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Linear Strength Vortex Panel Method for NACA 4412 Airfoil

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Abstract. The objective of this article is to formulate numerical models for two-dimensional potential flow over the NACA 4412 Airfoil using linear vortex panel methods. By satisfying the no penetration boundary condition and Kutta condition, the circulation density on each boundary points (end point of every panel) are obtained and according to which, surface pressure distribution and lift coefficients of the airfoil are predicted and validated by Xfoil, an interactive program for the design and analysis of airfoil. The sensitivity of results to the number of panels is also investigated in the end, which shows that the results are sensitive to the number of panels when panel number ranges from 10 to 160. With the increasing panel number (N>160), the results become relatively insensitive to it.

1. Introduction

The development of airfoil theory to predict lift and pressure estimates for a given airfoil has gone through several stages. The first successful airfoil theory, which based on conformal transformation, was developed by Joukowski [1]. He represented a potential flow by a complex potential and maps the complex potential flow around the circle in ζ plane to the corresponding flow around the airfoil in the z plane, which makes it possible to use the results for the cylinder with circulation to calculate the flow around an airfoil. However, it can only apply to a particular family of airfoil shape and all members of this family have a cusped trailing edge, which is inconsistent with the practical situation that has trailing edges with finite angles. Though Karman and Trefftz devised a more general conformal transformation giving the airfoils with trailing edges of finite angle, it still cannot be widely used in aerodynamic design since this technique cannot be extended to three-dimensional or high-speed flows. The second airfoil theory is the thin airfoil theory. In thin airfoil theory, the airfoil is replaced with its mean camber line. The flow pattern is built up by placing a bound vortex sheet on the camber line and adjusting its strength so that the camber line becomes a streamline of the flow. Within this framework, the theory adequately predicts lift and moment for thin airfoil. Nevertheless, its drawback is also obvious – it cannot be applied to arbitrarily thick airfoils because of the ignored thickness effects [2]. With the advent of digital computers offers the attractive alternative of a numerical rather than an analytical solution, a new method in aerodynamic design is widely used nowadays – the panel method. It relies on the distribution of singularities on discrete segments of the airfoil surface. By satisfying no penetration condition and Kutta condition, a system of linear algebraic equations to be solved for the unknown singularity-strength is created, with which, the lift coefficients and pressure distribution can be easily predicted. Panel method can be applied to airfoil section with any thickness and camber [3]. Computational Fluid Dynamics (CFD) is another option to provide accurate and reliable results but is more computationally expensive and complicated than panel methods. For preliminary stage, panel methods are accurate and less time consuming. Therefore, due to these advantages, it is necessary to

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investigate its principle for future use.

Panel methods have one key feature that distinguishes themselves from each other: the type of singularity element used to describe the flow field around the airfoil: a source, doublet or vortex element. As vortex element can model both pressure and lift, this article focuses on formulating numerical models for two-dimensional potential flow over the NACA 4412 Airfoil using linear vortex panel methods in the Matlab environment. By satisfying the no penetration boundary condition and Kutta condition, the circulation density on each boundary points (end point of every panel) are obtained and according to which, surface pressure distribution and lift coefficients of the airfoil are predicted and validated by Xfoil, an interactive program for the design and analysis of airfoil created by MIT. The sensitivity of results to the number of panels is also investigated in the end, which shows that the results are sensitive to the number of panels when panel number ranges from 10 to 160. With the increasing panel number (N>160), the results become relatively insensitive to it.

2. Methodology

For incompressible, inviscous and irrotational flow, the vector velocity can be represented as the gradient of a scalar velocity potential, $\vec{V} = \nabla\phi$, and the resulting flow is referred to as potential flow. According to continuity equation $\nabla \cdot \vec{V} = 0$, velocity potential satisfies the Laplace's equation $\nabla^2\phi = 0$. It is a liner partial differential equation and can be solved subject to no penetration boundary condition that no flow can cross the surface of the object [4].

2.1. To generate grid and establish NACA 4412 Airfoil geometry

a) Grid generation

Cosine Spacing can be used as the discretization schemes of airfoil. Its expression

$$\frac{x}{c} = \frac{1}{2}(\cos\zeta + 1) \quad , \zeta = 0 - 2\pi \tag{1}$$

This scheme provides a fine discretization at the leading and trailing edge of the airfoil.

b) Establishment of airfoil geometry

The NACA 4-digit series is defined by four digits. First digit describing maximum camber as percentage of the chord. Second digit describing the distance of maximum camber from the airfoil leading edge in tens of percents of the chord. Last two digits describing maximum thickness of the airfoil as percent of the chord. E.g. NACA 4412, m=4%, p=40%, t=12%. The formula for the shape of a NACA 4-digit airfoil can be given by:

$$y_t = \frac{t}{0.2}(0.2969\sqrt{x} - 0.126x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4) \tag{2}$$

$$y_c = \frac{m}{p^2}(2px - x^2) \text{ for x<p} \quad \text{or} \quad y_c = \frac{m}{(1-p^2)}((1-2p) + 2px - x^2) \text{ for x>p} \tag{3}$$

