Let $M = P(x_p - \frac{1}{2}, y_p + 1)$,

$F(x,y) = x^2 + y^2 - r^2$

$D = F(M) = (x_p - \frac{1}{2})^2 + (y_p + 1)^2 - r^2$

$= x_p^2 - x_p + \frac{1}{4} + y_p^2 + 2y_p + 1 - r^2$

Multiply equation by 4 to remove floating point computation.

$\Rightarrow \underline{4x_p^2 - 4x_p + 5 + 4y_p^2 + 8y_p - 4r^2}$

$\underline{D_{start}} = D(r, 0)$

$= 4r^2 - 4r + 5 - 4r^2$

$= \underline{5 - 4r}$

Let $P(x_{p+1}, y_{p+1})$ be the new pixel chosen (either N or NW).

$D_{new} = D(x_{p+1}, y_{p+1}) =$

$4x_{p+1}^2 - 4x_{p+1} + 4y_{p+1}^2 + 8y_{p+1} - 4r^2 + 5$

Compute $D_{new} - D_{old}$

$= 4x_{p+1}^2 - 4x_{p+1} + 4y_{p+1}^2 + 8y_{p+1} - 4r^2 + 5$

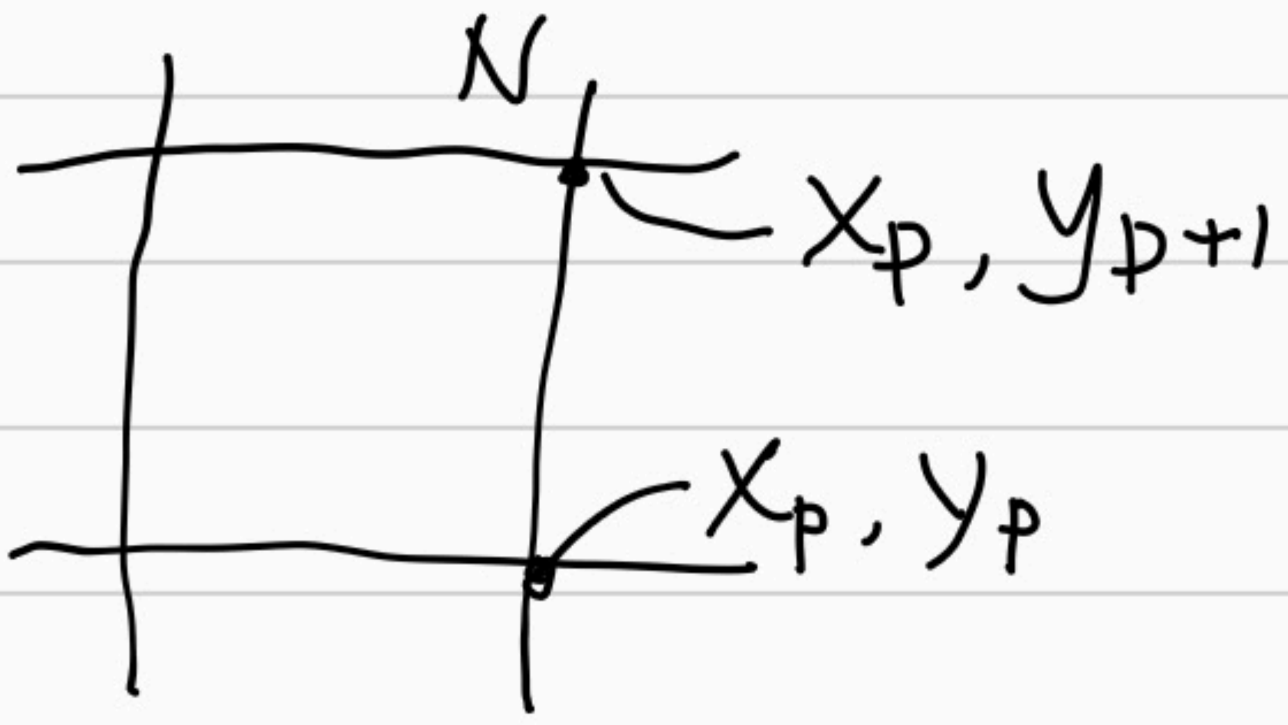
$- 4x_p^2 + 4x_p - 4y_p^2 - 8y_p + 4r^2 - 5$

$= 4(x_{p+1}^2 - x_p^2) - 4(x_{p+1} - x_p) + 4(y_{p+1}^2 - y_p^2)$

$+ 8(y_{p+1} - y_p)$

$$D_{new} = D_{old} + 4(x_{p+1}^2 - x_p^2) - 4(x_{p+1} - x_p)$$
$$+ 4(y_{p+1}^2 - y_p^2) + 8(y_{p+1} - y_p) \quad ①$$

