Targeted Persuasion

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March 31, 2025

Abstract

Sender maximizes how many receivers buy a widget of uncertain quality. She optimally chooses to target a receiver with whom to communicate. After communication, the target chooses whether to buy the widget and his choice is observed by other receivers with probability increasing in his popularity. We show that, independently of the Sender-target communication protocol, Sender optimally communicates with the target as if no other receiver exists, and a target's popularity is a double-edged sword for Sender. We characterize the optimal choice of target under multiple protocols and establish that Sender can benefit from protocols that constrain her communication more.

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Verba movent, exempla trahunt.
(Words persuade, examples compel.)

Latin proverb

1 Introduction

In many markets and organizations, one agent, call her "Sender," wishes to persuade the greatest possible number of "receivers." For example, a product salesperson wishes to persuade more consumers in a neighborhood to buy her product. A startup entrepreneur wishes to persuade more investors in a financial market to fund her startup. A lobbyists wishes to persuade more members of Congress to support a bill. However, persuasion often requires complex explanations and lengthy demonstrations so that Sender is limited to privately communicate with only those receivers with whom she has built stronger connections. For example, a salesperson may only have access to one customer that she familiarized with during her networking activities. However, she may reasonably hope that, because of her client's popularity in the neighborhood, many more customers will be convinced to buy when they see that her client does so. Similarly, an entrepreneur or a lobbyist may only be able to build connections with a famous investor or a popular senator, but may hope that, were they to persuade them to invest in their startup or to vote in favor of a particular policy, their example will compel others to do the same. Thus, Sender's choice of who to *target* in her networking activities crucially depends on her expectations about how persuading her target will allow her to persuade other receivers through the target's positive *example*.

In all these cases, Sender needs to choose *who to target*, as well as *how to persuade him*. Her choice of how to persuade a target depends on how she expects the target's example to influence other receivers who only observe the target's choice, but not what Sender communicated to him. Because Sender communicates with the target privately, whether and how the the target's example influences other receivers naturally depends on other receivers' expectation about how Sender privately tries to persuade the target. In turn, Sender's choice of who to target depends on her anticipation of how she will optimally persuade him and how many other receivers may observe his example. Sender may prefer to target receivers who are easier to persuade, perhaps because they are *fans* of Sender's products or the policies she advocates for. Or she may choose to target a more *popular* influencer, maximizing the visibility of his example. Targeting a fan may maximize the probability of persuading the target and setting a positive example for other receivers. But this choice is problematic. Because a fan is easy to persuade, his example is unlikely to compel other receivers to buy. Intuitively, persuading a more *skeptical* target would set a stronger, more compelling example. Similarly, a more popular target maximizes the chances that others will observe his example. But such a choice of target also carries risks. If a popular target does not buy the widget, many receivers may be compelled *not* to buy by such a negative example.

In this paper we study a one-Sender, many-receivers model of this problem of targeted persuasion. In our model, Sender's objective is to convince as many receivers as possible to buy a widget of uncertain quality. Receivers differ in two dimensions. First, some are more *skeptical* than others, in the sense that they buy the widget only when they have greater beliefs that the widget is good. In fact, some are *fans* in the sense that they would buy the widget in the absence of any further information. Second, some are more *popular* than others, in the sense that other receivers observe their choice with greater probability. Sender's problem consists in choosing a target receiver and privately communicating with the target about the widget's quality. After Sender communicates with the chosen target, the target chooses whether to buy the widget and, depending on his popularity, other receivers observe the target's choice before they make their own.

In Section 3, we study Sender's problem of how to persuade a chosen target. We establish a key result that holds independently of the communication protocol between Sender and target. Our Example-unraveling theorem says that, in any equilibrium, and independently of the choice of target, Sender's optimal private communication maximizes the probability that the target buys the widget. That is, Sender tries to persuade the target *as if* no other receiver exists, even though Sender's objective is, in fact, to persuade as many receivers as possible. Because of her objective, Sender would ideally credibly induce other receivers to believe she communicates with the target in the way that maximizes the compelling power of the target's example. However, in equilibrium, other receivers correctly anticipate that, in the privacy of the Sender-target communication, Sender would renege such a commitment and simply try to persuade the target so as to maximize the probability that he will set a positive example. Therefore, any Sender's plan to fool other receivers will "unravel."

In Section 4, we exploit this feature of our model to derive the optimal choice of target for Sender. We establish two general results that do not depend on the specific communication protocol between Sender and the target. First, we show that all equilibria are of one of two types. In *only fans* equilibria, Sender optimally chooses a target and communicates with him in such a way that the target buys the widget if and only if he is a fan. Thus, in equilibrium, the target's example carries no information for other receivers, and they too buy the widget if and only if they are fans. In contrast, in *informative* equilibria, Sender's optimal choice of target and communication induces the target to buy with probability strictly between 0 and 1. Thus, in equilibrium, the target's example has the power to influence other receivers. In particular, a positive example will (generically) compel some skeptical receivers to buy and a negative example will compel some fans not to buy. This feature of equilibrium behavior underpins the second result: what makes a target valuable for Sender crucially depends on his popularity in both positive and negative ways. In fact, an increase in a target's *popularity is a double-edged sword* for Sender. On the one hand, a more popular target is more likely, via a positive example, to persuade other receivers to buy the widget. On the other hand, he is also more likely to persuade them, via a negative example, *not* to buy the widget.

We show how these results allow us to fully characterize equilibrium behavior under different communication protocols. We focus on two canonical cases: *Bayesian persuasion* à la Kamenica and Gentzkow (2011) and (generalized) *information disclosure* (Dye, 1985; Milgrom, 1981). When Sender's communication with the target takes the form of Bayesian persuasion, Sender optimally targets a skeptical receiver only if he is not too skeptical. Intuitively, Sender anticipates that targeting a skeptical receiver entails a trade-off: if the target is persuaded to buy, his positive example will compel all receivers less skeptical than he is to buy the widget; but if he is not persuaded, his negative example will compel all receivers, including all fans, *not* to buy the widget. Because the probability to persuade a target is decreasing in the target's skepticism, some targets are just too skeptical for Sender. If all skeptical receivers are too skeptical, an only fans equilibrium arises in which Sender gives up on the idea of persuading anybody other than fans who are already willing to buy the widget.

What makes a skeptic "too skeptical" for Sender crucially depends on the double-edged sword of his popularity. Intuitively, the positive effect of popularity dominates when the target is less skeptical, when there are more skeptical receivers who are not as skeptical as he is, or when there are fewer fans. We show that in fact the negative effect of popularity may dominate for Sender's equilibrium choice of a skeptical target. I.e., in equilibrium, a marginal increase in the target's popularity may induce Sender to optimally switch to a less popular target, or even to targeting a fan (thus inducing an only fans equilibrium), to avoid losing too many fans.

We compare these results with the case when Sender-target communication takes the form of information disclosure. We characterize how Sender optimally communicates with her target and how other receivers interpret the target's example in Sender's preferred equilibrium. Importantly, we show that the less freedom afforded to Sender to "tailor" her communication to the target under information disclosure may in fact benefit Sender. Intuitively, recall that in our setting Sender would ideally credibly induce other receivers to believe she communicates with the target in the way that would maximize the power of the target's example. However, our Example-unraveling theorem says that such a commitment is not credible. Under Bayesian persuasion this forces Sender to choose the target based on the target's skepticism, because the target's positive example can only affect less skeptical receivers. In contrast, under information disclosure Sender is in part (mechanically) committed to how she communicates with the target. Therefore, the effect of a target's positive example is in part independent of the target's skepticism and his example may affect even receivers more skeptical than he is. The lesser freedom in choosing how to communicate affords Sender greater freedom in choosing the optimal target based on his popularity, in turn increasing the probability that the target's example compels more receivers. Thus, when popularity has a positive effect on a target's value, Sender's expected payoff may be strictly greater under information disclosure than under Bayesian persuasion.

Our work connects the Sender-Receiver communication literature with models of observational learning. In our model, the action of the target is observed by other receivers and, because the target in equilibrium possesses greater information, his example may persuade other receivers. We share this observational structure with models of social learning (e.g., Banerjee, 1992; Bikhchandani et al., 1992) and our specific one-to-many process is reminiscent of the classic two-step flow model of Katz and Lazarsfeld (1955). Our simple observational structure allows us to analyze how the target's popularity affects the Sender's choice of who to target.

Our concept of popularity can be viewed as a reduced form of a network game where each receiver's popularity is a public signal about the agent's communication centrality in the sense of Banerjee et al. (2013), i.e., a measure of how many other agents are likely to observe the target's action in the presence of exogenous and independent link failures.[1] We think of this reduced-form approach as capturing a feature of many real-world networks in which agents lack complete information about the network structure (who "follows" who), but can rely on proxies that measure influence, such as follower counts or ratings.

[1]Banerjee et al. (2013) develop "diffusion centrality" as a proxy for communication centrality, so we can also interpret popularity as a signal about diffusion centrality.

Works that combine social learning (or social experimentation[2]) with the presence of a sender include Caminal and Vives (1996) and Welch (1992). Arieli et al. (2023) study optimal information design to persuade a sequence of receivers who observe predecessors' actions. Recent work on Bayesian persuasion in networks (Candogan et al., 2020; Kerman and Tenev, 2021) emphasizes how network topology shapes the optimal information structure. These models do not tackle the question of who Sender should target to start such a process of social learning.

Our focus on who Sender should target is shared by an extensive literature on targeting "influencers" in networks (e.g., Ballester et al., 2006; Galeotti and Goyal, 2009). This literature often abstracts away from persuasion by assuming that targets take the desired action. In contrast, we capture influence in a reduced form but allow the target's example to be endogenous in the sense that Sender needs to persuade the target to buy.

Caillaud and Tirole (2007) and Schnakenberg (2017) combine the question of who Sender should communicate to with the idea that the target of this communication may in turn talk to other receivers. Caillaud and Tirole (2007) study a model in which Sender chooses who to target and the target's example can persuade others. However, in their model Sender cannot choose how to communicate to the target and other receivers do not need to make conjectures about what type of information Sender communicated. Schnakenberg (2017) models lobbying through cheap talk (Crawford and Sobel, 1982) with heterogeneous legislators, showing how Sender can target allied legislators to act as intermediaries. Awad (2020) and Awad and Minaudier (2025) instead allow Sender to communicate hard information. In these models, targeted legislators choose whether to relay Sender's message or endorse a policy via cheap talk.

In contrast, in our model indirect persuasion occurs through the action (example) of the target and we study how different receivers' skepticism and their popularity among other receivers combine in determining Sender's optimal choice of target.

In these political applications, Sender wishes to persuade a sufficiently large set of receivers—typically, a majority. In our model, Sender wishes to maximize the number of receivers who are persuaded. Nevertheless, our central results, the Example-unraveling theorem and The double-edged sword of popularity, only rely on the assumption that Sender's payoff is increasing in the expected number of receivers who are persuaded and therefore applies to majority settings if voting is probabilistic (Lindbeck and Weibull, 1987).

[2]In models of social experimentation (e.g., Board and Meyer-ter Vehn, 2024; Meyer-ter Vehn and Board, 2025) agents observe *outcomes* of other agents rather than the *actions* they take, so there is an informational externality but no informational asymmetry(Gale and Kariv, 2003).

Egorov and Sonin (2019) also study a problem in which Sender wishes to persuade many receivers and receivers may indirectly learn Sender's message from other receivers. However, their focus is on who Sender should "attract" in designing messages that receivers can only access at a cost. Other receivers are aware of how Sender communicates, but may not be willing pay the cost of observing the actual message. In contrast, in our model Sender can choose who to target directly and then can choose how to communicate with him, while the problem for other receivers is that they do not necessarily know how and what Sender communicated.

