Superconducting Quantum Technologies

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Lecture 9

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- Resonator – qubit system with decoherence
- Concept of strong coupling
- Fabrication
- Some experiments with artificial atoms

Atom-resonator
Strong coupling

Dissipation in resonators (photon decay)

$$H_{JC} = -\frac{\Delta E}{2}\sigma_z + \hbar\omega_r a^+ a + g_0\left(a\sigma^+ + a^+\sigma^-\right)$$

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] + L$$

No pure dephasing: $\Gamma_2 = \frac{\Gamma_1}{2}$

$$L = \begin{pmatrix} \Gamma_1\rho_{11} & -\frac{\Gamma_1}{2}\rho_{01} \\ -\frac{\Gamma_1}{2}\rho_{10} & -\Gamma_1\rho_{11} \end{pmatrix}$$

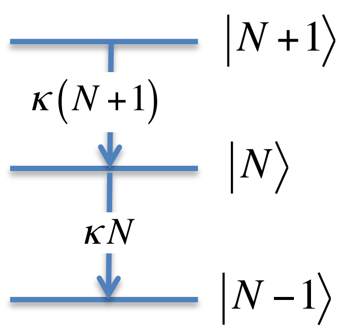
$$L = \frac{\Gamma_1}{2}\left(2\sigma^-\rho\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-\right)$$

Density matrix of a harmonic oscillator:

$$\rho^{(r)} = \sum_{M=0}^{\infty}\sum_{N=0}^{\infty}\rho_{NM}^{(r)}|N\rangle\langle M|$$

Lindblad operator of a harmonic oscillator:

$$L^{(r)} = \frac{\kappa}{2}\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$



N-th element of the resonator Lindblad operator:

$$\dot{\rho}_{NN}^{(r)} \rightarrow \kappa\left(N+1\right)\rho_{N+1,N+1}^{(r)} - \kappa N\rho_{NN}^{(r)}$$

κ is the photon decay rate

Line width of the resonator (quantum mechanical approach)

$$H = \hbar\omega_r a^\dagger a + \hbar\Omega\left(a + a^\dagger\right)\cos\omega t$$

$$H' = UHU^\dagger - i\hbar U\dot{U}^\dagger \qquad\qquad U = e^{i\omega t a^\dagger a}$$

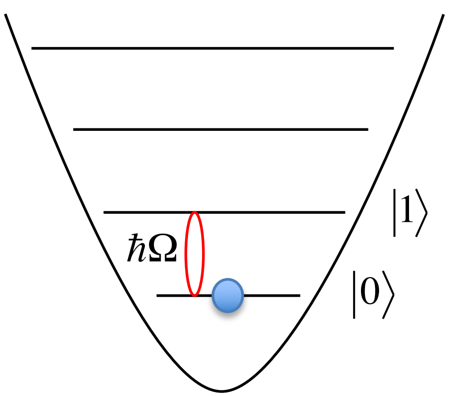
$$H' = \hbar\left(\omega_r - \omega\right)a^\dagger a + \hbar\Omega e^{i\omega t a^\dagger a}\left(a + a^\dagger\right)e^{-i\omega t a^\dagger a}\cos\omega t = \hbar\delta\omega a^\dagger a + \hbar\Omega\left(e^{-i\omega t}a + e^{i\omega t}a^\dagger\right)\frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

$$H' = -\hbar\delta\omega a^\dagger a + \frac{\hbar\Omega}{2}\left(a + a^\dagger\right) \qquad\qquad \delta\omega = \omega - \omega_r$$

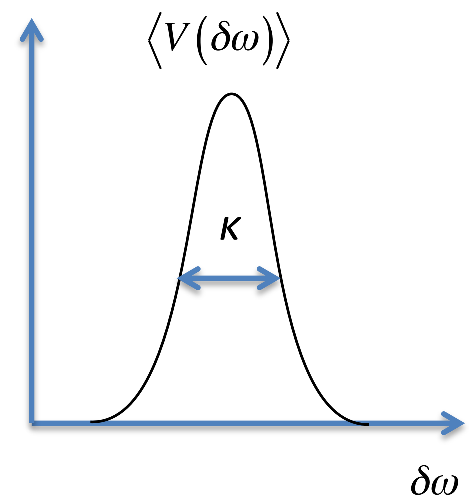
In a weak driving regime, when population of the $|1\rangle$ is low ($\rho_{11} << 1$) we can truncate the photon space by 0, 1

$$H\rho - \rho H + L = 0$$

$$L = \begin{pmatrix} \kappa\rho_{11} & -\frac{\kappa}{2}\rho_{01} \\ -\frac{\kappa}{2}\rho_{10} & -\kappa\rho_{11} \end{pmatrix}$$



$$\left\langle a + a^\dagger \right\rangle = \mathrm{tr}\left[\left(a + a^\dagger\right)\rho\right] \approx \frac{2\Omega}{\kappa + 2\delta\omega}$$



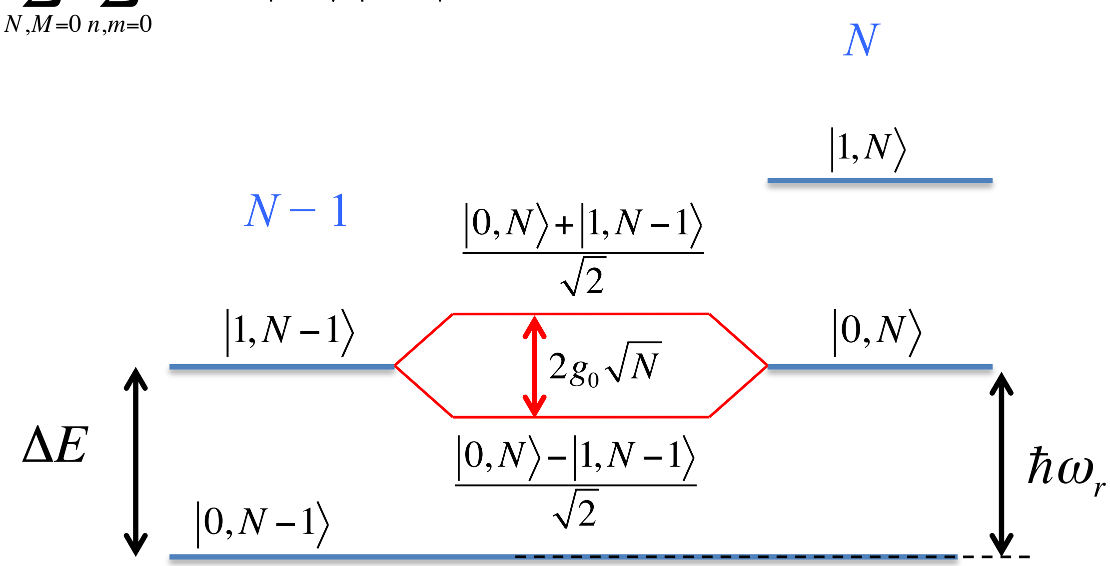
$$H_{JC} = -\frac{\Delta E}{2}\sigma_z + \hbar\omega_r a^+ a + g_0\left(a\sigma^+ + a^+\sigma^-\right)$$

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}\left[H,\rho\right] + L^{(r)} + L^{(a)}$$

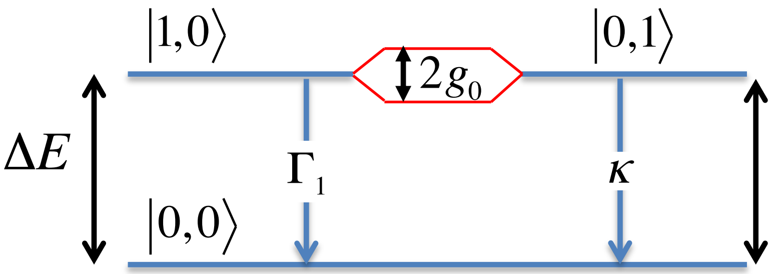
$$L^{(r)} = \frac{\kappa}{2}\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$

$$L^{(a)} = \frac{\Gamma_1}{2}\left(2\sigma^-\rho\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-\right)$$

$$\rho^{(ra)} = \sum_{N,M=0}^{\infty}\sum_{n,m=0}^{1}\rho_{Nn,Mm}\left|Nn\right\rangle\left\langle Mm\right|$$



Single-photon interaction



$$H' = g\left(|01\rangle\langle 10| + |10\rangle\langle 01|\right)$$

$$\rho = \begin{pmatrix} \rho_{01,01} & \rho_{01,10} \\ \rho_{10,01} & \rho_{10,10} \end{pmatrix}$$

