.imshow(z.T, extent=(-plot_lim, plot_lim, -plot_lim, plot_lim), \
466                     interpolation='gaussian', origin='lower', cmap=cmap, norm=plot_norm)
467     #interpolation='gaussian', origin='lower', cmap=cmap)
468     ax.locator_params(nbins=6)
469     if draw_circle:
470         ax.add_patch(Circle((hx, hy), r, fc="None", ec="black", lw=1))
471     if draw_contours:
472         x_midpoints = (xedges[:-1] + xedges[1:]) / 2.0
473         y_midpoints = (yedges[:-1] + yedges[1:]) / 2.0
474         X, Y = np.meshgrid(x_midpoints, y_midpoints)
475         ax.contour(X, Y, z.T, 2, colors='black', linewidths=4)
476         ax.contour(X, Y, z.T, 2, colors='white', linewidths=2)
477     if label_colorbar:
478         if log_scale_projections:
479             log_format = mpl.ticker.LogFormatterMathtext(10, labelOnlyBase=False)
480             ax.cax.colorbar(im, format=log_format)
481         else:
482             ax.cax.colorbar(im)
483     else:
484         bar = ax.cax.colorbar(im, ticks=[])
485         bar.ax.set_yticklabels([])
486         #plt.setp(bar.ax.get_yticklabels(), visible=False)
487
488 def draw_density_profile(ax, r, rho, err=None, color='black', label=None):
489     im = ax.loglog(r, rho, linestyle='steps-mid', color=color, label=label)

```

```

486     line1 = ax.axvline(res_limit, color='black', linestyle=':')
487     ax.set_xlim(r[0] - (r[1]-r[0]), r[-1] + (r[-1]-r[-2]))
488     #ax.set_xlabel(xlabel_prof)
489     #ax.set_ylabel(ylabel_prof)
490     if err != None:
491         err_bars = ax.errorbar(r, rho, yerr=err, linestyle='None', color=color)
492     if label != None:
493         ax.legend(fontsize='x-small')
494     return ax
495
496
497 def draw_nfw_profile(ax, r, rho, R_s=None, color='black'):
498     ax.loglog(r, rho, linestyle='-', color=color)
499     if R_s != None:
500         line = ax.axvline(R_s, color='purple', linestyle='-.')
501     return ax
502
503
504 def draw_parameters(ax, header, params1, params2):
505     strlen = 12
506     header = [str(item)[:strlen] for item in header]
507     params1 = [str(item)[:strlen] for item in params1]
508     params2 = [str(item)[:strlen] for item in params2]
509     header.insert(0, 'simulation')
510     params1.insert(0, '--_2lpt_--')
511     params2.insert(0, '--_za_--')
512     header = '\n'.join(header)
513     params1 = '\n'.join(params1)
514     params2 = '\n'.join(params2)
515     ax.text(0.05, 0.5, header, horizontalalignment="left", verticalalignment="center", transform=ax.transAxes)
516     ax.text(0.40, 0.5, params1, horizontalalignment="left", verticalalignment="center", transform=ax.transAxes)
517     ax.text(0.75, 0.5, params2, horizontalalignment="left", verticalalignment="center", transform=ax.transAxes)
518     ax.axis('off')
519     return ax
520
521
522 def hide_axes(ax):
523     ax.spines['top'].set_color('none')
524     ax.spines['bottom'].set_color('none')
525     ax.spines['left'].set_color('none')
526     ax.spines['right'].set_color('none')
527     ax.tick_params(labelcolor='w', top='off', bottom='off', left='off', right='off')
528     return ax
529
530
531
532
533 nhalos = 1
534 sort_col = 9 # density_profile 2lpt halo mass
535 #sort_col = 47 # rockstar 2lpt halo mass (M200c)
536
537 nbins = 40
538 nfit = 100
539 npixels = 30
540 #npixels = 100
541 smoothing_radius = 0.9
542 remove_nonfit_halos = True
543 global_filter_halos = True
544 column_filter_halos = True
545 log_scale_projections = True
546 wrap_box = False
547 label_colorbar = False
548 label_projection = True
549 zoom_projections = True
550 zoom_scale = 18.0 # kpc
551 draw_circle = False
552 draw_contours = True
553 extra_smoothing = True
554 label_proj = True
555 label_2lpt_z = True
556 equal_profile_axes = True
557 print_text = True
558
559 box_size = 10000.0 # kpc
560
561 id_col_2lpt = 0
562 id_col_z = 1
563
564 print_cols_2lpt = [43, 57, 6, 9, 17, 23, 31, 47, 51, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 91, 93, 97, 99,
101, 103, 105, 107, 111, 163, 201, -2]
565 print_cols_z = [44, 58, 6, 10, 18, 24, 32, 48, 52, 60, 62, 64, 66, 68, 70, 72, 74, 76, 78, 92, 94, 98, 100,
102, 104, 106, 108, 112, 164, 202, -1]
566
567 Rv1_col = 53
568 Rv2_col = 54
569 Rs1_col = 55
570 Rs2_col = 56
571
572 c_2lpt_col = 17
573 c_z_col = 18
574
575 # c_2lpt, c_z, chi2_2lpt, chi2_z
```

```

576 lt_cols = [17, 18, 37, 38]
577 lt_vals = [100.0, 100.0, 10.0, 10.0]
578
579 # c_2lpt, c_za, rho_0_2lpt, rho_0_za, chi2_2lpt, chi2_za
580 gt_cols = [17, 18, 31, 32, 37, 38]
581 gt_vals = [1.0, 1.0, 0.0, 0.0, 0.0, 0.0]
582
583 eq_cols = []
584 eq_vals = []
585
586 ne_cols = []
587 ne_vals = []
588
589 # bgc2 halo array columns
590 halo_id_col = 0
591 halo_r_col = 4
592 halo_mass_col = 5
593 halo_pos_cols = [6,7,8]
594
595 # bgc2 particle array columns
596 particle_mass_col = 0
597 particle_pos_cols = [1,2,3]
598 particle_vel_cols = [4,5,6]
599
600 mass_scale = 1.0
601 common_mass = 5.33423e5
602 dist_scale = 1.0e3
603 res_limit = 0.5 #changed from 4.0 to 0.5 to match density_profile.py <-- maybe check why it was 4.0?
604 nfit = 500
605
606 dist_units = 'kpc'
607 xlabel_proj = r'X Position (%s h$^{\{-1\}})' % (dist_units), r'X Position (%s h$^{\{-1\}})' % (dist_units), r'Y
   Position (%s h$^{\{-1\}})' % (dist_units)
608 ylabel_proj = r'Y Position (%s h$^{\{-1\}})' % (dist_units), r'Z Position (%s h$^{\{-1\}})' % (dist_units), r'Z
   Position (%s h$^{\{-1\}})' % (dist_units)
609 proj_xlabel = r'Position_(kpc_h$^{\{-1\}})'
610 proj_ylabel = r'Position_(kpc_h$^{\{-1\}})'
611 prof_xlabel = r'Radius_(%suh$^{\{-1\}})' % (dist_units)
612 prof_ylabel = r'Density_(M$_{\odot}$%s$\cdot$%s$^{-3}$uh$^{\{2\}}$)' % (dist_units)
613
614 colormap = 'ocean_r'
615 colormap = 'rainbow'
616 plot_base = 'plots/halo_pair_'
617 proj_name = '_proj'
618 dens_name = '_dens'
619 plot_ext = '.eps'
620
621 make_stats = False
622 make_projection = False
623 make_density_profile = True
624
625 plot_dest_type = 'paper'
626 if plot_dest_type == 'paper':
627     mpl.rcParams['font.family'] = 'serif'
628     mpl.rcParams['font.size'] = 16
629     mpl.rcParams['axes.linewidth'] = 3
630     mpl.rcParams['lines.linewidth'] = 4
631     mpl.rcParams['patch.linewidth'] = 4
632     mpl.rcParams['xtick.major.width'] = 3
633     mpl.rcParams['ytick.major.width'] = 3
634     mpl.rcParams['xtick.major.size'] = 8
635     mpl.rcParams['ytick.major.size'] = 8
636
637
638 if __name__ == '__main__':
639     main()

```

## Appendix H

### Concentration Comparison Code (Python)

```
1 #!/usr/bin/env python
2
3 import sys
4 import numpy as np
5 from ipdb import set_trace
6
7 def main():
8     # Read in particle files
9     header, halos = read_files(sys.argv[1:], header_line = 3)
10
11    if remove_nonfit_halos:
12        print 'Removing NaNs...'
13        halos = halos[np.isfinite(halos[:,c_lpt_col])]
14        halos = halos[np.isfinite(halos[:,c_za_col])]
15
16    if global_filter_halos:
17        print 'Filtering data...'
18        for col, val in zip(glob_lt_cols, glob_lt_vals):
19            halos = halos[halos[:, col] <= val]
20        for col, val in zip(glob_gt_cols, glob_gt_vals):
21            halos = halos[halos[:, col] >= val]
22        for col, val in zip(glob_eq_cols, glob_eq_vals):
23            halos = halos[halos[:, col] == val]
24        for col, val in zip(glob_ne_cols, glob_ne_vals):
25            halos = halos[halos[:, col] != val]
26
27
28    if sort_col != None:
29        halos = sort_by_column(halos, sort_col)
30    if (nhalos != None) or (nhalos != 0):
31        halos = halos[:nhalos]
32    #if (nhalos == 'perc25'):
33    #    halos = halos[:len(halos)/10]
34    if bad_halo_pairs != None:
35        mask = np.arange(len(halos))
36        mask = np.in1d(mask, bad_halo_pairs)
37        mask = np.invert(mask)
38        halos = halos[mask]
39
40    c_rockstar_2lpt = halos[:, Rv1_col] / halos[:, Rsi_col]
41    c_rockstar_zu = halos[:, Rv2_col] / halos[:, Rs2_col]
42
43    if use_klypin:
44        mask = (halos[:,4] < 100)
45        print "changed %d halos" % (mask.sum())
46        print "c_2lpt_before", c_rockstar_2lpt[mask][0]
47        c_rockstar_2lpt[mask] = halos[mask, Rv1_col] / halos[mask, 79]
48        print "c_2lpt_klypin", c_rockstar_2lpt[mask][0]
49        mask = (halos[:,5] < 100)
50        print "changed %d halos" % (mask.sum())
51        print "c_zu_before", c_rockstar_zu[mask][0]
52        c_rockstar_zu[mask] = halos[mask, Rv2_col] / halos[mask, 80]
53        print "c_zu_klypin", c_rockstar_zu[mask][0]
54    c_diff_2lpt = 2.0 * (c_rockstar_2lpt - halos[:, c_lpt_col]) / (c_rockstar_2lpt + halos[:, c_lpt_col])
55    c_diff_zu = 2.0 * (c_rockstar_zu - halos[:, c_za_col]) / (c_rockstar_zu + halos[:, c_za_col])
56    #halos = np.column_stack((halos, c_rockstar_2lpt, c_rockstar_zu, c_diff_2lpt, c_diff_zu))
57    #header.append('c_rockstar')
58    #header.append('c_rockstar')
59    #header.append('c_diff')
60    #header.append('c_diff')
61
62    c_diff_2lpt = c_diff_2lpt[np.isfinite(c_diff_2lpt)]
63    c_diff_zu = c_diff_zu[np.isfinite(c_diff_zu)]
64    c_diff_tot = np.append(c_diff_2lpt, c_diff_zu)
65
66    c_diff_2lpt_frac = (np.abs(c_diff_zu) <= cutoff_diff_frac).sum() / float(len(c_diff_2lpt))
67    c_diff_zu_frac = (np.abs(c_diff_zu) <= cutoff_diff_frac).sum() / float(len(c_diff_zu))
68    c_diff_tot_frac = (np.abs(c_diff_tot) <= cutoff_diff_frac).sum() / float(len(c_diff_tot))
69
70    with open(c_diff_file, 'w') as fd:
71        fd.write("%g %g %g\n" % (c_diff_tot_frac, c_diff_zu_frac, c_diff_2lpt_frac))
72
73    print 'Finished snapshot.'
74
75 def read_files(files, header_line = None, comment_char = '#'):
76     header = None
77     data = None
78     if type(files) == str:
79         files = [files]
80
81     if header_line != None:
82         with open(files[0], 'r') as fd:
```

```

83         for line in range(header_line):
84             fd.readline()
85             header = fd.readline()
86             if header[0] != comment_char:
87                 print "Header must start with a '%s'" % comment_char
88                 sys.exit(4)
89             header = header[1:]
90             header = header.split()
91
92     for file in files:
93         print 'Reading file %s...' % (file)
94         if data == None:
95             data = np.genfromtxt(file, comments=comment_char)
96         else:
97             data = np.append(data, np.genfromtxt(file, comments=comment_char), axis=0)
98
99     print 'Finished reading files.'
100    if header_line == None:
101        return data
102    else:
103        return header, data
104
105
106 def sort_by_column(halos, col):
107     print 'Sorting halos...'
108     mask = np.argsort(halos[:, col])
109     mask = mask[::-1]
110     halos = halos[mask]
111     return halos
112
113
114
115 remove_nonfit_halos = False
116 global.filter_halos = True
117 use_klypin = False
118
119 nhalos = 100
120 #nhalos = 'perc25'
121 #sort_col = None
122 sort_col = 9
123
124 cutoff_diff_frac = 0.2
125
126
127 Rv1_col = 53
128 Rv2_col = 54
129 Rs1_col = 55
130 Rs2_col = 56
131
132 c_lpt_col = 17
133 c_zs_col = 18
134
135
136 lt_cols = [17, 18]
137 lt_vals = [100.0, 100.0]
138
139 gt_cols = [17, 18, 31, 32]
140 gt_vals = [1.0, 1.0, 0.0, 0.0]
141
142 eq_cols = []
143 eq_vals = []
144
145 ne_cols = []
146 ne_vals = []
147
148
149 # global filters
150 glob_lt_cols = []
151 glob_lt_vals = []
152
153 glob_gt_cols = [4, 5]
154 glob_gt_vals = [100, 100]
155
156 glob_eq_cols = [109, 110]
157 glob_eq_vals = [-1, -1]
158
159 glob_ne_cols = []
160 glob_ne_vals = []
161
162 bad_halo_pairs = None
163
164 c_diff_file = 'stats/c_diff.dat'
165
166
167
168 if __name__ == '__main__':
169     main()

```

# Appendix I

## Differential Histogram Code

### I.1 Histogram Generation and Fitting (Python)