Arieli and Babichenko (2019), Bardhi and Guo (2018), Chan et al. (2019), and Wang (2015) also study private Bayesian persuasion of a group. However, in their context, Sender can communicate with all receivers. Therefore, these models are not well suited to understand who Sender should communicate with or how she should communicate with him when his example may persuade others.

2 A model of targeted persuasion

There is a Sender ("she") and $R \geq 2$ receivers ("he"), indexed by $r \in \mathcal{R} \equiv \{1, \dots, R\}$.

Sender. Sender wishes to maximize the number of receivers who buy a widget of uncertain quality, $\theta \in \{G, B\}$. The widget is *good* ($\theta = G$) with probability $\mu \in (0, 1)$. Otherwise it is *bad* ($\theta = B$). Let $a_r = 1$ if receiver $r \in \mathcal{R}$ buys the widget and $a_r = 0$ otherwise. Sender maximizes $\sum_{r \in \mathcal{R}} a_r$.

Receivers. Each receiver has a unit demand for the widget and buys the widget if and only if he believes it is good with sufficiently high probability. Formally, let p_r be receiver r's (posterior) belief that the widget is good. Receiver r buys the widget if and only if $p_r \geq \sigma_r$, where $\sigma_r \in [0, 1]$ is receiver r's publicly known *skepticism*. Without loss of generality, we order receivers by their skepticism: $\sigma_1 \leq \sigma_2 \leq \ldots \leq \sigma_R$. We say that receiver r is a *fan* if $\sigma_r \leq \mu$, i.e., he chooses to buy in the absence of any further information; otherwise, he is a *skeptic*. Let $F \equiv \max\{r \in \mathcal{R} : \sigma_r \leq \mu\}$ be the number of fans. To avoid uninteresting cases, we assume that there is at least one fan and at least one skeptic: $\sigma_1 \leq \mu < \sigma_R$.

Targeted persuasion. A targeted persuasion game plays out of as follows. First, Sender chooses a *target* $t \in \mathcal{R}$. Second, nature chooses quality θ. Third, Sender privately communicates with t. We discuss below what information Sender may obtain about the widget and how she may communicate it to the target. Fourth, t purchases the widget if and only if $p_t \geq \sigma_t$ and nature determines whether each non-targeted receiver observes t's choice: each receiver $r \neq t$ observes receiver t's choice with probability equal to t's publicly known *popularity*, $\pi_t \in (0,1)$. Finally, each receiver r buys the widget if and only if $p_r \geq \sigma_r$.

Private communication. Communication between Sender and her target is private. The specific *communication protocol* specifies how the interaction between Sender and target affects: (i) what information Sender obtains about the widget and (ii) how it is communicated to her target. We model a generic communication protocol as a publicly known triple $\mathcal{CP} = (M, A, I)$, where M are the possible messages privately observed by Sender and 2^M are the possible messages the target may observe from Sender's communication. $A(m_S) \subseteq 2^M$ are the *allowable* messages that Sender may communicate to the target, conditional on observing $m_S \in M$. I is a collection of *information structures*, i.e., families of conditional distributions over M of the form $i = \{i(\cdot \mid \theta)\}_{\theta \in \{G,B\}}$.

A communication protocol $\mathcal{CP}$ induces a communication game between Sender and the target as follows. First, Sender chooses an information structure $i \in I$, observed by the target. Conditional on θ, Sender privately observes a message m_S drawn with probability $i(m_S \mid \theta)$. Finally, Sender chooses which allowable message $m_t \in A(m_S)$ to communicate to the target. Notice that the communication between Sender and the target is private so that non-targeted receivers do not observe Sender's choice of information structure, nor do they observe which message Sender chooses to communicate to the target.

Our generic communication protocol encompasses many examples of interest, including two canonical cases we study in greater detail in Section 4. First, when I includes all possible information structures and $A(m_S) = \{m_S\}$ for all $m_S \in M$, so that Sender commits to truthfully communicate the message received, the communication game is one of *Bayesian persuasion* (Kamenica and Gentzkow, 2011). Second, when I is a singleton, so that Sender is simply endowed with information, and $A(m_S) \equiv \{m_t \in 2^M : m_S \in m_t\}$, so that Sender cannot lie about the message received, the communication game is one of *information disclosure* à la Milgrom (1981).[3]

Solution concept. A strategy for Sender is a triple $(t, \{i_t\}_{t\in\mathcal{R}}, \{c_t\}_{t\in\mathcal{R}})$, where t is the Sender's choice of target, and, for each possible t, i_t is Sender's choice of information

[3]We allow for Sender to be partially, fully, or not informed, and for her information endowment to be unknown to the target. This accommodates other well-known extensions of Milgrom's setting, e.g., in Dye (1985) and Hart et al. (2017).

structure and

$$c_t \in C \equiv \{c : M \to \Delta(2^M) \mid \forall m_S \in M,\ \text{supp}(c(\cdot|m_S)) \subseteq A(m_S)\}$$

is Sender's communication strategy for each possible observed message m_S.

For each receiver $r \in \mathcal{R}$, if he is the chosen target, t, his posterior $p_t(i_t, m_t)$ is a function of both Sender's choice of information structure i_t and the observed message m_t; if he is not the target, his posterior $p_r(t \mid o_t)$ for each possible target $t \neq r$ is a function of the target's identity t, and r's (private and independent) observation $o_t \in \{0, 1, \emptyset\}$ of the target's action, where $o_t = 0$ when r observes that t does not buy, $o_t = 1$ when he observes that t buys, and $o_t = \emptyset$ when he does not observe t's action.[4]

It is useful to define Sender's expected payoff (the number of widgets sold) given a choice of target t, $V_t(m_t, p_t, \boldsymbol{p}_{-t})$, as a function of the message communicated to the target m_t and posterior functions p_t and $\boldsymbol{p}_{-t} \equiv \{p_r(t)\}_{r \neq t}$:

$$\begin{aligned} V_t(m_t, p_t, \boldsymbol{p}_{-t}) \equiv \mathbb{1}[p_t(i_t, m_t) \geq \sigma_t] &\left(1 + \sum_{r \neq t} (\pi_t \mathbb{1}[p_r(t \mid 1) \geq \sigma_r] + (1 - \pi_t)\mathbb{1}[p_r(t \mid \emptyset) \geq \sigma_r]\right) \\ + \mathbb{1}[p_t(i_t, m_t) < \sigma_t] &\left(\sum_{r \neq t} (\pi_t \mathbb{1}[p_r(t \mid 0) \geq \sigma_r] + (1 - \pi_t)\mathbb{1}[p_r(t \mid \emptyset) \geq \sigma_r]\right) \end{aligned}$$

In what follows, we characterize the set of perfect Bayesian equilibria (Fudenberg and Tirole, 1991)—henceforth "equilibrium." In our context, an assessment $(t, \{i_t\}_{t \in \mathcal{R}}, \{c_t\}_{t \in \mathcal{R}}, \{p_t, \boldsymbol{p}_{-t}\}_{t \in \mathcal{R}})$ is an equilibrium if:

1. For each target $t \in \mathcal{R}$, the posterior $p_r(t)$ of each non-targeted receiver $r \neq t$ is derived using Bayes' rule, Sender's strategy, and the target's posterior, p_t:[5]

$$p_r(t \mid 0) = \frac{\sum_{\substack{m_t \in A(m_S): \\ p_t(i_t, m_t) < \sigma_t}} \mu \sum_{m_S \in M} i_t(m_S|\theta = 1) c_t(m_t \mid m_S)}{\sum_{\substack{m_t \in A(m_S): \\ p_t(i_t, m_t) < \sigma_t}} \sum_{\theta \in \{G, B\}} \Pr(\theta) \sum_{m_S \in M} i_t(m_S|\theta) c_t(m_t \mid m_S)}; \tag{1}$$

[4]We abuse notation slightly. Formally, for each possible target t and non-targeted receiver r's observation o_t, in equilibrium a target's posterior p_t equals the value of the function $p_t(i_t, m_t)$; a non-targeted receiver's posterior p_r equals the value of the function $p_r(t \mid o_t)$.

[5]Equation (3) is a version of the "no signaling what you don't know" condition (see Fudenberg and Tirole, 1991).

$$p_r(t \mid 1) = \frac{\sum_{\substack{m_t \in A(m_S):\\ p_t(i_t, m_t) \geq \sigma_t}} \mu \sum_{m_S \in M} i_t(m_S|\theta = 1) c_t(m_t \mid m_S)}{\sum_{\substack{m_t \in A(m_S):\\ p_t(i_t, m_t) \geq \sigma_t}} \sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{m_S \in M} i_t(m_S|\theta) c_t(m_t \mid m_S)}; \tag{2}$$

$$p_r(t \mid \emptyset) = \mu. \tag{3}$$

2. For each target $t \in \mathcal{R}$, his posterior p_t is derived using Bayes' rule and Sender's strategy: for all m_t Sender communicates with positive probability,

$$p_t(i_t, m_t) = \frac{\mu \sum_{m_S \in M} i_t(m_S|\theta = 1) c_t(m_t \mid m_S)}{\sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{m_S \in M} i_t(m_S|\theta) c_t(m_t \mid m_S)}. \tag{4}$$

   and, if $m_t = \{m'_S\} \notin A(m_S)$ for all $m_S \neq m'_S$, then[6]

$$p_t(i_t, \{m_S\}) = \frac{\mu\, i_t(m_S|\theta = 1)}{\sum_{\theta \in \{G,B\}} \Pr(\theta) i_t(m_S|\theta)}. \tag{5}$$

3. For each target $t \in \mathcal{R}$, Sender's choice of information structure i_t and communication strategy c_t are optimal given the posterior beliefs of the target, p_t, and of non-targeted receivers, $\boldsymbol{p}_{-t}$:

$$i_t \in \arg\max_{i \in I} \sum_{\theta} \Pr(\theta) \sum_{m_S} i(m_S \mid \theta) \sum_{m_t} c_t(m_t \mid m_S) V_t(m_t, p_t, \boldsymbol{p}_{-t}) \tag{6}$$

   and, for each observed message $m_S \in \mathrm{supp}(i_t(\cdot \mid \theta))$ and each allowable message $m_t \in A(m_S)$,

$$c_t(m_t \mid m_S) > 0 \Rightarrow m_t \in \arg\max_{m \in A(m_S)} V_t(m, p_t, \boldsymbol{p}_{-t}). \tag{7}$$

4. Sender's choice of target is sequentially optimal:

$$t \in \arg\max_{t' \in \mathcal{R}} \sum_{\theta} \Pr(\theta) \sum_{m_S} i_r(m_S \mid \theta) \sum_{m_t} c_r(m_t \mid m_S) V_r(m_t, p_{t'}, \boldsymbol{p}_{-t'}). \tag{8}$$

All proofs are in Appendix A.

[6] This last requirement says that, if the communication protocol allows Sender to credibly communicate the message he observes, then the target's belief upon observing such credible communication must coincide with Sender's belief. This is also a version of the "no signaling what you don't know condition".

3 How to optimally persuade a target

We now study how Sender optimally chooses to communicate with the target. To do so, we fix the identity of the target t and study how receivers' equilibrium behavior affects Sender's private communication with t.

We begin by studying how the target's example—his choice to buy—affects other receivers' beliefs regarding the quality of the widget, and therefore their choice of whether to buy it. Lemma 1 says that a positive example—the target buys—induces other receivers to hold more positive beliefs about the quality of the widget.

Lemma 1 (The power of examples)**.** *In any equilibrium $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$, and for each receiver $t \in \mathcal{R}$, if Sender targets t, then receiver r's posterior belief is greater when he observes that t buys the widget than when he observes that t does not: for all $r \neq t$, $p_r^*(t \mid 1) \geq p_r^*(t \mid 0)$.*

Lemma 1 says that the target's example has the power to influence other receivers. Intuitively, if other receivers observe that the target t buys the widget, then they infer that the target's posterior belief, p_t, is at least equal to his skepticism, σ_t. Conversely, if they observe that the target does not buy, then they infer that his posterior belief is less than his skepticism. Since the target has access to more information through his communication with Sender, other receivers who observe his example also make inference about the quality of the widget. They infer that the the widget is more likely to be good if they observe that the target buys the widget than if they observe that the target does not buy it.

