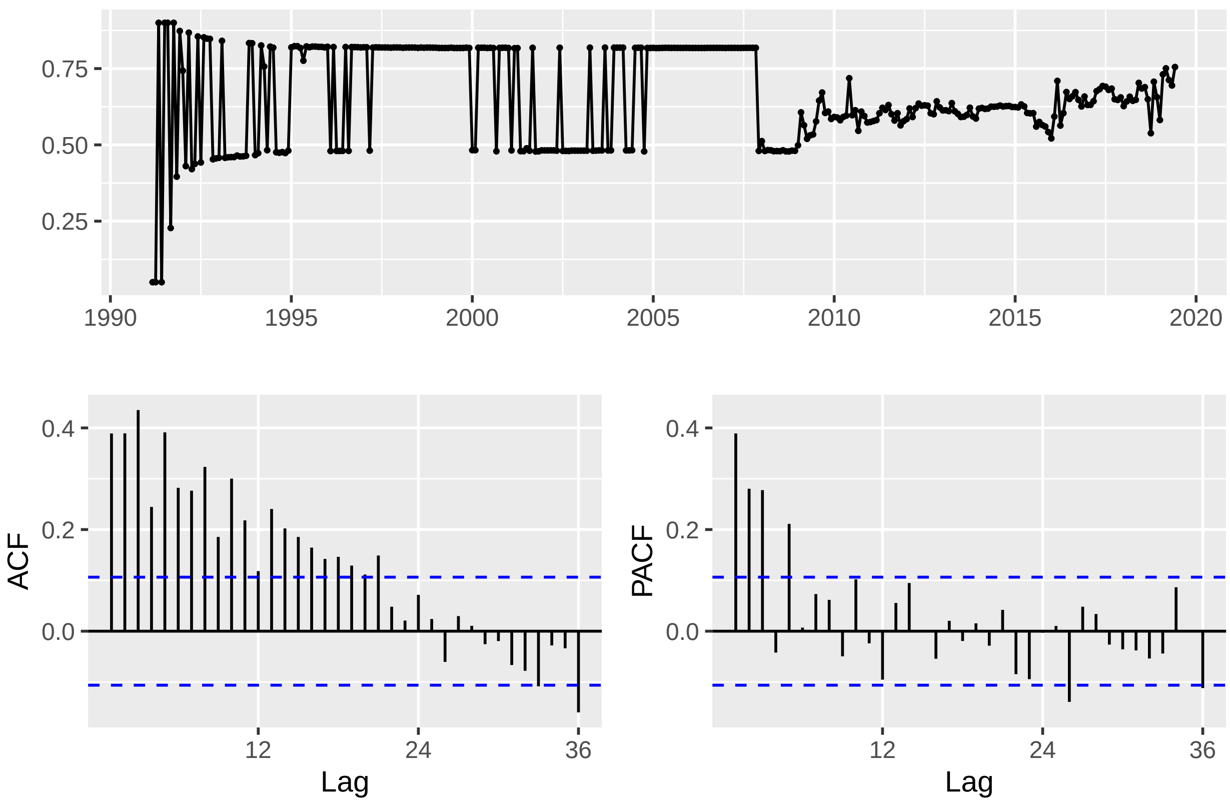
Investigating Portfolio Weights

I examined the possibility of ARIMA-GARCH to capture insights about the estimated portfolio weights. Although the fitted ARIMA(1,0,1)-GARCH(1,1) model showed significant p-values, residuals failed normality tests raising concerns about stationarity/ergodicity of the random variable.