```
1 #!/usr/bin/env python
2
3 import sys
4 import numpy as np
5 import matplotlib as mpl
6 mpl.use('Agg')
7 import matplotlib.pyplot as plt
8 import matplotlib.gridspec as gridspec
9 from scipy import stats
10 from scipy.special import gamma as gamma_func
11 from scipy.optimize import curve_fit
12 import statsmodels.sandbox.distributions.extras as extrastats
13 from ipdb import set_trace
14
15 def main():
16     # Read in particle files
17     header, halos = read_files(sys.argv[1:], header_line = 3)
18
19     if remove_nonfit_halos:
20         print 'Removing NaNs...'
21         halos = halos[np.isfinite(halos[:,c_lpt_col])]
22         halos = halos[np.isfinite(halos[:,c_za_col])]
23
24     if global_filter_halos:
25         print 'Filtering data...'
26         for col, val in zip(glob_lt_cols, glob_lt_vals):
27             halos = halos[halos[:, col] <= val]
28         for col, val in zip(glob_gt_cols, glob_gt_vals):
29             halos = halos[halos[:, col] >= val]
30         for col, val in zip(glob_eq_cols, glob_eq_vals):
31             halos = halos[halos[:, col] == val]
32         for col, val in zip(glob_ne_cols, glob_ne_vals):
33             halos = halos[halos[:, col] != val]
34
35
36     if sort_col != None:
37         halos = sort_by_column(halos, sort_col)
38     if (nhalos != None) or (nhalos != 0):
39         halos = halos[:nhalos]
40     if bad_halo_pairs != None:
41         mask = np.arange(len(halos))
42         mask = np.in1d(mask, bad_halo_pairs)
43         mask = np.invert(mask)
44         halos = halos[mask]
45
46     c_rockstar_2lpt = halos[:, Rv1_col] / halos[:, Rs1_col]
47     c_rockstar_za   = halos[:, Rv2_col] / halos[:, Rs2_col]
48     if use_klypin:
49         mask = (halos[:,4] < 100)
50         print "changed %d halos" % (mask.sum())
51         print "c_2lpt_before", c_rockstar_2lpt[mask][0]
52         c_rockstar_2lpt[mask] = halos[mask, Rv1_col] / halos[mask, 79]
53         print "c_2lpt_klypin", c_rockstar_2lpt[mask][0]
54         mask = (halos[:,5] < 100)
55         print "changed %d halos" % (mask.sum())
56         print "c_za_before", c_rockstar_za[mask][0]
57         c_rockstar_za[mask] = halos[mask, Rv2_col] / halos[mask, 80]
58         print "c_za_klypin", c_rockstar_za[mask][0]
59         c_diff_2lpt = 2.0 * (c_rockstar_2lpt - halos[:, c_lpt_col]) / (c_rockstar_2lpt + halos[:, c_lpt_col])
60         c_diff_za   = 2.0 * (c_rockstar_za - halos[:, c_za_col]) / (c_rockstar_za + halos[:, c_za_col])
61         halos = np.column_stack((halos, c_rockstar_2lpt, c_rockstar_za, c_diff_2lpt, c_diff_za))
62     header.append('c_rockstar')
63     header.append('c_rockstar')
64     header.append('c_diff')
65     header.append('c_diff')
66
67     if mass_quartiles and len(halos) > 50:
68         start_fracs = [0.0, 0.25, 0.50, 0.75, 0.0]
69         end_fracs   = [0.25, 0.50, 0.75, 1.0, 1.0]
70     else:
71         start_fracs = [0.0]
72         end_fracs   = [1.0]
73
74     for start_frac, end_frac in zip(start_fracs, end_fracs):
75         halos_to_pass = halos[start_frac * len(halos) : end_frac * len(halos)]
76         if use_alt_frac and (start_frac == 0.0) and (end_frac == 1.0):
77             alt_halos_to_pass = halos[alt_start_frac * len(halos) : alt_end_frac * len(halos)]
78         else:
79             alt_halos_to_pass = None
```

```

81     if len(halos_to_pass) > 0:
82         for (lpt_col, za_col, fancy_x_label) in zip(lpt_log_cols, za_log_cols, fancy_log_x_labels):
83             make_plot(halos_to_pass, alt_halos_to_pass, lpt_col, za_col, start_frac, end_frac, fancy_x_label,
84             header, use_log=True)
85         for (lpt_col, za_col, fancy_x_label) in zip(lpt_cols, za_cols, fancy_x_labels):
86             make_plot(halos_to_pass, alt_halos_to_pass, lpt_col, za_col, start_frac, end_frac, fancy_x_label,
87             header, use_log=False)
88
89     print 'Finished all plots.'
90
91 def read_files(files, header_line = None, comment_char = '#'):
92     header = None
93     data = None
94     if type(files) == str:
95         files = [files]
96
97     if header_line != None:
98         with open(files[0], 'r') as fd:
99             for line in range(header_line):
100                 fd.readline()
101             header = fd.readline()
102             if header[0] != comment_char:
103                 print 'Header must start with %s' % comment_char
104                 sys.exit(4)
105             header = header[1:]
106             header = header.split()
107
108     for file in files:
109         print 'Reading file %s...' % (file)
110         if data == None:
111             data = np.genfromtxt(file, comments=comment_char)
112         else:
113             data = np.append(data, np.genfromtxt(file, comments=comment_char), axis=0)
114
115     print 'Finished reading files.'
116     if header_line == None:
117         return data
118     else:
119         return header, data
120
121 def sort_by_column(halos, col):
122     print 'Sorting halos...'
123     mask = np.argsort(halos[:, col])
124     mask = mask[::-1]
125     halos = halos[mask]
126     return halos
127
128
129 def make_plot(halos, alt_halos, lpt_col, za_col, start_frac, end_frac, fancy_x_label, header=None, use_log=False):
130     print 'start=%s, start_frac=%s' % (start_frac, end_frac)
131     x_lpt = halos[:, lpt_col]
132     x_za = halos[:, za_col]
133     x_lpt, x_za = filter(x_lpt, x_za, lpt_col, za_col)
134
135     if alt_halos != None:
136         alt_x_lpt = alt_halos[:, lpt_col]
137         alt_x_za = alt_halos[:, za_col]
138         alt_x_lpt, alt_x_za = filter(alt_x_lpt, alt_x_za, lpt_col, za_col)
139
140     if header != None:
141         header_lpt = header[lpt_col]
142         header_za = header[za_col]
143         if header_lpt == header_za:
144             xlabel = header_lpt
145             xlabel = xlabel.replace('/', '_over_')
146         else:
147             print 'column mismatch... exiting'
148             set_trace()
149             sys.exit(123)
150
151     if len(x_lpt) == 0 or len(x_za) == 0:
152         print "Skipping range %f-%f for %s plot. No halos found." % (start_frac, end_frac, xlabel)
153         return
154     #set_trace()
155
156     if perc_diff:
157         print 'Finding percent difference stats...'
158         x_perc_diff = (x_lpt - x_za) / x_za
159         perc_diff_file = "%s%s%0.3d%s%0.3d%s%s_(%s-%s)%s" % \
160                         (perc_diff_base, '(', lpt_col, ',', za_col, ')', xlabel, start_frac, end_frac,
161                         stats_ext)
162         perc_diff_stats(x_perc_diff, perc_diff_file, use_log=use_log)
163         print 'done.'
164
165     x = 2.0 * (x_lpt - x_za) / (x_lpt + x_za)
166     x[np.logical_and(x_lpt == 0, x_za == 0)] = 0
167
168     if alt_halos != None:

```

```

169     alt_x = 2.0 * (alt_x_lpt - alt_x_za) / (alt_x_lpt + alt_x_za)
170     alt_x[np.logical_and(alt_x_lpt == 0, alt_x_za == 0)] = 0
171
172 # set_trace()
173
174 if x_lim == None:
175     #x_max = max(abs(x.max()), abs(x.min()))
176     if lpt_col == 47:
177         x_max = x.mean() + x.std() * 1.5
178         x_min = x.mean() - x.std() * 1.5
179     else:
180         x_max = np.std(x) * 3.0
181         x_min = -x_max
182 else:
183     x_max = x_lim
184     x_min = -x_lim
185
186 # get stats
187 data_mean = x.mean()
188 data_stdev = x.std()*2
189 data_skew = stats.skew(x)
190 data_kurt = stats.kurtosis(x)
191 data_rms = np.sqrt(np.mean(x**2))
192 data_gt_epsilon = float(len(x[np.abs(x) >= 0.1])) / float(len(x))
193
194 # Generate plot
195 print 'generating', xlabel, 'plot...'
196 fig = plt.figure(figsize=(9.0, 6.0))
197 if add_residuals_panel:
198     grid = gridspec.GridSpec(2, 1, height_ratios=[1,4])
199     ax = fig.add_subplot(grid[1])
200 else:
201     ax = fig.add_subplot(111)
202 ax, n, bins, patches = draw_hist(ax, x, x_min=x_min, x_max=x_max, \
203                                     use_log=use_log, color='blue', fill=None)
204
205 p0 = [1.0, data_mean, data_stdev, 2.0]
206 ax, fit_height, fit_mean, fit_stdev, fit_skew, fit_kurt, fit_height_err, fit_mean_err, fit_stdev_err,
207     fit_skew_err, fit_kurt_err, chi2, pval = draw_fit(ax, n, bins, p0)
208
209 if draw_data_fit:
210     ax = draw_data_gaussian(ax, x, n, bins)
211
212 if alt_halos != None:
213     ax, n_alt, bins_alt, patches_alt = draw_hist(ax, alt_x, x_min=x_min, x_max=x_max, \
214                                                 use_log=use_log, color='green', fill="0.75")
215     #ax = draw_fit(ax, n, bins)
216
217 #ax.grid(color='gray', linestyle='dashed')
218 ax.set_xlim([x_min, x_max])
219 #ax.set_xlabel('(' + xlabel + '_lpt - ' + xlabel + '_za) / ' + xlabel + '_avg')
220 #ax.set_ylabel(ylabel)
221 if label_axes:
222     ax.set_xlabel(fancy_x_label, fontsize="xx-large")
223     ax.set_ylabel(fancy_y_label, fontsize="xx-large")
224 #ax.legend()
225
226 if add_residuals_panel:
227     ax = fig.add_subplot(grid[0])
228     ax = draw_residuals(ax, n, bins, fit_height, fit_mean, fit_stdev, fit_kurt)
229     ax.tick_params(axis='x', labelbottom='off')
230
231 fig.tight_layout()
232 plot_name = "%s%0.3d%s%0.3d%s(%s-%s)%s" % \
233     (plot_base, '(', lpt_col, ',', za_col, ')', xlabel, start_frac, end_frac, plot_ext)
234 fig.savefig(plot_name, bbox_inches='tight')
235
236 if save_stats:
237     statsfile = "%s%0.3d%s%0.3d%s(%s-%s)%s" % \
238         (stats_base, '(', lpt_col, ',', za_col, ')', xlabel, start_frac, end_frac, stats_ext)
239     with open(statsfile, 'w') as fd:
240         if bin_test:
241             for ntestbins in range(nbins_min, nbins_max+1, 5):
242                 fit_mean, fit_stdev = rebin_stats(ntestbins, x, x_min=x_min, x_max=x_max, use_log=use_log)
243                 fd.write("%d%g%g%g%g%g%g%g%g%g%g%g%g%g%g%g%g%g%g\n" % \
244                     (ntestbins, data_mean, data_stdev, data_skew, data_kurt, \
245                      fit_height, fit_height_err, fit_mean, fit_mean_err, fit_stdev, fit_stdev_err, fit_skew,
246                      fit_skew_err, fit_kurt_err, \
247                      data_rms, data_gt_epsilon, chi2, pval))
248
249 print 'finished plot' + plot_name
250 return
251
252
253 def perc_diff_stats(x, filename, use_log=False):
254     data_mean = x.mean()
255     data_stdev = x.std()*2
256     data_skew = stats.skew(x)
257     data_kurt = stats.kurtosis(x)
258     data_rms = np.sqrt(np.mean(x**2))

```