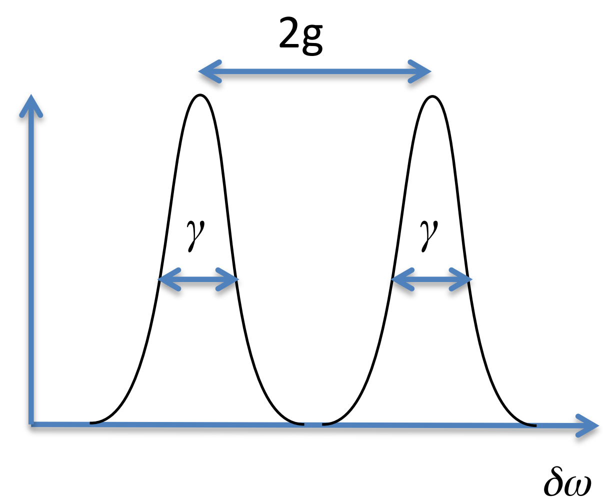
$$L = \begin{array}{c} \begin{array}{ccc} \quad|00\rangle\quad & \quad|01\rangle\quad & \quad|10\rangle\quad \end{array} \\ \begin{pmatrix} \kappa\rho_{01,01} + \Gamma_1\rho_{10,10} & -\frac{\kappa}{2}\rho_{00,01} & -\frac{\Gamma_1}{2}\rho_{00,10} \\ -\frac{\kappa}{2}\rho_{01,00} & -\kappa\rho_{01,01} & -\frac{\Gamma_1+\kappa}{2}\rho_{01,10} \\ -\frac{\Gamma_1}{2}\rho_{10,00} & -\frac{\Gamma_1+\kappa}{2}\rho_{10,01} & -\Gamma_1\rho_{10,10} \end{pmatrix} \end{array}$$

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] + L$$

The states decay with rate: $\gamma = \dfrac{\kappa + \Gamma_1}{2}$

To exchange energy between the qubit and the resonator the following condition should be fulfilled:

$$\boxed{(\Gamma_1,\ \Gamma_2,\ \kappa) << \frac{g}{\hbar}}$$



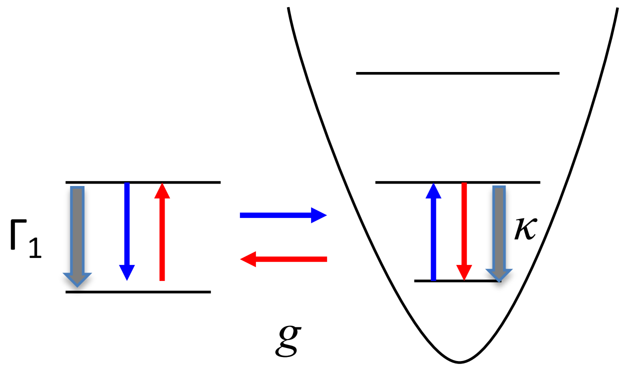
Matlab Quantum optics toolbox: qotoolbox

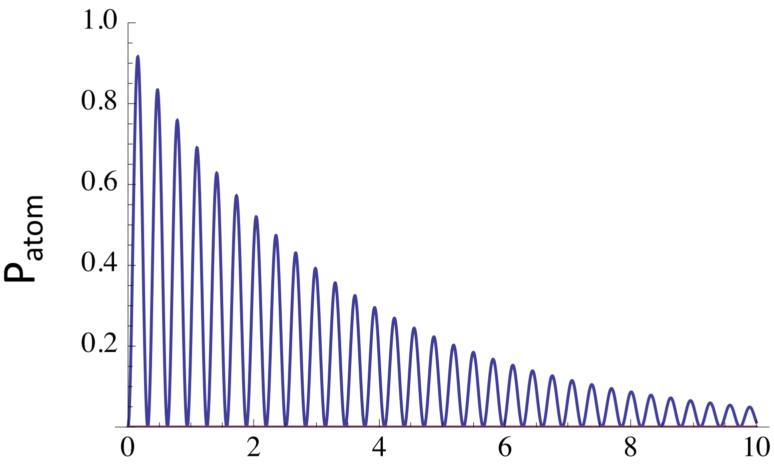
Energy exchange. Strong coupling.

Strong coupling regime is achieved, when the characteristic energy is higher than incoherent processes

For an atom coupled to a resonator the coupling energy g

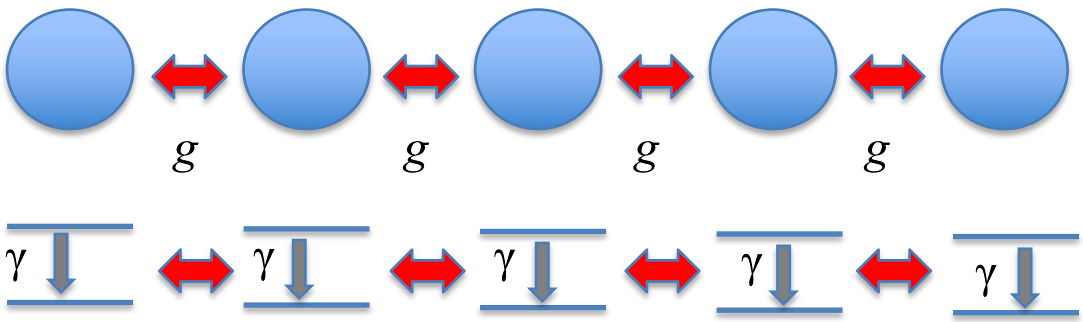
$$\left(\hbar\Gamma_1,\ \hbar\Gamma_2,\ \hbar\kappa\right) << g$$





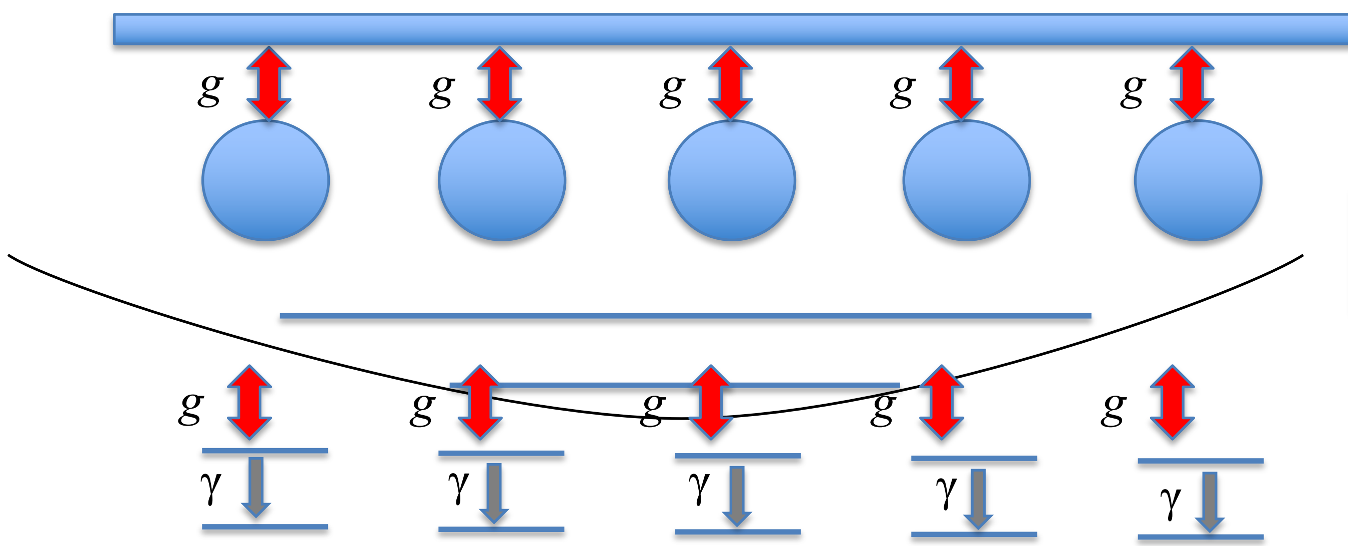
The strong coupling is necessary condition to manipulate with quantum states; to exchange information between different systems.

Possible geometries of quantum systems



$g >> \gamma$

Qubits coupled to quantum baths



$g >> \gamma$

Two-level atom in a resonator

Strong coupling:

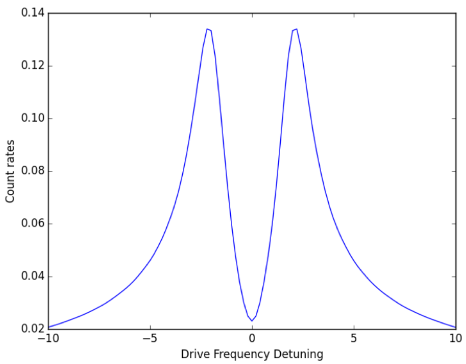
$\kappa = 0.5,\ \Gamma_1 = 0.5,\ g = 1$

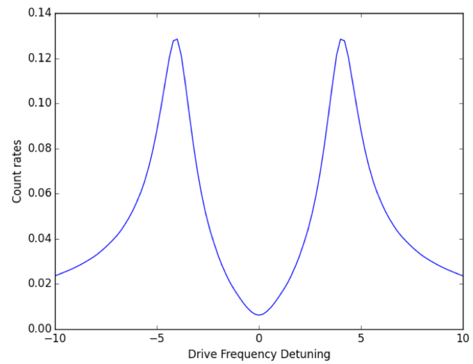
Strong coupling:

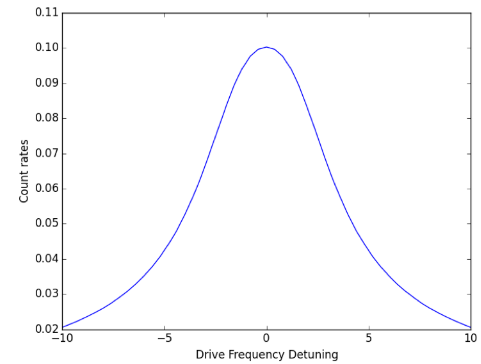
$\kappa = 0.5,\ \Gamma_1 = 0.5,\ g = 2$

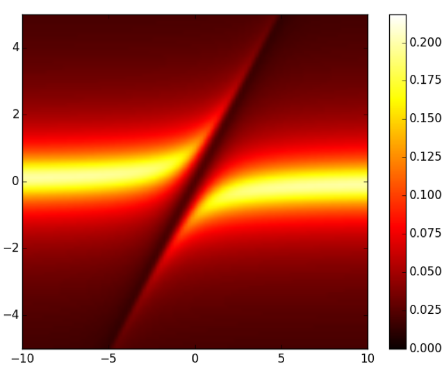
Weak coupling:

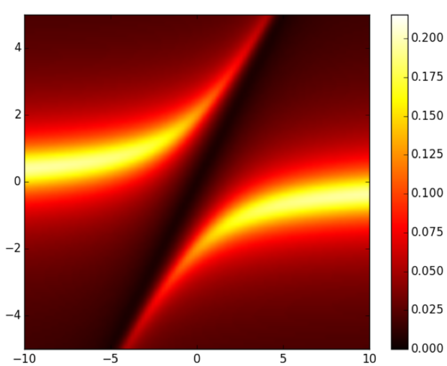
$\kappa = 0.5,\ \Gamma_1 = 4,\ g = 1$

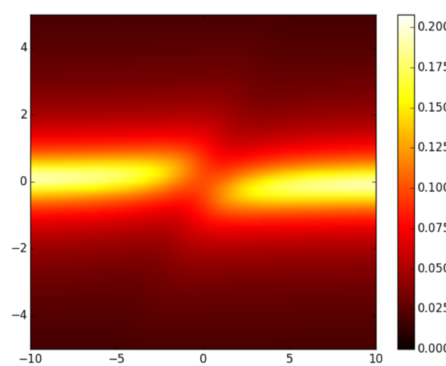






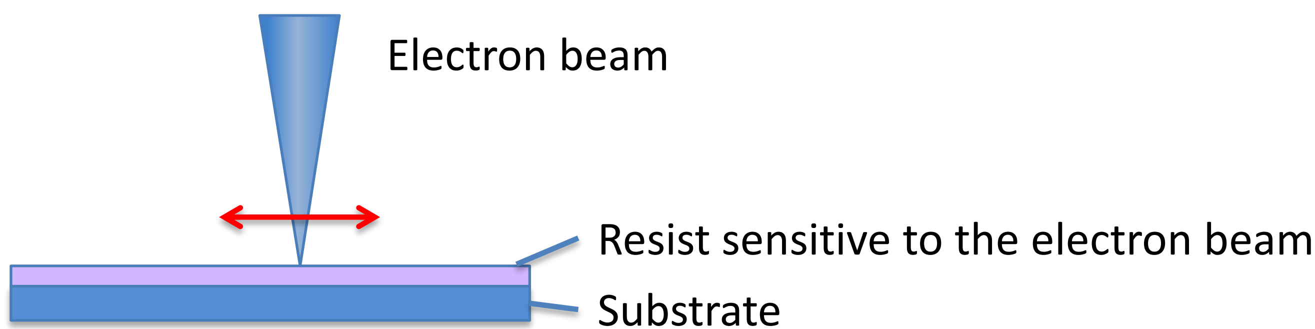




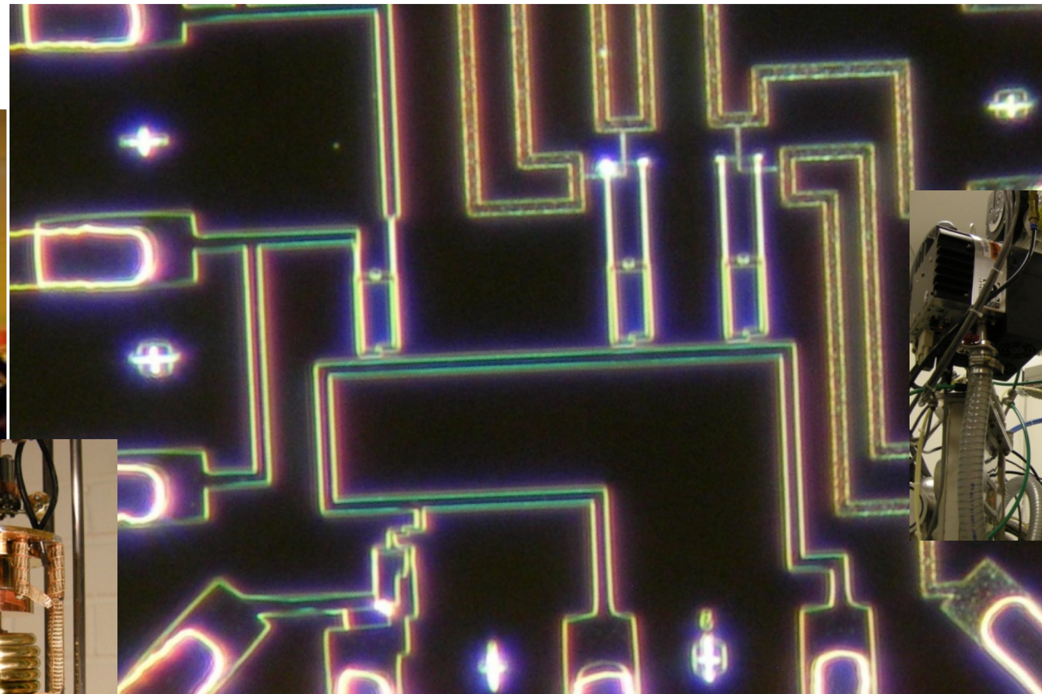


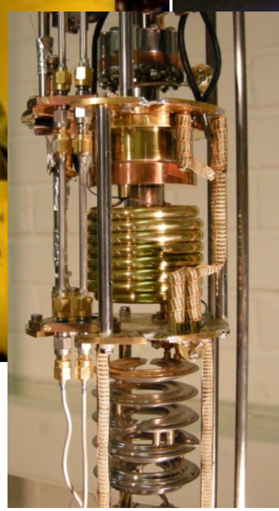
Sample Fabrication

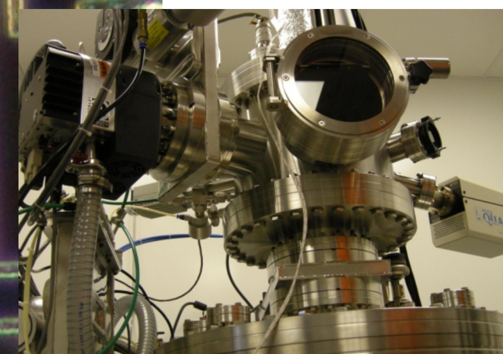
Electron-beam lithography systems (EBL)

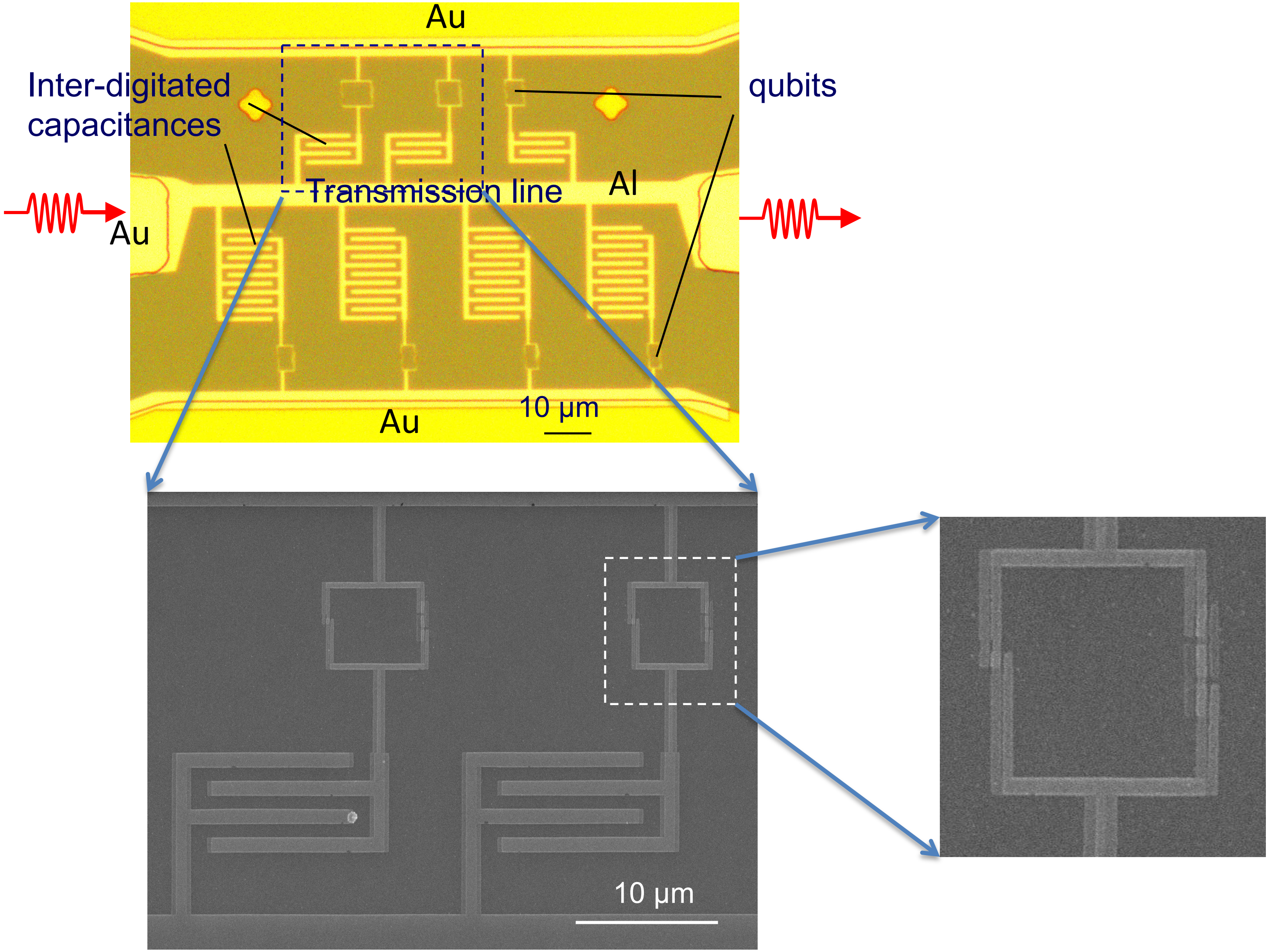


Hybrid nanostructures fabrication








Au
Inter-digitated
capacitances
qubits
Transmission line
Al
Au
Au
10 μm
10 μm

Lithography

Resists are organic materials, which change their properties under deep UV or electron beams