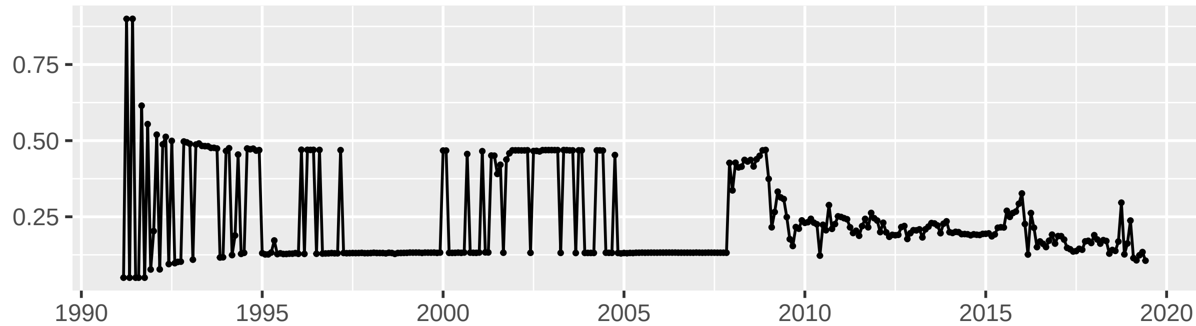
ACF and PACF for Stocks and Bonds

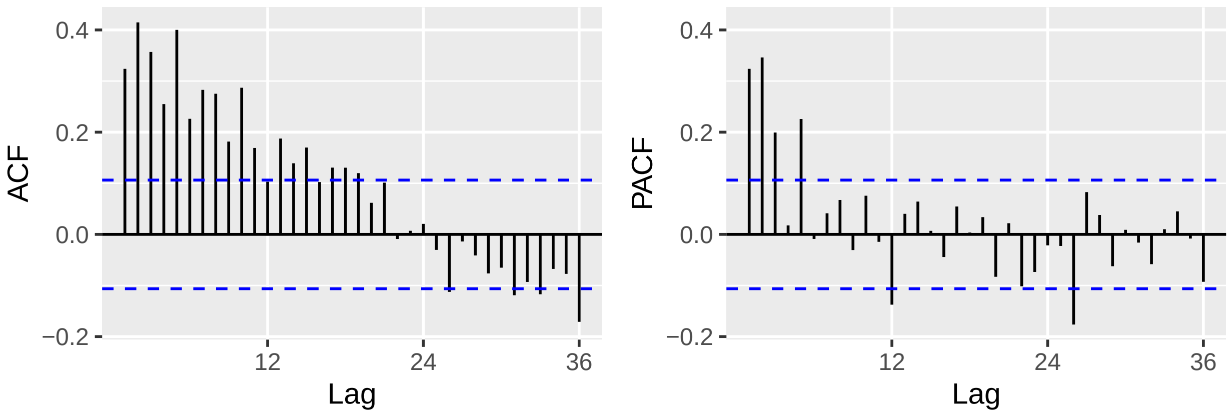
1) Observed there is an evidence of autocorrelation/partial-autocorrelation.
2) Variance changed over time suggesting heteroskedasticity.

```
w_stock <- ts (weights_data['w_stock'], frequency = 12, start = c(1991, 3))
w_bond <- ts (weights_data['w_bond'], frequency = 12, start = c(1991, 3))
w_stock %>%  ggtsdisplay(main="")
```




```
w_bond %>%  ggtsdisplay(main="")
```





Thus, I firstly fitted ARIMA-GARCH

```
spec = ugarchspec(
  variance.model=list(model="sGARCH", garchOrder=c(1,1),include.mean=FALSE),
  mean.model=list(armaOrder=c(1,1),include.mean=TRUE),distribution.model="norm")
```

```
## Warning: unidentified option(s) in variance.model:
##  include.mean
```

```
fit_stock=ugarchfit(spec=spec,data=w_stock)
fit_stock
```

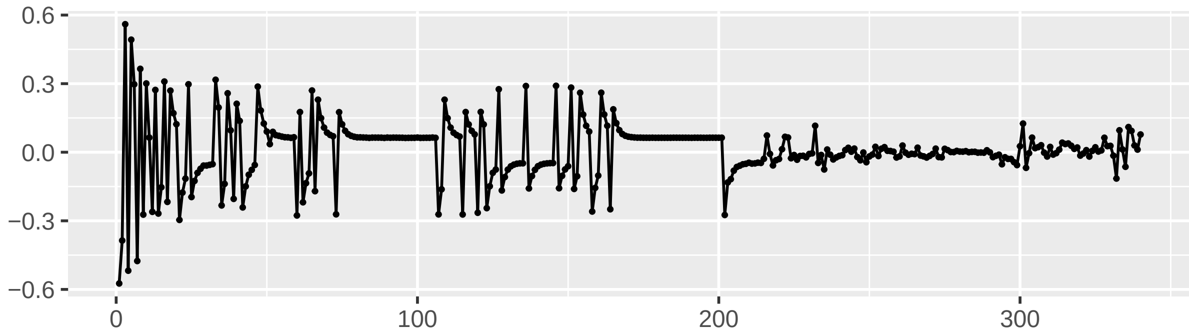
```
##
## *---------------------------------*
## *          GARCH Model Fit        *
## *---------------------------------*
##
## Conditional Variance Dynamics
## -----------------------------------
## GARCH Model  : sGARCH(1,1)
## Mean Model   : ARFIMA(1,0,1)
## Distribution : norm
##
## Optimal Parameters
## ------------------------------------
##         Estimate  Std. Error  t value Pr(>|t|)
## mu       0.62399    0.006496  96.0523 0.000000
## ar1      0.84111    0.034155  24.6260 0.000000
## ma1     -0.51421    0.087902  -5.8498 0.000000
## omega    0.00003    0.000017   1.7140 0.086536
```

