PHY-4810
COMPUTATIONAL PHYSICS

LECTURE 3: TRIALS AND ERRORS

ON THE MENU TODAY

- ☐ **Trials and Errors**
  - ☐ **Bisection Algorithm**
  - ☐ **Newton-Raphson Algorithm**
  - ☐ **NRA with Backtracking**
- ☐ **Short Introduction to LaTeX**

PART I: TRIALS AND ERRORS

ZERO FINDING

10
7.5
5
y
2.5
0
2.5
5
7.5
x
10
12.5
-2.5
-5

EXAMPLE OF ZERO FINDING

- A **transcendental function** is...
  ...a <u>function</u> that does not satisfy a <u>polynomial</u> equation whose <u>coefficients</u> are themselves polynomials, in contrast to an <u>algebraic function</u>, which does satisfy such an equation

- Example:

$$f(x) = x \tan x - (x - 1)$$

HOW CAN WE FIND THE ZEROS?

THE "BISECTION" ALGORITHM

Webster Dictionary:
ORIGIN: from **bi-** ***'two'*** + Latin ***sect-*** (from ***secare 'to cut'***).

BISECTION ALGORITHM

- **Zeros of f?** $$f(x) = 0$$
- **Starting from an interval $[x_{1-},x_{1+}]$ ($f(x_{1-})<0$ and $f(x_{1+})>0$)**
- **Trial: midpoint of current interval $x_2=(x_{1-}+x_{1+})/2$**
  - **if $|f(x_2)|<$ *epsilon* => STOP, the root is x_2**
  - **else if $f(x_2)>0$ => replace x_{1+} by x_2 and work on interval $[x_{1-},x_2]=[x_{2-},x_{2+}]$**
  - **else if $f(x_2)<0$ => replace x_{1-} by x_2 and work on interval $[x_2,x_{1+}]=[x_{2-},x_{2+}]$**
- **Repeat unless stop**

SIZE OF INTERVAL

A GRAPHICAL REPRESENTATION OF THE STEPS INVOLVED IN SOLVING FOR A ZERO OF F (X) USING THE BISECTION ALGORITHM.

THE BISECTION ALGORITHM TAKES THE MIDPOINT OF THE INTERVAL AS THE NEW GUESS FOR X, AND SO EACH STEP REDUCES THE INTERVAL SIZE BY ONE HALF.

10
7.5
5
2.5
0
-2.5
-5
y
2.5
5
7.5
10
12.5
x

$$f(x) = x\tan x - (x-1)$$

y
3.7735
3.774
3.7745
3.775
3.7755
3.776
3.7765
3.777
3.7775
x

	x_-	f_-	x_+	f_+
1	2.5	-3.37	4	1.631
2	3.25	-1.9	4	1.631
3	3.625	-0.72	4	1.631
4	3.625	-0.72	3.813	0.214
5	3.719	-0.3	3.813	0.214
6	3.766	-0.05	3.813	0.214
7	3.766	-0.05	3.789	0.076
8	3.766	-0.05	3.777	0.01
9	3.771	-0.02	3.777	0.01
10	3.774	-0.01	3.777	0.01
11	3.774	-0.01	3.776	0.002
12	3.775	-0	3.776	0.002

SOLUTION: 3.77551

NEWTON-RAPHSON

TANGENT TO THE CURVE
y
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
x
$f(x) = \sin(x^2 + x)$

NEWTON-RAPHSON: ALGORITHM

- **Start from x_0 "close" to the actual zero**
- **Draw a tangent of equation *mx+b*; to *f(x)* passing through $(x_0, f(x_0))$**
  - $m=df/dx;\ b=f(x_0)-mx_0$
- **Use, *x=-b/m* (intercept on *x*-axis) as your new guess**
- **Repeat until abs(f(x0))<epsilon**

