Homework 2: Random Numbers, Monte Carlo, and Multidimensional Integration

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Assignment 2

Random Numbers

2.1 Psuedo random

1. Use your computer's default pseudo-RNG to generate a sequence $\{x_i\}_{i=1,N}$ of random numbers in the range between 0.0 and 1.0 (choose an appropriate value for N, such as 10,000 or more).

   - **Use random numbers to calculate the integral in five dimensions:**

Make sure your report is self-contained with sufficient details and clear plots. Feel free to add listings of your code (or use pseudo-code). Collaborative work is allowed but each student will turn in an individual report.

Below is a suggestion for presentation. You can adapt it to your personal vision of the problem.

2.2 Multidimensional Integration

1. Use your computer's default pseudo-RNG to generate a sequence $\{x_i\}_{i=1,N}$ of random numbers in the range between 0.0 and 1.0 (choose an appropriate value for N, such as 10,000 or more).

$$\int_0^1 \int_0^1 \int_0^1 \int_0^1 \int_0^1 (x_1 + x_2 + x_3 + x_4 + x_5)^3 dx_1 dx_2 dx_3 dx_4 dx_5 \approx \tag{2.1}$$

The formula for a multidimensional Monte Carlo integral is as follows: $\int_a^b \int_c^d \int_e^f f(x_1, x_2, x_3) dx_1 dx_2 dx_3 \approx (b-a)(d-c)(f-e)\frac{1}{N} \sum_{i=0}^{n} i^3 f(x_i)$

Here, f is a function of three dimensions, the variables in the product of differences correlate directly to the limits of integration and N represents the quantity of random numbers used to determine a mean value for f.

As an example, consider the following 5D integral:

$$I = \int_0^1 \int_0^1 \int_0^1 \int_0^1 \int_0^1 (x_1 + x_2 + x_3 + x_4 + x_5)^3 dx_1 dx_2 dx_3 dx_4 dx_5 = 18.75$$

Using multidimensional Monte-Carlo integration, chose $N = 5 \times 10^4$ random numbers to approximate the integral as:

$\int_0^1 \int_0^1 \int_0^1 \int_0^1 \int_0^1 f(x_1 ... x_5) dx_1 ... dx_5 \approx (1-0)(1-0)(1-0)(1-0)(1-0) \left(\frac{1}{5 \times 10^4} \right) \sum_{i=0}^{5 \times 10^4} f(x_{i1} + x_{i2} + x_{i3} + x_{i4} + x_{i5})^3$

At the $(5 \times 10^{-4})^{th}$ iteration, the Monte-Carlo integration yields ≈ 18.7862. Off the cuff, we might say to increase the number of iterations to improve accuracy, but this is not the primary reason for why the approximation is inaccurate.
Note that for each iteration of the above summation, the number of $rand()$ calls grows as $5(1 + \sum_{i=2}^{5 \times 10^4} i)$. Therefore, RAND_MAX may be reached.

1×10^{-4}

- **Use random numbers to calculate the integral in five dimensions:**

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2.3 Implementation

2.4 Results

2.5 Discussion

2.6 References (if any)