For this cambered airfoil, the coordinates (x_U, y_U) and (x_L, y_L), of respectively the upper and lower airfoil surface, become:

$$x_U = x - y_t \sin\theta \quad , \quad y_U = y_c + y_t \cos\theta \tag{4}$$

$$x_L = x + y_t \sin\theta \quad , \quad y_L = y_c - y_t \cos\theta \tag{5}$$

where

$$\theta = \arctan(\frac{dy_c}{dx}) \tag{6}$$

2.2. To calculate influence coefficients and solve the linear algebraic equations

The linear strength vortex panel method introduced here has the feature that the circulation density on each panel varies linearly from one corner to the other and is continuous across the corner [5]. The

sequence of boundary points (end point of each panel) named in the clockwise direction, starting from the trailing edge. The condition that the airfoil be a streamline is met approximately by applying the condition of zero normal velocity components at control point, specified as the mid-point of the panels.

The velocity potential at the ith control point (x_i , y_i) can be described as:

$$\phi(x_i, y_i) = V_\infty(x_i \cos\alpha + y_i \sin\alpha) - \sum_{j=1}^{m} \int_j \frac{\gamma(s_j)}{2\pi} \tan^{-1}(\frac{y_i - y_j}{x_i - x_j}) ds_j \tag{7}$$

where

$$\gamma(s_j) = \gamma_j + (\gamma_{j+1} - \gamma_j)\frac{s_j}{S_j} \tag{8}$$

in which

V_∞ – velocity of uniform flow, in this article, V_∞ = 1 m/s;

α – angle of attack;

m – vortex panel numbers

(x_j, y_j) – arbitrary point on the jth panel;

S_j – length of jth panel;

s_j – distance measured from the leading edge of the panel;

The no penetration boundary condition requires that the normal velocity of the ith panel at the ith control point be zero, so that

$$\frac{\partial}{\partial n_i}\phi(x_i, y_i) = 0 \quad \text{i=1,2,…,m} \tag{9}$$

Carrying out the involved differentiation and integration, we obtain

$$\sum_{j=1}^{m}(C_{n1_{ij}}\gamma_j^{'} + C_{n2_{ij}}\gamma_{j+1}^{'}) = \sin(\theta_i - \alpha) \quad \text{i= 1, 2, … , m} \tag{10}$$

in which $\gamma^{'} = \gamma / 2\pi V_\infty$ is a dimensionless circulation density, θ_i is the orientation angle between the ith panel and x axis. The coefficients above can be expressed by

$$C_{n1_{ij}} = 0.5DF + CG - C_{n2_{ij}} \tag{11}$$

$$C_{n2_{ij}} = D + 0.5QF / S_j - (AC + DE)G / S_j \tag{12}$$

in which

$$A = -(x_i - X_j)\cos\theta_i - (y_i - Y_j)\sin\theta_j \tag{13}$$

$$B = (x_i - X_j)^2 + (y_j - Y_j)^2 \tag{14}$$

$$C = \sin(\theta_i - \theta_j) \tag{15}$$

$$D = \cos(\theta_i - \theta_j) \tag{16}$$

$$E = (x_i - X_j)\sin\theta_j - (y_i - Y_j)\cos\theta_j \tag{17}$$

$$F = \ln(1 + \frac{S_j^2 + 2AS_j}{B}) \tag{18}$$

$$G = \tan^{-1}(\frac{ES_j}{B + AS_j}) \tag{19}$$

$$P = (x_i - X_j)\sin(\theta_i - 2\theta_j) + (y_i - Y_j)\cos(\theta_i - 2\theta_j) \tag{20}$$

$$Q = (x_i - X_j)\cos(\theta_i - 2\theta_j) + (y_i - Y_j)\sin(\theta_i - 2\theta_j) \tag{21}$$

To ensure a smooth flow at the trailing edge, the Kutta condition is applied that,

$$\gamma_1^{'} + \gamma_{m+1}^{'} = 0 \tag{22}$$

Now, there are (m+1) equations and they are sufficient to solve for the (m+1) unknown $\gamma_j^{'}$ values. We may rewrite the linear equation:

$$\sum_{j=1}^{m+1} A_{n_{ij}} \gamma_j^{'} = (RHS)_i \quad \text{i= 1, 2, …, m+1} \tag{23}$$

in which, for i<m+1:

$$A_{n_{i1}} = C_{n1_{i1}} \tag{24}$$

$$A_{n_{ij}} = C_{n1_{ij}} + C_{n2_{ij-1}} \quad \text{j= 2, 3, …, m} \tag{25}$$

$$A_{ni_{m+1}} = C_{n2_{im}} \tag{26}$$

$$(RHS)_i = \sin(\theta_i - \alpha) \tag{27}$$

for i=m+1:

$$A_{n_{i1}} = A_{n_{im+1}} = 1 \tag{28}$$

$$A_{n_{ij}} = 0 \quad \text{j= 2, 3, …, m} \tag{29}$$

$$(RHS)_i = 0 \tag{30}$$

This system of linear equation is easy to solve by Matlab.

