HoloSim Variables and Equations Quick Reference

Step 1: Transmitter Placement

$$R_F > \frac{2D^2}{\lambda} \tag{1}$$

$$R_{min} \approx 5D \tag{2}$$

$$R_{react} < 0.62\sqrt{\frac{D^3}{\lambda}} \tag{3}$$

Parameters:

- D is the diameter of the antenna aperture
- λ is the wavelength at which holography is being done
- ν is the frequency at which holography is being done
- $c = 3 \cdot 10^8$ is the speed of light

Step 2: Spatial Resolution

$$\delta d < \frac{a\sqrt{2}}{2} \tag{4}$$

Parameters:

- δd is the spatial resolution
- a is the distance between corner adjusters

Step 3: Grid Point Integration Time and Total Map Time

$$\Delta t_{map} = \frac{171768 f_{osr} f_1 f_{apo}^2 D(m)}{\dot{\theta}(\text{arcsec/sec})\nu(\text{GHz})(\delta d(\text{cm}))^2}\text{hours} \tag{5}$$

$$t_{int} = \frac{6.2 \cdot 10^4 f_{osr} f_1 f_{apo}^2}{\dot{\theta}(\text{arcsec/sec})\nu(\text{GHz})D(\text{m})}\text{seconds} \tag{6}$$

Parameters:

- f_1 is the primary beam taper factor
- f_{apo} is an apodization smoothing factor used in holography imaging to dampen ringing on the edge of the aperture (equal to 1.3)
- f_{osr} is the oversampling factor between rows
- f_{oss} is the oversampling factor along a row
- $\dot{\theta}$ is the chosen rotation rate of the dish antenna
- t_{row} is the time it takes to measure one holography row
- t_{int} is the integration time per image grid point
- Δt_{map} is the total time it takes to measure one holography map

Step 4: Angular Extent of Map and Sampling Intervals

$$\delta d = \frac{D}{N_{row}} = \frac{f_1 f_{apo} c}{\nu \theta_{ext}} \tag{7}$$

$$\theta_{ext} = \frac{f_1 f_{apo} c}{\nu \delta_d}\text{deg} \tag{8}$$

$$\theta_b = \frac{61836.6 f_1}{\nu(\text{GHz})D(\text{m})}\text{arcsec} \tag{9}$$

$$\theta_{sr} = \frac{\theta_b}{f_{osr}}\text{arcsec} \tag{10}$$

$$\theta_{ss} = \frac{\theta_b}{f_{oss}}\text{arcsec} \tag{11}$$

Parameters:

- θ_{ext} is the angular extent of the holography map
- θ_b is the primary angular beam size of a single grid point on the aperture
- θ_{sr} is the angular sampling interval between map rows
- θ_{ss} is the sampling interval along a row

Step 5: Pointing Accuracy and SNR Requirement

$$\theta_{\text{point}} < \frac{180\delta_z\nu}{c} = (6 \cdot 10^{-4})\delta_z(\mu m)\nu(GHz)\,\text{deg} \tag{12}$$

$$N_{row} = \frac{Df_{apo}f_{osr}}{\delta d} \tag{13}$$

$$\delta_z = \frac{\lambda}{16\sqrt{2}}\sqrt{\frac{N_{row}^2}{f_{osr}f_{oss}}}\frac{1}{\text{SNR}}\frac{1}{\sqrt{f_{apo}}} \tag{14}$$

Parameters:

- θ_{point} is the phase accuracy requirement of the transmitter
- N_{row} is the number of final grid points in a row of the holography map
- SNR is the signal-to-noise-ratio
- δ_z is the surface deformation of the holography map

Step 6: Transmitter Output Power

$$\text{Noise Floor} = 10\log\left(\frac{kBT_{sys}}{1\text{mW}}\right) \tag{15}$$

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2} \tag{16}$$

$$z_R = \frac{\pi w_0^2 c}{\nu} \tag{17}$$

$$\eta = \frac{\left| \int_{\text{aperture}} E_{TX}(\mathbf{r}) E_{RX}^*(\mathbf{r})\, d\mathbf{r} \right|^2}{\int_{\text{aperture}} |E_{TX}(\mathbf{r})|^2\, d\mathbf{r} \int_{\text{aperture}} |E_{RX}(\mathbf{r})|^2\, d\mathbf{r}} \tag{18}$$

$$\text{Overlap} = \int_0^{D/2} r \exp\left(-\frac{2r^2}{w(z)^2}\right)\, dr \tag{19}$$

$$P_{\text{total}} = \frac{\pi w(z)^2}{2} \tag{20}$$

$$\eta = 10 \log \left(\frac{\text{Overlap}}{P_{\text{total}}}\right)^2 \tag{21}$$

$$P_{\text{dB}} = \text{Noise Floor} + \text{SNR} + \eta + G \tag{22}$$

Parameters:

- B is the detector bandwidth
- T_{sys} is the system temperature
- k is the Boltzmann's constant
- z distance between the transmitter and receiver
- $w(z)$ the beam radius at a distance z from the transmitter
- D_t is the transmitter diameter
- w_0 is the beam waist at the transmitter ($w_0 = D_t/2$)
- z_R is the Rayleigh range, which is the distance over which the beam radius approximately doubles
- $E(r)$ is the electric field strength as a function of radial distance r from the beam's center for the transmitter (TX) and receiver (RX)

- η is the beam-coupling efficiency, which quantifies how effectively the transmitted beam's energy is captured by the receiving aperture
- P_{total} is the total power in a Gaussian beam
- P_{dB} is the transmitter output power
- G is the system gain