```

259     data_gt_epsilon = float(len(x[np.abs(x) >= 0.1])) / float(len(x))
260
261     if x_lim == None:
262         x_max = min((x.mean() + x.std() * 3.0), x.max())
263         x_min = max((x.mean() - x.std() * 3.0), x.min())
264     else:
265         x_max = x_lim
266         x_min = -x_max
267
268     global nbins
269     if nbins <= 0:
270         nbins = np.sqrt(len(x))
271         if nbins % 2 == 0:
272             nbins = nbins - 1
273     if nbins < nbins_min:
274         nbins = nbins_min
275     elif nbins > nbins_max:
276         nbins = nbins_max
277
278     if use_log:
279         xbins = np.logspace(np.log10(x_min), np.log10(x_max), num=nbins+1)
280         mid_bins = 10.0**((0.5 * (np.log10(xbins[1:]) + np.log10(xbins[:-1]))))
281     else:
282         xbins = np.linspace(x_min, x_max, num=nbins+1)
283         mid_bins = 0.5 * (xbins[1:] + xbins[:-1])
284
285     hist, bin_edges = np.histogram(x, bins=xbins)
286     x_peak = mid_bins[hist == hist.max()][0]
287
288     x_sorted = np.sort(x)
289     n_halos = len(x_sorted)
290
291     x_vals = []
292     for frac in fractions:
293         x_vals.append(x_sorted[len(x_sorted)*frac])
294     x_vals = np.array(x_vals)
295
296     sum_frac_halos = []
297     for diff_val in diff_vals:
298         n_gt_val = (x_sorted >= diff_val).sum()
299         sum_frac_halos.append(float(n_gt_val) / float(n_halos))
300     sum_frac_halos = np.array(sum_frac_halos)
301
302     doublesum_frac_halos = []
303     for right_diff_val in diff_vals:
304         left_diff_val = (1.0 / (right_diff_val + 1.0)) - 1.0
305         n_gt_val = (x_sorted >= right_diff_val).sum() + (x_sorted <= left_diff_val).sum()
306         doublesum_frac_halos.append(float(n_gt_val) / float(n_halos))
307     doublesum_frac_halos = np.array(doublesum_frac_halos)
308
309     with open(filename, 'w') as fd:
310         fd.write("%d\n" % nbins, x_peak, \
311                 nbins, x_peak, \
312                 '\n'.join(["%g" % x for x in x_vals]), \
313                 '\n'.join(["%g" % x for x in sum_frac_halos]), \
314                 '\n'.join(["%g" % x for x in doublesum_frac_halos]), \
315                 data_mean, data_stdev, data_skew, data_kurt, \
316                 data_rms, data_gt_epsilon)
317
318     return
319
320
321 def find_frac_bounds(hist, start_bin, frac):
322     n_tot = hist.sum()
323     n_sum = hist[start_bin]
324
325     left_tot = hist[:start_bin].sum() + hist[start_bin]/2.0
326     right_tot = hist[start_bin+1:].sum() + hist[start_bin]/2.0
327
328     if float(left_tot) / float(n_tot) <= frac / 2.0:
329         right_only = True
330     if float(right_tot) / float(n_tot) <= frac / 2.0:
331         left_only = True
332
333     left_bound = start_bin
334     right_bound = start_bin
335     while(float(n_sum) / float(n_tot) < frac):
336
337         pass
338
339     return left_bound, right_bound
340
341
342 def filter(x_lpt, x_za, lpt_col, za_col):
343     mask = np.isfinite(x_lpt)
344     x_lpt = x_lpt[mask]
345     x_za = x_za[mask]
346     mask = np.isfinite(x_za)
347     x_lpt = x_lpt[mask]
348     x_za = x_za[mask]
349
350     if column_filter_halos:

```

```

351     x_lpt, x_za = filter_columns(lpt_col, x_lpt, x_za)
352     x_za, x_lpt = filter_columns(za_col, x_za, x_lpt)
353
354     return x_lpt, x_za
355
356
357 def filter_columns(x_col, x1, x2):
358     print 'Filtering data...'
359
360     mask = np.isfinite(x1)
361     x1 = x1[mask]
362     x2 = x2[mask]
363
364     mask = (x1 != -9999)
365     x1 = x1[mask]
366     x2 = x2[mask]
367
368     if x_col in lt_cols:
369         val = lt_vals[lt_cols.index(x_col)]
370         mask = (x1 <= val)
371         x1 = x1[mask]
372         x2 = x2[mask]
373     if x_col in gt_cols:
374         val = gt_vals[gt_cols.index(x_col)]
375         mask = (x1 >= val)
376         x1 = x1[mask]
377         x2 = x2[mask]
378     if x_col in eq_cols:
379         val = eq_vals[eq_cols.index(x_col)]
380         mask = (x1 == val)
381         x1 = x1[mask]
382         x2 = x2[mask]
383     if x_col in ne_cols:
384         val = ne_vals[ne_cols.index(x_col)]
385         mask = (x1 != val)
386         x1 = x1[mask]
387         x2 = x2[mask]
388
389     return x1, x2
390
391 def draw_hist(ax, x, x_min=None, x_max=None, use_log=False, color=None, fill=None, label=None):
392     global nbins
393     if nbins <= 0:
394         nbins = np.sqrt(len(x))
395         if nbins % 2 == 0:
396             nbins = nbins - 1
397     if nbins < nbins_min:
398         nbins = nbins_min
399     elif nbins > nbins_max:
400         nbins = nbins_max
401
402     if use_log:
403         xbins = np.logspace(np.log10(x_min), np.log10(x_max), num=nbins+1)
404         ax.set_xscale('log')
405     else:
406         xbins = np.linspace(x_min, x_max, num=nbins+1)
407
408     if fill == None:
409         type='step'
410     else:
411         type='stepfilled'
412
413     n, bins, patches = ax.hist(x, bins=xbins, histtype=type, facecolor=fill, normed=hist_normed, cumulative=
414         hist_cumulative, log=ylog, edgecolor=color, label=label)
415
416
417 def draw_fit(ax, hist, bin_edges, p0):
418     bin_centers = (bin_edges[:-1] + bin_edges[1:]) / 2.0
419
420     if ignore_central_bin:
421         mask = (np.abs(bin_centers) > 0.000001)
422         bin_centers = bin_centers[mask]
423         hist = hist[mask]
424
425     hist[hist==0] = 1 #fix devide by zero error
426
427     try:
428         if poisson_weight:
429             sigma=np.sqrt(hist)/hist
430             sigma = sigma / float(hist.max())
431         else:
432             sigma=None
433
434         if fit_in_log:
435             #if sigma != None:
436             #    sigma = np.log10(sigma)
437
438             coeffs, var_matrix = curve_fit(log_generalized_normal, bin_centers, np.log10(hist/float(hist.max())),
439             p0=p0, sigma=sigma)
440             coeffs[0] = coeffs[0]**2

```

```

441         var_matrix[0,0] = var_matrix[0,0]**2
442     else:
443         coeffs, var_matrix = curve_fit(generalized_normal, bin_centers, hist/float(hist.max()), p0=p0, sigma=
444             sigma)
445         if prevent_small_shape_param and coeffs[3] < 1.0:
446             coeffs[3] = 1.0 / coeffs[3]
447         print 'coeffs=', coeffs
448
449     except RuntimeError:
450         print '*****curve_fit failed!'
451         return ax, np.nan, np.nan
452
453 height, mean, stdv, skew, kurt = coeffs[0] * hist.max(), coeffs[1], coeffs[2], 0.0, coeffs[3]
454 height_err, mean_err, stdv_err, skew_err, kurt_err = np.sqrt(var_matrix[0,0]*hist.max()), np.sqrt(var_matrix
455 [1,1]), np.sqrt(var_matrix[2,2]), 0.0, np.sqrt(var_matrix[3,3])
456
457 fit_x = np.linspace(bin_edges[0], bin_edges[-1], nfitpoints+1)
458 hist_fit = generalized_normal(fit_x, height, mean, stdv, kurt)
459 ax.plot(fit_x, hist_fit, color='red', linestyle='--')
460
461 chi2_fit = generalized_normal(bin_centers, height, mean, stdv, kurt)
462 chi2, pval = stats.chisquare(hist / hist.max(), chi2_fit / hist.max())
463
464 return ax, height, mean, stdv, skew, kurt, height_err, mean_err, stdv_err, skew_err, kurt_err, chi2, pval
465
466 def draw_residuals(ax, hist, bin_edges, fit_height, fit_mean, fit_stdev, fit_kurt):
467     bin_centers = (bin_edges[:-1] + bin.edges[1:]) / 2.0
468     fit = generalized_normal(bin_centers, fit_height, fit_mean, fit_stdev, fit_kurt)
469     ratio = (hist - fit) / hist.max()
470     #ax.plot(bin_centers, ratio, linestyle='steps-mid')
471     ax.plot(bin_centers, ratio, linestyle='steps-mid')
472     return ax
473
474
475 def draw_data_gaussian(ax, x, hist, bins):
476     bin_centers = (bins[:-1] + bins[1:]) / 2.0
477     x_min = bins[0]
478     x_max = bins[-1]
479
480     mean = np.mean(x)
481     stdv = np.std(x)**2
482     skew = stats.skew(x)
483     kurt = stats.kurtosis(x)
484
485     print "data.stats:mean=%g stdv=%g skew=%g kurt=%g" % (mean, stdv, skew, kurt)
486
487     coeffs, var_matrix = curve_fit(gaussian_height(mean, stdv, skew, kurt), bin_centers, hist, p0=[hist.max()])
488     height = coeffs[0]
489
490     fit_x = np.linspace(x_min, x_max, nfitpoints+1)
491     hist_fit = gaussian(fit_x, height, mean, stdv, skew, kurt)
492     ax.plot(fit_x, hist_fit, color='0.25', linestyle='-.')
493     return ax
494
495
496 #def gaussian(x, A, mu, sigma, skew, kurtosis):
497 #    pdf_function = extrastats.pdf_mvsk([mu, sigma, skew, kurtosis])
498 #    return A * pdf_function(x)
499
500
501 def double_gaussian(x, A, mu, sigma, skew, kurtosis, A2, mu2, sigma2, skew2, kurtosis2):
502     return gaussian(x, A, mu, sigma, skew, kurtosis) + gaussian(x, A2, mu2, sigma2, skew2, kurtosis2)
503
504
505 def gaussian_height(mu, sigma, skew, kurtosis):
506     def func(x, A):
507         pdf_function = extrastats.pdf_mvsk([mu, sigma, skew, kurtosis])
508         return A * pdf_function(x)
509     return func
510
511
512 #def log_gaussian(x, A, mu, sigma, skew=0.0, kurtosis=0.0):
513 def log_gaussian(x, A, mu, sigma):
514     A = A**2 # remember to also square fit value for A
515     y = gaussian(x, A, mu, sigma)
516     #y = gaussian(x, A, mu, sigma, skew, kurtosis)
517     if (y <= 0).any():
518         #y[y<=0] = -y[y<=0] + 1
519         y[y<=0] = (y[y<=0] + 0.0001)**2
520     return np.log10(y)
521
522
523 #def log_double_gaussian(x, A1, mu1, sigma1, skew1, kurtosis1, A2, sigma2, skew2, kurtosis2):    # for common
524     mean
525 #def log_double_gaussian(x, A1, mu1, sigma1, skew1, kurtosis1, A2, mu2, sigma2, skew2, kurtosis2):
526 def log_double_gaussian(x, A1, mu1, sigma1, skew1, kurtosis1, A2, mu2, sigma2, skew2, kurtosis2):
527     #mu2 = mu1 # for common mean
528     A1 = A1**2 # remember to also square fit value for A
529     A2 = A2**2
530     skew1 = 0.0

```

```

530     skew2 = 0.0
531     kurtosis1 = 0.0
532     kurtosis2 = 0.0
533     y = double_gaussian(x, A1, mu1, sigma1, skew1, kurtosis1, A2, mu2, sigma2, skew2, kurtosis2)
534     if (y <= 0.0).any():
535         #y[y<=0] = -y[y<=0] + 1
536         y[y<=0] = (y[y<=0] + 0.0001)**2
537     return np.log10(y)
538
539
540 def gaussian(x, A, mu, sigma):
541     return A * np.exp(-(x - mu)**2 / (2.0 * sigma**2))
542
543
544 def generalized_normal(x, A, mu, alpha, beta):
545     if prevent_small_shape_param and beta < 1.0:
546         beta = 1.0 / beta
547     return A * (beta / (2.0 * alpha * gamma_func(1.0 / beta))) * np.exp(-(np.abs(x - mu)/alpha)**beta)
548
549
550 def log_generalized_normal(x, A, mu, alpha, beta):
551     A = A**2
552     y = generalized_normal(x, A, mu, alpha, beta)
553     if (y <= 0.0).any():
554         #y[y<=0] = -y[y<=0] + 1.0
555         y[y<=0] = (y[y<=0] + 0.0001)**2
556     return np.log10(y)
557
558
559 def add_text(fig, ax, textstr):
560     #props = dict(boxstyle='round', facecolor='white', alpha=0.25)
561     props = dict(edgecolor='none', facecolor='none')
562     ax.text(0.02, 0.16, textstr, transform=ax.transAxes, fontsize=14,
563             verticalalignment='top', bbox=props)
564     return fig, ax
565
566
567 def rebin_stats(ntestbins, x, x_min=None, x_max=None, use_log=False):
568     if use_log:
569         xbins = np.logspace(np.log10(x_min), np.log10(x_max), num=ntestbins+1)
570     else:
571         xbins = np.linspace(x_min, x_max, num=ntestbins+1)
572
573     hist, bin_edges = np.histogram(x, bins=xbins)
574
575     bin_centers = (bin_edges[:-1] + bin_edges[1:]) / 2.0
576     if ignore_central_bin:
577         mask = (np.abs(bin_centers) > 0.000001)
578         bin_centers = bin_centers[mask]
579         hist = hist[mask]
580     p0 = [hist.max(), 0.0, 0.2]
581     p0 = [hist.max(), hist.mean(), hist.std(), stats.skew(hist), stats.kurtosis(hist)]
582     hist[hist==0] = 1 #fix devide by zero error
583     try:
584         if poisson_weight:
585             coefffs, var_matrix = curve_fit(gaussian, bin_centers, hist, p0=p0, sigma=(np.sqrt(hist)/hist))
586         else:
587             coefffs, var_matrix = curve_fit(gaussian, bin_centers, hist, p0=p0)
588     except RuntimeError:
589         print '*****curve_fit failed!'
590     return np.nan, np.nan
591
592     mean, stdev = coefffs[1], coefffs[2]
593     return mean, stdev
594
595
596 nbins = 35
597 #nbins = 25
598 #nbins = -1
599 nbins_min = 15
600 nbins_max = 200
601 #nbins_max = 200
602 nfitpoints = 100
603 remove_nonfit_halos = False
604 global_filter_halos = True
605 column_filter_halos = True
606 use_klypin = False
607 label_axes = True
608 ignore_central_bin = False
609 save_stats = True
610 bin_test = False
611 poisson_weight = True
612 fit_in_log = True
613 draw_data_fit = False
614 mass_quartiles = False
615 prevent_small_shape_param = False
616 add_residuals_panel = False
617 perc_diff = True
618
619 hist_normed = False
620 hist_cumulative = False
621 ylog = False

```

