Math 135 - Exam 2

Please do the following:

a.) read each problem and its directions CAREFULLY;
b.) write neatly and legibly in the space provided, making sure that your final answer is clearly indicated;
c.) show all work.

For questions 1 and 2, you may leave your answer in either slope/intercept form or point/slope form.

1. Give an equation for the line that passes through $(2, 5)$ and $(4, 15)$.

Slope $= \dfrac{\Delta y}{\Delta x} = \dfrac{15-5}{4-2} = \dfrac{10}{2} = 5.$

So, in point/slope form, our line has the equation:

$$y - 5 = 5(x - 2)$$

Or, we can manipulate this to rewrite in slope/intercept form:

$$y - 5 = 5x - 10$$

$$\Longrightarrow y = 5x - 5$$

2. Give an equation for the line that is perpendicular to the line

$$y = \frac{1}{7}x + 3$$

and goes through $(2, -10)$.

The given line has a slope of $\dfrac{1}{7}$.

*If two lines are perpendicular, then the slope of one is the* ***negative reciprocal*** *of the other.*

So, our line has a slope of $-7$.
In point/slope form, our line has the equation:

$$y + 10 = -7(x - 2)$$

Or, we can manipulate this to rewrite in slope/intercept form:

$$y + 10 = -7x + 14$$

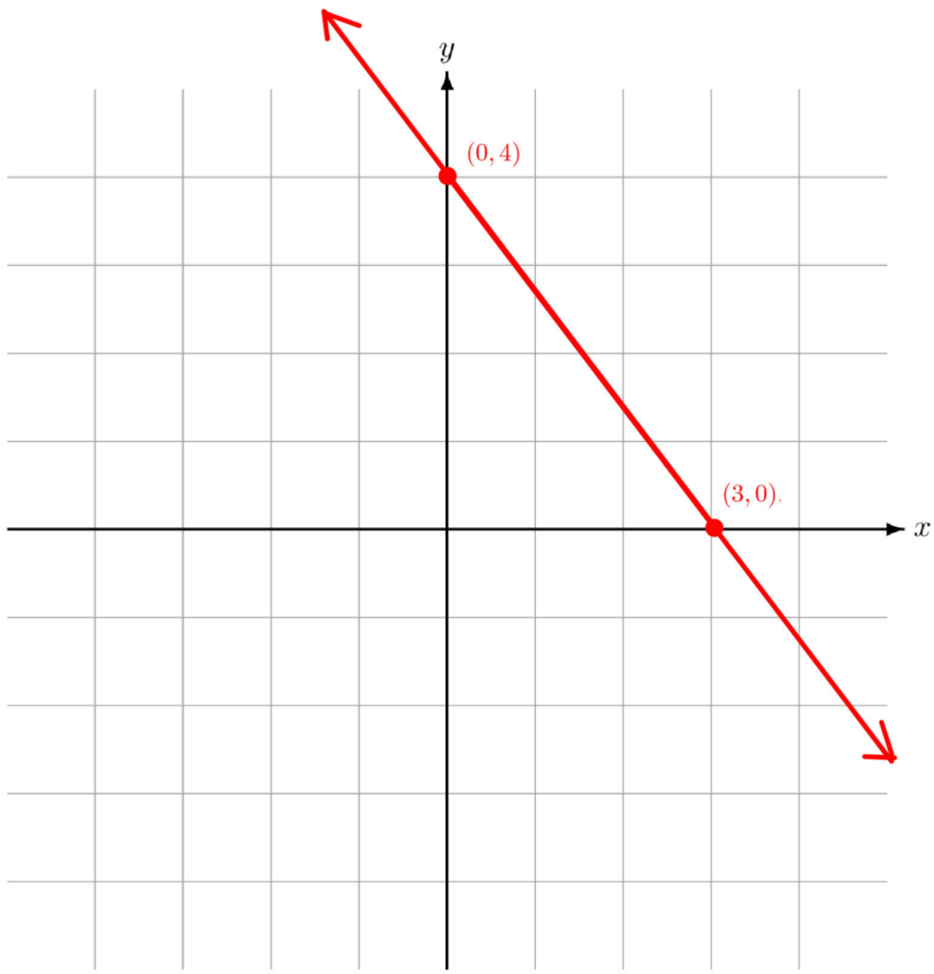
$$\Longrightarrow y = -7x + 4$$

3. a.) Graph the following linear equation:

$$4x + 3y = 12$$

You may graph this on the grid provided, or draw your own grid.

b.) Label the x-intercept and the y-intercept, and show their coordinates.




At $x = 0$, then $4(0) + 3y = 12 \implies 3y = 12 \implies y = 4$

So this line goes through $(0, 4)$.

At $y = 0$, then $4x + 3(0) = 12 \implies 4x = 12 \implies x = 3$

So this line goes through $(3, 0)$.

For questions 4 and 5, refer to the following information:

Tina's GPA closely follows a LINEAR relationship with the average number of hours she studies per week in any given semester. Several semesters ago, she recorded an average of 20.1 hours of studying per week. That semester her GPA was a 3.74. Two semesters later, her weekly studying average dropped to 14.7 hours, and for that semester she achieved a 2.84 GPA.