Case 1) $D_{old} < 0 \rightarrow$ chose N

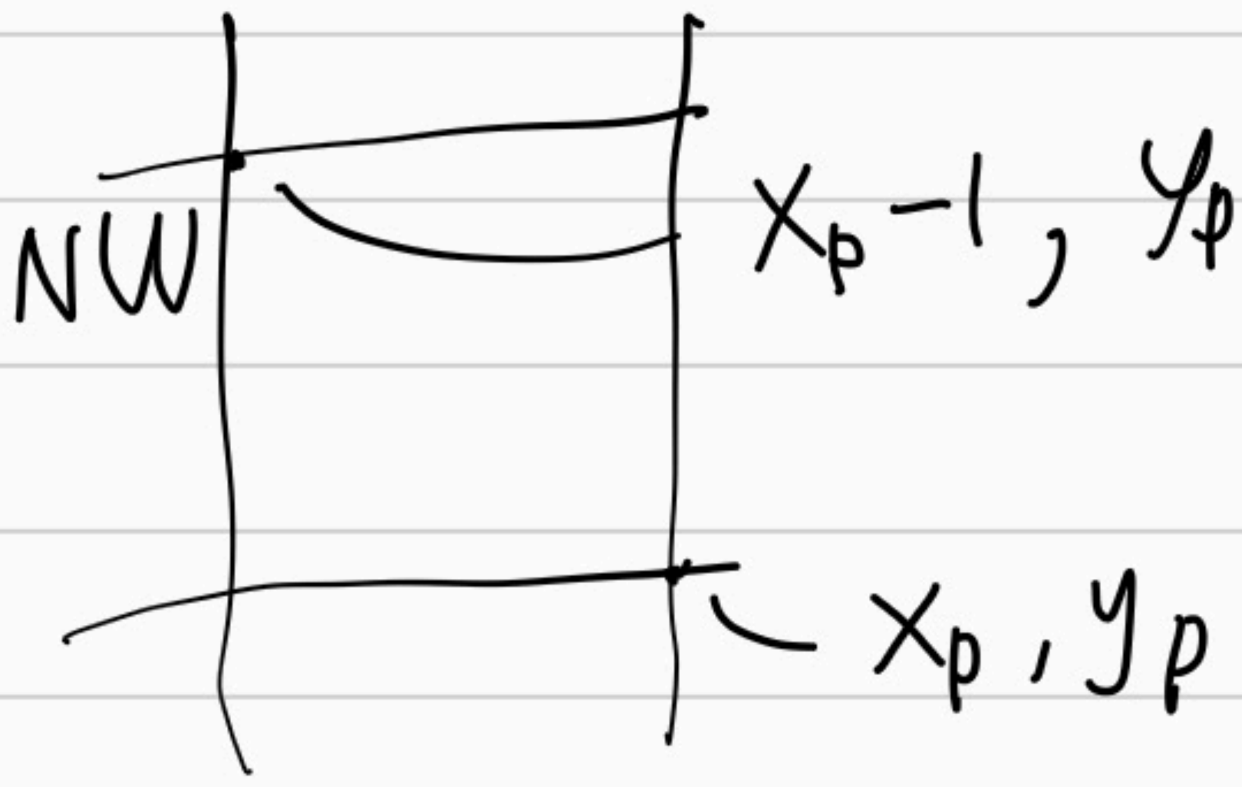


$$x_{p+1} = x_p$$
$$y_{p+1} = y_p + 1$$

Substitute ①

$$D_{new} = D_{old} + 4(x_p^2 - x_p^2) - 4(x_p - x_p)$$
$$+ 4((y_p + 1)^2 - y_p^2) + 8(y_p + 1 - y_p)$$
$$= D_{old} + 4(y_p^2 + 2y_p + 1 - y_p^2) + 8$$
$$= D_{old} + 4y_p^2 + 8y_p + 4 - 4y_p^2 + 8$$
$$= D_{old} + 8y_p + 12$$

Case 2) $D_{old} \geq 0 \rightarrow$ choose NW



$$x_{p+1} = x_p - 1$$
$$y_{p+1} = y_p + 1$$

Substitute ①

$$D_{new} = D_{old} + 4((X_p - 1)^2 - X_p^2) - 4(X_p - 1 - X_p)$$

$$+ 4((y_p + 1)^2 - y_p^2) + 8(y_p + 1 - y_p)$$

$$= D_{old} + 4(X_p^2 - 2X_p + 1 - X_p^2) + 4$$

$$+ 4(y_p^2 + 2y_p + 1 - y_p^2) + 8$$

$$= D_{old} + 4(-2X_p + 1) + 4 + 4(2y_p + 1) + 8$$

$$= D_{old} - 8X_p + 8y_p + 20$$

Summary: $D_{START} = -4r + 5$

$$D_{new} = D_{old} + \begin{cases} 8y_p + 12 & (D_{old} < 0) \\ -8X_p + 8y_p + 20 & (D_{old} \geq 0) \end{cases}$$