NR: PROOF

TAYLOR: $$f(x_0 + \delta x) \approx f(x_0) + \frac{df}{dx}\delta_x$$

CHANGE IN X: $$\delta x = -f(x_0)/\frac{df}{dx}$$

NEW X: $$x_{\text{new}} = x_0 + \delta x$$

$$x_{\text{new}} = x_0 + -f(x_0)/\frac{df}{dx}$$

	x_0	f_-		
1	2.5	-3.368		
2	4.0676	2.34		
3	3.8656	0.55		
4	3.7841	0.048		
5	3.7756	0.0004		

```
cout << "Enter initial guess : ";
cin >> x0;
f0=function(x0);
cout << x0 << "\t" << f0 <<endl;

do {
    df0=Dfunction(x0);
    x0=-(f0-df0*x0)/df0;
    f0=function(x0);
    cout << x0 << "\t" << f0 <<endl;
} while (fabs(f0)>delta);
```

```
#include <iostream>
#include <math.h>
using namespace std;
float h=0.01;
float function(float x){
   return x*tan(x)-(x-1);
}
float Dfunction(float x){
   return ((function(x+h/2)-function(x-h/
2))/h);
}
int main (int argc, char * const argv[]) {
   float x_m_new, x_p_new;
   float x0, f0, df0;
   float x_temp;
   float delta=0.001;
   float f_m, f_p, f_temp;
   int i, n=1000;
//

//newton raphson
   cout << "Enter initial guess : ";
   cin >> x0;
   f0=function(x0);
   cout << x0 << "\t" << f0 <<endl;

   do {
       df0=Dfunction(x0);
       x0=-(f0-df0*x0)/df0;
       f0=function(x0);
       cout << x0 << "\t" << f0 <<endl;
   } while (fabs(f0)>delta);

    cout << "Solution NR=" << x_temp;
```

```
   //bisection
   cout << "Enter initial guess (-): ";
   cin >> x_m_new;
   cout << "Enter initial guess (+): ";
   cin >> x_p_new;
   f_m=function(x_m_new);
   f_p=function(x_p_new);
   if(f_m*f_p>0.0) {
       cout << "F_m=" << f_m<<endl;
       cout << "F_p=" << f_p<< endl;
       cout << "Wrong starting bracket,
try again\n";
       return 1;
   }
   cout << x_m_new << " " << f_m << endl;
   cout << x_p_new << " " << f_p << endl;

   do {
       x_temp=(x_m_new+x_p_new)/2;
       f_temp=function(x_temp);
       cout << x_m_new << "\t" << f_m <<
"\t" << x_p_new << "\t" << f_p<<endl;
       if(f_temp*f_m >0) {
           x_m_new=x_temp;
       }
       else
       {
           x_p_new=x_temp;
       }
       f_m=function(x_m_new);
       f_p=function(x_p_new);
   } while (fabs(f_temp)>delta);
```

POSSIBLE PROBLEMS

PROBLEM 1: **GUESS ENDS AT A LOCAL OPTIMUM/ MAXIMUM, THE GUESS IS SENT TO +/- INFINITY**

PROBLEM 2: **INFINITE LOOP**

NEWTON-RAPHSON WITH BACKTRACKING

- **Solution 1**
  - **If the new guess leads to a larger value of the function, backtrack and try a smaller step (like half)**
  - **If the problem continues, try with a yet smaller step**
  - **Eventually there should be a good increment, since you follow the function along the tangent.**
- **Solution2**
  - **Use bisection first to “corner” the right solution first.**

LECTURE 3: SUMMARY

- **Simple Bisection algorithm: easy to code but quite slow**
- **Newton-Raphson: smart use of extrapolation using numerical derivative leads to great simplification and numerical gains**
- **<u>Exercise:</u> write a simple bisection algorithm**

PART II: INTRODUCTION TO LATEX

LATEX: OVERVIEW

- LATEX is quite different from the WYSIWYG approach that most modern word processors, such as MS Word, take.
  - With these applications, authors specify the document layout interactively while typing text into the computer.
  - They can see on the screen how the final work will look when it is printed.
- When using LATEX it is not normally possible to see the final output while typing the text, but the final output can be previewed on the screen after processing the file with LATEX. Then corrections can be made before actually sending the document to the printer.

ADVANTAGES

- Professionally crafted layouts are available, which make a document really look as if “printed.”
- The typesetting of mathematical formulae is supported in a convenient way.
- Users only need to learn a few easy-to-understand commands that specify the logical structure of a document.
- They almost never need to tinker with the actual layout of the document.
- Even complex structures such as footnotes, references, table of contents, and bibliographies can be generated easily.
- Free add-on packages exist for many typographical tasks not directly supported by basic LATEX. For example, packages are available to include PostScript graphics or to typeset bibliographies conforming to exact standards.
- TEX, the formatting engine of LATEX2ε, is highly portable and free. Therefore the system runs on almost any hardware platform available.