Let the variable $x$ be Tina's average weekly study time in hours for any one semester. Let $y$ be her GPA for that corresponding semester.

4. Write an equation in $x$ and $y$ that models Tina's GPA as a function of her average weekly study time in hours. (*Hint: I am NOT looking for a number for the answer to 4; I am looking for an equation relating $x$ to $y$.)*

| $x\,(hours)$ | $y\,(GPA)$ |
|---|---|
| 20.1 | 3.74 |
| 14.7 | 2.84 |

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{2.84 - 3.74}{14.7 - 20.1} = \frac{-.9}{-5.4} = \frac{9}{54} = \frac{1}{6} \approx 0.167.$$

So, in point/slope form, our line has the equation:

$$y - 2.84 = 0.167(x - 14.7)$$

Or, we can manipulate this to rewrite in slope/intercept form:

$$y - 2.84 = 0.167x - 2.45$$

$$\Longrightarrow y = 0.167x + 0.39$$

5. Tina wants to achieve a 4.0 next semester. Calculate the number of hours she would need to spend studying on average per week to do this.

$$GPA = 4.0 \implies y = 4.0$$

$$Hours = ? \implies \text{We don't know } x.$$

$$4.0 = 0.167x + 0.39 \implies 3.61 = 0.167x \implies x = 21.66\,hours$$

For questions 6 and 7, refer to the following information:

The demand curve for hot dogs sold at food trucks in a certain area is given by the equation:

$$p = 10 - \frac{1}{3}q$$

where $p$ is the price of hot dogs, and $q$ is quantity demanded.

The supply curve for those same hot dogs is given by:

$$p = 4 + q$$

where $q$ is now quantity supplied.

6. According to demand, how much should a vendor charge if he wants to sell 21 hot dogs in a period?

$$D : p = 10 - \frac{1}{3}q$$

At $q = 21$, then

$$p = 10 - \frac{1}{3}(21) = 10 - 7 = 3$$

$$\implies p = \$3$$

7. Calculate BOTH the equilibrium price per hot dog and the equilibrium quantity in this environment.

Equilibrium $\implies$ *Demand = Supply.*

$$D : p = 10 - \frac{1}{3}q, \ \ and \ \ S : p = 4 + q$$

Then

$$10 - \frac{1}{3}q = 4 + q \implies 6 - \frac{1}{3}q = q \implies 6 = q + \frac{1}{3}q$$

$$q = \frac{3}{4}(6) = 4.5$$

At $q = 4.5$, then $p = 4 + q = 4 + 4.5 = 8.5$.

i.e. The equilibrium price is \$8.5, and the equilibrium quantity is 4.5.

For questions 8, 9, and 10, refer to the following information:

A certain company manufactures a virtual reality device that it sells for \$800 per unit. The company's total monthly costs can be modeled by the equation:

$$C(x) = 325x + 15200$$

where $x$ is the number of units produced and sold per month, and $C(x)$ is in dollars.

8. a.) What is the cost to the company of making one more device?

*For C(x)= $mx$ + $b$, then $m$ = **marginal cost** = cost of producing one more item*

Then $m = \$325$

b.) What is $R(x)$, the company's monthly revenue, as a function of $x$?

*For $p$ = price of an item, then $R(x) = px$*

Then $R(x) = 800x$

9. What is the $P(x)$, the company's monthly profit, as a function of $x$?

*$P(x)$ = $R(x)$ - $C(x)$*

Then $P(x) = 800x - (325x + 15200) = 800x - 325x - 15200$

$\Longrightarrow P(x) = 475x - 15200$

10. How many devices must the company sell in a month's time to break even?

*Break-even occurs when **P(x)=0** (i.e. when **R(x)=C(x)** )*

We want $x$ that makes $P(x) = 0$.

$\Longrightarrow 475x - 15200 = 0 \Longrightarrow 475x = 15200 \Longrightarrow x = 32$

Bonus Question (7 points)

Below is a chart that gives the average number of gallons of milk produced in a week by cows of different ages in a county in Wisconsin:

| **Age** | **Gallons per Week** |
|---|---|
| 4 | 37.2 |
| 5 | 35.2 |
| 6 | 33.6 |
| 7 | 33.2 |
| 7 | 32.8 |
| 8 | 33.1 |
| 9 | 31.6 |
| 10 | 29.8 |

Let us make the assumption that there is an underlying linear relationship between cows' ages ($x$) and their average weekly milk production in gallons ($y$).

*Hint: use Desmos!*

a.) Find the equation in $x$ and $y$ that is the best linear fit to this relationship.

Using $DESMOS - Statistics : Linear Regression$, if we plug in the above numbers, with $x = Age (in\, years)$, and $y = Output\ (in\ gallons\ per\ week)$, then the line of best fit is:

$$y = -1.06786x + 40.7875$$

b.) What is the correlation coefficient ($r$) of this relationship?

$$r = -0.9641$$

c.) What two things can we say from $r$?

Since $r$ is close to $-1$ then we can conclude:

1.) We are more confident than not that there *IS* an underlying linear relationship between these two variables, and

2.) this relationship is decreasing (negatively sloped).