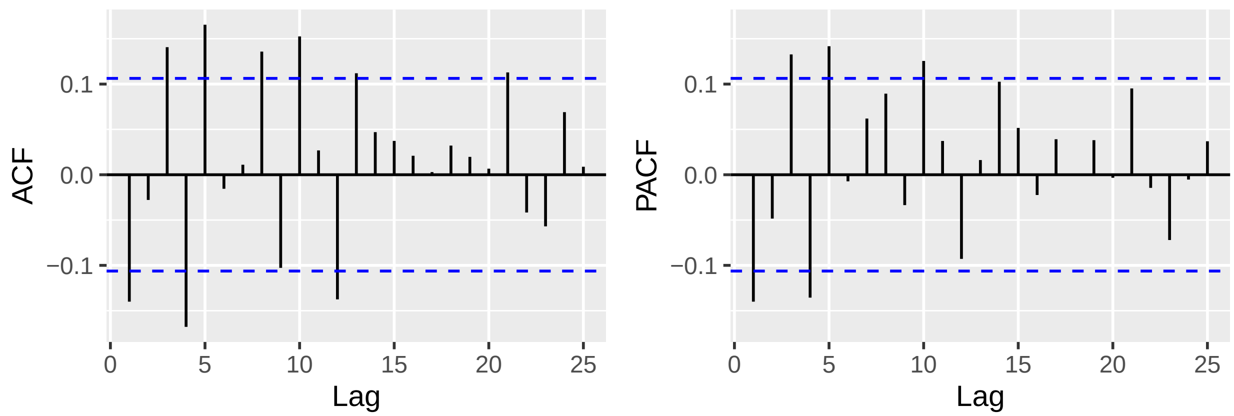
```
## alpha1   0.18080    0.033795   5.3499 0.000000
## beta1    0.81820    0.027207  30.0736 0.000000
##
## Robust Standard Errors:
##         Estimate  Std. Error  t value Pr(>|t|)
## mu       0.62399    0.008878 70.28663 0.000000
## ar1      0.84111    0.064113 13.11920 0.000000
## ma1     -0.51421    0.121001 -4.24965 0.000021
## omega    0.00003    0.000038  0.78506 0.432420
## alpha1   0.18080    0.043290  4.17658 0.000030
## beta1    0.81820    0.038355 21.33244 0.000000
##
## LogLikelihood : 363.1202
##
## Information Criteria
## ------------------------------------
##
## Akaike       -2.1007
## Bayes        -2.0331
## Shibata      -2.1013
## Hannan-Quinn -2.0738
##
## Weighted Ljung-Box Test on Standardized Residuals
## ------------------------------------
##                         statistic   p-value
## Lag[1]                      4.449 3.492e-02
## Lag[2*(p+q)+(p+q)-1][5]     8.567 5.748e-10
## Lag[4*(p+q)+(p+q)-1][9]    11.908 1.283e-03
## d.o.f=2
## H0 : No serial correlation
##
## Weighted Ljung-Box Test on Standardized Squared Residuals
## ------------------------------------
##                         statistic p-value
## Lag[1]                     0.1581  0.6910
## Lag[2*(p+q)+(p+q)-1][5]    0.6738  0.9279
## Lag[4*(p+q)+(p+q)-1][9]    1.3290  0.9684
## d.o.f=2
##
## Weighted ARCH LM Tests
## ------------------------------------
##             Statistic Shape Scale P-Value
## ARCH Lag[3]    0.1803 0.500 2.000  0.6711
## ARCH Lag[5]    0.7298 1.440 1.667  0.8144
## ARCH Lag[7]    1.2332 2.315 1.543  0.8729
##
## Nyblom stability test
## ------------------------------------
## Joint Statistic:  1.3584
## Individual Statistics:
## mu     0.04688
## ar1    0.02705
## ma1    0.06270
## omega  0.08798
```

```
## alpha1 0.15644
## beta1  0.14131
##
## Asymptotic Critical Values (10% 5% 1%)
## Joint Statistic:          1.49 1.68 2.12
## Individual Statistic:     0.35 0.47 0.75
##
## Sign Bias Test
## ------------------------------------
##                    t-value   prob sig
## Sign Bias           0.4238 0.6720
## Negative Sign Bias  0.2585 0.7962
## Positive Sign Bias  0.2690 0.7881
## Joint Effect        0.4942 0.9202
##
##
## Adjusted Pearson Goodness-of-Fit Test:
## ------------------------------------
##   group statistic p-value(g-1)
## 1    20     135.5    1.301e-19
## 2    30     144.2    3.077e-17
## 3    40     164.5    2.400e-17
## 4    50     205.6    4.458e-21
##
##
## Elapsed time : 0.145999
```