DISADVANTAGES

- LATEX does not work well for people who have sold their souls . . .
- Although some parameters can be adjusted within a predefined document layout, the design of a whole new layout is difficult and takes a lot of time.
- It is very hard to write unstructured and disorganized documents.
- Your hamster might, despite some encouraging first steps, never be able to fully grasp the concept of Logical Markup.

LATEX: SPACES

- The input for LATEX is a plain text file.
- "Whitespace" characters, such as blank or tab, are treated uniformly as "space" by LATEX.
  - Several consecutive whitespace characters are treated as one "space."
- An empty line between two lines of text defines the end of a paragraph.
  - Several empty lines are treated the same as one empty line.

SPECIAL CHARACTERS: #$%^&_{}~\

```
\# \$ \% \^{} \& \_ \{ \} \~{}
\textbackslash
```

$ % ^ & _ { } ~ \

LATEX COMMAND

$$\backslash command[optinal\ parameter]\{parameter\}$$

- **LATEX commands are case sensitive, and take one of the following two formats:**
  - **They start with a backslash \ and then have a name consisting of letters only. Command names are terminated by a space, a number or any other 'non-letter.'**
  - **They consist of a backslash and exactly one non-letter.**
  - **Many commands exist in a 'starred variant' where a star is appended to the command name.**

INPUT FILE STRUCTURE

```
\documentclass{...}
\usepackage{...}
\begin{document}

...

\end{document}
```

EXAMPLE

```
\documentclass[a4paper,11pt]{article}
% define the title
\author{H.~Partl}
\title{Minimalism}
\begin{document}
% generates the title
\maketitle
% insert the table of contents
\tableofcontents
\section{Some Interesting Words}
Well, and here begins my lovely article.
\section{Good Bye World}
\ldots{} and here it ends.
\end{document}
```

Minimalism

H. Partl

January 30, 2012

Contents

1 Some Interesting Words

Well, and here begins my lovely article.

2 Good Bye World

... and here it ends.

LAYOUT: DOCUMENT CLASS

`\documentclass[`*options*`]{`*class*`}`

```
\documentclass[11pt,twoside,a4paper]{article}
```

Table 1.1: Document Classes.

`article`	for articles in scientific journals, presentations, short reports, program documentation, invitations, ...
`proc`	a class for proceedings based on the article class.
`minimal`	is as small as it can get. It only sets a page size and a base font. It is mainly used for debugging purposes.
`report`	for longer reports containing several chapters, small books, PhD theses, ...
`book`	for real books
`slides`	for slides. The class uses big sans serif letters. You might want to consider using the Beamer class instead.

Table 1.2: Document Class Options.

`10pt`, `11pt`, `12pt` Sets the size of the main font in the document. If no option is specified, `10pt` is assumed.

`a4paper`, `letterpaper`, ... Defines the paper size. The default size is `letterpaper`. Besides that, `a5paper`, `b5paper`, `executivepaper`, and `legalpaper` can be specified.

`fleqn` Typesets displayed formulae left-aligned instead of centred.

`leqno` Places the numbering of formulae on the left hand side instead of the right.

`titlepage`, `notitlepage` Specifies whether a new page should be started after the document title or not. The `article` class does not start a new page by default, while `report` and `book` do.

`onecolumn`, `twocolumn` Instructs LaTeX to typeset the document in one column or two columns.

`twoside, oneside` Specifies whether double or single sided output should be generated. The classes `article` and `report` are single sided and the `book` class is double sided by default. Note that this option concerns the style of the document only. The option `twoside` does *not* tell the printer you use that it should actually make a two-sided printout.

`landscape` Changes the layout of the document to print in landscape mode.

`openright, openany` Makes chapters begin either only on right hand pages or on the next page available. This does not work with the `article` class, as it does not know about chapters. The `report` class by default starts chapters on the next page available and the `book` class starts them on right hand pages.