Lemma 1 immediately allows us to establish a key result towards our main theorem below. Lemma 2 says that, on average, a positive example weakly increases the *probability* that non-targeted receivers buy the widget. That is, a positive example may compel other receivers to buy.

Lemma 2 (Examples compel)**.** *In any equilibrium $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$, and for each receiver $t \in \mathcal{R}$, if Sender targets t, then receiver r's probability of buying the widget is greater if the target buys the widget: for all $r \neq t$,*

$$\pi_t \mathbb{1}[p_r^*(t \mid 1) \geq \sigma_r] + (1-\pi_t)\mathbb{1}[p_r^*(t \mid \emptyset) \geq \sigma_r] \geq \pi_t \mathbb{1}[p_r^*(t \mid 0) \geq \sigma_r] + (1-\pi_t)\mathbb{1}[p_r^*(t \mid \emptyset) \geq \sigma_r]. \tag{9}$$

We can now establish our central result. Theorem 1 says that, in any equilibrium, Sender optimally communicates with the target as if the target was the only receiver: Sender simply maximizes the probability that the target buys the widget.

Theorem 1 (Example-unraveling). *In any equilibrium* $(t^*,\ \{i_t^*\}_{t\in\mathcal{R}},\ \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, and for each receiver* $t \in \mathcal{R}$*,*

$$i_t^* \in \arg\max_{i\in I} \sum_\theta \Pr(\theta) \sum_{m_S} i(m_S \mid \theta) \sum_{m_t} c_t^*(m_t \mid m_S) \mathbb{1}[p_t^*(i, m_t) \geq \sigma_t] \tag{10}$$

and, for each observed message $m_S \in \operatorname{supp}(i_t^*(\cdot \mid \theta))$ *and each allowable message* $m_t \in A(m_S)$*,*

$$c_t^*(m_t \mid m_S) > 0 \Rightarrow m_t \in \arg\max_{m\in A(m_S)} \mathbb{1}[p_t^*(i_t^*, m) \geq \sigma_t]. \tag{11}$$

Theorem 1 is our central result regarding how Sender should persuade the target. Importantly, this result is independent of the specific communication protocol between Sender and the target. Intuitively, Theorem 1 says that, in equilibrium, Sender communicates with the target as if other receivers did not exist, even though Sender's objective is to maximize how many receivers buy the widget and she knows that, with probability π_t, each non-targeted receiver will observe the target's example. In fact, because of Lemma 2, Sender knows that the probability that a non-targeted receiver buys the widget is greater if the target buys the widget. Therefore, maximizing the probability that the target buys the widget also maximizes the probability that each other receiver buys it.

A—perhaps subtle—detail of Theorem 1 is that Sender maximizes the *equilibrium* probability that the target buys, $\Pr(p_t^*(i_t, m_t) \geq \sigma_t)$. Therefore, Sender's optimal strategy depends crucially on the target's equilibrium belief as a function of the messages that Sender communicates to him. This detail will turn out to be important when the communication protocol allows for multiple equilibrium beliefs for the target (see Section 4.2).

This result has key implications for the scope of choosing a target to set an example for other receivers. Sender may wish to communicate with the target with the objective of maximizing how many other receivers will buy the widget. With this in mind, Sender would ideally credibly induce other receivers to believe she communicates with the target in the way that would maximize the power of the target's example. However, Theorem 1 says that, in equilibrium, other receivers correctly anticipate that, in the privacy of the Sender-target communication, Sender would renege such a commitment and simply try to persuade the target so to maximize the probability that he will set a positive example. Therefore, any Sender's plan to fool other receivers will "unravel".

In the next section we show how Theorem 1 underpins Sender's optimal choice of target and provide two examples which illustrate the consequences of Theorem 1 under specific communication protocols, namely (i) Bayesian persuasion and (ii) information disclosure.[7]

4 Optimal targeted persuasion

We now turn to the question of who Sender should target. While our key result about how to persuade a target is independent of the specific communication protocol, the optimal choice of target depends on it. Nevertheless, we now show that Sender's key tradeoffs are the same across communication protocols.

We first note that the set of possible equilibria can be divided according to whether the target's example has the power to influence other receivers. Proposition 1 says that, independent of the communication protocol, all equilibria are of one of two types. In one, Sender chooses a target and communicates with him in such a way that, in equilibrium, the target buys the widget if and only if he is a fan—i.e., he would have bought the widget in the absence of any further information. Therefore, the target's example provides no useful information to other receivers, and each receiver buys the widget if and only if he is a fan. In contrast, in the other type of equilibria, Sender optimally chooses a target and communicates with him in such a way that, in equilibrium, the target buys the widget with probability strictly between 0 and 1. In turn, this means that the target's example is informative for other receivers—i.e., in equilibrium, the target's example has (strict) power.

Proposition 1 (Only fans and informative equilibria)**.** *In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, either:*

1. ***(Only fans equilibrium.)*** *Each receiver* r*, including the target, buys the widget if and only if he is a fan* ($\sigma_r \leq \mu$)*, or*

2. ***(Informative equilibrium.)*** *Sender's target,* t^**, buys the widget with probability strictly between* 0 *and* 1 *and, for every receiver* $r \neq t^*$*, his posterior belief when he observes that the target buys (respectively, does not buy) the widget is strictly greater (respectively, smaller) than the prior:* $p_r^*(t^* \mid 1) > \mu > p_r^*(t^* \mid 0)$.

We now characterize Sender's equilibrium value of choosing a specific target. To do so, we first recall that, by Theorem 1, in equilibrium Sender communicates with a target—i.e., she chooses i_t^* and c_t^*, given the target's posterior belief p_t^*—so as to maximize the

[7] Another canonical example one might consider here is *cheap talk* (Crawford and Sobel, 1982). In our setting cheap talk turns out to be trivial: the only equilibrium is a babbling one.

target's probability of buying the widget. For a given choice of i_t^*, the messages that may be observed by Sender, $M(i_t^*) \equiv \{m_S \in M : \exists \theta \in \{G, B\},\ i_t^*(m_S \mid \theta) > 0\}$, are divided into two sets: those that allow Sender to communicate a message m_t that induces the target to buy ($p_t^*(i_t^*, m_t) \geq \sigma_t$) and those not allowing such a message. Theorem 1 says that whenever Sender observes a message of the first type, she always prefers to induce the target to buy. Whenever she observes a message of the second type, she cannot do anything else than induce the target not to buy. Therefore, Sender-target equilibrium private communication is completely characterized by an information structure i_t^* and a set $Y_t^*(i_t^*, p_t^*) \subseteq M(i_t^*)$ of observable messages for Sender that, in equilibrium, result in the target buying the widget.

Lemma 3 (Characterizing Sender-target communication)**.** *In any equilibrium* (t^*, $\{i_t^*\}_{t\in\mathcal{R}}$, $\{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}}$)*, and for each receiver* $t \in \mathcal{R}$*, there exists a set of messages* $Y_t^*(i_t^*, p_t^*) \subseteq M(i_t^*)$ *such that, if Sender targets* t*, then* t *buys the widget if and only if Sender observes a message in* $Y_t^*(i_t^*, p_t^*)$*.*

In an informative equilibrium, $Y_{t^*}^* \neq \emptyset$ and $Y_{t^*}^* \subsetneq M(i_{t^*}^*)$, so that a non-targeted receiver $r \neq t^*$ who observes that the target buys the widget will have posterior belief

$$p_r^*(t^* \mid 1) = \Pr(\theta = G \mid m_S \in Y_t^*(i_{t^*}^*, p_{t^*}^*), i_{t^*}^*) > \mu; \tag{12}$$

while one who observes that the target does not buy will have posterior belief

$$p_r^*(t^* \mid 0) = \Pr(\theta = G \mid m_S \notin Y_t^*(i_{t^*}^*, p_{t^*}^*), i_{t^*}^*) < \mu. \tag{13}$$

In contrast, in an only fans equilibrium, $Y_{t^*}^* \in \{\emptyset, M(i_{t^*}^*)\}$, so that non-targeted receivers' posterior always equals the prior, μ.

Sender's value of choosing a target is given by the expected number of receivers that will buy the widget if Sender chooses him. Because powerful examples can compel receivers to buy and not to buy (Lemma 2), Sender is optimally choosing a risky lottery. If the target buys the widget, then his positive example may compel even a skeptical receiver to buy, adding to Sender's sales. Therefore, intuitively, a more popular target is valuable to Sender because his example is bound to compel more receivers. However, such a lottery also carries risks: because Senders has fans, a negative example may compel some of them *not* to buy, reducing Sender's sales. Therefore, a more popular target also carries greater risks.

To formally analyze this tradeoff and study how the equilibrium value of targeting a specific receiver depends on his popularity, it is useful to define the following equilibrium quantities. First, given an information structure i_t^* and a set of messages $Y_t^* \subseteq M(i_t^*)$, let

$$G_t^*(i_t^*, p_t^*) \equiv |\{r \in \mathcal{R} \setminus \{t\} : \mu < \sigma_r \leq \Pr(\theta = G \mid m_S \in Y_t^*(i_t^*, p_t^*), i_t^*)\}| \geq 0 \tag{14}$$

denote the number of skeptical receivers (not equal to t) who, in (any continuation-game) equilibrium, are compelled to buy the widget if they observe t's positive example. This captures the potential *gains* from the target's example. Similarly, let

$$L_t^*(i_t^*, p_t^*) \equiv |\{r \in \mathcal{R} \setminus \{t\} : \Pr(\theta = G \mid m_S \notin Y_t^*(i_t^*, p_t^*), i_t^*) < \sigma_r \leq \mu\}| \geq 0 \tag{15}$$

be the number of fans (not equal to t) who, in (any continuation-game) equilibrium, are compelled not to buy the widget if they observe t's negative example. This captures the potential *losses* from the target's example. Finally, let

$$F_t \equiv \max\{r \in \mathcal{R} \setminus \{t\} : \sigma_r \leq \mu\} \in \{F-1, F\}$$

be the number of fans (not equal to t).

Notice that, because of Theorem 1, the equilibrium probability of Sender observing a message in Y_t^* under i_t^*,

$$P_t^*(i_t^*, p_t^*) \equiv \Pr(m_S \in Y_t^*(i_t^*, p_t^*), i_t^*)$$

is independent of the target's popularity π_t. It follows that $G_t^*(i_t^*, p_t^*)$ and $L_t^*(i_t^*, p_t^*)$ are also independent of the target's popularity. Therefore, we can characterize the equilibrium value of targeting receiver t as a linear function of his popularity, π_t.

Lemma 4 (The equilibrium value of a targeting a receiver)**.** *In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, and for each receiver* $t \in \mathcal{R}$*, Sender's value* $\mathbb{E}[V_t^*(i_t^*, p_t^*)]$ *of targeting* t *is given by:*

$$\mathbb{E}[V_t^*(i_t^*, p_t^*)] = P_t^*(i_t^*, p_t^*)(1 + \pi_t G_t^*(i_t^*, p_t^*)) - (1 - P_t^*(i_t^*, p_t^*))\pi_t L_t^*(i_t^*, p_t^*) + F_t. \tag{16}$$

An immediate implication of (16) is that the popularity π_t of a target t is a double-edged sword for Sender—whether Sender benefits from a more popular target depends on the potential gains and losses from his example.