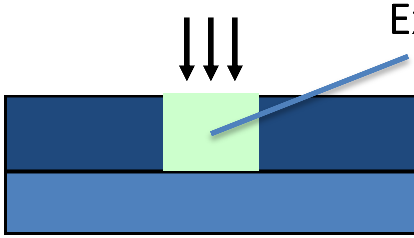
Positive resists: Exposed areas are washed away during development processes

Negative resists: Unexposed areas are washed away during development processes

Positive resist processing

Exposure

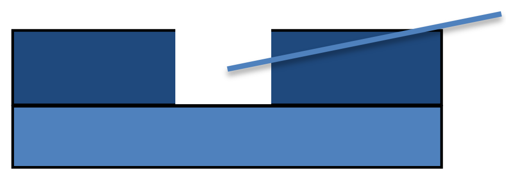
Exposed resist



Spin coating

Thin film of resist (0.05 – 1 µm)

Development in liquids (developers)

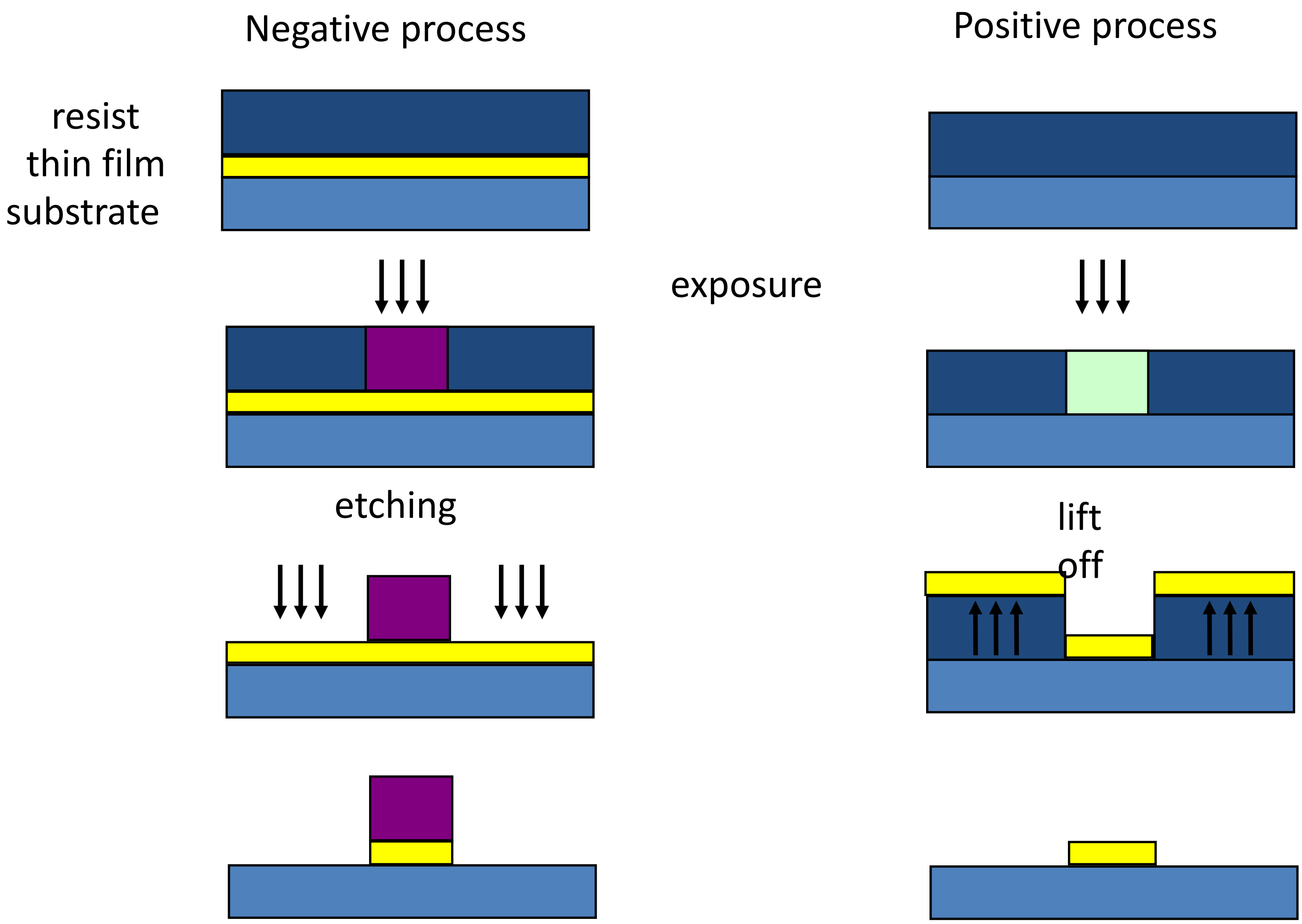


Developed resist is washed away after development

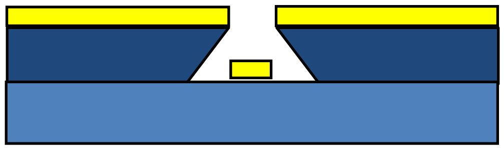
The wafer is ready for metal deposition or etching

Resolution:
Photo-lithography: 0.5 µm
EB-lithography: 10 nm

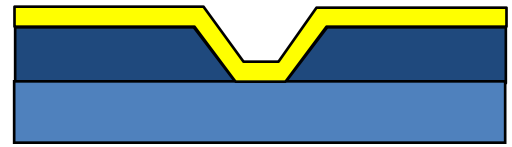
Lithography steps



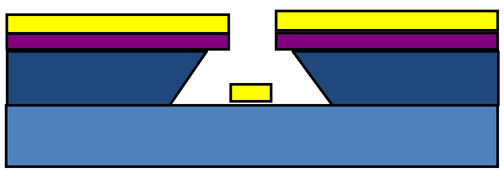
undercut



overcut



double layers resist



PMMA / Copolimer e-beam resists
S1813 / LOR5B photoresists

Bottom layer “softer’’ than top layer
Selective developers

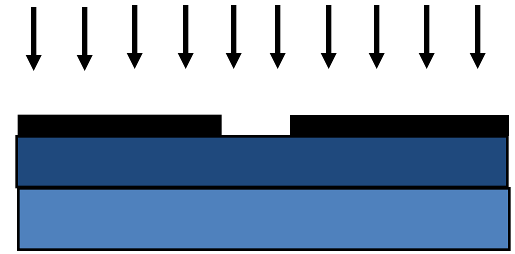
Photolithography

UV light changes the properties of the

resist in the openings of the mask

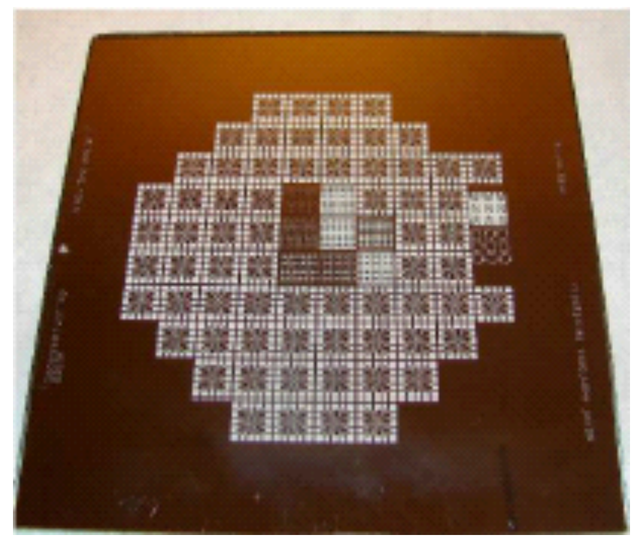
Resist mask formed after development.

