Math 135 - Final Exam

Please do the following:

a.) read each problem and its directions CAREFULLY;
b.) write neatly and legibly in the space provided, making sure that your final answer is clearly indicated;
c.) show all work.

A. **Simplifying Expressions**

5 pts 1. Perform the operation and reduce to lowest terms.

Factor.

$$\frac{r^2+5r+6}{r^2+r-12}\cdot\frac{r^2+5r+4}{r^2+4r+3} = \frac{(r+2)(r+3)}{(r+4)(r-3)}\cdot\frac{(r+1)(r+4)}{(r+1)(r+3)}$$

$$= \frac{r+2}{r-3}$$

5 pts 2. Rationalize the denominator.

Multiply the numerator and the denominator by the conjugate of the denominator.

$$\frac{7}{3-\sqrt{5}}\cdot\left(\frac{3+\sqrt{5}}{3+\sqrt{5}}\right) = \frac{7(3+\sqrt{5})}{3^2-(\sqrt{5})^2}$$

$$= \frac{21+7\sqrt{5}}{9-5}$$

$$= \frac{21+7\sqrt{5}}{4}$$

For 3 and 4, remove as many factors as possible from under the radical, and combine any like-terms. Assume that all variables represent positive real numbers.

5 pts 3. $\sqrt[4]{243a^8b^7c^{17}}$

$$= \sqrt[4]{3^5a^8b^7c^{17}}$$

$$= \sqrt[4]{3^4a^8b^4c^{16}} \cdot \sqrt[4]{3b^3c}$$

$$= \sqrt[4]{3^4 \cdot (a^2)^4 \cdot b^4 \cdot (c^4)^4} \cdot \sqrt[4]{3b^3c}$$

$$= 3a^2bc^4\sqrt[4]{3b^3c}$$

5 pts 4. $4\sqrt{63} - 3\sqrt{28}$

$$= 4\sqrt{9 \cdot 7} - 3\sqrt{4 \cdot 7}$$

$$= 4\sqrt{9}\sqrt{7} - 3\sqrt{4}\sqrt{7}$$

$$= 4 \cdot 3\sqrt{7} - 3 \cdot 2\sqrt{7}$$

$$= 12\sqrt{7} - 6\sqrt{7} = 6\sqrt{7}$$

For 5 and 6, write answers with positive exponents only. Assume that all variables represent positive real numbers

5 pts 5. $\dfrac{4q^{-7}(4q^2)^{-2}}{q^{-5}}$

$$= \frac{4q^{-7}4^{-2}q^{-4}}{q^{-5}}$$

$$= \frac{4^{1+(-2)}q^{-7+(-4)}}{q^{-5}}$$

$$= \frac{4^{-1}q^{-11}}{q^{-5}} = 4^{-1}q^{-11-(-5)}$$

$$= 4^{-1}q^{-6} = \frac{1}{4q^6}$$

5 pts 6. $\dfrac{x^{\frac{7}{3}}y^{-\frac{2}{7}}z^{\frac{3}{5}}}{x^{-\frac{2}{3}}y^{\frac{5}{7}}z^{-\frac{7}{5}}}$

$$= x^{\frac{7}{3}-(-\frac{2}{3})}y^{-\frac{2}{7}-\frac{5}{7}}z^{\frac{3}{5}-(-\frac{7}{5})}$$

$$= x^{\frac{9}{3}}y^{-\frac{7}{7}}z^{\frac{10}{5}}$$

$$= x^3y^{-1}z^2$$

$$= \frac{x^3z^2}{y}$$

B. **Solving Equations**

For 7, 8, and 9, solve each equation. Write all possible values for the given variable. If the solution is irrational, make sure it is simplified.

5 pts 7. $t(2t-13)=7$

$$\implies 2t^2 - 13t = 7$$

$$\implies 2t^2 - 13t - 7 = 0$$

$$\implies (2t+1)(t-7) = 0$$

$$\implies 2t+1 = 0 \ \ or \ \ t-7=0$$

$$\implies t = -\frac{1}{2} \ \ or \ \ t = 7$$

5 pts 8. $5x^2+2=8x$

$$\implies 5x^2 - 8x + 2 = 0$$

$$For\ \ ax^2+bx+c=0,\ \ then\ \ x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

$a=5,\ b=-8,\ c=2$

$$Then \qquad x = \frac{8 \pm \sqrt{64-4(5)(2)}}{2(5)} = \frac{8 \pm \sqrt{64-40}}{10} = \frac{8 \pm \sqrt{24}}{10}$$

$$= \frac{8 \pm \sqrt{4}\sqrt{6}}{10} = \frac{8 \pm 2\sqrt{6}}{10} = \frac{2(4 \pm \sqrt{6})}{10} = \frac{4 \pm \sqrt{6}}{5}$$