Proposition 2 (The double-edged sword of popularity)**.** *In any equilibrium,* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, Sender's value* $\mathbb{E}[V_t^*(i_t^*, p_t^*)]$ *of targeting receiver* $t \in \mathcal{R}$ *is:*

1. *increasing with his popularity if the expected gains from his example are greater than the*

expected losses from it: $P_t^*(i_t^*, p_t^*)G_t^*(i_t^*, p_t^*) > (1 - P_t^*(i_t^*, p_t^*))L_t^*(i_t^*, p_t^*)$,

2. *decreasing with his popularity if the expected gains from his example are less than the expected losses from it:* $P_t^*(i_t^*, p_t^*)\ G_t^*(i_t^*, p_t^*) < (1 - P_t^*(i_t^*, p_t^*))L_t^*(i_t^*, p_t^*)$.

The relative importance of gains and losses from a target's example crucially depend on (i) the probability $P_t^*(i_t^*, p_t^*)$ with which the target buys the widget under Sender's optimal communication and (ii) the distribution of skepticism among other receivers. Therefore, the Example-unraveling theorem and The double-edged sword of popularity jointly drive the choice of the optimal target. In fact, we will show that, in equilibrium, Sender may optimally choose a target "despite" his popularity—i.e., an increase in the target's popularity may induce Sender to switch to a less popular target.

We now turn to fully characterize equilibrium behavior in two canonical special cases of our model: when communication takes the form of Bayesian persuasion and when it takes the form of information disclosure. The full characterization of equilibrium behavior allows us to discuss (i) Sender's optimal choice of target, (ii) when the target's popularity is, in equilibrium, a positive or a negative feature from Sender's perspective, and (iii) why Sender may benefit from communication protocols that allow her *less* flexibility.

4.1 Bayesian Persuasion

We now study optimal targeted persuasion when Sender-target communication takes the form of Bayesian persuasion à la Kamenica and Gentzkow (2011). In this setting, Sender can choose any information structure, but is committed to truthfully communicate to the target the message she observes. I.e., $I = \{i : \Theta \to \Delta M\}$ and $A(m_S) = \{m_S\}$ for all $m_S \in M$. We assume that Sender has access to at least two messages: $|M| > 1$.

We begin by characterizing the value of a target for Sender. Lemma 5 says that, in any equilibrium, the probability that the target buys depends on the target's skepticism, σ_t.

Lemma 5 (The optimal Bayesian persuasion of a target.)**.** *Suppose that communication takes the form of Bayesian persuasion. In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, and for each receiver* $t \in \mathcal{R}$*, if Sender targets* t*, then* t *buys the widget with probability* $\min\{1, \mu/\sigma_t\}$*.*

Intuitively, because under Bayesian persuasion Sender must truthfully reveal her message, for a given information structure, the probability that the target buys the widget coincides with the probability that Sender's own posterior exceeds the target's threshold, σ_t. Then, by Theorem 1, in equilibrium Sender chooses the information structure which maximizes the probability her own posterior exceeds σ_t. If t is a skeptic, this involves choosing an information structure that induces only posteriors $p_t = 0$ (t is sure that the widget is bad) and $p_t = \sigma_t$ (the target is just indifferent between buying and not buying the widget). Bayes plausibility then yields that the probability of inducing $p_t = \sigma_t$ equals μ/σ_t. If instead t is a fan ($\sigma_t \leq \mu$), so that he buys when completely uninformed about the quality of the widget, then this involves choosing any information structure that always induces posteriors under which the target buys.

Lemma 6 characterizes the power of the target's example over non-targeted receivers' behavior. It says that a fan's example has no power, while a skeptic's positive example persuades all receivers who are less skeptical than he is.

Lemma 6 (The power of examples under Bayesian persuasion)**.** *Suppose that communication takes the form of Bayesian persuasion. In any equilibrium* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, and for each receiver* $t \in \mathcal{R}$*, if Sender targets* t *then, for all receivers* $r \neq t$*:*

1. ***(If the target is a fan.)*** *If* $\sigma_t \leq \mu$*, then* r *buys the widget if and only if he is a fan (*$\sigma_r \leq \mu$*).*
2. ***(If the target is a skeptic.)*** *If* $\sigma_t > \mu$*, then* r *buys the widget if and only if either he observes that* t *buys the widget and he is less skeptical than* t *(*$\sigma_r \leq \sigma_t$*) or he does not observe* t*'s choice and he is a fan (*$\sigma_r \leq \mu$*).*

Intuitively, when Sender targets a fan, all receivers correctly anticipate that the target will always buy the widget. Therefore, the target's example provides no useful information to other receivers. In contrast, when Sender targets a skeptic, all other receivers correctly anticipate that Sender will optimally persuade the target. Therefore, if they observe that the target buys the widget, they will correctly infer that the information provided to the target is such that the widget is good with probability exactly σ_t. Hence, Receiver r only buys if he is less skeptical than the target: $\sigma_r \leq \sigma_t$. Instead, if non-targeted receivers observe that the target does not buy the widget, they infer that the information provided to the target is such that the widget is surely bad. Hence, they do not buy.

It is useful to recall from Lemma 4 that the value of targeting receiver r depends on the potential gains $G_r^*(i_r^*, p_r^*)$ and losses $L_r^*(i_r^*, p_r^*)$ from his example. In the case of Bayesian Persuasion of a skeptical target, $P_r^*(i_r^*, p_r^*) = \mu/\sigma_r$, $G_r^*(i_r^*, p_r^*) = r - 1$, and $L_r^*(i_r^*, p_r^*) = F$. We can therefore compute the value of choosing a target as follows.

Lemma 7 (The value of a target under Bayesian persuasion)**.** *Suppose that communication takes the form of Bayesian persuasion. In any equilibrium,* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*, Sender's value* $\mathbb{E}[V_t^*]$ *of targeting receiver* $t \in \mathcal{R}$ *is equal to*

1. ***(If the target is a fan.)*** *the number of fans,* F*, if* $\sigma_t \leq \mu$*;*

2. ***(If the target is a skeptic.)***

$$\underbrace{\mu/\sigma_r}_{\text{Sale to } t} + \underbrace{\pi_t(t-1)\mu/\sigma_t}_{\text{Power of } t\text{'s example}} + \underbrace{(1-\pi_t)F}_{\text{Sales to fans.}}$$

if $\sigma_t > \mu$.

We can now characterize the set of equilibria. Proposition 3 says that if all skeptical receivers are excessively skeptical, then Sender optimally chooses to target a fan. The outcome is equivalent to Sender targeting nobody and only selling the widget to fans. Otherwise, Sender targets the skeptical receiver with the greatest value.

Proposition 3 (Optimal targeted Bayesian persuasion)**.** *Suppose that communication takes the form of Bayesian persuasion. In any equilibrium,* $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$*:*

1. ***(Only fans equilibrium.)*** *If for all skeptical receivers* r*,*

$$\sigma_r > \mu\frac{1+\pi_r(r-1)}{\pi_r F}, \tag{17}$$

then Sender targets a fan ($\sigma_t^* \leq \mu$) *and, along the equilibrium path, each receiver* r *buys if and only if he is a fan* ($\sigma_r \leq \mu$)*.*

2. ***(Bayesian persuasion of a skeptic.)*** *If there exists a skeptical receiver* r *such that*

$$\sigma_r < \mu\frac{1+\pi_r(r-1)}{\pi_r F}, \tag{18}$$

then Sender targets a skeptic $t^* \in \arg\max_{t\in\mathcal{R}}\{\mu/\sigma_t[1+\pi_t(t-1)]+(1-\pi_t)F\}$ *and, along the equilibrium path, each non-targeted receiver* r *buys if and only if either he observes that* t^* *buys the widget and he is less skeptical than* t^* ($\sigma_r \leq \sigma_{t^*}$) *or he does not observe* t^**'s choice and he is a fan* ($\sigma_r \leq \mu$)*.*

Intuitively, Sender avoids receivers who are too skeptical because they are too hard to persuade. Were they the only receivers, Sender would attempt to persuade them as such an attempt would not cost Sender any sale. But because Senders has fans, attempting to persuade skeptical receivers carries the cost of losing some sales to fans if the fans observe the negative example of skeptical targets. If skeptical receivers are very skeptical, then this cost is very likely to materialize as Sender is unlikely to persuade them. Therefore, Sender may prefer to avoid targeting skeptical receivers altogether.

What makes a skeptical receiver "not too skeptical" for Sender also depends on the receiver's popularity. As we discussed in Proposition 2, greater popularity is a double edge sword for Sender and may increase (or decrease) the value of targeting a specific receiver depending on the equilibrium expected gains and losses from his example. Corollary 1 precisely pins down, for the case of Bayesian Persuasion, when the expected gains from a target's example outweigh the expected losses from it.

Corollary 1 (The double-edged sword of popularity under Bayesian persuasion)**.** *Suppose that communication takes the form of Bayesian persuasion. In any equilibrium, Sender's value* $\mathbb{E}[V_t^*]$ *of targeting receiver* $t \in \mathcal{R}$ *increases with t's popularity,* π_t*, if* $\sigma_t < (t-1)\mu/F$*, and decreases with his popularity if* $\sigma_t > (t-1)\mu/F$*.*

Because (18) can hold for $\sigma_t > (t-1)\mu/F$, Corollary 1 and Proposition 3 imply that a marginal increase in the popularity of the *optimal* target may harm Sender. In fact, when these conditions hold together, an increase in the target's popularity may induce Sender to switch to a less popular target or to a fan.

Intuitively, an increase in the target's popularity entails a trade-off for Sender. On the one hand, a more popular target raises the probability that, if he is persuaded to buy, less skeptical (but not fans) receivers are persuaded to buy by his example. On the other hand, a more popular target also raises the probability that, if he is not persuaded to buy, fans—who would have bought the widget otherwise—are persuaded *not* to buy by his example. The first effect arises with the probability μ/σ_t that the target is persuaded to buy, and affects at most $t-1-F$ skeptic receivers. The second effect arises with probability $1-\mu/\sigma_t$ and affects F receivers. Hence, the first effect dominates when the target is less skeptical, when there are more skeptical receivers less skeptical than he is, or when there are fewer fans.

4.2 Information disclosure

We now study optimal targeted persuasion when Sender-target communication takes the form of (generalized) information disclosure à la Milgrom (1981). In this setting, Sender is endowed with a single information structure and can only choose what to (truthfully) disclose about the message she observes. That is, $I = \{i\}$, and, for all $m_S \in M$, $A(m_S) \equiv \{m_t \in 2^M : m_S \in m_t\}$. Notice that Sender does not have to communicate all of the information received (i.e., she can communicate $m_t \neq \{m_S\}$). However, he cannot lie about what she observes (i.e., she must choose m_t such that $m_S \in m_t$). We assume that $i(\cdot|\theta)$ has full support.[8] Furthermore, for ease of notation, we let $M \subseteq \mathbb{N}$ and associate smaller messages to smaller beliefs about the widget being good: $m_S < m'_S \Rightarrow \Pr(\theta = 1 \mid$

[8]That is, for all $m_S \in M$, there exists a $\theta \in \{G, B\}$ in which $i(m_S|\theta) > 0$.

$m_S) < \Pr(\theta = 1 \mid m'_S)$. As is common in this literature, we focus on the Sender-preferred equilibrium.

As with Bayesian persuasion, we first characterize the value of a target for Sender. Recall by Lemma 3 that, in any equilibrium, the messages Sender observe are partitioned so that there exists a set of observable message such that: when Sender observes a message in this set, her optimal communication induces the target to buy; when Sender observes a message outside of this set, her optimal communication induces the target not to buy. Lemma 8 precisely characterizes the set of possible equilibrium message partitions under information disclosure.