```

622 ylabel= 'Number\u00d7of\u00d7Halos'
623
624 #
625 fractions = [0.01, 0.05, 0.10, 0.25, 0.50, 0.75, 0.90, 0.95, 0.99]
626 diff_vals = [0.01, 0.05, 0.10, 0.25, 0.50, 0.75, 1.00, 2.00, 4.00]
627 #
628
629 #nhalos = 100
630 nhalos = None
631 sort_col = 9
632
633 #lpt_log_cols = [ 9, 23, 31, 47, 51, 57]
634 #za_log_cols = [10, 24, 32, 48, 52, 58]
635 #lpt_cols = [17, 77, 91, 93, 97, 99, 107, 111, -4, -2]
636 #za_cols = [18, 78, 92, 94, 98, 100, 108, 112, -3, -1]
637
638 lpt_log_cols = []
639 za_log_cols = []
640 #lpt_cols = [-4, 47, 91, 107, 111]
641 #za_cols = [-3, 48, 92, 108, 112]
642 #lpt_cols = [-4, 31, 47, 91, 107, 111]
643 #za_cols = [-3, 32, 48, 92, 108, 112]
644 #lpt_cols = [-4, 31, 47, 91, 111]
645 #za_cols = [-3, 32, 48, 92, 112]
646 lpt_cols = [-4, 47, 91, 93, 107]
647 za_cols = [-3, 48, 92, 94, 108]
648 # concentration, mass, x_off, v_off, T/|U|
649
650 fancy_log_x_labels = []
651 #fancy_x_labels = [r"\frac{c_{2LPT} - c_{ZA}}{c_{avg}}",
652 #                   r"\frac{\rho_0, 2LPT} - \rho_0, ZA}{\rho_0, avg}}",
653 #                   r"\frac{M_{vir}, 2LPT} - M_{vir, ZA}}{M_{vir, avg}}",
654 #                   r"\frac{X_{off}, 2LPT} - X_{off, ZA}}{X_{off, avg}}",
655 #                   r"\frac{N_{subs}, 2LPT} - N_{subs, ZA}}{N_{subs, avg}}"]
656
657 fancy_x_labels = [r"\frac{c_{2LPT} - c_{ZA}}{c_{avg}}",
658                   r"\frac{M_{vir}, 2LPT} - M_{vir, ZA}}{M_{vir, avg}}",
659                   r"\frac{X_{off}, 2LPT} - X_{off, ZA}}{X_{off, avg}}",
660                   r"\frac{V_{off}, 2LPT} - V_{off, ZA}}{V_{off, avg}}",
661                   r"\frac{(T/|U|)_{2LPT} - (T/|U|)_{ZA}}{(T/|U|)_{avg}}"]
662
663 fancy_y_label = r"N_{halos}"
664
665 Rv1_col = 53
666 Rv2_col = 54
667 Rs1_col = 55
668 Rs2_col = 56
669
670 c_lpt_col = 17
671 c_za_col = 18
672
673
674 # c_2lpt, c_za, chi2_2lpt, chi2_za
675 #lt_cols = [17, 18, 37, 38]
676 #lt_vals = [100.0, 100.0, 10.0, 10.0]
677 lt_cols = [17, 18]
678 lt_vals = [100.0, 100.0]
679
680 # c_2lpt, c_za, rho_0_2lpt, rho_0_za, chi2_2lpt, chi2_za
681 #gt_cols = [17, 18, 31, 32, 37, 38]
682 #gt_vals = [1.0, 1.0, 0.0, 0.0, 0.0, 0.0]
683 gt_cols = [17, 18, 31, 32]
684 gt_vals = [1.0, 1.0, 0.0, 0.0]
685
686 eq_cols = []
687 eq_vals = []
688
689 ne_cols = []
690 ne_vals = []
691
692
693 # global filters
694 glob_lt_cols = []
695 glob_lt_vals = []
696
697 glob_gt_cols = [4, 5]
698 glob_gt_vals = [100, 100]
699
700 glob_eq_cols = [109, 110]
701 glob_eq_vals = [-1, -1]
702
703 glob_ne_cols = []
704 glob_ne_vals = []
705
706
707
708 use_alt_frac = True
709 alt_start_frac = 0.75
710 alt_end_frac = 1.0
711
712 #x_lim = 0.5
713 #x_lim = 1.0

```

```

714 x_lim = None
715
716 bad_halo_pairs = None
717 #bad_halo_pairs = [9, 28, 39, 51, 59, 95]
718
719 perc_diff_base = 'plots/perc_diff_'
720 #statsfile = 'plots/stats.txt'
721 stats_base = 'plots/stats_'
722 stats_ext = '.txt'
723 plot_base = 'plots/hist_'
724 plot_ext = '.eps'
725
726 plot_dest_type = 'paper'
727 if plot_dest_type == 'paper':
728     mpl.rcParams['font.family'] = 'serif'
729     mpl.rcParams['font.size'] = 16
730     mpl.rcParams['axes.linewidth'] = 3
731     mpl.rcParams['lines.linewidth'] = 4
732     mpl.rcParams['patch.linewidth'] = 4
733     mpl.rcParams['xtick.major.width'] = 3
734     mpl.rcParams['ytick.major.width'] = 3
735     mpl.rcParams['xtick.major.size'] = 8
736     mpl.rcParams['ytick.major.size'] = 8
737
738 if __name__ == '__main__':
739     main()

```

## I.2 PBS Submission Script (Bash)

```

1 #!/usr/bin/env bash
2 #PBS -M djsissom@gmail.com
3 #PBS -m bae
4 #PBS -l nodes=1:ppn=1
5 #PBS -l pmem=4000mb
6 #PBS -l mem=4000mb
7 #PBS -l walltime=1:00:00
8 #PBS -o out.log
9 #PBS -j oe
10
11 minsnap=0
12 maxsnap=61
13
14 # Change to working directory
15 echo $PBS_NODEFILE
16 cd $PBS_O_WORKDIR
17
18 for ((snap=$minsnap; snap<=$maxsnap; snap++)); do
19
20     if [ $snap -lt 10 ]; then
21         j=0$snaps
22     elif [ $snap -lt 100 ]; then
23         j=0$snaps
24     fi
25
26     new_plot_dir=snap${j}_plots
27
28     if [ ! -e plots_all_snaps/${new_plot_dir} ]; then
29         mkdir plots_all_snaps/${new_plot_dir}
30     fi
31
32     echo "Starting box ${i} snap ${j}..."
33     ./hist.py ~/projects/simulations/rockstar/box{1,2,3}/crossmatch/snap${j}/halos.dat > plots/out.log 2>&1
34     mv plots/* plots_all_snaps/${new_plot_dir}/.
35     echo "Finished snap ${j}"
36
37 done
38
39 wait
40
41 # - end of script

```

## I.3 PBS Submission Script - Individual Boxes (Bash)

```

1 #!/usr/bin/env bash
2 #PBS -M djsissom@gmail.com
3 #PBS -m bae
4 #PBS -l nodes=1:ppn=1
5 #PBS -l pmem=4000mb
6 #PBS -l mem=4000mb
7 #PBS -l walltime=2:00:00
8 #PBS -o out.log
9 #PBS -j oe
10
11 minsnap=0
12 maxsnap=61
13
14 minbox=1
15 maxbox=3
16

```

```

17 # Change to working directory
18 echo $PBS_NODEFILE
19 cd $PBS_O_WORKDIR
20
21
22 for ((box=$minbox; box<=$maxbox; box++)); do
23
24     new_box_dir=plots_all_snaps_box${box}
25     if [ ! -e ${new_box_dir} ]; then
26         mkdir ${new_box_dir}
27     fi
28
29     for ((snap=$minsnap; snap<=$maxsnap; snap++)); do
30
31         if [ $snap -lt 10 ]; then
32             j=0$snap
33         elif [ $snap -lt 100 ]; then
34             j=0$snap
35         fi
36
37         new_plot_dir=snap{j}_plots
38
39         if [ ! -e ${new_box_dir}/${new_plot_dir} ]; then
40             mkdir ${new_box_dir}/${new_plot_dir}
41         fi
42
43         echo -n "Starting box${box} snap${j}... "
44         ./hist.py ~/projects/simulations/rockstar/box${box}/crossmatch/snap{j}/halos.dat > plots/out.log 2>&1
45         mv plots/* ${new_box_dir}/${new_plot_dir}/.
46         echo "Finished snap{j}"
47
48     done
49 done
50
51 # - end of script

```

## I.4 Statistics Collection Script (Bash)

```

1#!/usr/bin/env bash
2
3 if [ "$#" -ne 1 ]; then
4     echo "Please provide a directory as an argument."
5     exit -1
6 fi
7
8 parent_dir=$1
9
10 for stats_path in $parent_dir/snap061_plots/{stats_*,perc_diff_*}; do
11     stats_file=$(basename "$stats_path")
12     out_file=${stats_file/_allsnaps_}()
13     echo "Merging stats for $stats_file..."
14
15     for snap_dir in $parent_dir/snap*_plots; do
16         if [ -e $snap_dir/$stats_file ]; then
17             snap_num=$(basename "$snap_dir")
18             echo -n "${snap_num:5:2} "
19             cat $snap_dir/$stats_file | cut -d' ' -f 2-
20         fi
21     done | column -t > $parent_dir/$out_file
22
23     echo "Stats written to $out_file..."
24
25 done

```

## Appendix J

### Redshift Trends Code (Python)

```
1  #!/usr/bin/env python
2
3  import sys
4  import os
5  import numpy as np
6  import matplotlib as mpl
7  mpl.use('Agg')
8  import matplotlib.pyplot as plt
9  from scipy.special import gamma as Gamma
10 from scipy.special import psi as digamma
11 from ipdb import set_trace
12
13
14 def main():
15     #for filenum, file in enumerate(sys.argv[1:]):
16     if (len(sys.argv[1:]) == 4):
17         data1 = read_files(sys.argv[1], header_line = None)
18         data2 = read_files(sys.argv[2], header_line = None)
19         data3 = read_files(sys.argv[3], header_line = None)
20         rsnap_data = read_files(sys.argv[4], header_line = None)
21     else:
22         print 'need 4 files'
23         sys.exit(15)
24
25     if fit_mean_trend:
26         with open(statsfile, 'w') as fd:
27             fd.write("#plot_uslope_uslope_err_intercept_intercept_err\n")
28
29     if skew_err_boxes:
30         skew_err1 = get_skew_err(sys.argv[1])
31         skew_err2 = get_skew_err(sys.argv[2])
32         skew_err3 = get_skew_err(sys.argv[3])
33
34     if minsnap > 0:
35         #for data in data1, data2, data3:
36         #    data = data[data[:,0] >= minsnap]
37         data1 = data1[data1[:,0] >= minsnap]
38         data2 = data2[data2[:,0] >= minsnap]
39         data3 = data3[data3[:,0] >= minsnap]
40
41         if skew_err_boxes:
42             skew_err1 = skew_err1[-len(data1):]
43             skew_err2 = skew_err2[-len(data2):]
44             skew_err3 = skew_err3[-len(data3):]
45
46     if skew_err_col == -2:
47         data1 = np.column_stack((data1, skew_err1))
48         data2 = np.column_stack((data2, skew_err2))
49         data3 = np.column_stack((data3, skew_err3))
50
51     #if (mean_err_col == -2) or (var_err_col == -2) or (skew_err_col == -2) or (kurt_err_col == -2):
52     #    fake_err = np.zeros(len(data1))
53     #    data1 = np.column_stack((data1, fake_err))
54     #    data2 = np.column_stack((data2, fake_err))
55     #    data3 = np.column_stack((data3, fake_err))
56
57     z = 1.0 / rsnap_data[:,1] - 1.0
58     if (len(data1) == len(data2)) and (len(data1) == len(data3)):
59         z = z[-len(data1):]
60     else:
61         sys.exit(16)
62
63     data1 = np.column_stack((data1, z))
64     data2 = np.column_stack((data2, z))
65     data3 = np.column_stack((data3, z))
66
67     #data1[:, -1] = data1[:, -1] - 0.12
68     #data2[:, -1] = data2[:, -1] + 0.12
69
70     for data in [data1, data2, data3]:
71         if expand_error:
72             mask = (np.abs(data[:, data_mean_col] - data[:, mean_col]) > data[:, mean_err_col])
73             data[mask, mean_err_col] = np.abs(data[mask, data_mean_col] - data[mask, mean_col])
74
75         if transform_variance:
76             data[:, var_col] = data[:, var_col]**2 * Gamma(3.0 / data[:, beta_col]) / Gamma(1.0 / data[:, beta_col])
77             data[:, var_err_col] = data[:, var_err_col]**2 * Gamma(3.0 / data[:, beta_col]) / Gamma(1.0 / data[:, beta_col])
78
79         if transform_kurtosis:
80             #data[:, kurt_col] = (Gamma(5.0 / data[:, kurt_col]) * Gamma(1.0 / data[:, kurt_col]) / Gamma(3.0 / data[:, kurt_col])) - 3.0
81             beta = data[:, beta_col]
82             beta_err = data[:, beta_err_col]
83             kurtosis = (Gamma(5.0 / beta) * Gamma(1.0 / beta) / Gamma(3.0 / beta)) - 3.0
```