5 pts 9 $\dfrac{2y+3}{y+1} - \dfrac{y-2}{y-1} = \dfrac{2}{y^2-1}$

6 pts 10. Solve the inequality for the given variable. Give EACH of the following:

a.) a number line expression,

b.) an inequality, and

c.) a respresentation in interval notation.

$$\frac{4}{k} - \frac{5}{k-3} \geq -1 \qquad \Longrightarrow \frac{4}{k} - \frac{5}{k-3} + 1 \geq 0$$

$$\Longrightarrow \frac{4}{k}\left(\frac{k-3}{k-3}\right) - \frac{5}{k-3}\left(\frac{k}{k}\right) + 1\left(\frac{k(k-3)}{k(k-3)}\right) \geq 0$$

$$\Longrightarrow \frac{4k - 12 - 5k + k^2 - 3k}{k(k-3)} \geq 0$$

$$\Longrightarrow \frac{k^2 - 4k - 12}{k(k-3)} \geq 0$$

$$\Longrightarrow \frac{(k+2)(k-6)}{k(k-3)} \geq 0$$

"Points of interest" are values for the variable that make either the numerator or the denominator 0.

The numerator is 0 iff $x = -2$ or $x = 6$, and the denominator is 0 iff $x = 0$ or $x = 3$.

So we test AROUND these points, choosing convenient representatives from each interval.

At $k = -3$,

$$\frac{(-3+2)(-3-6)}{(-3)(-3-3)} = \frac{(-1)(-9)}{(-3)(-6)} = \frac{+}{+} = + \overset{?}{\geq} 0 \ \text{ YES } \ \Longrightarrow (-\infty, \, -2] \text{ is in the solution set.}$$

At $k = -1$,

$$\frac{(-1+2)(-1-6)}{(-1)(-1-3)} = \frac{(-1)(-7)}{(-1)(-4)} = \frac{-}{+} = - \overset{?}{\geq} 0 \ \text{ NO } \ \Longrightarrow (-2, \, 0) \text{ is NOT in the solution set.}$$

At $k = 1$,

$$\frac{(1+2)(1-6)}{(1)(1-3)} = \frac{(3)(-5)}{(1)(-2)} = \frac{-}{-} = + \overset{?}{\geq} 0 \ \text{ YES } \ \Longrightarrow [0, \, 3] \text{ is in the solution set.}$$

However, since both 0 and 3 make the denominator zero, we remove them both from the solution set.

$\Longrightarrow (0, \, 3)$ is in the solution set.

At $k = 4$,

$$\frac{(4+2)(4-6)}{(4)(4-3)} = \frac{(6)(-2)}{(4)(1)} = \frac{-}{+} = - \overset{?}{\geq} 0 \ \text{ NO } \ \Longrightarrow (0, \, 6) \text{ is NOT in the solution set.}$$

At $k = 7$,

$$\frac{(7+2)(7-6)}{(7)(7-3)} = \frac{(9)(1)}{(7)(4)} = \frac{+}{+} = + \overset{?}{\geq} 0 \ \text{ YES } \ \Longrightarrow [6, \, \infty) \text{ is in the solution set.}$$

Then our solution set for k is:

a.)

b.) $k \leq -2$ or $0 < k < 3$ or $6 \leq k$

c.) $(-\infty, -2] \cup (0, 3) \cup [6, \infty)$

C. **Linear Functions**

5 pts 11. Give an equation for the line parallel to the line

$$2x - 3y = 6$$

that goes through the point $(3, 5)$.

$$\implies y = \frac{2}{3}x - 2$$

Parallel lines have the same slope.

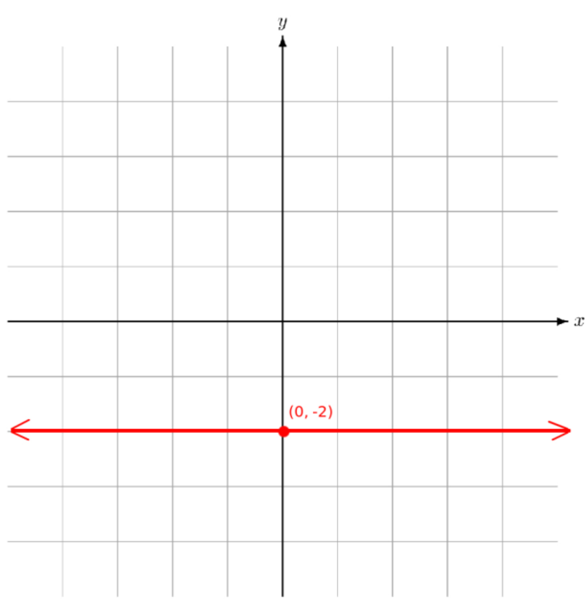
Then our line is:

$$y - 5 = \frac{2}{3}(x - 3) \qquad \text{or} \qquad y = \frac{2}{3}x + 3$$

6 pts 12. a.) Give an equation for the horizontal line that goes through the point $(4, -2)$.

$$y = -2$$

b.) Graph that line on the provided grid, and label any intercepts.