Lemma 8 (The optimal information disclosure to a target)**.** *Suppose that communication takes the form of information disclosure and Sender chooses target $t \in \mathcal{R}$. Let*

$$\mathcal{Y}(t) \equiv \left\{ Y \subseteq M : \begin{array}{l} \frac{\mu i(m_S|G)}{\mu i(m_S|G)+(1-\mu) i(m_S|B)} \geq \sigma_t \Rightarrow m_S \in B, \text{ and} \\ \frac{\mu \sum_{m_S \in B} i(m_S|G)}{\mu \sum_{m_S \in Y} i(m_S|G)+(1-\mu)\sum_{m_S \in Y} i(m_S|B)} \geq \sigma_t \end{array} \right\}$$

be the set of subsets Y of messages for Sender (i) containing all messages which, if fully revealed to the target, would induce him to buy, and (ii) that would induce the target to buy were he to know only that Sender has observed a message in Y. Take any pair (p^*, c^*) satisfying (4) and (5). Then, there exist an equilibrium, $(t^*, \{i^*_t\}_{t\in\mathcal{R}}, \{c^*_t\}_{t\in\mathcal{R}}, \{p^*_t, \boldsymbol{p}^*_{-t}\}_{t\in\mathcal{R}})$, with $c^*_t = c^*$ and $p^*_t = p^*$ if and only if there exists $Y \in \mathcal{Y}(t)$ such that the target buys the widget if and only if Sender observes $m_S \in Y$: for all $m_S \in Y$ and m_t such that $c^*(m_t \mid m_S) > 0$, $p^*(i, m_t) \geq \sigma_t$; for all $m_S \notin Y$ and m_t such that $c^*(m_t \mid m_S) > 0$, $p^*(i, m_t) < \sigma_t$.

Intuitively, for any target's posterior belief $p_t(i, m_t)$, messages observed by Sender are naturally divided into two sets: those that allow Sender to communicate a message m_t that induces the target to buy ($p_t(i, m_t) \geq \sigma_t$) and those not allowing such a message. Theorem 1 says that whenever Sender observes a message of the first type, she always prefers to induce the target to buy. Whenever she observes a message of the second type, she cannot do anything else than induce the target not to buy. Therefore, in any equilibrium, Sender-target private communication is completely characterized by a set Y of observable message for Sender that, in equilibrium, result in the target buying the widget.

Lemma 9 (The power of example under information disclosure)**.** *Suppose that communication takes the form of information disclosure and Sender chooses target $t \in \mathcal{R}$. For each $Y \in \mathcal{Y}(t)$,* *let*

$$p_Y(1) \equiv \Pr(\theta = G \mid m_S \in Y) = \frac{\mu \sum_{m_S \in Y} i(m_S|G)}{\mu \sum_{m_S \in Y} i(m_S|G) + (1-\mu) \sum_{m_S \in Y} i(m_S|B)}, \text{ and}$$
$$p_Y(0) \equiv \Pr(\theta = G \mid m_S \notin Y) = \frac{\mu \sum_{m_S \notin Y} i(m_S|G)}{\mu \sum_{m_S \notin Y} i(m_S|G) + (1-\mu) \sum_{m_S \notin Y} i(m_S|B)}.$$

For any Receiver $r \neq t$, in any equilibrium in which, if targeted, t buys the widget if and only if Sender observes $m_S \in Y$, r buys the widget if and only if he either: observes that t buys the widget and $\sigma_r \leq p_Y(1)$; observes that t does not buy the widget and $\sigma_r \leq p_Y(0)$; or he does not observe t's choice and he is a fan ($\sigma_r \leq \mu$).

Intuitively, non-targeted receivers correctly anticipate that, in equilibrium, Sender persuades the target to buy if and only if she observes a message in Y. Hence, upon observing that the target buys the widget, they must conclude that the widget is good with probability $p_Y(1)$—and hence they buy if they are not more skeptical than $p_Y(1)$. Instead, if they observe that he does not buy, they conclude that the widget is good with probability $p_Y(0)$—and hence they buy if they are not more skeptical than $p_Y(0)$.

We can now compute the value of choosing a target.

Lemma 10 (The value of a target under information disclosure)**.** *Suppose that communication takes the form of information disclosure. For each $Y \in \mathcal{Y}(t)$, let $i(Y)$ be the probability that Sender observes a message $m_S \in Y$. Then in any Sender-preferred equilibrium, Sender's value $\mathbb{E}[V_t]$ of choosing target $t \in \mathcal{R}$ equals*

$$\max_{Y \in \mathcal{Y}(t)} \left\{ \begin{array}{c} i(Y) + \pi_t \Big(i(Y) \Big| \{r \neq t : \sigma_r \leq p_Y(1)\} \Big| + \\ (1 - i(Y)) \Big| \{r \neq t : \sigma_r \leq p_Y(0)\} \Big| \Big) + (1-\pi_t)(F - \mathbb{1}[t \leq F]) \end{array} \right\}. \tag{19}$$

We can now characterize the set of Sender-preferred equilibria. Intuitively, only two types of equilibria may arise. First, Sender may choose a target and communicate with him so that the target either buys or does not buy with certainty ($Y \in \{\emptyset, M\}$). In this case, the target's example bears no information to other receivers and we have an *only fans* equilibrium: a receiver buys if and only if he is a fan.

Second, Sender may choose a target and communicate with him in such a way that the target buys the widget with probability strictly between 0 and 1 ($Y \notin \{\emptyset, M\}$)—i.e., an *informative* equilibrium. Lemma 11 says that, in any such equilibrium, the target's positive example is strong enough to persuade at least one skeptic to buy. Notably, the target himself may not be a skeptic. That is, unlike in Bayesian Persuasion (see Part 2 of

Proposition 3), Sender may now target a receiver with the aim of indirectly persuading receivers that are *more skeptical* than the target.

Lemma 11 (Sender aims to persuade skeptics)**.** *Suppose that communication takes the form of information disclosure. In any informative Sender-preferred equilibrium in which Sender targets $t^* \in \mathcal{R}$, t^* buys the widget if and only if Sender observes $m_S \in Y \notin \{\emptyset, M\}$. Moreover, there exists a skeptical receiver $r \geq t^*$ that buys the widget upon observing that the target does so: $p_r(t^* \mid 1) = p_Y(1) \geq \sigma_r$.*

Proposition 4 says that if all receivers are excessively skeptical, then Sender optimally chooses to sell only to fans. Otherwise, Sender chooses a target—but not necessarily a skeptical target—and communicates with him in such a way that the target's positive example suffices to compel at least one skeptical receiver to buy the widget.

Proposition 4 (Optimal targeted information disclosure)**.** *Suppose that communication takes the form of information disclosure. In equilibrium:*

1. ***(Only fans equilibrium.)*** *If for all receivers $t \in \mathcal{R}$ and all $Y \in \mathcal{Y}(t) \cap \cup_{r>F}\mathcal{Y}(r)$,*

$$i(Y)|\{r \neq t : \sigma_r \leq p_Y(1)\}| + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(0)\}| > \frac{F - \mathbb{I}[r \leq F] - i(Y)}{\pi_t} \tag{20}$$

   then, along the equilibrium path, each receiver r', including the target, buys if and only if he is a fan ($\sigma_{r'} \leq \mu$).

2. ***(Disclosure to persuade a skeptic.)*** *If there exists a receiver $t \in \mathcal{R}$ and $Y \in \mathcal{Y}(t) \cap \bigcup_{r>F}\mathcal{Y}(r)$ such that*

$$i(Y)|\{r \neq t : \sigma_r \leq p_Y(1)\}| + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(0)\}| > \frac{F - \mathbb{I}[r \leq F] - i(Y)}{\pi_t}, \tag{21}$$

   then there exists a skeptic $\overline{r} > F$ and a fan $\underline{r} \leq F$ such that, along the equilibrium path, the target t^* buys if and only if the sender observes $m_S \in Y$, and each non-targeted receiver r' buys if and only if either he observes that t^* buys the widget and he is less skeptical than $\overline{r}$ ($\sigma_{r'} \leq \sigma_{\overline{r}}$), he observes that t^* does not buy the widget and he is less skeptical than $\underline{r}$ ($\sigma_{r'} \leq \sigma_{\underline{r}}$), or he does not observe t^*'s choice and he is a fan ($\sigma_{r'} \leq \mu$).

Corollary 2 characterizes the double-edged sword of popularity (Proposition 2) when Sender-target communication takes the form of information disclosure.

Corollary 2 (The double-edged sword of popularity under information disclosure)**.** *Suppose that communication takes the form of information disclosure. In any equilibrium, Sender's value $\mathbb{E}[V_t^*]$ of targeting receiver $t \in \mathcal{R}$ increases with his popularity π_t if*

$$\min_{Y \in \mathcal{Y}(t)} \left\{ i(Y)|\{r \neq t : \sigma_r \leq p_Y(1)\}| + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(0)\}| \right\} \geq F - \mathbb{1}[t \leq F],$$

and decreases with his popularity if

$$\max_{Y \in \mathcal{Y}(t)} \left\{ i(Y)|\{r \neq t : \sigma_r \leq p_Y(1)\}| + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(0)\}| \right\} \leq F - \mathbb{1}[t \leq F].$$

The main difference between optimal targeted persuasion when communication takes the form of Bayesian persuasion or information disclosure is that, under information disclosure, Sender may choose a target and communicate with him so that the target's positive example compels even more skeptical receivers to buy. In contrast, under Bayesian persuasion, Theorem 1 implies that Sender can never persuade a more skeptical receiver than the target to buy the widget. We now explore the intuition behind this difference and its implications.

4.3 Comparing Bayesian persuasion and information disclosure.

We now compare Sender's payoff when communication takes the form of Bayesian Persuasion to when it takes the form of information disclosure. It may seem intuitive that Sender always prefers Bayesian persuasion because this communication protocol affords Sender has the flexibility choosing the information structure, i_t. Thus, Sender has greater freedom in choosing how to persuade the target. However, we show that this is not necessarily the case in our setting.

Under Bayesian persuasion, Sender optimally chooses both the information structure i_t and what to communicates to the target, and *both* are privately observed only by Sender and the target, but not by other receivers. Sender's objective is to persuade as many receivers as possible. Therefore, ideally, Sender would choose a very popular target and commit to an information structure that maximizes the power of his example. However, as discussed in Section 3, the Example-unraveling Theorem establishes that, in equilibrium, such a plan unravels. This is because holding fixed the equilibrium beliefs of non-targeted receivers, Sender strictly prefers to deviate to the information structure that maximizes the *equilibrium* probability that the target buys, so that a positive example will induce to buy only receivers less skeptical than the target and a negative example would induce even fans not to buy (see Lemma 6). Therefore, sometimes Sender may prefer to target a less popular receiver, or even a fan.

In contrast, when communication takes the form of information disclosure, Sender *is* committed to the only information structure available to her. Thus, even though they know Sender will maximize the probability that the target buys, other receivers do not need to conjecture what information structure was chosen by Sender—she must have used the only one available to her. It follows that a target's positive example induces non-targeted receivers to hold beliefs that are, generically, greater than the target's exact skepticism, thus compelling to buy even receivers more skeptical than he is. Therefore, in equilibrium, because Sender is less free to choose how to communicate to the target, Sender is more free to optimally choose a target who is more popular and therefore whose example may persuade more receivers to buy.

Proposition 5 gives sufficient conditions for the existence of an information disclosure structure i such that Sender's expected payoff is greater under information disclosure than under Bayesian persuasion.

Proposition 5 (When Sender strictly prefers information disclosure)**.** *Suppose that under Bayesian persuasion Sender optimally targets a skeptical receiver t. If*

1. *$\sigma_t < (t-1)\mu/F$, so that a marginal increase in t's popularity strictly increases the Sender's value of choosing t, and*
2. *there exists a skeptical Receiver $r < t$ with $\pi_r > \pi_t$, so that r is strictly more popular than t,*

then there exists an information structure i such that Sender's equilibrium expected payoff under information disclosure with $I = \{i\}$ is strictly greater than her equilibrium expected payoff under Bayesian persuasion.

We illustrate this result in an example with three receivers.