```

81         kurtosis_err = beta_err * (1.0 / beta**2) * (kurtosis + 3) * (6.0 * digamma(3.0/beta) - 5.0 * digamma
82             (5.0/beta) - digamma(1.0/beta))
83
84         data[:,kurt_col] = kurtosis[:]
85         data[:,kurt_err_col] = kurtosis_err[:]
86
87         data[:,var_col] = np.sqrt(data[:,var_col])           # var to stdev
88         data[:,var_err_col] = np.sqrt(data[:,var_err_col])    # var to stdev
89
90     if save_transformed_data:
91         for data, path in zip([data1, data2, data3], sys.argv[1:4]):
92             fname = transform_file_base + os.path.basename(path)
93             with open(fname, 'w') as fd:
94                 fd.write(transformed_data_header)
95                 np.savetxt(fd, np.column_stack((z, data)), fmt='%g')
96
97
98
99     ######
100    # make mean and stdv plots
101    ######
102
103    for (data, ylabel, color, label, name) in zip([data1, data2, data3], ylabels1, colors, labels1, names):
104        print "Making %s plot..." % (name)
105        fig = plt.figure(figsize=(9.0, 6.0))
106        ax = fig.add_subplot(111)
107
108        ax = make_plot(ax, data[:,z_col], data[:,mean_col], err = data[:,mean_err_col], color = 'blue', marker='o',
109            , label=label)
110        ax = make_plot(ax, data[:,z_col], data[:,mean_col] + data[:,var_col], color = 'black', linestyle='--')
111        ax = make_plot(ax, data[:,z_col], data[:,mean_col] - data[:,var_col], color = 'black', linestyle='--')
112
113        if add_rms_line:
114            ax = make_plot(ax, data[:,z_col], data[:,data_rms_col], color = 'green', linestyle=':')
115
116        if fit_mean_trend:
117            ax, slope, slope_err, intercept, intercept_err = add_fit(ax, data[:,z_col], data[:,mean_col], err =
118                data[:,mean_err_col], color='red')
119            save_fits(statsfile, name, slope, np.sqrt(slope_err), intercept, np.sqrt(intercept_err))
120
121        ax.legend(loc='lower right')
122        ax.set_xlim(z[0] + 1.0, z[-1] - 1.0)
123        #ax.invert_xaxis()
124
125        ax.set_xlabel(xlabel, fontsize='x-large')
126        ax.set_ylabel(ylabel, fontsize='x-large')
127
128        fig.tight_layout()
129        fig.savefig(plot_base + 'mean_stddev_' + name + plot_ext, bbox_inches='tight')
130
131    ######
132    # make skew and kurtosis plots
133    ######
134
135
136
137    for (data, ylabel_kurt, ylabel_skew, color, name, ylim_low1, ylim_high1, ylim_low2, ylim_high2) in zip([data1
138        , data2, data3], ylabels2_kurt, ylabels2_skew, colors, names, [-10.0, -10.0, -1.0], [20.0, 20.0, 1.5],
139        [-0.2, -1.5, -0.4], [0.5, 3.5, 0.1]):
140        print "Making %s plot..." % (name)
141        fig = plt.figure(figsize=(9.0, 6.0))
142        ax = fig.add_subplot(111)
143
144        #ax = make_plot(ax, data[:,z_col] - offset, data[:,kurt_col], err = data[:,kurt_err_col], color = 'red',
145            marker='o', linestyle='-', label='Kurtosis')
146        #ax = make_plot(ax, data[:,z_col] + offset, data[:,skew_col], err = data[:,skew_err_col], color = 'blue',
147            marker='o', linestyle='--', label='Skew')
148        ax = make_plot(ax, data[:,z_col] - offset, data[:,kurt_col], err = data[:,kurt_err_col], color = 'red',
149            marker='o', linestyle=':', label='Kurtosis')
150        legend_lines1, legend_labels1 = ax.get_legend_handles_labels()
151
152        ax.set_xlabel(xlabel, fontsize='x-large')
153        ax.set_ylabel(ylabel_kurt, fontsize='x-large')
154        ax.set_ylim(ylim_low1, ylim_high1)
155
156        if separate_skew_axes:
157            ax = ax.twinx()
158            ax = make_plot(ax, data[:,z_col] + offset, data[:,skew_col], err = data[:,skew_err_col], color = 'blue',
159                marker='o', linestyle=':', label='Skew')
160            legend_lines2, legend_labels2 = ax.get_legend_handles_labels()
161
162            ax.set_ylabel(ylabel_skew, fontsize='x-large')
163            ax.legend(legend_lines1 + legend_lines2, legend_labels1 + legend_labels2, loc='lower right')
164            ax.set_xlim(z[0] + 1.0, z[-1] - 1.0)
165            ax.set_ylim(ylim_low2, ylim_high2)
166            #ax.invert_xaxis()
167
168        fig.tight_layout()
169        fig.savefig(plot_base + 'skew_kurtosis_' + name + plot_ext, bbox_inches='tight')

```

```

164     #-----#
165     print 'Finished all plots.'
166
167     def make_plot(ax, x, y, err=None, color='black', marker='None', linestyle='None', label=None):
168         if err == None:
169             if label == None:
170                 ax.plot(x, y, color=color, marker=marker, linestyle=linestyle)
171             else:
172                 ax.plot(x, y, color=color, marker=marker, linestyle=linestyle, label=label)
173         else:
174             if label == None:
175                 ax.errorbar(x, y, yerr=err, color=color, marker=marker, linestyle=linestyle)
176             else:
177                 ax.errorbar(x, y, yerr=err, color=color, marker=marker, linestyle=linestyle, label=label)
178
179     return ax
180
181
182
183
184
185     def add_fit(ax, x, y, err=None, color='red'):
186         from scipy.optimize import curve_fit
187         p0 = [0.0, 0.0]
188
189         try:
190             coeffs, pcov = curve_fit(linear, x, y, sigma=err, p0=p0)
191         except RuntimeError:
192             print '*****Curve fit failed*****'
193             return np.nan, np.nan
194         xmin, xmax = ax.get_xlim()
195         x_fit = np.linspace(xmin, xmax, 20)
196         y_fit = linear(x_fit, coeffs[0], coeffs[1])
197         ax.plot(x_fit, y_fit, color=color, linestyle='--')
198
199     return ax, coeffs[0], coeffs[1], pcov[0,0], pcov[1,1]
200
201
202
203
204     def linear(x, slope, intercept):
205         return slope * x + intercept
206
207
208
209
210     def read_files(files, header_line = None, comment_char = '#'):
211         header = None
212         data = None
213
214         if type(files) == str:
215             files = [files]
216
217         if header_line != None:
218             with open(files[0], 'r') as fd:
219                 for line in range(header_line):
220                     fd.readline()
221                 header = fd.readline()
222                 if header[0] != comment_char:
223                     print "Header must start with a %s" % comment_char
224                     sys.exit(4)
225                 header = header[1:]
226                 header = header.split()
227
228         for file in files:
229             print 'Reading %s...' % (file)
230             if data == None:
231                 data = np.genfromtxt(file, comments=comment_char)
232             else:
233                 data = np.append(data, np.genfromtxt(file, comments=comment_char), axis=0)
234
235         print 'Finished reading files.'
236         if header_line == None:
237             return data
238         else:
239             return header, data
240
241
242
243
244     def get_skew_err(filebase):
245         z = None
246         skew = None
247         for i in range(3):
248             filename = filebase.replace('plots_all_snaps', 'plots_all_snaps_box'+str(i+1))
249             data = read_files(filename, header_line = None)
250
251             if i == 0:
252                 min_length = len(data)
253             elif len(data) < min_length:
254                 min_length = len(data)
255
256             if z == None:
257                 z = data[-min_length:,snap_col]
258             else:
259                 z = np.column_stack((z[-min_length:], data[-min_length:,snap_col]))
260
261             if skew == None:
262                 skew = data[-min_length:,skew_col]
263             else:
264                 skew = np.column_stack((skew[-min_length:], data[-min_length:,skew_col]))

```

```

256     if (z[:,0] != z[:,1]).all() or (z[:,0] != z[:,2]).all():
257         print 'Need matching snapshots for skew error from individual boxes.'
258         print z
259         sys.exit(-1)
260
261     skew_err = np.std(skew, axis=1) / np.sqrt(3.0)
262     return skew_err
263
264
265
266 def save_fits(file, name, slope, slope_err, intercept, intercept_err):
267     with open(file, 'a') as fd:
268         fd.write("%s %g %g %g\n" % (name, slope, slope_err, intercept, intercept_err))
269
270
271 plot_dest_type = 'paper'
272 if plot_dest_type == 'paper':
273     mpl.rcParams['font.family'] = 'serif'
274     mpl.rcParams['font.size'] = 16
275     mpl.rcParams['axes.linewidth'] = 3
276     mpl.rcParams['lines.linewidth'] = 4
277     mpl.rcParams['patch.linewidth'] = 4
278     mpl.rcParams['xtick.major.width'] = 3
279     mpl.rcParams['ytick.major.width'] = 3
280     mpl.rcParams['xtick.major.size'] = 8
281     mpl.rcParams['ytick.major.size'] = 8
282
283 #colors = ['red', 'green', 'blue']
284 colors = ['black', 'black', 'black']
285 labels1 = [r'$c$', r'$M_{\mathrm{vir}}$' , r'$X_{\mathrm{off}}$']
286 names = ['c_rockstar', 'Mvir', 'Xoff']
287 xlabel = 'Redshift'
288 ylabel1 = [r'$\mu_{\Delta c}$', r'$\sigma_{\Delta c}$', r'$\Delta M_{\mathrm{vir}}$' , r'$\Delta X_{\mathrm{off}}$']
289 ylabel2_kurt = ['Kurtosis for $\Delta c$', 'Kurtosis for $\Delta M_{\mathrm{vir}}$' , 'Kurtosis for $\Delta X_{\mathrm{off}}$']
290 ylabel2_skew = ['Skew for $\Delta c$', 'Skew for $\Delta M_{\mathrm{vir}}$' , 'Skew for $\Delta X_{\mathrm{off}}$']
291 plot_base = 'plots/'
292 plot_ext = '.eps'
293
294 statsfile = 'plots/stats.dat'
295 transform_file_base = 'plots/'
296 transformed_data_header = '#z_snap data_mean data_stdev data_skew data_kurt fit_height +/-errufit_mean +/-errufit_stdev +/-errufit_skew +/-errufit_kurt +/-errufit_rms data_gt_epsilon_chi2_pval skew_erruz\n'
297
298 z_col      = -1
299 snap_col   = 0
300 mean_col   = 7
301 mean_err_col = 8
302 var_col    = 9
303 var_err_col = 10
304 skew_col   = 3
305 skew_err_col = -2
306 #skew_col   = 7
307 #skew_err_col = 8
308 #kurt_col   = 4
309 #kurt_err_col = -2
310 kurt_col   = 13
311 kurt_err_col = 14
312 beta_col   = 13
313 beta_err_col = 14
314
315 data_mean_col = 1
316 data_rms_col  = 15
317
318 #z_col      = -1
319 #snap_col   = 0
320 #mean_col   = 1
321 #mean_err_col = -2
322 #var_col    = 2
323 #var_err_col = -2
324 #skew_col   = 3
325 #skew_err_col = -2
326 #kurt_col   = 4
327 #kurt_err_col = -2
328
329 offset = 0.06
330 #offset = 0.0
331
332 minsnap = 39
333 #minsnap = None
334
335 transform_variance = True
336 transform_kurtosis = True
337 expand_error       = True
338 fit_mean_trend    = True
339 separate_skew_axes = True
340 skew_err_boxes     = True
341 add_rms_line       = True
342 save_transformed_data = True

```

```
343
344
345 if __name__ == '__main__':
346     main()
```

## Appendix K

### Mass Trends Code

#### K.1 Mass and Concentration vs. Mass (Python)