MACROS: USEPACKAGE

`\usepackage[`*options*`]{`*package*`}`

EXAMPLE OF TEXT

```
% Example 1
\ldots when Einstein introduced his formula
\begin{equation}
  e = m \cdot c^2 \; ,
\end{equation}
which is at the same time the most widely known
and the least well understood physical formula.


% Example 2
\ldots from which follows Kirchhoff's current law:
\begin{equation}
  \sum_{k=1}^{n} I_k = 0 \; .
\end{equation}

Kirchhoff's voltage law can be derived \ldots


% Example 3
\ldots which has several advantages.

\begin{equation}
  I_D = I_F - I_R
\end{equation}
is the core of a very different transistor model. \ldots
```

...when Einstein introduced his formula

$$e = m \cdot c^2 \; , \tag{1}$$

which is at the same time the most widely known and the least well understood physical formula. ...from which follows Kirchhoffs current law:

$$\sum_{k=1}^{n} I_k = 0 \; . \tag{2}$$

Kirchhoffs voltage law can be derived .....which has several advantages.

$$I_D = I_F - I_R \tag{3}$$

is the core of a very different transistor model. ...

BREAKS...

`\\` or `\newline`

`\newpage`

READ-MADE STRINGS

Command	Example	Description
`\today`	April 6, 2011	Current date
`\TeX`	TEX	Your favorite typesetter
`\LaTeX`	LATEX	The Name of the Game
`\LaTeXe`	LATEX 2_ε	The current incarnation

ACCENTS

ò	\‘o	ó	\’o	ô	\^o	õ	\~o
ō	\=o	ȯ	\.o	ö	\"o	ç	\c c
ŏ	\u o	ǒ	\v o	ő	\H o	ǫ	\c o
ọ	\d o	o̱	\b o	o͡o	\t oo		
œ	\oe	Œ	\OE	æ	\ae	Æ	\AE
å	\aa	Å	\AA				
ø	\o	Ø	\O	ł	\l	Ł	\L
ı	\i	ȷ	\j	¡	!‘	¿	?‘

TITLES/CHAPTERS/SECTIONS

```
\maketitle
```

```
\chapter{...}
```

```
\section{...}
\subsection{...}
\subsubsection{...}
\paragraph{...}
\subparagraph{...}
```

FOOTNOTES

`\footnote{`*footnote text*`}`

WORDS EMPHASIZING

`\underline{`*text*`}`

`\emph{`*text*`}`

ENVIRONMENTS

`\begin{`*environment*`}` *text* `\end{`*environment*`}`

ENVIRONMENTS : ITEMIZE

```
\flushleft
\begin{enumerate}
\item You can next the list
environments to your taste:
\begin{itemize}
\item But it might start to
look silly.
\item[-] With a dash.
\end{itemize}
\item Therefore remember:
\begin{description}
\item[Stupid] things will not
become smart because they are
in a list.
\item[Smart] things, though,
can be presented beautifully
in a list.
\end{description}
\end{enumerate}
```

1. You can next the list environments to your taste:
   - But it might start to look silly.
   - With a dash.
2. Therefore remember:

Stupid things will not become smart because they are in a list.

Smart things, though, can be presented beautifully in a list.

ENVIRONMENT: ALIGN

```
\begin{flushleft}
This text is\\ left-aligned.
\LaTeX{} is not trying to make
each line the same length.
\end{flushleft}
```

This text is
left-aligned. LaTeX is not trying to make
each line the same length.

```
\begin{flushright}
This text is right-\\aligned.
\LaTeX{} is not trying to make
each line the same length.
\end{flushright}
```

This text is right-
aligned. LaTeX is not trying to make each
line the same length.

```
\begin{center}
At the centre\\of the earth
\end{center}
```

At the centre
of the earth

ENVIRONMENT: VERBATIM

```
The \verb|\ldots| command \ldots

\begin{verbatim}
10 PRINT "HELLO WORLD ";
20 GOTO 10
\end{verbatim}
```

The \ldots command ...

```
10 PRINT "HELLO WORLD ";
20 GOTO 10
```

```
\begin{verbatim*}
the starred version of
the      verbatim
environment emphasizes
the spaces   in the text
\end{verbatim*}
```

```
the␣starred␣version␣of
the␣␣␣␣␣␣verbatim
environment␣emphasizes
the␣spaces␣␣␣in␣the␣text
```

MATH: SINGLE EQUATIONS: X

```
Add $a$ squared and $b$ squared
to get $c$ squared. Or, using
a more mathematical approach:
$a^2 + b^2 = c^2$
```

