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Gravitational radiation in high-speed black-hole collisions

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The geometry of a single black hole which has been boosted to the speed of light is well-known [1]. Assuming that the mass is scaled so that the energy μ remains fixed as the speed tends to c, the limiting metric is a plane-fronted shock wave. This geometry describes two portions of Minkowski space, in $\hat{u} < 0$ and $\hat{u} > 0$, which have joined across the null shock surface $\hat{u} = 0$ with a warp, given by sliding along the null generators by an amount $8\mu \ln \rho$, where ρ measures distance transverse to the motion.

Now consider the collision of two such shocks, each with energy μ. This can be regarded as the limit of the physically more realistic collision of two black holes moving in opposite directions with large γ-factors [2], where the geometry is somewhat smoother. Such shock-wave space-times have attracted interest recently since they help in modelling quantum scattering processes at very high energies, where gravitational effects dominate [3,4]. A speed-of-light collision space-time consists of four regions separated by shocks: a flat region I ahead of both incoming plane shocks, flat regions II and III behind each incoming shock, and a curved space-time region IV produced after the shocks have collided. The boundary of region IV consists of two curved shock surfaces, found by following the null shock generators through the collision.

For simplicity, let us consider the axisymmetric head-on collision. Penrose [5] has found an apparent horizon on the union of the two incoming shocks, joined at the 2-surface where they collide. The area is $32\pi\mu^2$, and assuming the cosmic censorship hypothesis, and that the collision produces a single Schwarzschild black hole plus outgoing gravitational radiation, the final Schwarzschild mass must be $\geqslant \sqrt{2}\mu$, and the energy in gravitational waves $\leqslant (1 - 1/\sqrt{2})2\mu \simeq 29\% \times 2\mu$.

There are two closely related perturbation approaches to calculation of the gravitational radiation. One can apply a large Lorentz boost to the speed-of-light collision, and consider the scattering of a weak shock 2 off a strong shock 1, with energies λ, ν obeying $\lambda \ll \nu$ [6,7]. Alternatively, one can study the collision with large finite γ-factors [2]. In either case, one obtains analytic information about the gravitational radiation emitted near the axis of symmetry $\hat{\theta} = 0, \ \pi$ in the centre-of-mass frame. These collision space-times provide the only known examples with the remarkable property that strong-field radiation can be computed analytically. Physically, this occurs because gravitational radiation with small $\hat{\theta}$ or $(\pi - \hat{\theta})$ propagates near the far-field curved shocks, which have been deflected by small angles; however, the generators of the far-field curved shocks have been given a logarithmic head start over their near-field counterparts (the logarithmic 'warp' of paragraph 1), and so can propagate to future null infinity without interference from the region of the space-time.

The gravitational radiation at retarded time $\hat{\tau}$ and angle $\hat{\theta}$ is described by the Bondi news function $c_0(\hat{\tau}, \hat{\theta}) = \partial c/\partial\hat{\tau}$, which determines the rate of change of the Bondi mass $m(\hat{\tau})$ according to [8]

$$\frac{dm}{d\hat{\tau}} = -\frac{1}{2}\int_0^\pi (c_0)^2 \sin\hat{\theta}\, d\hat{\theta}\ . \tag{1}$$

For the speed-of-light collision, c_0 is expected to admit the convergent expansion

$$c_0(\hat{\tau}, \hat{\theta}) = \sum_{n=0}^{\infty} a_{2n}(\hat{\tau}/\mu)\sin^{2n}\hat{\theta}\ . \tag{2}$$

First-order perturbation theory gives $a_0(\hat{\tau}/\mu)$ as an integral expression [2,7]. If the radiation were isotropic (i.e. if a_{2n} equalled zero for $n \geqslant 1$), the energy emitted in waves would be $1/2\mu$, corresponding to 25% efficiency for gravitational wave generation. The calculation can be continued to second order [7], using the property that there is a conformal symmetry at each order in perturbation theory, along orbits corresponding to a Lorentz boost. Hence $a_2(\hat{\tau}/\mu)$ can be found by a perturbation calculation involving only functions of two variables.

Because one has detailed knowledge of the news function near the axis, one expects similar knowledge of the mass aspect [8] $M(\hat{\tau}, \hat{\theta})$, which obeys

$$\frac{\partial M}{\partial\hat{\tau}} = -\left(\frac{\partial c}{\partial\hat{\tau}}\right)^2 + \frac{1}{2}\frac{\partial}{\partial\hat{\tau}}\left(\frac{\partial^2 c}{\partial\hat{\theta}^2} + 3\cot\hat{\theta}\frac{\partial c}{\partial\hat{\theta}} - 2c\right)\ . \tag{3}$$

At early times $M(\hat{\tau} = -\infty,\ \hat{\theta})$ is known, depending only on the incoming 4-momenta and on $\hat{\theta}$, and at late times one expects $M(\hat{\tau} = +\infty,\ \hat{\theta}) = m_{\text{final}}$, the final Schwarzschild mass. If one assumes that the series (2) obeys a suitable uniformity condition with respect to $\hat{\tau}$, then by integrating (3) with respect to $\hat{\tau}$ one obtains for the speed-of-light collision

$$m_{\text{final}} = -\int_{-\infty}^{\infty}[a_0(\hat{\tau}/\mu)]^2\, d\hat{\tau} + \int_{-\infty}^{\infty}[4a_2(\hat{\tau}/\mu) - a_0(\hat{\tau}/\mu)]\, d\hat{\tau}\ . \tag{4}$$

Thus one might expect to find the final Schwarzschild mass by this approach.

It turns out, from numerical calculation, that this gives a nonsensical result $m_{\text{final}} > 3.5\mu$, which exceeds the incoming energy 2μ. The explanation must be that the uniformity condition for (2) fails. This is quite reasonable, since the perturbation theory is singular (non-uniform) at late retarded times. In fact, the metric perturbations at third and higher order are expected to grow as $\exp(N\hat{\tau}/4\mu)$ at late retarded times, for suitable integers N. The series (2) has to describe both a 'first burst' of radiation, generated in the far field, centred (say) around retarded time $\hat{\tau} = 0$ for small angles $\hat{\theta}$, and a 'second burst' generated in the central region, centred around $\hat{\tau} = |8\mu\ \ln\ \hat{\theta}|$; the 'bursts' merge together at larger angles $\hat{\theta}$.

Nevertheless, one can obtain a crude estimate $\Delta m = 0.328\mu$ from the Bondi formula (1) for the mass loss Δm in gravitational waves, by keeping only the first two terms a_0, a_2 in (2). This would give an efficiency of 16.4%.

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