```
1 #!/usr/bin/env python
2
3 import sys
4 import numpy as np
5 import matplotlib as mpl
6 mpl.use('Agg')
7 import matplotlib.pyplot as plt
8 from matplotlib import cm
9 from scipy import interpolate
10 from scipy.ndimage.filters import gaussian_filter
11 from scipy.optimize import curve_fit
12 #from ipdb import set_trace
13
14
15
16 def main():
17     # Read in particle files
18     header, halos = read_files(sys.argv[1:], header_line = 3)
19
20     if c_source == 'density_profile':
21         print 'len(halos)=', len(halos)
22         halos = halos[np.isfinite(halos[:,c_2lpt_col])]
23         halos = halos[np.isfinite(halos[:,c_zs_col])]
24         print 'len(halos)=', len(halos)
25
26     print 'Filtering data...'
27     for col, val in zip(lt_cols, lt_vals):
28         halos = halos[halos[:, col] <= val]
29     for col, val in zip(gt_cols, gt_vals):
30         halos = halos[halos[:, col] >= val]
31     for col, val in zip(eq_cols, eq_vals):
32         halos = halos[halos[:, col] == val]
33     for col, val in zip(ne_cols, ne_vals):
34         halos = halos[halos[:, col] != val]
35
36     m_avg = (halos[:,47] + halos[:,48])/2.0
37     halos = np.column_stack((halos, m_avg))
38     header = np.append(header, 'M_avg')
39
40     if x_min_lim > 0:
41         print 'nhalos=', len(halos)
42         mask = (m_avg >= x_min_lim)
43         halos = halos[mask]
44         print 'nhalos=', len(halos)
45
46     if c_source == 'rockstar':
47         c1 = halos[:, Rv1_col] / halos[:, Rs1_col]
48         c2 = halos[:, Rv2_col] / halos[:, Rs2_col]
49         if use_klypin:
50             mask = (halos[:,4] < 100)
51             c1[mask] = halos[mask, Rv1_col] / halos[mask, 79]
52             mask = (halos[:,5] < 100)
53             c1[mask] = halos[mask, Rv2_col] / halos[mask, 80]
54     if c_source == 'density_profile':
55         c1 = halos[:, c_2lpt_col]
56         c2 = halos[:, c_zs_col]
57
58     dc = 2.0 * (c1 - c2) / (c1 + c2)
59     #dc = c1 - c2
60
61     m1 = halos[:,47]
62     m2 = halos[:,48]
63     dm = 2.0 * (m1 - m2) / (m1 + m2)
64
65     for x_col, xlabel in zip(x_cols, xlabels):
66         make_plot(halos[:, x_col], dm, x_col, header[x_col], xlabel, ylabel_m, plot_base_m, stats_file_m, y_lim_m
67         , use_log=False)
68         make_plot(halos[:, x_col], dc, x_col, header[x_col], xlabel, ylabel_c, plot_base_c, stats_file_c, y_lim_c
69         , use_log=False)
70     for x_col, xlabel in zip(x_log_cols, xlabels_log):
71         make_plot(halos[:, x_col], dm, x_col, header[x_col], xlabel, ylabel_m, plot_base_m, stats_file_m, y_lim_m
72         , use_log=True)
73         make_plot(halos[:, x_col], dc, x_col, header[x_col], xlabel, ylabel_c, plot_base_c, stats_file_c, y_lim_c
74         , use_log=True)
75 def read_files(files, header_line = None, comment_char = '#'):
76     header = None
```

```

77     data = None
78     if type(files) == str:
79         files = [files]
80
81     if header_line != None:
82         with open(files[0], 'r') as fd:
83             for line in range(header_line):
84                 fd.readline()
85             header = fd.readline()
86         if header[0] != comment_char:
87             print "Header must start with a '%s'" % comment_char
88             sys.exit(4)
89         header = header[1:]
90         header = header.split()
91
92     for file in files:
93         print 'Reading file %s...' % (file)
94         if data == None:
95             data = np.genfromtxt(file, comments=comment_char)
96         else:
97             data = np.append(data, np.genfromtxt(file, comments=comment_char), axis=0)
98
99     print 'Finished reading files.'
100    if header_line == None:
101        return data
102    else:
103        return header, data
104
105
106 def make_plot(x, y, x_col, header, xlabel, ylabel, plot_base, stats_file, y_lim, use_log):
107     print 'generating plot...'
108     fig = plt.figure(figsize=(9.0,6.0))
109     ax = fig.add_subplot(1,1,1)
110     ax = draw_hist2d(ax, x, y, y_lim)
111     if fit_to_data:
112         ax = draw_data_fit(ax, x, y, x.min(), x.max(), use_log=use_log)
113     if fit_to_binned_data:
114         mid_bins, mean, stdev, n = get_bin_avgs(x, y, use_log=use_log)
115         ax = draw_bin_fit(ax, mid_bins, mean, stdev/np.sqrt(n), x.min(), x.max(), stats_file, use_log=use_log)
116         ax = draw_bin_avgs(ax, mid_bins, mean, stdev, n, use_log=use_log)
117
118     ax.set_xlim([x.min(), x.max()])
119     #ax.set_yscale("log")
120     ax.set_xlabel(xlabel, fontsize="x-large")
121     ax.set_ylabel(ylabel, fontsize="x-large")
122
123     fig.tight_layout()
124     header = header.replace("/", "over")
125     plot_name = "%s%0.3d%s%s" % (plot_base, '(', x_col, ')', header, plot_ext)
126     plt.savefig(plot_name, bbox_inches='tight')
127     print 'finished plot' + plot_name
128
129
130 def draw_hist2d(ax, x, y, y_lim):
131     if use_log:
132         xbins = np.logspace(np.log10(x.min()), np.log10(x.max()), num=nbins+1)
133     else:
134         xbins = np.linspace(x.min(), x.max(), num=nbins+1)
135
136     ybins = np.linspace(y.min(), y.max(), num=nbins+1)
137
138     if use_log:
139         ax.set_xscale("log")
140         im = my_hist2d(ax, x, y, bins=[xbins, ybins], zorder=-50)
141     else:
142         im = ax.hist2d(x, y, bins=[xbins, ybins], cmap=cmap, zorder=-50)
143
144     if y_lim > 0.0:
145         ax.set_ylim([-y_lim, y_lim])
146
147     line = ax.plot([x.min(), x.max()], [0.0, 0.0], color='0.65', linestyle='--', linewidth=1, zorder=-20)
148     return ax
149
150
151 def my_hist2d(ax, x, y, bins=10, range=None, normed=False, weights=None,
152               cmin=None, cmax=None, **kwargs):
153     import matplotlib as mpl
154
155     bin_range = range
156     range = mpl.axes._builtins__["range"]
157     h, xedges, yedges = np.histogram2d(x, y, bins=bins, range=bin_range,
158                                         normed=normed, weights=weights)
159
160     if cmin is not None:
161         h[h < cmin] = None
162     if cmax is not None:
163         h[h > cmax] = None
164
165     if z_log:
166         h[h<1.0] = 0.5
167         h = np.log10(h)
168

```

```

169     h = gaussian_filter(h, len(h) / 75.0)
170
171     pc = ax.imshow(h[:, ::-1].T, cmap=c colormap, extent=[x.min(), x.max(), y.min(), y.max()], interpolation='gaussian', **kwargs)
172     ax.set_xlim(xedges[0], xedges[-1])
173     ax.set_ylim(yedges[0], yedges[-1])
174     return h, xedges, yedges, pc
175
176
177 def get_bin_avgs(x, y, use_log):
178     if use_log:
179         fit_bins = np.logspace(np.log10(x.min()), np.log10(x.max()), num=nfit_bins+1)
180     else:
181         fit_bins = np.linspace(x.min(), x.max(), num=nfit_bins+1)
182
183     mid_bins = (fit_bins[:-1] + fit_bins[1:]) / 2.0
184
185     mean = np.array([])
186     stdev = np.array([])
187     n = np.array([])
188     for xmin, xmax in zip(fit_bins[:-1], fit_bins[1:]):
189         mask = np.logical_and(x > xmin, x <= xmax)
190         if mask.sum() > 0:
191             mean_el = y[mask].mean()
192             #stdev_el = y[mask].std() / np.sqrt(len(y))
193             stdev_el = y[mask].std()
194             #stdev_el = stdev / np.sqrt(len(y[mask]))
195             n_el = len(y[mask])
196         else:
197             mean_el = 0.0
198             stdev_el = -1.0
199             n_el = 0
200         mean = np.append(mean, mean_el)
201         stdev = np.append(stdev, stdev_el)
202         n = np.append(n, n_el)
203
204     mask = (n > 0)
205     mean = mean[mask]
206     stdev = stdev[mask]
207     n = n[mask]
208     mid_bins = mid_bins[mask]
209
210     return mid_bins, mean, stdev, n
211
212
213 def draw_bin_avgs(ax, mid_bins, mean, stdev, n, use_log):
214     ax.errorbar(mid_bins, mean, yerr=stdev/np.sqrt(n), fmt='o', color='black', linewidth=2)
215
216     if draw_stdev_lines:
217         ax.plot(mid_bins, mean + stdev, color='black', linestyle=':', linewidth=3, zorder=-15)
218         ax.plot(mid_bins, mean - stdev, color='black', linestyle=':', linewidth=3, zorder=-15)
219     return ax
220
221
222 def draw_bin_fit(ax, mid_bins, mean, stdev, x_min, x_max, stats_file, use_log):
223     stdev[stdev == 0.0] = 0.1
224
225     #fit data
226     if use_log:
227         #coefs, res, rank, singvals, rcond = np.polyfit(np.log10(mid_bins), mean, 1, full=True)
228         coefs, pcov = curve_fit(linear, np.log10(mid_bins), mean, sigma=stdev, p0=[0.0, 0.0])
229     else:
230         #coefs, stats = np.polynomial.polynomial.polyfit(mid_bins, mean, 1, full=True)
231         coefs, pcov = curve_fit(linear, mid_bins, mean, sigma=stdev, p0=[0.0, 0.0])
232     print 'coefs=%u', coefs
233
234     m = coefs[0]
235     b = coefs[1]
236     m_err = pcov[0,0]
237     b_err = pcov[1,1]
238
239     if use_log:
240         x = np.logspace(np.log10(x_min), np.log10(x_max), 100)
241         y = m * np.log10(x) + b
242     else:
243         x = np.linspace(x_min, x_max, 100)
244         y = m * x + b
245     #y = x**m + b
246     #line = ax.plot(x, y, color='white', linewidth=8) # to avoid blending with colormap background
247     line = ax.plot(x, y, color='magenta', zorder=-10)
248
249     if print_fit_params:
250         if use_log:
251             textstr = '$y = %u \\log x + %u$\\n$%u = %u g \\n%u = %u g$' % (m, b)
252         else:
253             textstr = '$y = %u x + %u$\\n$%u = %u g \\n%u = %u g$' % (m, b)
254         props = dict(boxstyle='round', facecolor='wheat', alpha=0.5)
255         ax.text(0.75, 0.95, textstr, transform=ax.transAxes, fontsize=14,
256                 verticalalignment='top', bbox=props)
257
258     if save_fit_params:
259         with open(stats_file, "a") as fd:

```

```

260         fd.write("%g %g %g\n" % (m, m_err, b, b_err))
261     return ax
263
264
265 def linear(x, slope, intercept):
266     return slope * x + intercept
267
268
269 def draw_data_fit(ax, x, y, x_min, x_max, use_log):
270     if remove_zero_strip:
271         mask = (np.abs(y) >= y_epsilon)
272         x = x[mask]
273         y = y[mask]
274
275     #fit data
276     if use_log:
277         coefs, residual, rank, singular_values, rcond = np.polyfit(np.log10(x), y, 1, full=True)
278         # coefs, stats = np.polynomial.polynomial.polyfit(np.log10(mid_bins), mean, 1, w=1.0/stdev, full=True)
279         # coefs, res, rank, singvals, rcond = np.polyfit(np.log10(mid_bins), mean, 1, full=True)
280     else:
281         coefs, residual, rank, singular_values, rcond = np.polyfit(x, y, 1, full=True)
282         # coefs, stats = np.polynomial.polynomial.polyfit(mid_bins, mean, 1, w=1.0/stdev, full=True)
283         # coefs, stats = np.polynomial.polynomial.polyfit(mid_bins, mean, 1, full=True)
284     print 'coefs=%s' % coefs, '+/-', residual
285
286
287     m = coefs[0]
288     b = coefs[1]
289     if use_log:
290         x = np.logspace(np.log10(x_min), np.log10(x_max), 100)
291         y = m * np.log10(x) + b
292     else:
293         x = np.linspace(x_min, x_max, 100)
294         y = m * x + b
295     y = x**m + b
296     line = ax.plot(x, y, color='red')
297
298     if print_fit_params:
299         if use_log:
300             textstr = '$y = {} \log x + {}$\n $m = {}$ \n $b = {}$'.format(m, b)
301         else:
302             textstr = '$y = {} x + {}$\n $m = {}$ \n $b = {}$'.format(m, b)
303         props = dict(boxstyle='round', facecolor='wheat', alpha=0.5)
304         ax.text(0.75, 0.95, textstr, transform=ax.transAxes, fontsize=14,
305                 verticalalignment='top', bbox=props)
306
307     if save_fit_params:
308         with open("fits_to_data.dat", "a") as fd:
309             fd.write("%g %g\n" % (m, b, residual))
310
311     return ax
312
313
314 use_log = True
315 #use_log = False
316 z_log = True
317
318 #fit_bins = True
319 #fit_data = True
320
321 print_fit_params = False
322 save_fit_params = True
323
324 use_klypin = True
325
326 remove_zero_strip = False
327 y_epsilon = 0.01
328
329 y_lim_m = 0.5
330 y_lim_c = 1.0
331 x_min_lim = 5.33e5 * 100
332
333 #if use_log:
334 #    x_cols = [4, 5, 6, 9, 10, 23, 24, 31, 32, 47, 48, 51, 52, 57, 58] # log10 columns
335 #else:
336 #    x_cols = [17, 18, 77, 78, 91, 92, 93, 94, 97, 98, 99, 100, 107, 108, 111, 112] # nolog columns
337
338 x_cols = []
339 x_log_cols = [-1]
340 #x_log_cols = [47, 48, -1]
341
342 xlabelss = []
343 xlabelss_log = [r"$M_{\mathrm{vir,avg}}$, $M_{\mathrm{\dot{odot}}}$"]
344 #xlabelss_log = [r"$M_{\mathrm{2LPT}}$, $M_{\mathrm{\dot{odot}}}$",
345 #                 r"$M_{\mathrm{ZA}}$, $M_{\mathrm{\dot{odot}}}$",
346 #                 r"$M_{\mathrm{avg}}$, $M_{\mathrm{\dot{odot}}}$"]
347
348 ylabel = r"$M_{\mathrm{2LPT}} - M_{\mathrm{ZA}} / M_{\mathrm{avg}}$"
349 ylabel_m = r"$(M_{\mathrm{vir,2LPT}} - M_{\mathrm{ZA}}) / M_{\mathrm{avg}}$"
350 ylabel_c = r"$(c_{\mathrm{2LPT}} - c_{\mathrm{ZA}}) / c_{\mathrm{avg}}$"
351
```