This allows the substrate to be selectively exposed to etching or deposition



Wavelength < 1μm for Hg lamps;
180-280nm for deep UV lamps

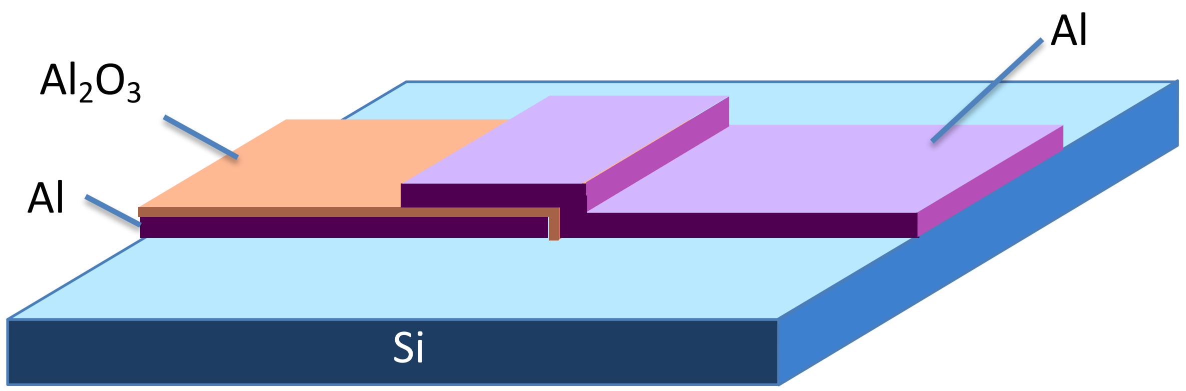
Direct printing resolution $w=(k\ \lambda\ g)^{1/2}$
k constant depending on resist
λ wavelength
g gap between resist and mask



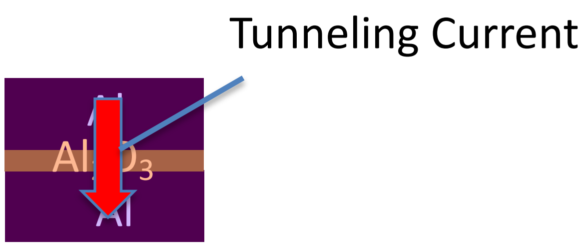
Photomask for 3 inch wafer

Fabrication of Josephson junctions

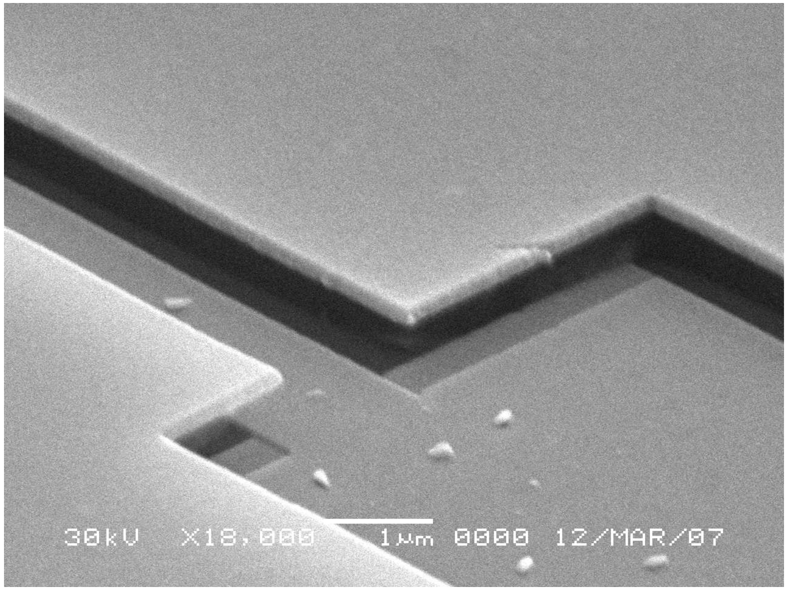
1. Bare silicon substrate
2. Aluminium deposition
3. Aluminium oxidation
4. Aluminium deposition (layer 2)