Add a squared and b squared to get c squared. Or, using a more mathematical approach: $a^2 + b^2 = c^2$

```
\TeX{} is pronounced as
$\tau\epsilon\chi$\\[5pt]
100~m$^{3}$ of water\\[5pt]
This comes from my $\heartsuit$
```

TEX is pronounced as $\tau\epsilon\chi$

100 m^3 of water

This comes from my $\heartsuit$

MATH: LARGE EQUATIONS (+ REFERENCE)

```
Add $a$ squared and $b$ squared
to get $c$ squared. Or, using
a more mathematical approach
 \begin{equation}
   a^2 + b^2 = c^2
 \end{equation}
Einstein says
 \begin{equation}
   E = mc^2 \label{clever}
 \end{equation}
He didn't say
 \begin{equation}
  1 + 1 = 3 \tag{dumb}
 \end{equation}
This is a reference to
\eqref{clever}.
```

Add a squared and b squared to get c squared. Or, using a more mathematical approach

$$a^2 + b^2 = c^2 \tag{3.1}$$

Einstein says

$$E = mc^2 \tag{3.2}$$

He didn't say

$$1 + 1 = 3 \tag{dumb}$$

This is a reference to (3.2).

```
This is text style:
$\lim_{n \to \infty}
 \sum_{k=1}^n \frac{1}{k^2}
 = \frac{\pi^2}{6}$.
And this is display style:
 \begin{equation}
  \lim_{n \to \infty}
  \sum_{k=1}^n \frac{1}{k^2}
  = \frac{\pi^2}{6}
 \end{equation}
```

This is text style: $\lim_{n \to \infty}
 \sum_{k=1}^n \frac{1}{k^2}
 = \frac{\pi^2}{6}$.
And this is display style:

$$\lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{k^2} = \frac{\pi^2}{6} \tag{3.3}$$

```
A $d_{e_{e_p}}$ mathematical
expression  followed by a
$h^{i^{g^h}}$ expression. As
opposed to a smashed
\smash{$d_{e_{e_p}}$} expression
followed by a
\smash{$h^{i^{g^h}}$} expression.
```

A $d_{e_{e_p}}$ mathematical expression followed by a $h^{i^{g^h}}$ expression. As opposed to a smashed $d_{e_{e_p}}$ expression followed by a $h^{i^{g^h}}$ expression.

MATH MODE

```
$\forall x \in \mathbf{R}:
 \qquad x^{2} \geq 0$
```

$\forall x \in \mathbf{R}: \qquad x^{2} \geq 0$

```
$x^{2} \geq 0\qquad
 \text{for all }x\in\mathbf{R}$
```

$x^{2} \geq 0 \qquad \text{for all }x\in\mathbf{R}$

GREEK LETTERS IN MATH

```
$\lambda,\xi,\pi,\theta,
 \mu,\Phi,\Omega,\Delta$
```

$\lambda, \xi, \pi, \theta, \mu, \Phi, \Omega, \Delta$

EXPONENT, SUBSCRIPTS, SUPERSCRIPTS

```
$p^3_{ij} \qquad
 m_\text{Knuth}\qquad
\sum_{k=1}^3 k \\[5pt]
 a^x+y \neq a^{x+y}\qquad
 e^{x^2} \neq {e^x}^2$
```

$$p^3_{ij} \qquad m_\text{Knuth}\qquad \sum_{k=1}^3 k$$

$$a^x+y \neq a^{x+y}\qquad e^{x^2} \neq {e^x}^2$$

SQUARE ROOT

```
$\sqrt{x} \Leftrightarrow x^{1/2}
 \quad \sqrt[3]{2}
 \quad \sqrt{x^{2} + \sqrt{y}}
 \quad \surd[x^2 + y^2]$
```

$$\sqrt{x} \Leftrightarrow x^{1/2} \quad \sqrt[3]{2} \quad \sqrt{x^{2} + \sqrt{y}} \quad \surd[x^2 + y^2]$$