Example 1 (Sender strictly prefers information disclosure)**.** *Suppose $F = 1$ and $R = 3$, so there is exactly one fan and two skeptics, $M = \{0, 1\}$, so the message space is binary, and*

$$\pi_2 > \pi_3; \tag{22}$$

$$\frac{\mu}{\sigma_3}(1 + 2\pi_3) + (1 - \pi_3) \geq \max\left\{1, \frac{\mu}{\sigma_2}(1 + \pi_2) + (1 - \pi_2)\right\}; \tag{23}$$

$$\frac{\mu}{\sigma_3} \geq \frac{1}{2}. \tag{24}$$

Notice that equation (22) says that the less skeptical Receiver 2 is more popular than the more skeptical Receiver 3. Equation (23) says that, under Bayesian persuasion, the value of targeting

Receiver 3 is greater than the value of targeting Receiver 2 or the value of targeting the only fan. Equation (24) says that (by Corollary 1) a marginal increase in Receiver 3's popularity would increase Sender's value of targeting him.

Finally, suppose that, under information disclosure, Sender is endowed with $i = i^*$ such that $i^*(0|G) = 0$, $i^*(1|G) = 1$, and

$$i^*(1|B) = \frac{\mu(1-\sigma_2)}{(1-\mu)\sigma_2}.$$

Bayesian persuasion. *Suppose communication takes the form of Bayesian Persuasion. By Lemma 7 and (23), Sender optimally targets the most skeptical Receiver 3, and her expected payoff is*

$$\frac{\mu}{\sigma_3}(1+2\pi_3) + (1-\pi_3). \tag{25}$$

Notice that in this case Sender optimally chooses $i_3 = i^*$.

Information disclosure. *Suppose communication takes the form of information disclosure. It is straightforward to see that Sender can target Receiver 3, optimally communicate with him under i^*, and expect the same payoff as under Bayesian persuasion. However, we now show that Sender can target the less skeptical—but more popular—Receiver 2, and expect a greater payoff.*

To see this, suppose Sender chooses $t = 2$. By Lemma 8, in the unique (continuation-game) equilibrium the target buys the widget if and only if Sender observes $m_S = 1$. Furthermore, by Lemma 10, Sender's (continuation-equilibrium) expected payoff is given by

$$\frac{\mu}{\sigma_3}(1+2\pi_2) + (1-\pi_2) > \frac{\mu}{\sigma_3}(1+2\pi_3) + (1-\pi_3) \tag{26}$$

where the last inequality follows from (22) and (24).

The key to the example is that, because Sender can credibly commit to use i^*, under information disclosure she can, by targeting Receiver 2, also persuade receiver $r = 3$ if he observes 2's example.[9] In contrast, under Bayesian persuasion, by targeting Receiver 2, Sender is giving up any chance to persuade the more skeptical Receiver 3. The only way to persuade both is to target Receiver 3 and hope that Receiver 2 will observe 3's example. But because Receiver 2 is more popular than Receiver 3, all else equal it is more likely that Receiver 3 will observe 2's example than vice versa. Thus, the extra opportunity of targeting 2 and affect 3 via the example afforded by information disclosure increases Sender's equilibrium payoff. Finally, it is instructive to notice that this result crucially depends on popularity being a positive feature of a target (i.e., it relies on (24)).

[9]By a continuity argument, this holds even for information structures "close" to i^*.

5 Conclusions

When choosing who to target in networking's activities, salespersons, entrepreneurs, or lobbyists need to anticipate how persuading a target will affect other potential customers, investors, or politicians. Our Example-unraveling theorem allows us to characterize optimal targeted persuasion independently of the specific assumptions about how the communication with a target takes place. An implication of this result captures the intuitive idea that the popularity of a target is a double-edged sword for Sender. We showed how to employ these result to characterize the optimal choice of target under two canonical communication models: Bayesian persuasion and (generalized) information disclosure.

We remark that our central result—the Example-unraveling theorem—extends to a broader class of models than the one we studied here.[10] For example, in many persuasion environments there are strategic complementarities to adoption. It is easy to see that Lemmas 1 and 2 continue to hold in many environments that capture this idea because receivers who observe the target buy have both a higher posterior belief that the widget is good *and* greater beliefs about the probability that other receivers buy. Therefore, the Example-unraveling theorem holds and Sender communicates with the target as if the target is the sole possible buyer. In other contexts, Sender's target is a committee of receivers, rather than an individual, but the logic behind our central result holds: other receivers will conjecture that, in the privacy of Sender-target communication, Sender will maximize the probability that the committee will buy the widget, thus maximizing the probability of a positive example. Similarly, our central result extends to games on networks in which each receiver can only observe the choice of his predecessor before choosing his own action. Finally, we focused on Sender's choice of target *before* the quality of the widget is realized—a natural assumption in the context of networking strategies for future sales. This assumption allows us to isolate the choice of target and communication from signaling incentives. However, once Sender has chosen a target (and therefore signaling motivations have exhausted their effects) the logic of the Example-unraveling theorem still holds.

One of our key results is that the popularity of a target is a double-edged sword. Thus, the optimal target may not be the customer or investor with the greatest visibility among other customers or investors. However, when popularity positively affects the value of a target, we showed that Sender may benefit from communication forms that

[10] Nevertheless, we note that our result crucially hinges on the assumption that receivers have common values in the sense that all prefer buying a good widget over not buying any widget and prefer both over buying a bad widget. Violating this assumption "inverts" the meaning of the target's example and therefore Sender may not prefer to maximize the probability that the target buys.

afford her *less* flexibility but allow her to more freely choose who to target based on her popularity. Thus, for example, when hard information is produced independently of her choice, a salesperson will more likely optimally target a very popular customer with high visibility within his neighborhood. In contrast, when hard information is produced through experiments and demonstrations designed by her, a startup entrepreneur will more likely base her targeting choice on the target's skepticism, even at the expense of losing some visibility among other potential investors.

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A Appendix: Proofs

Proof of Lemma 1. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and t be any receiver. If t is targeted by Sender, then, by (1), for any receiver $r \neq t$,

$$\begin{aligned}
p_r^*(t \mid 1) &= \frac{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \mu \sum_{m_S\in M} i_t^*(m_S|\theta=1)c_t(m_t \mid m_S)}{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \sum_{\theta\in\{G,B\}} \Pr(\theta)\sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)} \\
&= \sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \left(p_t^*(i_t,m_t) \times \frac{\sum_{\theta\in\{G,B\}} \Pr(\theta)\sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)}{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \sum_{\theta\in\{G,B\}} \Pr(\theta)\sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)} \right) \\
&\geq \sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \left(\sigma_t \times \frac{\sum_{\theta\in\{G,B\}} \Pr(\theta)\sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)}{\sum_{\substack{m_t\in A(m_S):\\ p_t^*(i_t,m_t)\geq\sigma_t}} \sum_{\theta\in\{G,B\}} \Pr(\theta)\sum_{m_S\in M} i_t^*(m_S|\theta)c_t(m_t \mid m_S)} \right) \\
&= \sigma_t,
\end{aligned}$$

where the inequality holds because, conditional on observing message m_t, if t buys, then her posterier belief satisfies $p_t^*(i_t, m_t) \geq \sigma_t$. Similarly, by (2),

$$\begin{aligned}
p_r^*(t \mid 0) &= \frac{\sum_{\substack{m_t \in A(m_S): \\ p_t^*(i_t,m_t)<\sigma_t}} \mu \sum_{m_S \in M} i_t^*(m_S|\theta=1) c_t(m_t \mid m_S)}{\sum_{\substack{m_t \in A(m_S): \\ p_t^*(i_t,m_t)<\sigma_t}} \sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{m_S \in M} i_t^*(m_S|\theta) c_t(m_t \mid m_S)} \\
&= \sum_{\substack{m_t \in A(m_S): \\ p_t^*(i_t,m_t)<\sigma_t}} \left(p_t^*(i_t, m_t) \times \frac{\sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{m_S \in M} i_t^*(m_S|\theta) c_t(m_t \mid m_S)}{\sum_{\substack{m_t \in A(m_S): \\ p_t^*(i_t,m_t)<\sigma_t}} \sum_{\theta \in \{G,B\}} \Pr(\theta) \sum_{m_S \in M} i_t^*(m_S|\theta) c_t(m_t \mid m_S)} \right) \\
&\leq \sigma_t
\end{aligned}$$

where the inequality holds because, conditional on observing message m_t, if t does not buy, then her posterior belief satisfies $p_t^*(i_t, m_t) < \sigma_t$. Combining these two results yields $p_r^*(t \mid 1) \geq p_r^*(t \mid 0)$. □

Proof of Lemma 2. Let $(t^*, \{i_t^*\}_{t \in \mathcal{R}}, \{c_t^*\}_{t \in \mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t \in \mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ be any receiver. Notice that (9) is satisfied if and only if $p_r^*(t \mid 1) \geq p_r^*(t \mid 0)$. By Lemma 1, this is satisfied for all $r \neq t$. □

Proof of Theorem 1. Let $(t^*, \{i_t^*\}_{t \in \mathcal{R}}, \{c_t^*\}_{t \in \mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t \in \mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ be any receiver. We will show that

$$\arg\max_{m \in A(m_S)} V_t(m_t, p_t^*, \boldsymbol{p}_{-t}^*) \subseteq \arg\max_{m \in A(m_S)} \mathbb{1}[p_t^*(i_t^*, m_t) \geq \sigma_t]. \tag{27}$$

If so, by (7), we conclude that (11) holds; by (6), we conclude that (10) holds.

There are two cases to consider. First, suppose that, for all $m_S \in \operatorname{supp}(i_t^*(\cdot|\theta))$ and $m_t \in A(m_S)$, $p_t^*(i_t, m_t) < \sigma_t$. Then every message induces the same action from the target. Hence, the left and right hand sides of (27) coincide.

Second, suppose that there exists at least one $\overline{m}_S \in \operatorname{supp}(i_t^*(\cdot|\theta))$ and $\overline{m}_t \in A(m_S)$ for which $p_t^*(i_t, \overline{m}_t) \geq \sigma_t$. Take any $m_t \in \arg\max_{m \in A(m_S)} V_t(m, p_t^*, \boldsymbol{p}_{-t}^*)$. Then,

$$
\begin{aligned}
V_t(m_t, p_t^*, \boldsymbol{p}_{-t}^*) - V_t(\overline{m}_t, p_t^*, \boldsymbol{p}_{-t}^*) = & \\
&= \left(\mathbb{1}[p_t^*(i_t^*, m_t) \geq \sigma_t] - \mathbb{1}[p_t^*(i_t^*, \overline{m}_t) \geq \sigma_t]\right)\left(1 + \sum_{r \neq t} \pi_t \mathbb{1}[p_r^*(t \mid 1) \geq \sigma_r]\right) \\
&\quad + \left(\mathbb{1}[p_t^*(i_t^*, m_t) < \sigma_t] - \mathbb{1}[p_t^*(i_t^*, \overline{m}_t) < \sigma_t]\right) \sum_{r \neq t} \pi_t \mathbb{1}[p_r^*(t \mid 0) \geq \sigma_r] \\
&= \left(\mathbb{1}[p_t^*(i_t, m_t) \geq \sigma_t] - 1\right)\left(1 + \sum_{r \neq t} \pi_t \mathbb{1}[p_r^*(t \mid 1) \geq \sigma_r]\right) \\
&\quad + \mathbb{1}[p_t^*(i_t^*, m_t) < \sigma_t] \sum_{r \neq t} \pi_t \mathbb{1}[p_r^*(t \mid 0) \geq \sigma_r] \\
&\geq \mathbb{1}[p_t^*(i_t, m_t) \geq \sigma_t] - 1 \\
&\quad + \left(\mathbb{1}[p_t^*(i_t, m_t) \geq \sigma_t] + \mathbb{1}[p_t^*(i_t, m_t) < \sigma_t] - 1\right) \sum_{r \neq t} \pi_t \mathbb{1}[p_r^*(t \mid 0) \geq \sigma_r] \\
&= \mathbb{1}[p_t^*(i_t, m_t) \geq \sigma_t] - 1,
\end{aligned}
$$

where the inequality on the second last line holds as, by Lemma 2, $\mathbb{1}[p_r^*(t \mid 1) \geq \sigma_r] \geq \mathbb{1}[p_r^*(t \mid 0) \geq \sigma_r]$. Since the last term must be positive, $p_t^*(i_t, m_t) \geq \sigma_t$ holds. Therefore, $m_t \in \arg\max_{m \in A(m_S)} \mathbb{1}[p_r^*(t \mid 1) \geq \sigma_r]$. □

Proof of Proposition 1. Let $(t^*, \{i_t^*\}_{t \in \mathcal{R}}, \{c_t^*\}_{t \in \mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t \in \mathcal{R}})$ be an equilibrium. If t^* buys upon observing message m_t (respectively, does not buy), then $p_{t^*}(i_{t^*}^*, m_{t^*}) \geq \sigma_{t^*}$ (respectively, $p_{t^*}(i_{t^*}^*, m_{t^*}) < \sigma_{t^*}$). Note first that, for any $r \neq t^*$, the posterior belief $p_r^*(t^* \mid 1)$ (respectively, $p_r^*(t^* \mid 0)$) conditional on observing the target buying (respectively, not buying) equals t^*'s average posterior belief across all messages inducing him to buy (respectively, not buy). Therefore, $p_r^*(t^* \mid 1) \geq \mu \geq p_r^*(t^* \mid 0)$.