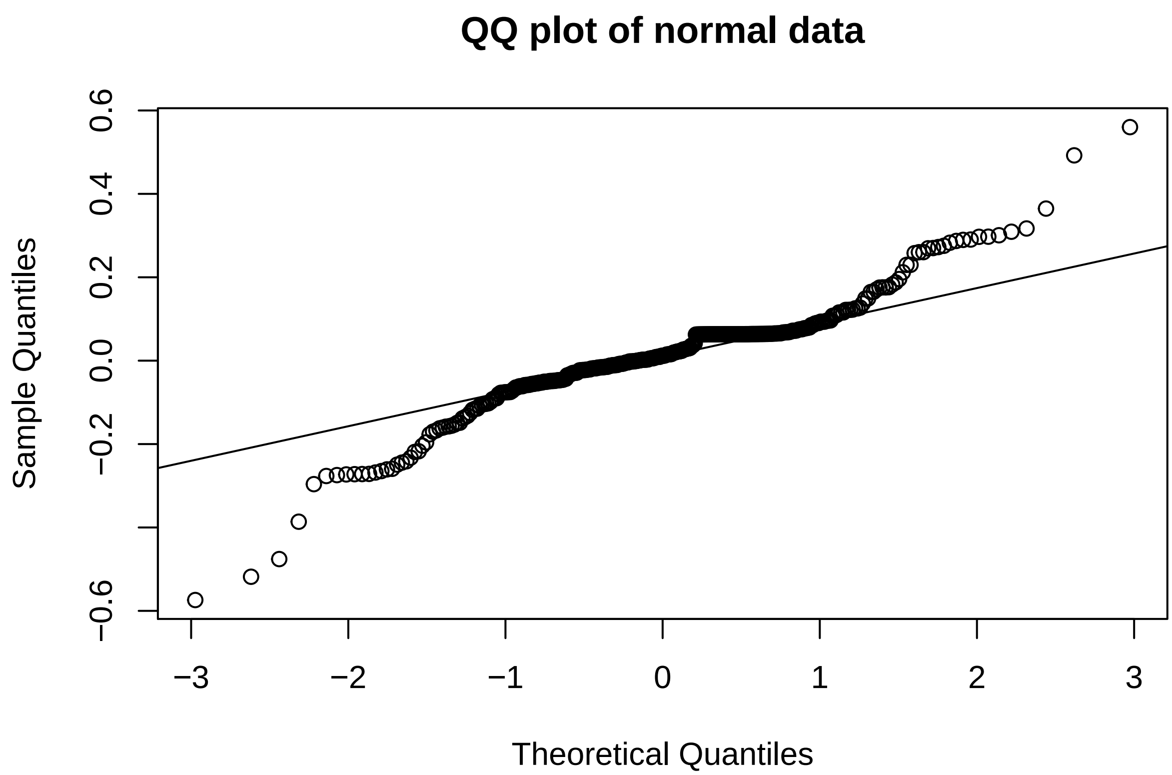
I tested normality of stock weights based on Q-Q plot and Then, I tested normality of residuals to see whether statistical tests would be valuable. For example, the significance of coefficients would be based on asymptotic normality of the random variabble. If the random variable is not stationary then it may be the case that more complicated considerations such as non-ergodicity play a role in asymptotic convergence.

```
residuals(fit_stock) %>%  ggtsdisplay(main="")
```






```r
residuals(fit_stock) %>% qqnorm(,main="QQ plot of normal data")
qqline(residuals(fit_stock))
```



Combining results on QQ-plot and Anderson-Darling test for normality, it is difficult to conclude that the random variable is normally distributed. The QQ-plot is far from linear.