INTEGRALS, SUMS,...

```
\begin{equation*}
\sum_{i=1}^n \qquad
\int_0^{\frac{\pi}{2}} \qquad
\prod_\epsilon
\end{equation*}
```

$$\sum_{i=1}^{n} \qquad \int_{0}^{\frac{\pi}{2}} \qquad \prod_{\epsilon}$$

PARENTHESES

```
$\Big((x+1)(x-1)\Big)^{2}$\\
$\big( \Big( \bigg( \Bigg( \quad
\big\} \Big\} \bigg\} \Bigg\} \quad
\big\| \Big\| \bigg\| \Bigg\| \quad
\big\Downarrow \Big\Downarrow
\bigg\Downarrow \Bigg\Downarrow$
```

$$\Big((x+1)(x-1)\Big)^{2}$$

$$\big(\Big(\bigg(\Bigg(\quad \big\} \Big\} \bigg\} \Bigg\} \quad \big\| \Big\| \bigg\| \Bigg\| \quad \big\Downarrow \Big\Downarrow \bigg\Downarrow \Bigg\Downarrow$$

MULTILINE EQUATION

```
\begin{multline}
  a + b + c + d + e + f
  + g + h + i
  \\
  = j + k + l + m + n
\end{multline}
```

$$\begin{multline} a + b + c + d + e + f + g + h + i \\ = j + k + l + m + n \end{multline} \tag{3.4}$$

MULTIPLE EQUATIONS

```
\begin{eqnarray}
  a & = & b + c \\
  & = & d + e + f + g + h + i
  + j + k + l \nonumber \\
  && +\: m + n + o \\
  & = & p + q + r + s
\end{eqnarray}
```

$$
\begin{aligned}
a &= b + c && (3.12)\\
&= d + e + f + g + h + i + j + k + l \\
&\quad + m + n + o && (3.13)\\
&= p + q + r + s && (3.14)
\end{aligned}
$$

MORE MATH SYMBOLS

$\hat{a}$	`\hat{a}`	$\check{a}$	`\check{a}`	$\tilde{a}$	`\tilde{a}`
$\grave{a}$	`\grave{a}`	$\dot{a}$	`\dot{a}`	$\ddot{a}$	`\ddot{a}`
$\bar{a}$	`\bar{a}`	$\vec{a}$	`\vec{a}`	$\widehat{AAA}$	`\widehat{AAA}`
$\acute{a}$	`\acute{a}`	$\breve{a}$	`\breve{a}`	$\widetilde{AAA}$	`\widetilde{AAA}`
$\mathring{a}$	`\mathring{a}`				

α	`\alpha`	θ	`\theta`	o	`o`	υ	`\upsilon`
β	`\beta`	ϑ	`\vartheta`	π	`\pi`	ϕ	`\phi`
γ	`\gamma`	ι	`\iota`	ϖ	`\varpi`	φ	`\varphi`
δ	`\delta`	κ	`\kappa`	ρ	`\rho`	χ	`\chi`
ϵ	`\epsilon`	λ	`\lambda`	ϱ	`\varrho`	ψ	`\psi`
ε	`\varepsilon`	μ	`\mu`	σ	`\sigma`	ω	`\omega`
ζ	`\zeta`	ν	`\nu`	ς	`\varsigma`		
η	`\eta`	ξ	`\xi`	τ	`\tau`		
Γ	`\Gamma`	Λ	`\Lambda`	Σ	`\Sigma`	Ψ	`\Psi`
Δ	`\Delta`	Ξ	`\Xi`	Υ	`\Upsilon`	Ω	`\Omega`
Θ	`\Theta`	Π	`\Pi`	Φ	`\Phi`		

$<$	`<`	$>$	`>`	$=$	`=`
$\leq$	`\leq` or `\le`	$\geq$	`\geq` or `\ge`	$\equiv$	`\equiv`
$\ll$	`\ll`	$\gg$	`\gg`	$\doteq$	`\doteq`
$\prec$	`\prec`	$\succ$	`\succ`	$\sim$	`\sim`
$\preceq$	`\preceq`	$\succeq$	`\succeq`	$\simeq$	`\simeq`
$\subset$	`\subset`	$\supset$	`\supset`	$\approx$	`\approx`
$\subseteq$	`\subseteq`	$\supseteq$	`\supseteq`	$\cong$	`\cong`
$\sqsubset$ [a]	`\sqsubset` [a]	$\sqsupset$ [a]	`\sqsupset` [a]	$\bowtie$ [a]	`\Join` [a]
$\sqsubseteq$	`\sqsubseteq`	$\sqsupseteq$	`\sqsupseteq`	$\bowtie$	`\bowtie`
$\in$	`\in`	$\ni$	`\ni` , `\owns`	$\propto$	`\propto`
$\vdash$	`\vdash`	$\dashv$	`\dashv`	$\models$	`\models`
$\mid$	`\mid`	$\parallel$	`\parallel`	$\perp$	`\perp`
$\smile$	`\smile`	$\frown$	`\frown`	$\asymp$	`\asymp`
$:$	`:`	$\notin$	`\notin`	$\neq$	`\neq` or `\ne`