We divide the proof in two complementary cases. First, suppose that t^* buys with probability 1 or 0. By the law of iterated expectations, the average posterior over all messages equals the prior μ. Therefore, we have that t^* buys if and only if he is a fan. Furthermore, $p_r^*(t^* \mid 1) = p_r^*(t^* \mid 0) = p_r^*(t^* \mid \emptyset) = \mu$, so that receiver r buys if and only if he is a fan. This is an only fans equilibrium. Second, suppose that t^* buys with probability $P \in (0, 1)$. Using (1)–(3), we have $p_r^*(t^* \mid 1) > \mu > p_r^*(t^* \mid 0)$. This is an informative equilibrium. □

Proof of Lemma 3. Let $(t^*, \{i_t^*\}_{t \in \mathcal{R}}, \{c_t^*\}_{t \in \mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t \in \mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ be any receiver. Let $Y_t^*(i_t^*, p_t^*)$ denote the subset of messages in M drawn with positive probability under i_t^* for which there is an allowable message that persuades the target to buy under belief p_t^*:

$$Y_t^*(i_t^*, p_t^*) \equiv \left\{ m_S \in M : \begin{array}{c} i(m_S|G)\mu + i(m_S|B)(1-\mu) > 0, \text{ and} \\ \exists m_t \in A(m_S) \text{ s.t. } p_t(i_t^*, m_t) \geq \sigma_t \end{array} \right\}.$$

By Theorem 1, for all $m_S \in Y_t^*(i_t^*, p_t^*)$, in equilibrium, when she observes m_S, any message that Sender sends to t with positive probability persuades the target to buy. Thus, the target buys with probability 1 conditional on Sender observing any such m_S. Furthermore, for all $m_S \notin Y_t^*(i_t^*, p_t^*)$, either Sender never observes the message under i_t^*, or Sender observes the message, but there exists no message Sender can send to the target to persuade the target to buy. Hence, the target buys with probability zero conditional on Sender observing any such m_S. $\square$

Proof of Lemma 4. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ any receiver. If Sender targets t, then conditional on not observing t's action, F_t non-targeted receivers buy the widget. Conditional on observing t's action, if receiver t buys and a non-target receiver $r \neq t$ observes receiver t buy (with probability π_t), receiver r's posterior is $\Pr(\theta = G|m_S \in Y_t^*(i_t^*, p_t^*), i_t^*)$. Hence, the non-target receiver buys if and only if $\Pr(\theta = G|m_S \in Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r$. An analogous argument implies that conditional on observing the target receiver not buy, a non-targeted receiver r buys if and only if $\Pr(\theta = G|m_S \notin Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r$. Hence, Sender's value from targeting receiver r is

$$\begin{aligned}
\mathbb{E}[V_t^*(i_t^*, p_t^*)] &= P_t^*(i_t^*, p_t^*) \times \left(\begin{array}{c} 1 + (1-\pi_t) \times F_t \\ +\pi_t \times |\{r \neq t : Pr(\theta = G|m_S \in Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r\}| \end{array} \right) \\
&\quad + (1 - P_t^*(i_t^*, p_t^*)) \times \left(\begin{array}{c} 1 + (1-\pi_t) \times F_t \\ +\pi_t \times |\{r \neq t : Pr(\theta = G|m_S \notin Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r\}| \end{array} \right) \\
&= P_t^*(i_t^*, p_t^*)(1 + \pi_t(G_t^*(i_t^*, p_t^*))) - (1 - P_t^*(i_t^*, p_t^*))L_t^*(i_t^*, p_t^*)
\end{aligned}$$

where the second equality holds as

$$\begin{aligned}
|\{r \neq t : Pr(\theta = G|m_S \in Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r\}| - F_t &= G_t^*(i_t^*, p_t^*) \\
F_t - |\{r \neq t : Pr(\theta = G|m_S \notin Y_t^*(i_t^*, p_t^*), i_t^*) \geq \sigma_r\}| &= L_t^*(i_t^*, p_t^*)
\end{aligned}$$

$\square$

Proof of Proposition 2. Follows immediately from Lemma 4 and noting that, by Theorem 1, the equilibrium quantities $P_t^*(i_t^*, p_t^*)$, $G_t^*(i_t^*, p_t^*)$, $L_t^*(i_t^*, p_t^*)$, and F_t do not depend on π_t. $\square$

Proof of Lemma 5. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ any receiver. Under Bayesian persuasion, Sender must truthfully communicate whichever message is observed under their chosen information structure $i_t^* \in I$. Hence conditional on Sender observing a message $m_S \in M$, the target's posterior belief coincides with the Sender's posterior belief. This implies that Sender's problem can be reduced to choosing a distribution over posterior beliefs, where the Sender's payoff conditional on posterior belief q is given by $\mathbb{1}[q \geq \sigma_t]$. By Kamenica and Gentzkow (2011), the maximum payoff the Sender can achieve is equal to the concavification of $\mathbb{1}[q \geq \sigma_t]$ evaluated at the prior belief μ, which is $\min\{1, \frac{\mu}{\sigma_t}\}$, and that this can be achieved if Sender can choose an information structure with at least two messages. That the Sender in our setting indeed has access to at least two messages completes our proof. $\square$

Proof of Lemma 6. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ any receiver. Recall by Lemma 5 that the Sender's payoff is achieved by the concavification of $\mathbb{1}[q \geq \sigma_t]$ evaluated at the prior belief μ. Any information structure i_t^* which achieves this must have the following properties.

1. The set of messages that Sender observes under i_t^* with strictly positive probability can be divided into two: $2^M = M_1 \cup M_0$, where $M_1 \cap M_0 = \emptyset$.

2. For all $m_S \in M_0$, $i_t^*(m_S|G) = 0$, so the target (and Sender) is certain the quality is bad under m_S.

3. For all $m_S \in M_1$, the posterior induced $p_t^*(i_t^*, \{m_S\})$ satisfies $p_t^*(i_t^*, \{m_S\}) \geq \sigma_t$, and

$$\sum_{\theta\in\{G,B\}} \sum_{m_S\in M} p_t^*(i_t^*, \{m_S\}) i^*(m_S|\theta) \Pr(\theta) = \min\{\mu, \sigma_t\},$$

so the target always has a belief of at least σ_t that the quality is good upon observing $m_S \in M_1$, and his average belief coincides with $\min\{\mu, \sigma_t\}$.

By (1) and (2), a non-target receiver r's beliefs that the widget is good conditional on observing that the target buys ($o_t = 1$) and does not buy ($o_t = 0$) are, respectively, $p_r^*(t \mid 1) = \min\{\mu, \sigma_t\}$ and $p_r^*(t \mid 0) = 0$. Parts 1 and 2 of Lemma 6 are then easily verified by comparing $p_r^*(t \mid 1)$ and $p_r^*(t \mid 0)$ and $p_r^*(t \mid \emptyset)$ against r's level of skepticism σ_r. $\square$

Proof of Lemma 7. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be an equilibrium and $t \in \mathcal{R}$ any receiver. First, suppose t is a fan. By Lemma 5, in the continuation equilibrium where t is targeted, t buys the widget with probability 1, and by Lemma 6 non-targeted receivers $r \neq t$ with $\sigma_r \leq \mu$ buy the widget. So the value $V_t(m_t, p_t^*, \boldsymbol{p}_{-t}^*)$ to the Sender must be

$1 + (F - 1) = F$ for any message m_t sent with positive probability in equilibrium, and it follows that $\mathbb{E}[V_t] = F$.

Now suppose t is a skeptic. By Lemma 5, the target buys the widget with probability μ/σ_t, and, by Lemma 6, each non-targeted receiver $r \neq t$ buys the widget either if they observe that t buys the widget and $\sigma_r \leq \sigma_t$, or if they do not observe that t buys the widget and $\sigma_r \leq \mu$. In particular, $p_r^*(t \mid 1) = \sigma_t$ and $p_r^*(t \mid 0) = 0$. So the expected value, $\mathbb{E}[V_t(m_t, p_t^*, \boldsymbol{p}_{-t}^*)]$, to the Sender is

$$\begin{aligned}
\mathbb{E}[V_t(m_t, p_t^*, \boldsymbol{p}_{-t}^*)] &= \frac{\mu}{\sigma_t}\left(1 + \pi_t(t - 1 - F) + F\right) + \left(1 - \frac{\mu}{\sigma_t}\right)(1 - \pi_t)F \\
&= \frac{\mu}{\sigma_t} + \frac{\mu}{\sigma_t}\left((1 - \pi_t)F + \pi_t(t - 1)\right) + (1 - \pi_t)F - \frac{\mu}{\sigma_t}(1 - \pi_t)F \\
&= \frac{\mu}{\sigma_t} + \pi_t(t - 1)\frac{\mu}{\sigma_t} + (1 - \pi_t)F,
\end{aligned}$$

as claimed. □

Proof of Proposition 3. By Lemma 7, Sender weakly prefers targeting skeptic r over every fan if and only if $\mu/\sigma_r + \pi_r(r-1)\mu/\sigma_t + (1-\pi_r)F \geq F$. Rearranging this yields $\sigma_r \leq \mu(1 + \pi_r(r-1))/\pi_r F$. Hence: if (17) holds for all $r > F$, then Sender strictly prefers targeting some fan over every skeptic; if (18) holds for at least one $r > F$, then Sender prefers targeting some skeptic over every fan. The remainder of the proposition then follows directly from Lemma 6. □

Proof of Corollary 1. Take any receiver $t \in \mathcal{R}$. If t is a fan, then $V_t = F$, which is constant in π_t. If t is a skeptic, then by Lemma 7, $V_t = \mu/\sigma_t + \pi(t-1)\mu/\sigma_t + (1-\pi_t)F$, which is increasing in π_t if and only if $\sigma_t \leq (t-1)\mu/F$. □

Proof of Lemma 8. We prove necessity and sufficiency separately.

Necessity: Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be any equilibrium with $c_t^* = c^*$ and $p_t^* = p^*$, and $t \in \mathcal{R}$ any receiver. Let $Y \equiv \{m_S : \exists m_T \in A(m_S) \text{ s.t. } c^*(m_T|m_S) > 0 \text{ and } p^*(i, m_t) \geq \sigma_t\}$ denote the set of messages received by the Sender in which, conditional on targeting t, Sender persuades t to buy the widget with positive probability.

First, we claim that t buys the widget if and only if Sender observes $m_S \in Y$. This holds because, by the Example-unraveling Theorem, if the Sender observes some $m_S \in Y$, so there exists a message that Sender could send which persuades the target to buy, then every message sent by the Sender must persuade the target to buy.