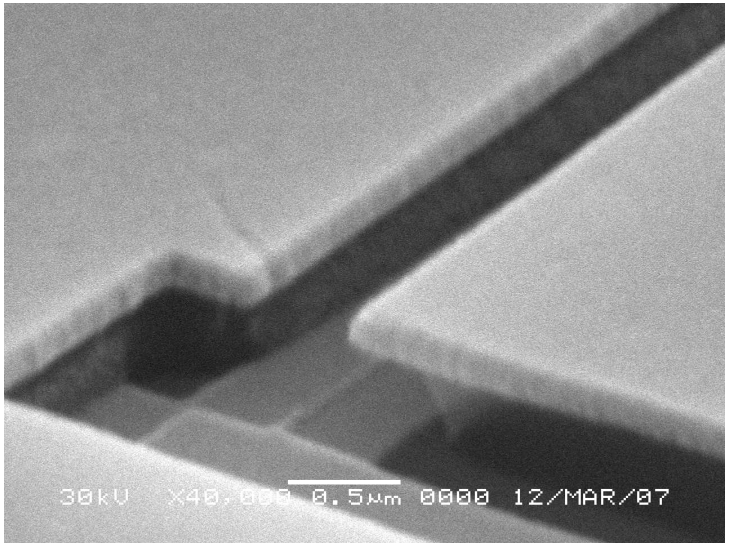


Crossection of the Josephson junction

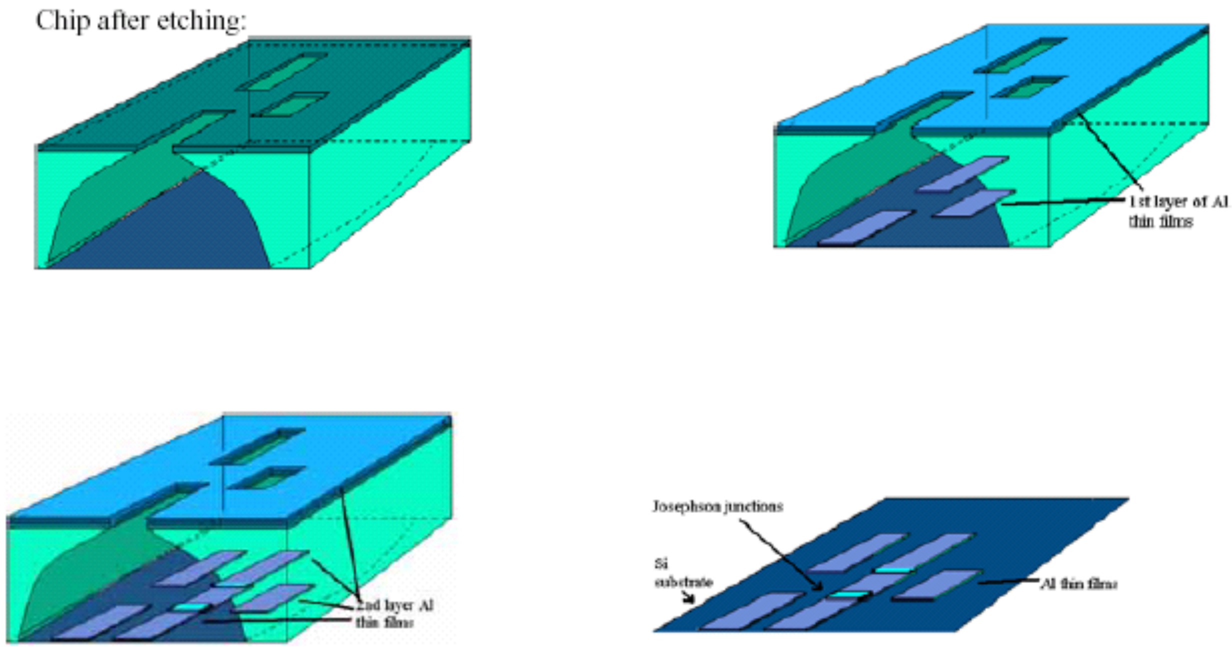


Au film pictured after evaporation on double layer resist



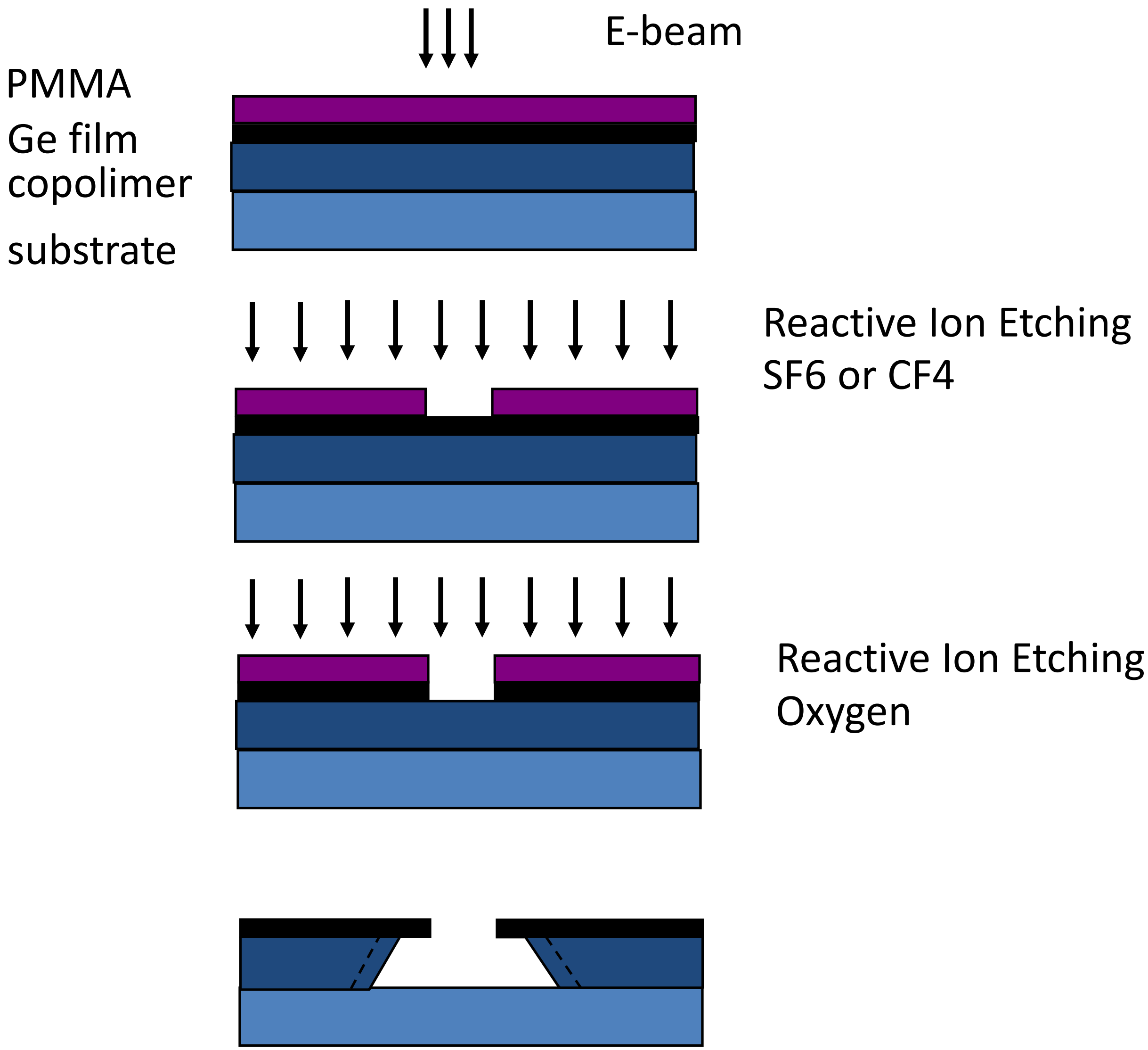


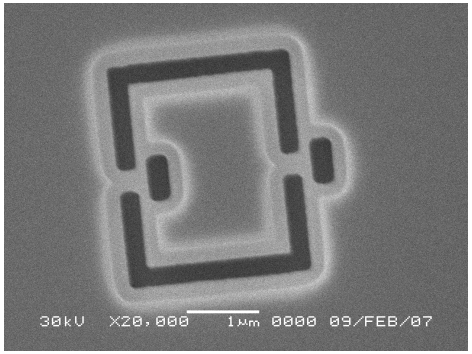
Angular evaporation + oxidation

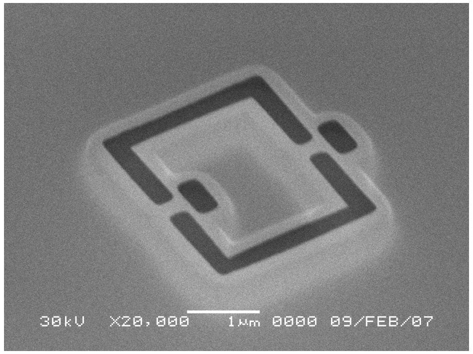


Soft mask
PMMA / Copolimer double layer resist
ZEP / Copolimer resist layer

Hard mask


E-beam
PMMA
Ge film
copolimer
substrate
Reactive Ion Etching
SF6 or CF4
Reactive Ion Etching
Oxygen

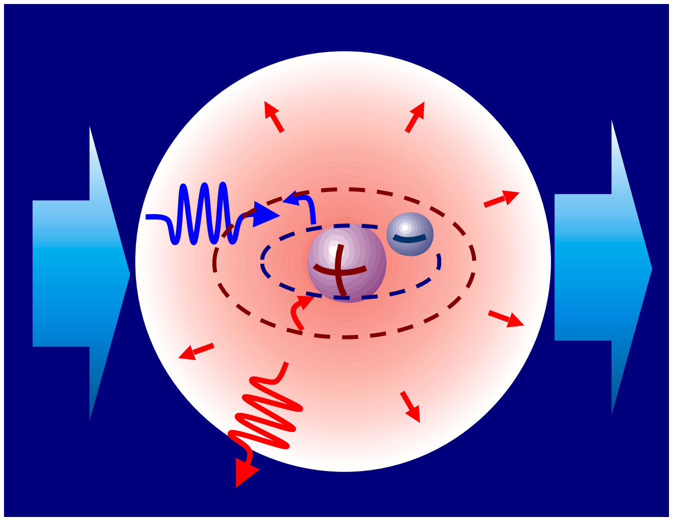

30kV X20,000 1µm 0000 09/FEB/07

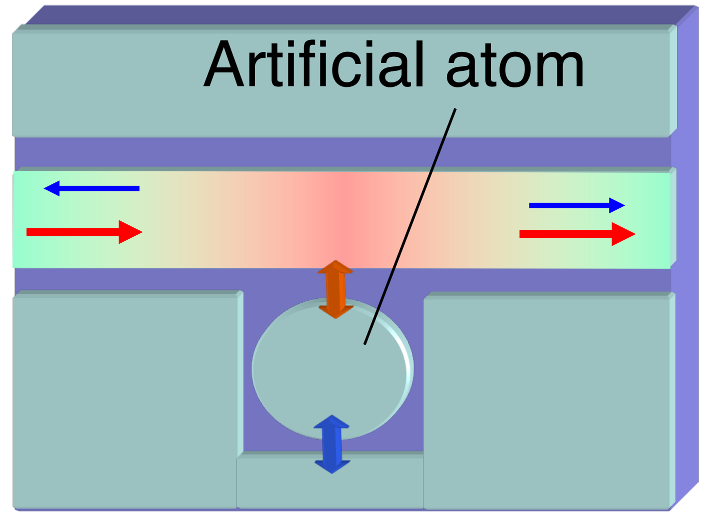

30kV X20,000 1µm 0000 09/FEB/07

Artificial atom in the open space

Atom in open space

Light scattering by an atom





Artificial atoms are strongly coupled to electromagnetic waves
Natural atoms are weakly coupled to electromagnetic waves (weak scattering)

Strong scattering of propagating waves

A series of very promising applications

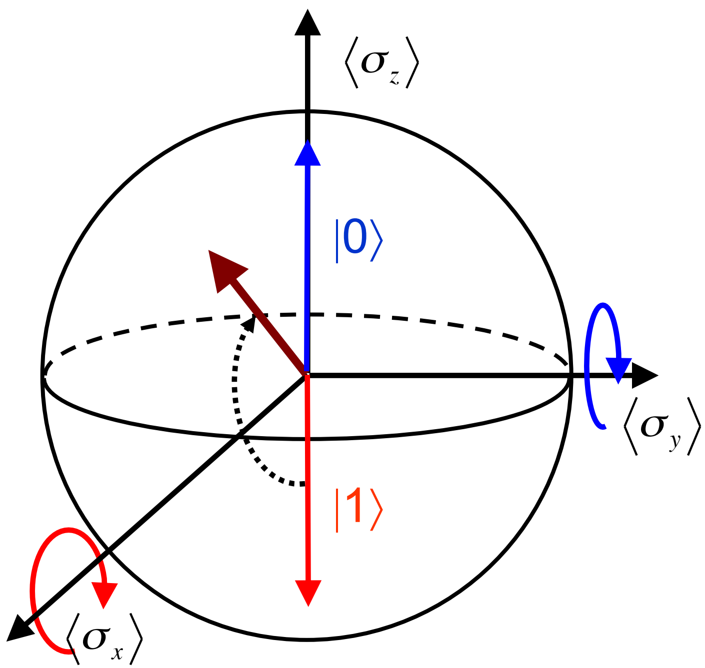
O. Astafiev, A. M. Zagoskin, A.A. Abdumalikov, Yu. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J.S. Tsai. Resonance fluorescence of a single artificial atom. *Science* **327** (2010).

Bloch Sphere for dissipative spin dynamics

$$H = -\frac{\hbar\omega_a}{2}\sigma_z + \hbar\Omega\cos(\omega t + \varphi) \qquad H = \frac{\hbar\delta\omega}{2}\sigma_z + \frac{\hbar\Omega}{2}(\sigma_x\cos\varphi + \sigma_y\sin\varphi) \qquad \delta\omega = \omega - \omega_a$$

$$\frac{\partial}{\partial t}\begin{pmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{pmatrix} = \begin{pmatrix}-\Gamma_2 & -\delta\omega & \Omega\sin\varphi\\ \delta\omega & -\Gamma_2 & \Omega\cos\varphi\\ -\Omega\sin\varphi & -\Omega\cos\varphi & -\Gamma_1\end{pmatrix}\begin{pmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{pmatrix} + \begin{pmatrix}0\\ 0\\ \Gamma_1\end{pmatrix} \qquad \frac{d\vec{\sigma}}{dt} = B\vec{\sigma} + \vec{b}$$

$$\vec{\sigma}(t) = e^{Bt}\vec{\sigma}(0) + B^{-1}\left(e^{Bt} - 1\right)\vec{b}$$