$+$	`+`	$-$	`-`		
$\pm$	`\pm`	$\mp$	`\mp`	$\triangleleft$	`\triangleleft`
$\cdot$	`\cdot`	$\div$	`\div`	$\triangleright$	`\triangleright`
$\times$	`\times`	$\setminus$	`\setminus`	$\star$	`\star`
$\cup$	`\cup`	$\cap$	`\cap`	$*$	`\ast`
$\sqcup$	`\sqcup`	$\sqcap$	`\sqcap`	$\circ$	`\circ`
$\vee$	`\vee` , `\lor`	$\wedge$	`\wedge` , `\land`	$\bullet$	`\bullet`
$\oplus$	`\oplus`	$\ominus$	`\ominus`	$\diamond$	`\diamond`
$\odot$	`\odot`	$\oslash$	`\oslash`	$\uplus$	`\uplus`
$\otimes$	`\otimes`	$\bigcirc$	`\bigcirc`	$\amalg$	`\amalg`
$\bigtriangleup$	`\bigtriangleup`	$\bigtriangledown$	`\bigtriangledown`	$\dagger$	`\dagger`
$\lhd$ [a]	`\lhd` [a]	$\rhd$ [a]	`\rhd` [a]	$\ddagger$	`\ddagger`
$\unlhd$ [a]	`\unlhd` [a]	$\unrhd$ [a]	`\unrhd` [a]	$\wr$	`\wr`

$\sum$	`\sum`	$\bigcup$	`\bigcup`	$\bigvee$	`\bigvee`
$\prod$	`\prod`	$\bigcap$	`\bigcap`	$\bigwedge$	`\bigwedge`
$\coprod$	`\coprod`	$\bigsqcup$	`\bigsqcup`	$\biguplus$	`\biguplus`
$\int$	`\int`	$\oint$	`\oint`	$\bigodot$	`\bigodot`
$\bigoplus$	`\bigoplus`	$\bigotimes$	`\bigotimes`		

$\leftarrow$	`\leftarrow` or `\gets`	$\longleftarrow$	`\longleftarrow`
$\rightarrow$	`\rightarrow` or `\to`	$\longrightarrow$	`\longrightarrow`
$\leftrightarrow$	`\leftrightarrow`	$\longleftrightarrow$	`\longleftrightarrow`
$\Leftarrow$	`\Leftarrow`	$\Longleftarrow$	`\Longleftarrow`
$\Rightarrow$	`\Rightarrow`	$\Longrightarrow$	`\Longrightarrow`
$\Leftrightarrow$	`\Leftrightarrow`	$\Longleftrightarrow$	`\Longleftrightarrow`
$\mapsto$	`\mapsto`	$\longmapsto$	`\longmapsto`
$\hookleftarrow$	`\hookleftarrow`	$\hookrightarrow$	`\hookrightarrow`
$\leftharpoonup$	`\leftharpoonup`	$\rightharpoonup$	`\rightharpoonup`
$\leftharpoondown$	`\leftharpoondown`	$\rightharpoondown$	`\rightharpoondown`
$\rightleftharpoons$	`\rightleftharpoons`	$\iff$	`\iff` (bigger spaces)
$\uparrow$	`\uparrow`	$\downarrow$	`\downarrow`
$\updownarrow$	`\updownarrow`	$\Uparrow$	`\Uparrow`
$\Downarrow$	`\Downarrow`	$\Updownarrow$	`\Updownarrow`
$\nearrow$	`\nearrow`	$\searrow$	`\searrow`
$\swarrow$	`\swarrow`	$\nwarrow$	`\nwarrow`
$\leadsto$	`\leadsto` [a]		