6 pts 13. A firm that manufactures laptop computers charges \$450 each for its M1 model. The marginal cost to the firm of producing one more M1 is \$150. The firm's fixed costs pertaining to prodution of the M1 are \$12,000 per month.

Let x be the number of M1's produced and sold in a month's time.

Assume that the firm's monthly cost of production of any of its models is a LINEAR function of the quantity produced of that model.

a.) What is $C(x)$, the firm's total monthly cost function in dollars for production of the M1?

Marginal cost = \$150

Fixed cost = \$12,000

$C(x) = 150x + 12000$

b.) What is $P(x)$, the firm's monthly profit function in dollars for production and sales of the M1?

$$P(x) = R(x) - C(X)$$

$$R(x) = px$$

$R(x) = 450x$

$P(x) = 450x - (150x + 12000) = 300x - 12000$

c.) What is the monthly profit to the firm in dollars from the production and sale of 70 M1's?

$Want\ P(70)$

$P(70) = 300(70) - 12000 = 21000 - 12000 = \$9000$

6 pts 14. The relationship between temperature measurement from Fahrenheit to Celsius is a LINEAR relationship. It is known that water freezes at 0° Celsius (32° Fahrenheit), and that water boils at 100° Celsius (212° Fahrenheit).

Let the variable C stand for temperature in degrees Celsius, and F stand for temperature in degrees Fahrenheit.

Temperature	**C**	**F**
Water freezes	0°	32°
Water boils	100°	212°

a.) Give an equation in C and F that models the relationship between degrees Fahrenheit and degrees Celsius.

(Hint: Your answer should be of the form $F = mC + b$.)

$$\textit{Slope} = \frac{\Delta F}{\Delta C} = \frac{212-31}{100-0} = \frac{180}{100} = \frac{9}{5}$$

So,

$$F - 32 = \frac{9}{5}(C-0)$$

i.e.

$$F = \frac{9}{5}C + 32$$

b.) The temperature must be much lower to freeze alcohol than to freeze water. Ethyl alcohol, for example, freezes at −114° Celsius. What temperature is this in degrees Fahrenheit?

At $C = -114°$,

$$F = \frac{9}{5}(-114) + 32 = -173.2°$$

6 pts 15. The following table gives information about ten students. It tells each student's study time in hours in preparation for the math portion of the SAT, and it gives each student's corresponding score on the test.

Hours Spent Studying	**Math SAT Score**
9	580
14	730
8	590
4	410
7	530
1	350
3	400
8	560
5	450
6	520

a.) Let the variable x stand for students' study time in hours, and let y be students' corresponding scores on the math SAT.

Using a linear regression application/calculator, give an equation in x and y for the line of best fit that relates study times in hours to scores.

Entering the above data into any linear regression calculator yields:

$$y = 30.63x + 312.89$$

b.) Using this equation, what score would you expect to achieve if you studied for 12 hours?

At $x = 12$,

$y = 30.63(12) + 312.89 = 680.45 \approx 680$

D. **Systems of Equations**

For questions 16 to 18, solve each sytem of equations, and express your answer as an ordered pair (e.g. if the solution is x=9 and y=1, write "(x,y) = (9,1)".)

If there is no solution, write "No solution."

If there is an infinite number of solutions, then express a parametrized solution (i.e. express the solution as the ordered pair (t, f(t))).

5 pts 16. $14x - 10y = 20$

$y = \frac{7}{5}x - 2$

$$\begin{array}{rrrrcr} & \frac{1}{2}\,(& 14x & - & 10y & = & 20\ ) \\ + & 5\ (& -\frac{7}{5}x & + & y & = & -2\ ) \\ \hline & & 7x & - & 5y & = & 10 \\ + & & -7x & + & 5y & = & -10 \\ \hline & & & & 0 & = & 0 \end{array}$$

There are an infinite number of solutions (the two equations represent the same line) of the form:

$(t, \frac{7}{5}t - 2)$, for all t.

5 pts 17. $9x + 3y = 4$

$y = -3x + 8$

$$\begin{array}{rrrrrcr} & & 9x & + & 3y & = & 4 \\ - & 3\ (& 3x & + & y & = & 8\ ) \\ \hline & & 9x & + & 3y & = & 4 \\ - & (& 9x & + & 3y & = & 8) \\ \hline & & & & 0 & = & -4 \end{array}$$

No solution (these lines are parallel).

