Homework 3: Ordinary Differential Equations (ODEs): physics at work

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Assignment

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1. Consider the problem of planetary motion around the Sun, as discussed during lecture 8.
   (a) Write a C++ code, using Runge-Kutta at fourth order (RK4) to describe the motion of Earth and Jupiter around the Sun. You will use the RK4 library provided in class.
      - First discuss the trajectories when Jupiter and Earth do not interact. (Hint: you will have 2 bodies, 2 spatial variables per body and no cross-terms)
      - Then, turn on the Jupiter-Earth gravitational interaction. (Hint: same as before but with a cross-term between Jupiter and Earth)
      - In both cases, use realistic numbers for the orbits (size of the orbit, velocity at aphelion or perihelion, etc...). Verify Kepler's laws and compute periodicity.
      - Repeat your calculations, by artificially increasing the mass of Jupiter by a factor of 1,000. Discuss the effect on the orbits. Make sure you make clear plots of the orbits.

      Note: it is a good idea to use astronomical units.
   (b) Study the precession of the perihelion of Mercury due to general relativity corrections to the $1/r^2$ classical gravitational law. Use the following equation of the modified force:
      $$F_G = \frac{GM_S M_M}{r^2}(1 + \frac{\alpha}{r^2})$$
      where M_M and M_S correspond to the mass of Mercury and the Sun, respectively. Use $\alpha \sim 10^{-8}$ as a small coefficient accounting to relativity corrections.
      - Plot the trajectory of Mercury for a given $\alpha = 0.01AU^2$. Draw the line between the Sun and the closest approach of Mercury for a few trajectories. The change in direction indicates the change of orientation of the perihelion.
      - Plot the orbit's orientation change with time for $\alpha = 0.0008AU^2$.

      In all these calculations, do not consider the effect of the planets on the Sun (Sun is static); choose initial conditions properly, test your step size h carefully.
2. Select a problem of your choice from any physics class (or book) where ODEs cannot be solved analytically. Present the physics of the problem carefully and make a case for the solution you obtained, in light of the conditions you selected.

Make sure your report is self-contained with sufficient details and clear plots. Feel free to add listings of your codes (or use pseudo-code).