[a] Use the latexsym package to access this symbol

$\dots$	`\dots`	$\cdots$	`\cdots`	$\vdots$	`\vdots`	$\ddots$	`\ddots`
$\hbar$	`\hbar`	$\imath$	`\imath`	$\jmath$	`\jmath`	ℓ	`\ell`
$\Re$	`\Re`	$\Im$	`\Im`	$\aleph$	`\aleph`	$\wp$	`\wp`
$\forall$	`\forall`	$\exists$	`\exists`	$\mho$ [a]	`\mho` [a]	∂	`\partial`
$'$	`'`	$\prime$	`\prime`	$\emptyset$	`\emptyset`	∞	`\infty`
∇	`\nabla`	$\triangle$	`\triangle`	$\Box$	`\Box` [a]	$\Diamond$	`\Diamond` [a]
$\bot$	`\bot`	$\top$	`\top`	$\angle$	`\angle`	$\surd$	`\surd`
$\diamondsuit$	`\diamondsuit`	$\heartsuit$	`\heartsuit`	$\clubsuit$	`\clubsuit`	$\spadesuit$	`\spadesuit`
$\neg$	`\neg` or `\lnot`	$\flat$	`\flat`	$\natural$	`\natural`	$\sharp$	`\sharp`

[a] Use the latexsym package to access this symbol

INCLUDING ENCAPSULATED POSTSCRIPTS (FIGURES)

`\usepackage[`*driver*`]{graphicx}`

`\includegraphics[`*key*=*value*, ...`]{`*file*`}`

`width`	scale graphic to the specified width
`height`	scale graphic to the specified height
`angle`	rotate graphic counterclockwise
`scale`	scale graphic

EPS: EXAMPLE

```
\begin{figure}
\centering
\includegraphics[angle=90,
                 width=0.5\textwidth]{test}
\caption{This is a test.}
\end{figure}
```

EXAMPLE I: SIMPLEEQUATIONS.TEX

Emacs@kalaupapa-01.dynamic.rpi.edu

```
\documentclass{report}
\begin{document}

\title{Simple equation}
\begin{verbatim}
$a^2+\frac{a}{b}=\sum_{i=1}^n\Gamma_i+\log \frac{x}{\zeta}-\cos(\pi x)\times \alpha$
\end{verbatim}
$a^2+\frac{a}{b}=\sum_{i=1}^n\Gamma_i+\log \frac{x}{\zeta}-\cos(\pi x) \times \alpha$
\end{document}
```

```
$a^2+\frac{a}{b}=\sum_{i=1}^n\Gamma_i+\log \frac{x}{\zeta}-\cos(\pi x)\times \alpha$
```

$$a^2 + \frac{a}{b} = \sum_{i=1}^{n} \Gamma_i + \log \frac{x}{\zeta} - \cos(\pi x) \times \alpha$$

EXAMPLE II: SIMPLEGRAPHICS

Emacs@kalaupapa-01.dynamic.rpi.edu

```latex
\documentclass{report}
\usepackage[pdftex]{color,graphicx}
\begin{document}

\title{Simple equation}
\begin{verbatim}

\end{verbatim}
\begin{figure}
\includegraphics[width=5cm]{LaTeX2e_logo.png}
\caption{First try}
\end{figure}

\begin{figure}
\includegraphics[width=5cm,angle=34]{LaTeX2e_logo.png}
\caption{Second try}
\end{figure}


\end{document}
```

LATEX2E_LOGO.PNG

THE FIGURE NEEDS TO BE FINDABLE!

USE: PDFLATEX SIMPLEGRAPHICS.TEX

RESULT

Figure 1: First try

Figure 2: Second try

```
\documentclass{report}
\usepackage[pdftex]{color,graphicx}
\begin{document}

\title{Simple equation}
\begin{verbatim}

\end{verbatim}
\begin{figure}
\includegraphics[width=5cm]{LaTeX2e_logo.png}
\caption{First try}
\end{figure}

\begin{figure}
\includegraphics[width=5cm,angle=34]{LaTeX2e_logo.png}
\caption{Second try}
\end{figure}


\end{document}
```

DISPLAY C++ CODES WITH SYNTAX HIGHLIGHT

EXAMPLES OF WORK FROM STUDENTS WHO DID NOT KNOW LATEX BEFORE CLASS