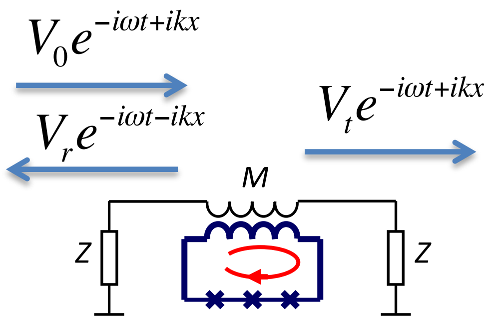


$$\langle\sigma_x\rangle = -\frac{\Omega}{\Gamma_2}\frac{\delta\omega/\Gamma_2}{1 + (\delta\omega/\Gamma_2)^2 + \Omega^2/\Gamma_1\Gamma_2}$$

$$\langle\sigma_y\rangle = -\frac{\Omega}{\Gamma_2}\frac{1}{1 + (\delta\omega/\Gamma_2)^2 + \Omega^2/\Gamma_1\Gamma_2}$$

$$\langle\sigma^-\rangle = \left\langle\frac{\sigma_x + i\sigma_y}{2}\right\rangle = -\frac{\Omega}{2\Gamma_2}\frac{1 + i\delta\omega/\Gamma_2}{1 + (\delta\omega/\Gamma_2)^2 + \Omega^2/\Gamma_1\Gamma_2}$$

Bloch Sphere for dissipative spin dynamics



Classically $V = \dot{\Phi} = M\dot{I}$

$\langle V \rangle = \langle \dot{\phi} \rangle$ $\quad \phi_p = MI_p$

$$\langle V^- \rangle = -i\omega \langle \phi^- \rangle = -i\omega\phi_p \langle \sigma^- \rangle \qquad \langle V \rangle = \langle V^- \rangle + cc$$

$$r = -\frac{i\omega\phi_p \langle \sigma^- \rangle}{2I_0 Z_0}$$

$$1 - r = t$$

$$I_0 - I_r = I_t \qquad I_0(1-r) = I_0 t$$

$$V_0 + V_r = V_t + \langle V \rangle \qquad V_0(1+r) = V_0 t + \langle V \rangle$$

$$1 + r = t + \frac{\langle V \rangle}{I_0 Z_0}$$

$$t = 1 + \frac{i\omega\phi_p \langle \sigma^- \rangle}{2I_0 Z_0}$$

$$\langle \sigma^- \rangle = -\frac{\Omega}{2\Gamma_2} \frac{1 + i\,\delta\omega/\Gamma_2}{1 + (\delta\omega/\Gamma_2)^2 + \Omega^2/\Gamma_1\Gamma_2} \qquad r = \frac{i\omega\phi_p}{2I_0 Z_0} \frac{\Omega}{\Gamma_2} \frac{1 + i\,\delta\omega/\Gamma_2}{1 + (\delta\omega/\Gamma_2)^2 + \Omega^2/\Gamma_1\Gamma_2}$$

$$r = \frac{i\omega\phi_p^2}{2\hbar\Gamma_2 Z_0} \frac{1 + i\,\delta\omega/\Gamma_2}{1 + (\delta\omega/\Gamma_2)^2 + \Omega^2/\Gamma_1\Gamma_2}$$

$$r = \frac{\Gamma_1}{2\Gamma_2} \frac{1 + i\delta\omega/\Gamma_2}{1 + (\delta\omega/\Gamma_2)^2 + \Omega^2/\Gamma_1\Gamma_2}$$

If $\delta\omega = 0$ and $\Gamma_2 = \frac{\Gamma_1}{2}$:

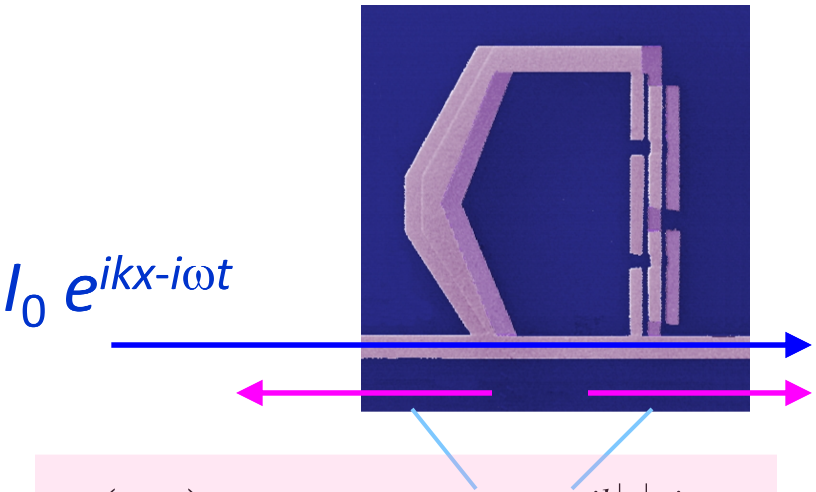
$r = 1,\ t = 0$

$$\hbar\Omega = \phi_p I_0 \qquad \Gamma_1 = \frac{2\phi_p^2\omega}{2\hbar Z_0}$$

$$t = 1 - \frac{\Gamma_1}{2\Gamma_2} \frac{1 + i\delta\omega/\Gamma_2}{1 + (\delta\omega/\Gamma_2)^2 + \Omega^2/\Gamma_1\Gamma_2}$$

At resonance and low drive full power is reflected

Driven artificial-atom in open space



$$I(x,t) = I_0 e^{ikx-i\omega t} + I_{sc} e^{ik|x|-i\omega t}$$

Wave equation:

$$\frac{\partial^2 I}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 I}{\partial t^2} = c\,\delta(x)\frac{\partial^2 \langle \phi \rangle}{\partial t^2}$$

Dipole moment:

$$\langle \phi(t) \rangle = \phi_p \langle \sigma^- \rangle e^{-i\omega t} \qquad \phi_p = M I_p$$

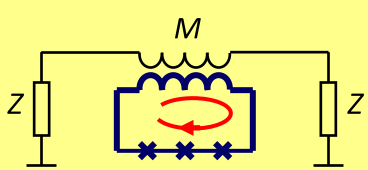
$$2ik\frac{I_{sc}}{2} = -\omega^2 c\,\phi_p \langle \sigma^- \rangle$$

RWA Hamiltonian:

$$H = -\frac{\hbar\delta\omega}{2}\sigma_z - \frac{\hbar\Omega}{2}\sigma_x \qquad \delta\omega = \omega - \omega_a$$

Relaxation produced by ohmic environment of Z-impedance:

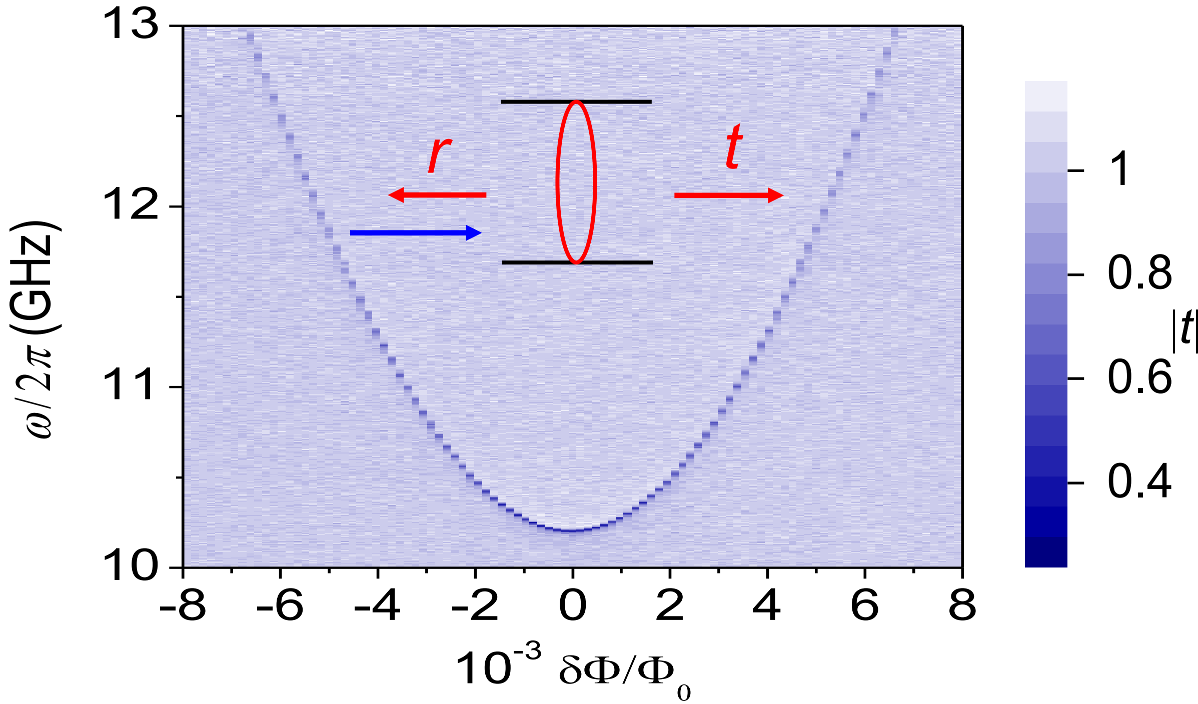
$$\Gamma_1 = \frac{\hbar\omega\phi_p^2}{\hbar^2 Z}$$



