Homework 2: Random Numbers, Monte Carlo, and Multidimensional Integration

X.Y.Z
Department of Physics, Astronomy, and Applied Physics,
Rensselaer Polytechnic Institute
name@rpi.edu

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Question 1

1. Use your computer's default pseudo-RNG to generate a sequence $\{x_i\}_{i=1,N}$ of random numbers in the range between 0.0 and 1.0 (choose an appropriate value for N, such as 10,000 or more).

   (a) Plot x_i as a function of sequence number i. Comment on the plot. Does it *look* random and uniform?

   (b) Plot x_i as a function of x_{i+k} for a small k of your choice. What can you learn from this plot?

   (c) Compute the k^{th} moment of your distribution. Compare to the theoretical value and comment.

   (d) Calculate the auto-correlation between x_i and x_{i+k} for a few values of k. Comment and discuss.

   (e) Prepare a plot with *bins* to assess the uniformity of the sequence. Use a reasonable number of bins (10, for instance). What does this plot tell you about the sequence?

(f) Using the information in (a)-(e), comment on the quality of your sequence of random numbers.

(g) Repeat with a sequence of numbers obtained from `www.random.org`.

Question 2

Choose *either* of the two following problems:

1. **Use random numbers to calculate the integral in five dimensions**

2. **Random walk in 3D**

Question 2a: Use random numbers to calculate:

$$\int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^1 dx_4 \int_0^1 dx_5 (x_1 + x_2 + x_3 + x_4 + x_5)^3$$

(a) Compare your result with the analytical result.
(b) Repeat the calculation for a varying number of random numbers.
(c) Study how the error behaves as you change the number of points. Plot this error and find if you can detect a $\sim 1/\sqrt{N}$ behavior of the error, where N is the number of points.
(d) Compare the order of magnitude of your error with (numerically obtained):

$$\sigma^2 = < f^2 > - < f >^2 \tag{1}$$

Question 2b: Random walk in 3D

(a) Repeat the 2D work presented in class for an equivalent 3D random walk.

(b) At each step, consider a constant step size (i.e. of length 1, thereby defining the unit of distance). This means you could use spherical coordinates with constant $r = 1$, while randomly picking values for azimuthal angle $\theta \in [0, 2\pi]$ and zenith angle $\phi \in [0, \pi]$. Feel free to use Cartesian coordinates if you prefer; but in that case make sure you normalize the displacement vector.

(c) Vary the total number N of steps and plot $\sqrt{< R^2 >}$ as a function of $\sqrt{N}$. $< R^2 >$ could be computed by averaging over, say, $n_{average} = 16$ different random walks (choose a different value for $n_{average}$ if you want). Comment on the general behavior of the $\sqrt{< R^2 >}$ versus $\sqrt{N}$ plot.

(d) Plot a few trajectories and discuss their shape.

Comments

- Due date: Wednesday 02/27/2013 at midnight
- Make sure your report is self-contained with sufficient details and clear plots.
- Feel free to add listings of your code (or use pseudo-code).
- Collaborative work is allowed but each student will turn in an individual report.

Technicalities: random numbers in C++

Function:

```
double randDouble(double a, double b) {
   double cd;
   d=b-a;
   c=((double)rand()/(static_cast<double>(RAND_MAX)+1.0))*d+a;
   return c;}
```

Seed:

```
srand( (unsigned)time(0) );
```

Call:

```
a=randDouble(-1,1);
```