```

352 #c_source = 'density_profile'
353 c_source = 'rockstar'
354
355 plot_base_m = 'plots/diff_M_--vs--'
356 plot_base_c = 'plots/diff_c_--vs--'
357 plot_ext = '.eps'
358
359 stats_file_m = 'fits_to_bins_m.dat'
360 stats_file_c = 'fits_to_bins_c.dat'
361
362 #plot_name = 'test.eps'
363 #plot_name = 'c_v_M200c_2lpt.eps'
364 fit_to_binned_data = True
365 fit_to_data = False
366 draw_stdev_lines = True
367
368 Rv1_col = 53
369 Rv2_col = 54
370 Rsi1_col = 55
371 Rs2_col = 56
372
373 c_2lpt_col = 17
374 c_za_col = 18
375
376 nbins = 100
377 nfit_bins = 10
378
379 ## c_2lpt, c_za, chi2_2lpt, chi2_za
380 #lt_cols = [17, 18, 37, 38]
381 #lt_vals = [100.0, 100.0, 10.0, 10.0]
382 #
383 ## c_2lpt, c_za, chi2_2lpt, chi2_za
384 #gt_cols = [17, 18, 37, 38]
385 #gt_vals = [1.0, 1.0, 0.0, 0.0]
386
387 lt_cols = []
388 lt_vals = []
389
390 gt_cols = [4, 5]
391 gt_vals = [100, 100]
392
393 eq_cols = [109, 110]
394 eq_vals = [-1, -1]
395
396 ne_cols = []
397 ne_vals = []
398
399 #colormap = cm.PuBuGn
400 #colormap = cm.cubehelix_r
401 #colormap = cm.ocean_r
402 #colormap = cm.rainbow
403 #colormap = cm.gnuplot2_r
404 #colormap = cm.CMRmap_r
405
406 def add_white(orig_map, num):
407     temp_cmap = cm.get_cmap(orig_map, num)
408     vals = temp_cmap(np.arange(num))
409     nfade = num / 7
410     vals[:nfade,0] = np.linspace(1., vals[nfade-1,0], nfade)
411     vals[:nfade,1] = np.linspace(1., vals[nfade-1,1], nfade)
412     vals[:nfade,2] = np.linspace(1., vals[nfade-1,2], nfade)
413     #vals[:nfade,3] = np.linspace(0., vals[nfade-1,3], nfade)
414     #vals[0] = [1.0, 1.0, 1.0, 1.0]
415     #vals[1] = (vals[1] + [1.0, 1.0, 1.0, 1.0]) / 2.0
416     newcmap = mpl.colors.LinearSegmentedColormap.from_list("custom_1", vals)
417     return newcmap
418
419 colormap = add_white('rainbow', 30)
420
421 plot_dest_type = 'paper'
422 if plot_dest_type == 'paper':
423     mpl.rcParams['font.family'] = 'serif'
424     mpl.rcParams['font.size'] = 16
425     mpl.rcParams['axes.linewidth'] = 3
426     mpl.rcParams['lines.linewidth'] = 4
427     #mpl.rcParams['lines.linewidth'] = 3
428     #mpl.rcParams['patch.linewidth'] = 4
429     #mpl.rcParams['patch.linewidth'] = 3
430     #mpl.rcParams['xtick.major.width'] = 3
431     #mpl.rcParams['ytick.major.width'] = 3
432     #mpl.rcParams['xtick.major.size'] = 8
433     #mpl.rcParams['ytick.major.size'] = 8
434     #mpl.rcParams['xtick.minor.width'] = 2
435     #mpl.rcParams['ytick.minor.width'] = 2
436     #mpl.rcParams['xtick.minor.size'] = 4
437     #mpl.rcParams['ytick.minor.size'] = 4
438     #mpl.rcParams['lines.antialiased'] = True
439
440
441 if __name__ == '__main__':
442     main()

```

## K.2 PBS Submission Script (Bash)

```

1 #!/usr/bin/env bash
2 #PBS -M djsissom@gmail.com
3 #PBS -m bae
4 #PBS -l nodes=1:ppn=1
5 #PBS -l pmem=40000mb
6 #PBS -l mem=40000mb
7 #PBS -l walltime=1:00:00
8 #PBS -o out.log
9 #PBS -j oe
10
11 minsnap=0
12 maxsnap=61
13
14 # Change to working directory
15 echo $PBS_NODEFILE
16 cd $PBS_O_WORKDIR
17
18 [ -e fits_to_bins_m.dat ] && rm -v fits_to_bins_m.dat
19 [ -e fits_to_bins_c.dat ] && rm -v fits_to_bins_c.dat
20 #rm -v fits_to_data.dat
21
22 echo "#snap slope slope_err intercept_err" > fits_to_bins_m.dat
23 echo "#snap slope slope_err intercept_err" > fits_to_bins_c.dat
24
25 for ((snap=$minsnap; snap<=$maxsnap; snap++)); do
26
27     if [ $snap -lt 10 ]; then
28         j=00$snap
29     elif [ $snap -lt 100 ]; then
30         j=0$snap
31     fi
32
33     new_plot_dir=snap{j}_plots
34
35     if [ ! -e plots_all_snaps/${new_plot_dir} ]; then
36         mkdir plots_all_snaps/${new_plot_dir}
37     fi
38
39     {
40         echo "Starting snap${j}..."
41         echo -n "${j}uuuu" >> fits_to_bins_m.dat
42         echo -n "${j}uuuu" >> fits_to_bins_c.dat
43         ./mass_plot.py ~/projects/simulations/rockstar/box{1,2,3}/crossmatch/snap{j}/halos.dat > plots/out.log
44         mv plots/* plots_all_snaps/${new_plot_dir}/.
45         echo "Finished snap${j}"
46     }
47
48 done
49
50 # - end of script

```

## Appendix L

### Alternate Differential Distribution Redshift Trends Code (Python)

```
1  #!/usr/bin/env python
2
3  import sys
4  import numpy as np
5  import matplotlib as mpl
6  mpl.use('Agg')
7  import matplotlib.pyplot as plt
8  from scipy.special import gamma as Gamma
9  from scipy.special import psi as digamma
10 from ipdb import set_trace
11
12
13 def main():
14     if (len(sys.argv[1:]) == 4):
15         data1 = read_files(sys.argv[1], header_line = None)
16         data2 = read_files(sys.argv[2], header_line = None)
17         data3 = read_files(sys.argv[3], header_line = None)
18         rsnap_data = read_files(sys.argv[4], header_line = None)
19     else:
20         print 'need 4 files'
21         sys.exit(15)
22
23     if fit_mean_trend:
24         with open(statsfile, 'w') as fd:
25             fd.write("#plot slope slope_err intercept intercept_err\n")
26
27     minsnap > 0:
28         data1 = data1[data1[:,0] >= minsnap]
29         data2 = data2[data2[:,0] >= minsnap]
30         data3 = data3[data3[:,0] >= minsnap]
31
32     z = 1.0 / rsnap_data[:,1] - 1.0
33     if (len(data1) == len(data2)) and (len(data1) == len(data3)):
34         z = z[-len(data1):]
35     else:
36         sys.exit(16)
37
38     data1 = np.column_stack((data1, z))
39     data2 = np.column_stack((data2, z))
40     data3 = np.column_stack((data3, z))
41
42
43     ######
44     # make mean and stdv plots
45     ######
46
47     for (data, ylabel, label, name) in zip([data1, data2, data3], ylabels1, labels1, names):
48         print "Making %s plot..." % (name + '_xvals')
49         fig = plt.figure(figsize=(9.0, 6.0))
50         ax = fig.add_subplot(111)
51
52         ax = make_plot(ax, data[:,z_col], data[:,peak_col], err = None, color = 'black', marker='o', linestyle='-', label=None)
53
54         for (x_val_col, color) in zip(x_val_cols, colors1):
55             ax = make_plot(ax, data[:,z_col], data[:,x_val_col], err = None, color = color, marker='o', linestyle='--', label=None)
56
57             #if add_rms_line:
58             #    ax = make_plot(ax, data[:,z_col], data[:,data_rms_col], color = 'green', linestyle=':')
59
60             #if fit_mean_trend:
61             #    ax, slope, slope_err, intercept, intercept_err = add_fit(ax, data[:,z_col], data[:,mean_col], err = data[:,mean_err_col], color='red')
62             #    save_fits(statsfile, name, slope, np.sqrt(slope_err), intercept, np.sqrt(intercept_err))
63
64             #ax.legend(loc='lower right')
65             ax.set_xlim(z[0] + 1.0, z[-1] - 1.0)
66             #ax.invert_xaxis()
67
68             ax.set_xlabel(xlabel, fontsize='x-large')
69             ax.set_ylabel(ylabel, fontsize='x-large')
70
71             fig.tight_layout()
72             fig.savefig(plot_base + name + '_xvals' + plot_ext, bbox_inches='tight')
73
74     ######
75
76     for (data, ylabel, label, name) in zip([data1, data2, data3], ylabels2, labels1, names):
77         print "Making %s plot..." % (name + '_sumfrac')
78         fig = plt.figure(figsize=(9.0, 6.0))
79         ax = fig.add_subplot(111)
```

```

80
81     for (sum_frac_col, color) in zip(sum_frac_cols, colors2):
82         ax = make_plot(ax, data[:,z_col], data[:,sum_frac_col], err = None, color = color, marker='o',
83         linestyle='--', label=None)
84         for (doublesum_frac_col, color) in zip(doublesum_frac_cols, colors2):
85             ax = make_plot(ax, data[:,z_col], data[:,doublesum_frac_col], err = None, color = color, marker='o',
86             linestyle='--', label=None)
87
88         ax.set_xlabel(xlabel, fontsize='x-large')
89         ax.set_ylabel(ylabel, fontsize='x-large')
90         ax.set_xlim(z[0] + 1.0, z[-1] - 1.0)
91         ax.set_yscale('log')
92
93     fig.tight_layout()
94     fig.savefig(plot_base + name + '_sumfrac' + plot_ext, bbox_inches='tight')
95
96
97
98 def make_plot(ax, x, y, err=None, color='black', marker='None', linestyle='None', label=None):
99     if err == None:
100         if label == None:
101             ax.plot(x, y, color=color, marker=marker, linestyle=linestyle)
102         else:
103             ax.plot(x, y, color=color, marker=marker, linestyle=linestyle, label=label)
104     else:
105         if label == None:
106             ax.errorbar(x, y, yerr=err, color=color, marker=marker, linestyle=linestyle)
107         else:
108             ax.errorbar(x, y, yerr=err, color=color, marker=marker, linestyle=linestyle, label=label)
109     return ax
110
111
112
113 def add_fit(ax, x, y, err=None, color='red'):
114     from scipy.optimize import curve_fit
115     p0 = [0.0, 0.0]
116
117     try:
118         coeffs, pcov = curve_fit(linear, x, y, sigma=err, p0=p0)
119     except RuntimeError:
120         print '*****Curve fit failed*****'
121         return np.nan, np.nan
122     xmin, xmax = ax.get_xlim()
123     x_fit = np.linspace(xmin, xmax, 20)
124     y_fit = linear(x_fit, coeffs[0], coeffs[1])
125     ax.plot(x_fit, y_fit, color=color, linestyle='--')
126     return ax, coeffs[0], pcov[0,0], coeffs[1], pcov[1,1]
127
128 def linear(x, slope, intercept):
129     return slope * x + intercept
130
131
132 def read_files(files, header_line = None, comment_char = '#'):
133     header = None
134     data = None
135     if type(files) == str:
136         files = [files]
137
138     if header_line != None:
139         with open(files[0], 'r') as fd:
140             for line in range(header_line):
141                 fd.readline()
142             header = fd.readline()
143             if header[0] != comment_char:
144                 print "Header must start with a %s" % comment_char
145                 sys.exit(4)
146             header = header[1:]
147             header = header.split()
148
149     for file in files:
150         print 'Reading file %s...' % (file)
151         if data == None:
152             data = np.genfromtxt(file, comments=comment_char)
153         else:
154             data = np.append(data, np.genfromtxt(file, comments=comment_char), axis=0)
155
156     print 'Finished reading files.'
157     if header_line == None:
158         return data
159     else:
160         return header, data
161
162
163 def save_fits(file, name, slope, slope_err, intercept, intercept_err):
164     with open(file, 'a') as fd:
165         fd.write("%s %g %g %g\n" % (name, slope, slope_err, intercept, intercept_err))
166
167
168 plot_dest_type = 'paper'
169 if plot_dest_type == 'paper':

```

