EQUILIBRIUM CALCULATIONS: INTERMEDIATE VALUES OF K

INITIAL [ ]
REACTION AND K $\longrightarrow$ "FINAL" [ ]'s
AT LONG TIME

$\uparrow$
AT EQUILIBRIUM, Q WILL EQUAL K

INITIAL [ ]'s : $[Fe^{+3}] = 4.0\,M$ $[L] = 3.0\,M$ $[Fe^{+3}L_2] = 2.0\,M$

REACTION : $Fe^{+3} + 2L \rightarrow FeL_2^{+3}$

K : $K = 19$

AT EQUILIBRIUM $Q = K$ $Q = \frac{[FeL_2^{+3}]}{[Fe^{+3}][L]^2}$

LET'S TRY A FEW CHANGES

		$1\,Fe^{+3}$	$+\,2L$	$\rightarrow Fe^{+3}L_2$
INITIAL	I	4.0	3.0	2.0
CHANGE	C	−1.0	−2.0	+1.0
RESULT	R	3.0	1.0	3.0

$Q = \frac{(2.0)}{(4.0)(3.0)^2} = 0.055$

$Q = \frac{(3.0)}{(3.0)(1.0)^2} = 1.0$

$Q = K = 19$ WILL LIE BETWEEN THESE TWO TRIAL CHANGES

		$1\,Fe^{+3}$	$+\,2L$	$\rightarrow FeL_2^{+3}$
INITIAL	I	4.0	3.0	2.0
CHANGE	C	−1.4	−2.8	+1.4
RESULT	R	2.6	0.2	3.4

$Q = \frac{(3.4)}{(2.6)(0.2)^2} = 33$

INTRODUCE A VARIABLE x TO DESCRIBE CHANGE IN [ ]'s

		Fe^{+3}	$+\,2L$	$\rightarrow FeL_2^{+3}$
INITIAL	I	4.0	3.0	2.0
CHANGE	C	−x	−2x	+x
RESULT	R	4.0−x	3.0−2x	2.0+x

$Q = \underbrace{\frac{(2.0+x)}{(4.0-x)(3.0-2x)^2} = K}_{\text{DETERMINE x FOR WHICH } Q = K}$

SOLVE Q = K FOR x

$$\frac{(2.0+x)}{(4.0-x)(3.0-2x)^2} = 19$$

$$(4-x)\,(9-12x+4x^2)$$

$$(36-57x+28x^2-4x^3)$$

$$\frac{(2+x)}{(36-57x+28x^2-4x^3)} = 19$$

CROSS MULTIPLY

$$(2+x) = 19\,(36-57x+28x^2-4x^3)$$

$$76x^3 - 532x^2 + 1084x - 682 = 0$$

HAS 3 SOLUTIONS (ROOTS) $\Rightarrow$
1. x = 3.987
2. x = 1.643
3. x = 1.370

ONLY ONE WILL GIVE REASONABLE RESULTS (ALL [ ] > 0)

TRY EACH OF THE 3 ROOTS

① x = 3.987

		$1\,Fe^{+3}$	$+\,2L$	$\rightarrow Fe^{+3}L_2$
INITIAL	I	4.0	3.0	2.0
CHANGE	C	−3.987	−2(3.987)	+3.987
RESULT	R	0.013	−4.975	+5.987

UNPHYSICAL
CAN'T HAVE A NEGATIVE [ ]

② x = 1.643

		$1\,Fe^{+3}$	$+\,2L$	$\rightarrow Fe^{+3}L_2$
INITIAL	I	4.0	3.0	2.0
CHANGE	C	−1.643	−2(1.643)	+1.643
RESULT	R	2.357	−0.285	3.643

UNPHYSICAL
CAN'T HAVE A NEGATIVE [ ]

③ x = 1.370

ICE TABLE

		$1\,Fe^{+3}$	$+\,2L$	$\rightarrow Fe^{+3}L_2$
INITIAL	I	4.0	3.0	2.0
CHANGE	C	−1.370	−2(1.370)	+1.370
EQUILIBRIUM	E	2.630	0.260	3.370

ONLY x = 1.370 GIVES ALL [ ] > 0, SO Q = K LEADS TO EQUILIBRIUM [ ]'s

$[Fe^{+3}] = 2.6\,M$ $[L] = 0.26\,M$ $[Fe^{+3}L_2] = 3.4\,M$

CONFIRM THAT Q = K $Q = \frac{[Fe^{+3}L_2]}{[Fe^{+3}][L]^2} = \frac{(3.4)}{(2.6)(0.26)^2} = 19$ ✓ Q = K