2.3. To predict lift coefficients and pressure distribution of the airfoil

With the known circulation densities, the velocity at each control point can be computed. Since there is only tangential velocity at the control point, the dimensionless velocity defined as $(\partial\phi / \partial t_i) / V_\infty$ can be obtained, which can be given by

$$V_i = \cos(\theta_i - \alpha) + \sum_{j=1}^{m} (C_{t1_{ij}} \gamma_j^{'} + C_{t2_{ij}} \gamma_{j+1}^{'}) \quad \text{i= 1, 2, …, m} \tag{31}$$

in which,

$$A_{t_{i1}} = C_{t1_{i1}} \tag{32}$$

$$A_{t_{ij}} = C_{t1_{ij}} + C_{t2_{ij-1}} \quad \text{j= 2, 3, …, m} \tag{33}$$

$$A_{ti_{m+1}} = C_{t2_{im}} \tag{34}$$

After the determination of velocity at control point, the pressure distribution can be computed as

$$C_{pi} = 1 - V_i^2 \tag{35}$$

The lift coefficients can be computed from the pressure distribution by integral below

$$C_l = -\oint C_p \vec{n} \cdot \vec{l} ds \tag{36}$$

3. Results

The results of linear strength vortex panel method for NACA 4412 Airfoil is validated by Xfoil. Figure 1 below shows a C_l vs. angle of attack for vortex panel method (Panel Number N= 160), Xfoil and experimental data. The results from vortex panel method agree reasonably well with those from Xfoil, indicating that the vortex panel code is working well. However, the inviscid solution from panel method becomes increasingly less accurate at higher angles of attack (when $\alpha > 15^\circ$) due to viscous effects. For experimental data, when lift coefficients reach its maximum, $(C_l)_{\max}$, the corresponding angle of attack, α, called the stall angle. After this angle, the lift coefficients from panel method still increase,

while the experimental lift coefficients begin to reduce and finally the airfoil stalls.

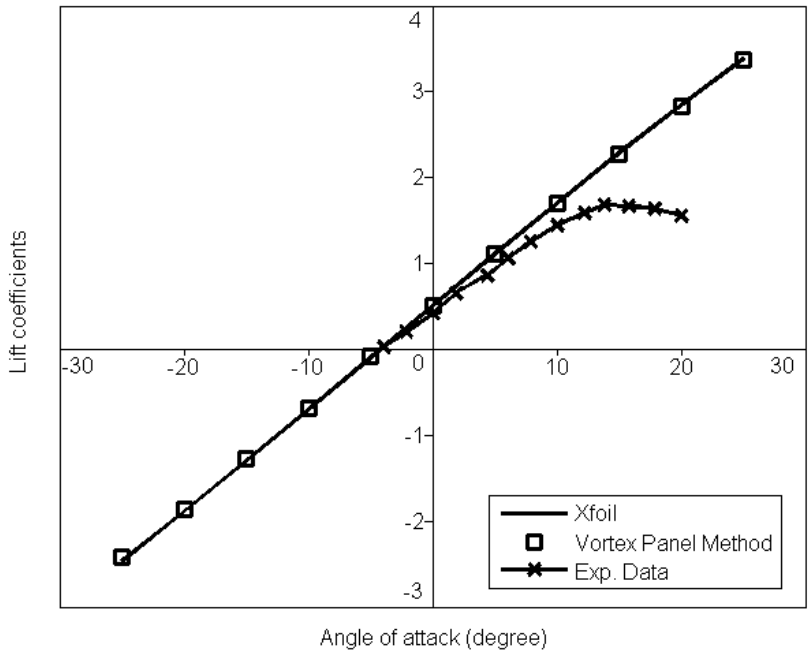

Figure 1. C_l vs. α for NACA 4412 Airfoil

Figure 2 shows the pressure distribution of NACA 4412 Airfoil from linear vortex panel code (Panel Number N= 160). The results are also validated against Xfoil and demonstrate expected shape, which confirms the validity of the code in further step.