```

170     mpl.rcParams['font.family'] = 'serif'
171     mpl.rcParams['font.size'] = 16
172     mpl.rcParams['axes.linewidth'] = 3
173     mpl.rcParams['lines.linewidth'] = 4
174     mpl.rcParams['patch.linewidth'] = 4
175     mpl.rcParams['xtick.major.width'] = 3
176     mpl.rcParams['ytick.major.width'] = 3
177     mpl.rcParams['xtick.major.size'] = 8
178     mpl.rcParams['ytick.major.size'] = 8
179
180 #colors = ['red', 'green', 'blue']
181 colors = ['black', 'black', 'black']
182 labels1 = [r'$\mathfrak{M}_{\mathrm{vir}}$', r'$X_{\mathrm{off}}$']
183 names = ['c_rockstar', 'Mvir', 'Xoff']
184 xlabel = 'Redshift'
185 ylabel1 = [r"$\Delta'_{\mathrm{c}}(f_{\mathrm{h}}, z)$", r"$\Delta'_{\mathrm{M}}(\mathrm{vir})(f_{\mathrm{h}}, z)$", r"$\Delta'_{\mathrm{M}}(\mathrm{vir}, \mathrm{peak})$",
186             r"$\Delta'_{\mathrm{X}}(\mathrm{off})$"]
187 ylabel2 = [r"$f_{\mathrm{h}}(\Delta'_{\mathrm{c}}, z)$", r"$f_{\mathrm{h}}(\Delta'_{\mathrm{M}}(\mathrm{vir}), z)$", r"$f_{\mathrm{h}}(\Delta'_{\mathrm{X}}(\mathrm{off}), z)$"]
188 plot_base = 'plots/'
189 plot_ext = '.eps'
190
191 statsfile = 'plots/stats.dat'
192
193 z_col = -1
194 snap_col = 0
195 mean_col = 7
196 mean_err_col = 8
197 var_col = 9
198 var_err_col = 10
199 skew_col = 3
200 skew_err_col = -2
201 #skew_col = 7
202 #skew_err_col = 8
203 #kurt_col = 4
204 kurt_col = 13
205 kurt_err_col = 14
206 beta_col = 13
207 beta_err_col = 14
208
209 data_mean_col = 1
210 data_rms_col = 15
211
212
213
214 peak_col = 1
215 x_val_cols = np.array([4, 6, 8]) + 2
216 sum_frac_cols = np.array([2, 4, 6, 8]) + 2 + 9
217 doublesum_frac_cols = sum_frac_cols + 9
218
219 colors1 = ['red', 'green', 'blue']
220 colors2 = ['blue', 'green', 'red', 'black']
221
222 offset = 0.06
223 #offset = 0.0
224
225 minsnap = 39
226 #minsnap = None
227
228 fit_mean_trend = False
229 add_rms_line = False
230
231
232 if __name__ == '__main__':
233     main()

```

## Appendix M

### Miscellaneous Scripts

#### M.1 Directory Structure Setup (Bash)

```
1 #!/usr/bin/env bash
2
3 minsnap=0
4 maxsnap=61
5
6 minbox=1
7 maxbox=3
8
9 for ((i=$minbox; i<=$maxbox; i++)); do
10   if [ ! -e ../box$i ]; then
11     mkdir -v ../box$i
12   fi
13   if [ ! -e ../box$i/2lpt ]; then
14     mkdir -v ../box$i/2lpt
15   fi
16   if [ ! -e ../box$i/za ]; then
17     mkdir -v ../box$i/za
18   fi
19   if [ ! -e ../box$i/crossmatch ]; then
20     mkdir -v ../box$i/crossmatch
21   fi
22
23 cp -v run_*.pbs ../box$i/.
24 cp -v postprocess.sh ../box$i/.
25
26 for ((snap=$minsnap; snap<=$maxsnap; snap++)); do
27   if [ $snap -lt 10 ]; then
28     j=0$snap
29   elif [ $snap -lt 100 ]; then
30     j=0$snap
31   fi
32
33   if [ ! -e ../box$i/2lpt/snap$j ]; then
34     mkdir -v ../box$i/2lpt/snap$j
35   fi
36   if [ ! -e ../box$i/za/snap$j ]; then
37     mkdir -v ../box$i/za/snap$j
38   fi
39
40 cp -v -r proto/* ../box$i/2lpt/snap$j/.
41 cp -v -r proto/* ../box$i/za/snap$j/.
42
43 ln -v -s ~/projects/data/2lpt/box$i/2lpt_512_z300_PM_$j ../box$i/2lpt/snap$j/particles/2lpt_512_z300_PM_$j
44 ln -v -s ~/projects/data/za/box$i/za_512_z300_PM_$j ../box$i/za/snap$j/particles/za_512_z300_PM_$j
45
46 echo /home/sissomdj/projects/simulations/rockstar/box$i/2lpt/snap$j/particles/2lpt_512_z300_PM_$j > ../box$i/2lpt/snap$j/particles/snapnames.lst
47 echo /home/sissomdj/projects/simulations/rockstar/box$i/za/snap$j/particles/za_512_z300_PM_$j > ../box$i/za/snap$j/particles/snapnames.lst
48
49 echo "BGC2_SNAPNAMES_=\"/home/sissomdj/projects/simulations/rockstar/box$i/2lpt/snap$j/particles/snapnames.lst\"">>> ../box$i/2lpt/snap$j/onenode.cfg
50 echo "BGC2_SNAPNAMES_=\"/home/sissomdj/projects/simulations/rockstar/box$i/za/snap$j/particles/snapnames.lst\"">>> ../box$i/za/snap$j/onenode.cfg
51
52 echo "FILENAME_=\"2lpt_512_z300_PM_$j\"">>> ../box$i/2lpt/snap$j/onenode.cfg
53 echo "FILENAME_=\"za_512_z300_PM_$j\"">>> ../box$i/za/snap$j/onenode.cfg
54 done
55
56 done
```

#### M.2 CrossMatch Setup (Bash)

```
1 #!/usr/bin/env bash
2
3 minsnap=0
4 maxsnap=61
5
6 minbox=1
7 maxbox=3
8
9 for ((i=$minbox; i<=$maxbox; i++)); do
10   if [ ! -e ../box$i/crossmatch ]; then
11     mkdir -v ../box$i/crossmatch
12   fi
13
14 cp -v run_crossmatch.pbs ../box$i/.
15
16 for ((snap=$minsnap; snap<=$maxsnap; snap++)); do
```

```

17     if [ $snap -lt 10 ]; then
18         j=0$snap
19     elif [ $snap -lt 100 ]; then
20         j=0$snaps
21     fi
22
23     if [ ! -e ../box$i/crossmatch/snap$j ]; then
24         mkdir -v ../box$i/crossmatch/snap$j
25     fi
26
27     cp -v -r crossmatch_proto/* ../box$i/crossmatch/snap$j.
28
29     echo "OUTPUT_DIR${snap} ${home}/sissomdj/projects/simulations/rockstar/box${i}/crossmatch/snap${j}" >> ../box${i}/
30     crossmatch/snap${j}/rockstar_2lpt.param
31     echo "FIRST_GROUPDIR${snap} ${home}/sissomdj/projects/simulations/rockstar/box${i}/2lpt/snap${j}/halos" >> ../box${i}/
32     crossmatch/snap${j}/rockstar_2lpt.param
33     echo "SECOND_GROUPDIR${snap} ${home}/sissomdj/projects/simulations/rockstar/box${i}/za/snap${j}/halos" >> ../box${i}/
34     crossmatch/snap${j}/rockstar_2lpt.param
35     echo "OUTPUT_DIR${snap} ${home}/sissomdj/projects/simulations/rockstar/box${i}/crossmatch/snap${j}" >> ../box${i}/
36     crossmatch/snap${j}/rockstar_za.param
37     echo "FIRST_GROUPDIR${snap} ${home}/sissomdj/projects/simulations/rockstar/box${i}/za/snap${j}/halos" >> ../box${i}/
38     crossmatch/snap${j}/rockstar_za.param
39     echo "SECOND_GROUPDIR${snap} ${home}/sissomdj/projects/simulations/rockstar/box${i}/2lpt/snap${j}/halos" >> ../box${i}/
40     crossmatch/snap${j}/rockstar_za.param
41
42     done
43
44 done

```

### M.3 Individual Snapshot Rockstar Run Script (Bash)

```

1 #!/bin/bash
2
3 echo "Cleaning old files..."
4 if [ -e out.log ]; then
5     mv -v out.log out.log.bak
6 fi
7 if [ -e server.out ]; then
8     mv -v server.out server.out.bak
9 fi
10 if [ -e clients.out ]; then
11     mv -v clients.out clients.out.bak
12 fi
13 if [ -e auto-rockstar.cfg ]; then
14     rm -v auto-rockstar.cfg
15 fi
16 if [ $(ls halos/* 2> /dev/null | wc -l) != "0" ]; then
17     rm -rv halos/*
18 fi
19
20 echo "Submitting run script..."
21 echo "qsub run_rockstar.pbs"
22 qsub run_rockstar.pbs

```

### M.4 All Snapshots Rockstar 2lpt PBS Submission Script (Bash)

```

1 #!/usr/bin/env bash
2
3 #PBS -M djsissom@gmail.com
4 #PBS -m bae
5 #PBS -l nodes=1:ppn=10
6 #PBS -l pmem=3000mb
7 #PBS -l mem=30000mb
8 #PBS -l walltime=6:00:00
9 #PBS -o out_2lpt.log
10 #PBS -j oe
11
12 echo $PBS_NODEFILE
13 cd $PBS_O_WORKDIR
14
15 for snapdir in 2lpt/*; do
16     # Change to working directory
17     echo Working on $snapdir...
18     cd $PBS_O_WORKDIR/$snapdir
19
20     # Start the server
21     rockstar -c onenode.cfg &> server.out &
22
23     # Wait for auto-rockstar.cfg to be created
24     perl -e 'sleep 1 while (!(-e "halos/auto-rockstar.cfg"))'
25     mv halos/auto-rockstar.cfg .
26
27     # Execute the reader processes
28     mpixexec -verbose -n 1 rockstar -c auto-rockstar.cfg >> clients.out 2>&1 &
29     sleep 20
30
31     # Execute the analysis processes
32     mpixexec -verbose -n 8 rockstar -c auto-rockstar.cfg >> clients.out 2>&1
33

```

```

34  # - end of script
35 done

1 #!/usr/bin/env bash
2
3 #PBS -M djsissom@gmail.com
4 #PBS -m bae
5 #PBS -l nodes=1:ppn=10
6 #PBS -l pmem=3000mb
7 #PBS -l mem=30000mb
8 #PBS -l walltime=6:00:00
9 #PBS -o out_za.log
10 #PBS -j oe
11
12 echo $PBS_NODEFILE
13 cd $PBS_O_WORKDIR
14
15 for snapdir in za/*; do
16   # Change to working directory
17   echo Working on $snapdir...
18   cd $PBS_O_WORKDIR/$snapdir
19
20   # Start the server
21   rockstar -c onenode.cfg &> server.out &
22
23   # Wait for auto-rockstar.cfg to be created
24   perl -e 'sleep 1 while (!(-e "halos/auto-rockstar.cfg"))'
25   mv halos/auto-rockstar.cfg .
26
27   # Execute the reader processes
28   mpixec -verbose -n 1 rockstar -c auto-rockstar.cfg >> clients.out 2>&1 &
29   sleep 20
30
31   # Execute the analysis processes
32   mpixec -verbose -n 8 rockstar -c auto-rockstar.cfg >> clients.out 2>&1
33
34 # - end of script
35 done

```

## M.6 All Snapshots Rockstar Post-Process Script (Bash)

```

1 #!/usr/bin/env bash
2
3 startdir=`pwd`
4
5 for snapdir in {2lpt,za}/*; do
6   echo Working on $snapdir...
7   cd $startdir/$snapdir
8
9   ./postprocess
10
11 done
12
13 # - end of script

```

## M.7 All Snapshots CrossMatch PBS Submission Script (Bash)

```

1 #!/usr/bin/env bash
2
3 #PBS -M djsissom@gmail.com
4 #PBS -m bae
5 #PBS -l nodes=62:ppn=1
6 #PBS -l pmem=3000mb
7 #PBS -l mem=186000mb
8 #PBS -l walltime=1:00:00
9 #PBS -o out_crossmatch.log
10 #PBS -j oe
11
12 echo $PBS_NODEFILE
13 cd $PBS_O_WORKDIR
14
15 for snapdir in crossmatch/*; do
16   # Change to working directory
17   echo Working on $snapdir...
18   cd $PBS_O_WORKDIR/$snapdir
19
20   {
21     mpixec -verbose -n 1 crossmatch rockstar_2lpt.param > out_2lpt_first.log 2>&1
22     mpixec -verbose -n 1 crossmatch rockstar_za.param > out.za_first.log 2>&1
23     echo "Finished:$snapdir"
24   } &
25
26 done
27
28 wait
29 # - end of script

```

## M.8 All Snapshots Density Profile PBS Submission Script (Bash)

```
1 #!/usr/bin/env bash
2
3 #PBS -M djsissom@gmail.com
4 #PBS -m bae
5 #PBS -l nodes=124:ppn=1
6 #PBS -l pmem=4000mb
7 #PBS -l mem=496000mb
8 #PBS -l walltime=1:00:00
9 #PBS -o out_density_profile.log
10 #PBS -j oe
11
12 echo $PBS_NODEFILE
13 cd $PBS_O_WORKDIR
14
15 for snapdir in {2lpt,za}/snap*/halos; do
16     # Change to working directory
17     echo Working on $snapdir...
18     cd $PBS_O_WORKDIR/$snapdir
19
20 {
21     mpiexec -verbose -n 1 density_profile halos_0.*.bgc2 > density_profile_out.log 2>&1
22     echo "Finished $snapdir"
23 } &
24
25 done
26
27 wait
28 # - end of script
```