Second, we claim that $Y \in \mathcal{Y}(t)$. Because $\{m'_S\} \notin A(m_S)$ for all $m'_S \neq m_S$, we have $p^*(i, \{m_S\}) = \mu i(m_S|G)/(\mu i(m_S|G) + (1-\mu)i(m_S|B))$. Hence, if $\mu i(m_S|G)/(\mu i(m_S|G) + (1-$

$\mu)i(m_S|B)) \geq \sigma_t$, so fully revealing the message to the target would persuade the target to buy the widget, then $m_S \in Y$. Furthermore, since for all $m_S \in Y$ and $m_T \in A(m_S)$, $c^*(m_T|m_S) > 0$ implies $p^*(i, m_t) \geq \sigma_t$,

$$\begin{aligned}
&\frac{\mu \sum_{m_S \in M} i(m_S|G)}{\mu \sum_{m_S \in M} i(m_S|G) + (1-\mu)\sum_{m_S \in M} i(m_S|B)} \\
&= \sum_{m_S \in M} \left(\sum_{m_t \in \mathcal{M}_T} p_t^*(i, m_t) c^*(m_t|m_S) \right) \frac{\mu i(m_S|G) + (1-\mu) i(m_S|B)}{\mu \sum_{m'_S \in M} i(m'_S|G) + (1-\mu)\sum_{m'_S \in M} i(m'_S|B)} \\
&\geq \sigma_t
\end{aligned}$$

as required. □

Sufficiency: Take any $Y \in \mathcal{Y}^*(t)$ such that, for all $m_S \in Y$ and m_T such that $c^*(m_t \mid m_S) > 0$, $p^*(i, m_t) \geq \sigma_t$; and for all $m_S \notin Y$ and m_t such that $c^*(m_t \mid m_S) > 0$, $p^*(i, m_t) < \sigma_t$.

Now consider the quadruple $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ defined as follows:

1. For receiver t, $i_t^* = i$, $c_t^* = c^*$, $p_t^* = p^*$ and $\boldsymbol{p}_{-t}^*$ is defined by (1), (2) and (3) using (p^*, c^*).

2. For receiver $r \neq t$, take any $Y_r \in \mathcal{Y}(r)$. Then, let $i_r^* = i$, p_r^* be defined such that if $m_S \in Y_r$ ($m_S \notin Y_r$), then $p_r(m_S|B)$ is the expected posterior of Sender conditional on observing a message in Y_r (not in Y_r), c_r^* be defined such that for all $m_S \in Y$, $c_r^*(Y|m_S) = 1$ while for all $m_S \notin Y$, $c_r^*(\{m_S\}|m_S) = 1$, and $\boldsymbol{p}_{-r}^*$ are defined by (4), (1), (2) and (3) using (p_r^*, c_r^*).

3. t^* is defined by (8) using the previously defined quantities.

Notice here that for any non-targeted receiver $r \neq t$, Sender's reporting strategy and the target's belief are defined in an analogous way to that for receiver t. Hence, to verify that $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ constitutes an equilibrium, it suffices to show that for all $m_S \in M$ and each $m_t \in A(m_S)$, Sender's reporting strategy for receiver t, c_t^*, satisfies (7). To see this, take any such m_t in which $c^*(m_S|m_t) > 0$. If $m_S \in Y$, then $p^*(i, m_t) \geq \sigma_t$ so sending the message maximizes the probability the target buys. By a similar argument to Lemma 2, sending the message also maximizes the probability the target buys. We thus conclude that (7) holds. □

Proof of Lemma 9. Let $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ be any equilibrium and $t \in \mathcal{R}$ any receiver. By (1) and (2), we see that for any non-target receiver $r \neq t$, $p_r^*(t \mid 1) = p_Y(1)$ and $p_r^*(t \mid 0) = p_Y(0)$. The claims in Lemma 9 are then easily verified by comparing $p_r^*(t \mid 1)$ and $p_r^*(t \mid 0)$ and $p_r^*(t \mid \emptyset)$ against a non-target r's level of skepticism σ_r. □

Proof of Lemma 10. Take any receiver $t \in \mathcal{R}$. By Lemma 8, every equilibrium in which Sender targets t can be associated to some $Y \in \mathcal{Y}(t)$ and vice versa, and under any equilibrium associated to Y, the target buys if and only if Sender observes $m_S \in Y$. This means that t buys the widget with probability $i(Y)$. Furthermore, by Lemma 9, the set of non-targets who buy upon observing the target take action $a_t \in \{0,1\}$ is $\{r \neq t : \sigma_t \leq p_Y(a_t)\}$, while the number of fans who buy upon not observing the target is $F - \mathbb{1}[t \leq F]$. Thus, the expected payoff from sales to non-targeted receivers is

$$\pi_t\bigg(i(Y)|\{r \neq t : \sigma_r \leq p_Y(1)\}| + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(1)\}|\bigg) + (1 - \pi_t)(F - \mathbb{1}[t \leq F]).$$

Adding these expressions together then yields the term inside the max operator of (19). Since Sender's payoff in the Sender-preferred equilibrium must be the maximum of these terms across all $Y \in \mathcal{Y}(t)$, this payoff is equal to (19). □

Proof of Lemma 11. Take any receiver $t \in \mathcal{R}$. First, suppose t is a skeptic. Then for all $Y \in \mathcal{Y}(t)$ in which $Y \neq \emptyset$, $p_Y(1) \geq \sigma_t$. Hence, the claim holds.

Next, suppose t is a fan. Take any $Y \in \mathcal{Y}(t)$ in which $Y \neq \{\emptyset, M\}$ and for all skeptics $r > F$, $p_Y(1) < \sigma_r$ holds. Then, Sender's equilibrium payoff is equal to

$$\begin{aligned}
&i(Y) + \pi_t\bigg(i(Y)\{r \neq t : \sigma_r \leq p_Y(1)\} + (1 - i(Y))|\{r \neq t : \sigma_r \leq p_Y(0)\}\bigg) \\
&\leq i(Y) + \pi_t\bigg(i(Y)(F-1) + (1 - i(Y))(F-1)\bigg) \\
&< 1 + \pi_t(F-1) \\
&\leq F.
\end{aligned}$$

The first inequality holds because, since no skeptic receiver buys upon observing that t buys, there is at most $F-1$ other receivers who buy upon observing t's action. The second (strict) inequality holds because, since $Y \neq \{\emptyset, M\}$, t buys with strictly interior probability. The final inequality holds as t's popularity is bounded above by 1. Hence, Sender is strictly better off in the equilibrium in which t always buys. I.e., when $Y = M$. □

Proof of Proposition 4. By Proposition 1, Sender's (Sender-preferred) equilibrium payoff under information disclosure across all targets is the maximum of the payoff across all only-fans equilibria and informative equilibria. The payoff in any only-fans equilibrium is F. To compute the maximum payoff across all informative equilibrium, we start by fixing a receiver $t \in \mathcal{R}$. Among all informative equilibria in which t is targeted, Lemma 11 says that the sender-preferred one is the one with Y being maximum among all $Y \in \mathcal{Y}(t)$

in which the target's example is strong enough to convince some skeptic to buy, i.e., $Y \in \cup_{r>F}\mathcal{Y}(r)$. Thus, the payoff from (ii) is

$$\max_t \max_{Y:Y\in\mathcal{Y}(t)\cap\cup_{r>F}\mathcal{Y}(r)} \left[\begin{array}{c} i(Y)+\pi_t \left(\begin{array}{c} i(Y)\Big|\{r\neq t:\sigma_r\leq p_Y(1)\}\Big|+ \\ (1-i(Y))\Big|\{r\neq t:\sigma_r\leq p_Y(0)\}\Big| \end{array} \right)+ \\ +(1-\pi_t)(F-\mathbb{1}[t\leq F]) \end{array} \right]. \tag{28}$$

Thus, if (20) holds, so that (28) is strictly less than F, the equilibrium is as described by Part 1 of Proposition 4. Analogously, if (21)) holds, so that (28) is strictly greater than F, then the equilibrium is as described by Part 2 of Proposition 4. □

Proof of Corollary 2. Take any equilibrium $(t^*, \{i_t^*\}_{t\in\mathcal{R}}, \{c_t^*\}_{t\in\mathcal{R}}, \{p_t^*, \boldsymbol{p}_{-t}^*\}_{t\in\mathcal{R}})$ and let $t\in\mathcal{R}$ be any receiver. By Lemma 10, Sender's equilibrium expected payoff of targeting t is

$$\max_{Y\in\mathcal{Y}(t)} \left\{ \begin{array}{c} i(Y)+\pi_t\Big(i(Y)\Big|\{r\neq t:\sigma_r\leq p_Y(1)\}\Big|+ \\ (1-i(Y))\Big|\{r\neq t:\sigma_r\leq p_Y(0)\}\Big|\Big)+(1-\pi_t)(F-\mathbb{1}[t\leq F]) \end{array} \right\}.$$

Hence a sufficient condition for Sender's payoff to be increasing in π_t is for

$$\pi_t\Big(i(Y)\Big|\{r\neq t:\sigma_r\leq p_Y(1)\}\Big|+(1-i(Y))\Big|\{r\neq t:\sigma_r\leq p_Y(0)\}\Big|\Big)-\pi_t(F-\mathbb{1}[t\leq F]) \tag{29}$$

to be increasing in π_t for all choices of Y. That is, (differentiating with respect to π_t):

$$\Big(i(Y)\Big|\{r\neq t:\sigma_r\leq p_Y(1)\}\Big|+(1-i(Y))\Big|\{r\neq t:\sigma_r\leq p_Y(0)\}\Big|\Big)-(F-\mathbb{1}[t\leq F])\geq 0.$$

Since this must hold for all Y, it suffices that it holds for the minimum, and rearranging gives the expression in Corollary 2. Similarly, if the maximum slope of (29) is ≤ 0, then the value of the target is decreasing with his popularity, as claimed. □

Proof of Proposition 5. First, notice that, by Lemma 7, Sender's value under Bayesian Persuasion of targeting t is $\mu/\sigma_t+\pi_t(t-1)\mu/\sigma_t+(1-\pi_t)F$.

Take any message $m_0\in M$. Consider the information structure i defined as follows: $i(m_0|G)=1$, $i(m_0|B)=(\mu(1-\sigma_t))/((1-\mu)\sigma_t)$, $i(m_S|B)=(1-(\mu(1-\sigma_t))/((1-\mu)\sigma_t))/(|M|-1)$ for $m_S\neq m_0$. Since receiver r is less skeptical than the target t, $\{m_1\}\in\mathcal{Y}(r)$. Hence, by

Lemma 10, under information disclosure, if Sender is endowed with information structure i, then Sender's preferred equilibrium payoff of targeting r is at least

$$\begin{aligned}
&i(\{m_0\}) + \pi_r\bigg(i(\{m_0\})|\{r' \neq r : \sigma_{r'} \leq p_{\{m_0\}}(1)\}| + (1 - i(\{m_0\}))|\{r' \neq r : \sigma_{r'} \leq p_Y(1)\}|\bigg) \\
&+ (1 - \pi_r)(F - \mathbb{1}[r \leq F]) \\
&\quad = \mu/\sigma_r + \pi_r(t-1)\mu/\sigma_r + (1 - \pi_r)F \\
&\quad \geq \mu/\sigma_t + \pi_r(t-1)\mu/\sigma_t + (1 - \pi_r)F \\
&\quad > \mu/\sigma_t + \pi_t(t-1)\mu/\sigma_t + (1 - \pi_t)F,
\end{aligned}$$

where the first inequality holds as r is less skeptical than t, and the second as r is strictly more popular than t and $\sigma_t < (t-1)\mu/F$. Thus, Sender is strictly better off under information disclosure than under Bayesian persuasion. □