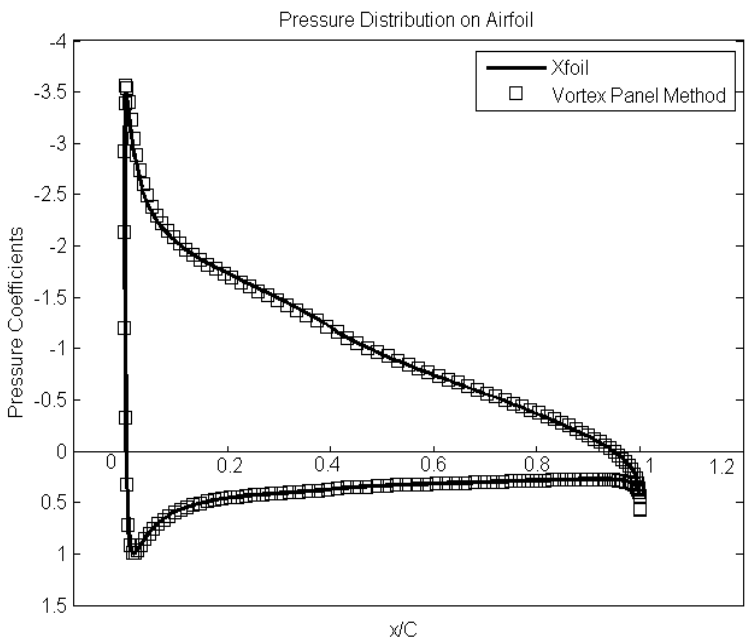

Figure 2. Pressure Distribution for NACA 4412 Airfoil When $\alpha = 8^{\circ}$

The sensitivity of results to the number of panels is showed below in Figure 3. It is obvious that when panel number N ranges from 10 to 160, C_l is sensitive to N. However, with the increasing panel number (N>160), the results become relatively insensitive to it.

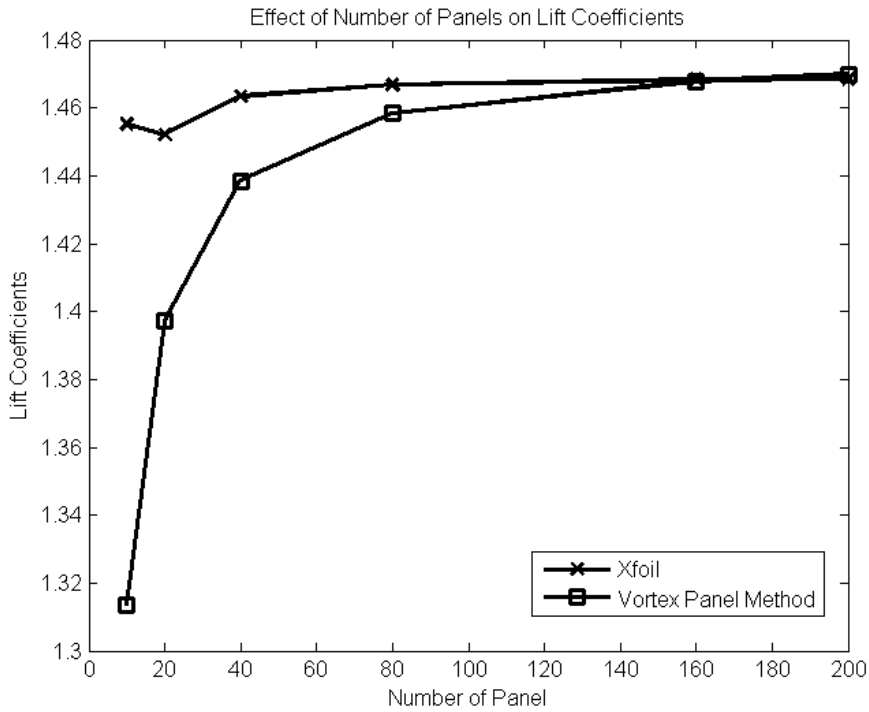

Figure 3. Sensitivity of Results to the Number of Panels When $\alpha = 8^\circ$

4. Conclusions

A numerical model for linear strength vortex panel method is developed. By analyzing NACA 4412 Airfoil, the lift coefficients, pressure distribution and sensitivity of results to panel number are studied. The results are validated against Xfoil program, which shows the validity of the vortex panel code.

The analysis results illustrate that panel method cannot account for viscous effects on lift when angle of attack increasing (typical when $\alpha > 15^\circ$), which can result in stall. Besides, sensitivity of results to the number of panels N is obvious when N ranging from 10 to 160. When N>160, the results become relatively stable. Thus, when panel method are used to compute complex models, such as large three dimensional model, the number of panels can be chosen appropriately to balance the precision of results and computing time.

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