Homework 2: Random Numbers, Monte Carlo, and Multidimensional Integration

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Assignment 2

Random Numbers

2.1 Assignment

1. Use your computer's default pseudo-RNG to generate a sequence $\{x_i\}_{i=1,N}$ of random numbers in the range between 0.0 and 1.0 (choose an appropriate value for N, such as 10,000 or more).

   (a) Plot x_i as a function of sequence number i. Comment on the plot. Does it *look* random and uniform?

   (b) Plot x_i as a function of x_{i+k} for a small k of your choice. What can you learn from this plot?

   (c) Compute the k^{th} moment of your distribution. Compare to the theoretical value and comment.

   (d) Calculate the auto-correlation between x_i and x_{i+k} for a few values of k. Comment and discuss.

   (e) Prepare a plot with *bins* to assess the uniformity of the sequence. Use a reasonable number of bins (10, for instance). What does this plot tell you about the sequence?

   (f) Using the information in (a)-(e), comment on the quality of your sequence of random numbers.

   (g) Repeat with a sequence of numbers obtained from random.org.

2. Choose one of the two following problems:

   - **Use random numbers to calculate the integral in five dimensions:**

   $$\int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^1 dx_4 \int_0^1 dx_5 (x_1+x_2+x_3+x_4+x_5)^3 \quad (2.1)$$

   (a) Compare your result with the analytical result.

(b) Repeat the calculation for a varying number of random numbers.

(c) Study how the error behaves as you change the number of points. Plot this error and find if you can detect a $\sim 1/\sqrt{N}$ behavior of the error, where N is the number of points.

(d) Compare the order of magnitude of your error with (numerically obtained):

$$\sigma^2 =< f^2 > - < f >^2 \tag{2.2}$$

where

$$f = (x_1 + x_2 + x_3 + x_4 + x_5)^2 \tag{2.3}$$

- **Random walk in 3D**

(a) Repeat the 2D work presented in class for an equivalent 3D random walk.

(b) At each step, consider a constant step size (i.e. of length 1.0, thereby defining the unit of distance). This means you could use spherical coordinates with constant $r = 1$, while randomly picking values for azimuthal angle $\theta \in [0, 2\pi]$ and zenith angle $\phi \in [0, \pi]$. Feel free to use Cartesian coordinates if you prefer; but in that case make sure you normalize the displacement vector.

(c) Vary the total number N of steps and plot $\sqrt{< R^2 >}$ as a function of $\sqrt{N}$. $< R^2 >$ could be computed by averaging over, say, $n_{average} = 16$ different random walks (choose a different value for $n_{average}$ if you want). Comment on the general behavior of the $\sqrt{< R^2 >}$ versus $\sqrt{N}$ plot.

(d) Plot a few trajectories and discuss their shape.

Make sure your report is self-contained with sufficient details and clear plots. Feel free to add listings of your code (or use pseudo-code). Collaborative work is allowed but each student will turn in an individual report.

Below is a suggestion for presentation. You can adapt it to your personal vision of the problem.

2.2 Algorithmic Considerations

2.3 Implementation

2.4 Results

2.5 Discussion

2.6 References (if any)