5 pts 18. $2x - 3y = -2$

$4x + y = 24$

$$\begin{array}{rrrrrcr} & 2\ (& 2x & - & 3y & = & -2\ ) \\ - & (& 4x & + & y & = & 24\ ) \\ \hline & & 4x & - & 6y & = & -4 \\ - & (& 4x & + & y & = & 24) \\ \hline & & & & -7y & = & -28 \end{array}$$

$\implies 4x + 4 = 24 \implies 4x = 20 \implies x = 5$

$\implies y = 4$

$(x, y) = (5, 4)$

5 pts

19. Using the Gauss/Jordan row reduction method, reduce the following system to solve for at least ONE variable (you may choose which variable).

Before each reduction, describe the row operation for that specific reduction.

$$x - 2y + 3z = 7$$
$$x + z = 3$$
$$-2x + z = -3$$

(Hint: The solution has only integer values. If you get any value other than an integer, you've made a mistake.)

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 1 & 0 & 1 & 3 \\ -2 & 0 & 1 & -3 \end{array}\right)$$

To solve for z,

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 1 & 0 & 1 & 3 \\ -2 & 0 & 1 & -3 \end{array}\right) \quad \tfrac{1}{3}R_3 + \tfrac{2}{3}R_2 \longrightarrow R_3 : \quad \left(\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 1 & 0 & 1 & 3 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\Longrightarrow \; z = 1$$

This is all that was required to solve, but if you picked a different variable or wished to solve the entire system:

$$\tfrac{1}{2}R_2 - \tfrac{1}{2}R_1 \longrightarrow R_2 : \quad \left(\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$R_2 + R_3 \longrightarrow R_2 : \quad \left(\begin{array}{ccc|c} 1 & -2 & 3 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$R_1 + 2R_2 \longrightarrow R_1 : \quad \left(\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$R_1 - 3R_3 \longrightarrow R_1 : \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\Longrightarrow \; (x, y, z) = (2, -1, 1)$$

20. Using a matrix operations calculator/application, solve the following system for EACH given variable. Express your answer as an ordered quadruplet (e.g. if your solution is ”w=8, x=7, y=6, z=5”, write ”(w,x,y,z) = (8,7,6,5)”).

4 pts (Bonus)

$$2w - x + 5y + z = -3$$
$$3w + 2x + 2y - 6z = -32$$
$$w + 3x + 3y - z = -47$$
$$5w - 2x - 3y + 3z = 49$$

(Hint: The solution has only integer values. If you get any value other than an integer, you've made a mistake.)

Using a matrix calculator as in class, we let

$$A = \begin{pmatrix} 2 & -1 & 5 & 1 \\ 3 & 2 & 2 & -6 \\ -1 & 3 & 3 & -1 \\ 5 & -2 & -3 & 3 \end{pmatrix}$$

and

$$v = \begin{pmatrix} -3 \\ -32 \\ -47 \\ 49 \end{pmatrix}$$

Then, for

$$u = \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix},$$

such that $Au = v$, if A^{-1} exists, we want to find $u = A^{-1}v$.

Then,

$$A^{-1} = \begin{pmatrix} \frac{16}{684} & \frac{44}{684} & \frac{40}{684} & \frac{96}{684} \\ -\frac{110}{684} & -\frac{64}{684} & \frac{238}{684} & \frac{24}{684} \\ \frac{107}{684} & -\frac{5}{684} & \frac{11}{684} & -\frac{42}{684} \\ \frac{7}{684} & -\frac{109}{684} & \frac{103}{684} & \frac{42}{684} \end{pmatrix}$$

So,

$$\begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = u = A^{-1}v = \begin{pmatrix} \frac{16}{684} & \frac{44}{684} & \frac{40}{684} & \frac{96}{684} \\ -\frac{110}{684} & -\frac{64}{684} & \frac{238}{684} & \frac{24}{684} \\ \frac{107}{684} & -\frac{5}{684} & \frac{11}{684} & -\frac{42}{684} \\ \frac{7}{684} & -\frac{109}{684} & \frac{103}{684} & \frac{42}{684} \end{pmatrix} \begin{pmatrix} -3 \\ -32 \\ -47 \\ 49 \end{pmatrix} = \begin{pmatrix} 2 \\ -12 \\ -4 \\ 1 \end{pmatrix}$$

$(w, x, y, z) = (2, -12, -4, 1)$

Bonus: (1pt each)

For each of the following intervals, classify each as one of the following:

CLOSED, *OPEN*, *NEITHER*, *or BOTH.*

(Hint: For any that you aren't sure of, I wouldn't mind terribly if you took a minute to look the answer up somewhere.)

(i) $(-1, 5) \cup (15, 20)$

open

(ii) $[0, 5]$

closed

(iii) $[0, \infty)$

closed

(iv) $(-\infty, 5)$

open

(v) $(0, 5]$

neither

(vi) $(-1, 5) \cup [15, 20]$

neither

(vii) $(-\infty, 5) \cup (9, \infty)$

open

(viii) $(-\infty, \infty)$

both