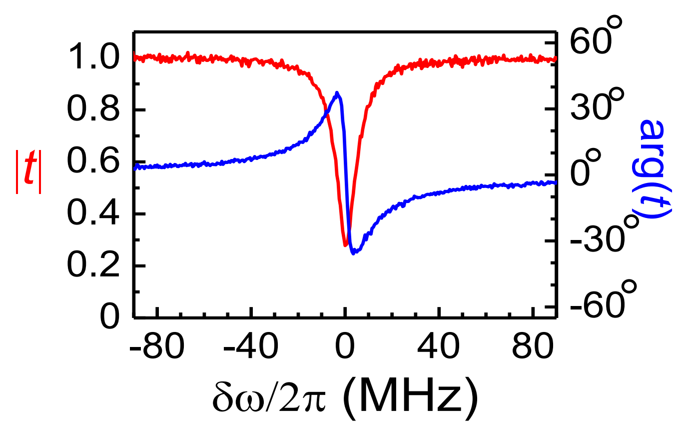
$$I_{sc}(x,t) = i\frac{\hbar\Gamma_1}{\phi_p}\langle \sigma^- \rangle e^{ik|x|-i\omega t}$$

$$r = \frac{I_{sc}}{I_0} = \frac{\Gamma_1}{\Gamma_2}\frac{1 + i\,\delta\omega/\Gamma_2}{1 + (\delta\omega/\Gamma_2)^2 + \Omega^2/\Gamma_1\Gamma_2}$$

Direct transmission spectroscopy
Elastic scattering (Rayleigh case)



Transmission at
the degeneracy point



Anomalous dispersion

$t = 1 - r$

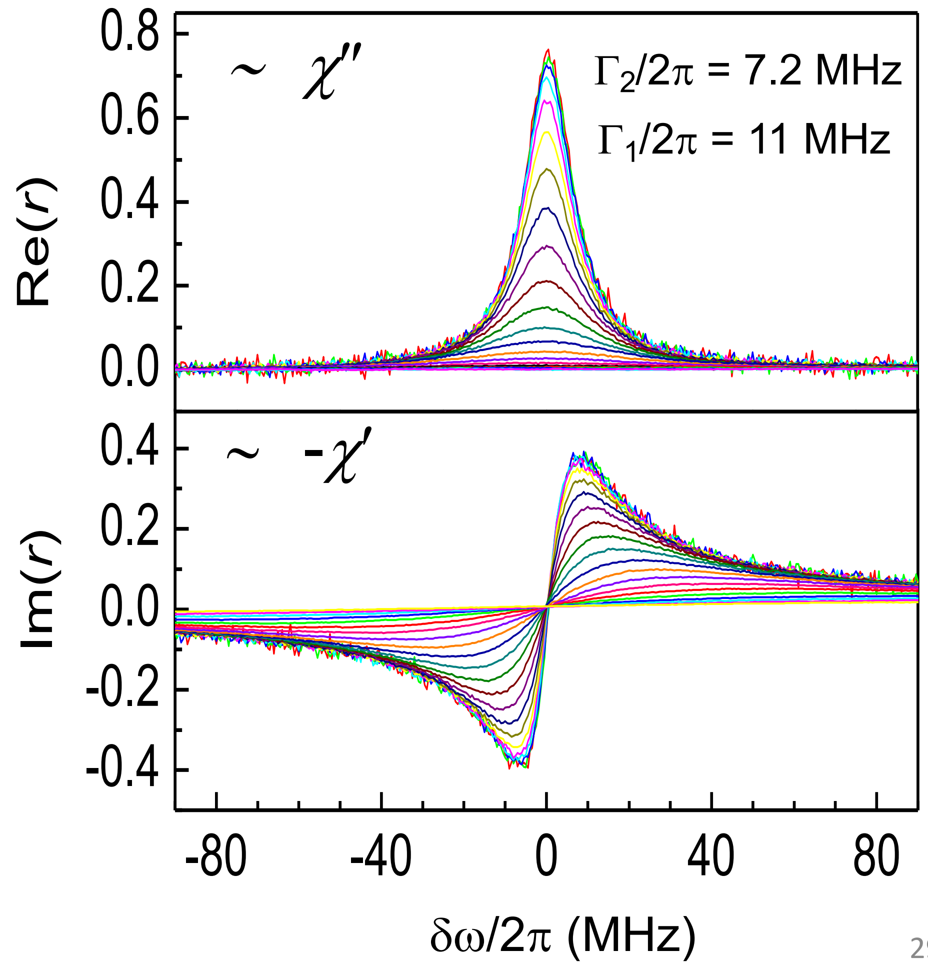
$$r = r_0 \frac{1 + i\delta\omega/\Gamma_2}{1 + (\delta\omega/\Gamma_2)^2 + \Omega^2/\Gamma_1\Gamma_2}$$

$$\Gamma_2 = \frac{\Gamma_1}{2} + \Gamma_\varphi$$

The peak height is the direct measure of dephasing:

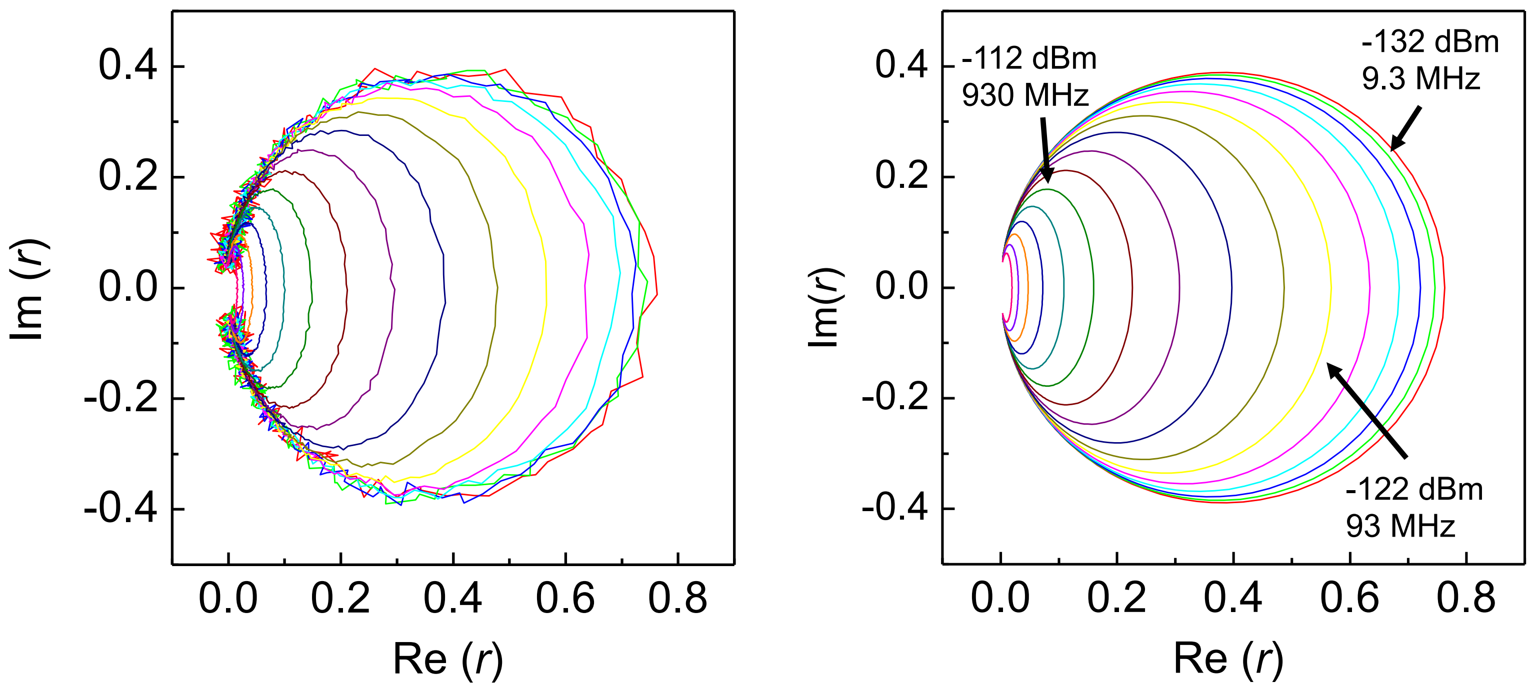
$$r_0 = \frac{\Gamma_1}{2\Gamma_2} = \frac{1}{1 + 2\Gamma_\varphi/\Gamma_2}$$

$$\chi = \chi' + i\chi''$$



- Strong interaction
- Anomalous dispersion
- Nonlinearity

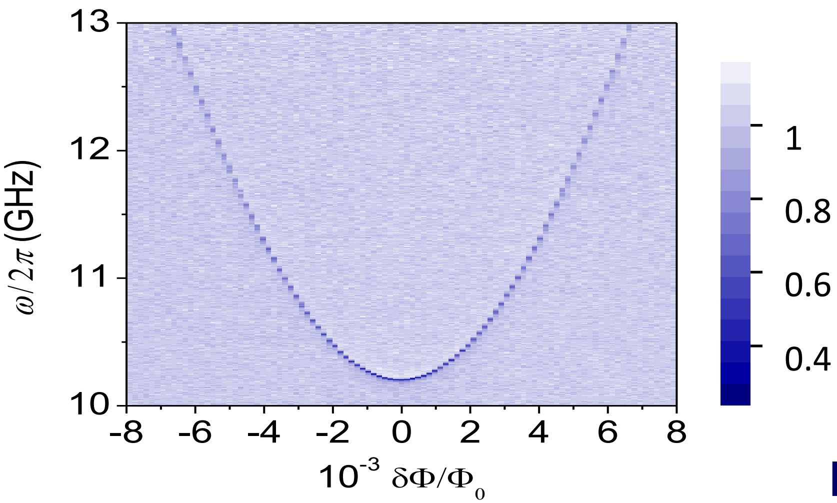
Smith chart of reflection coefficient vs power

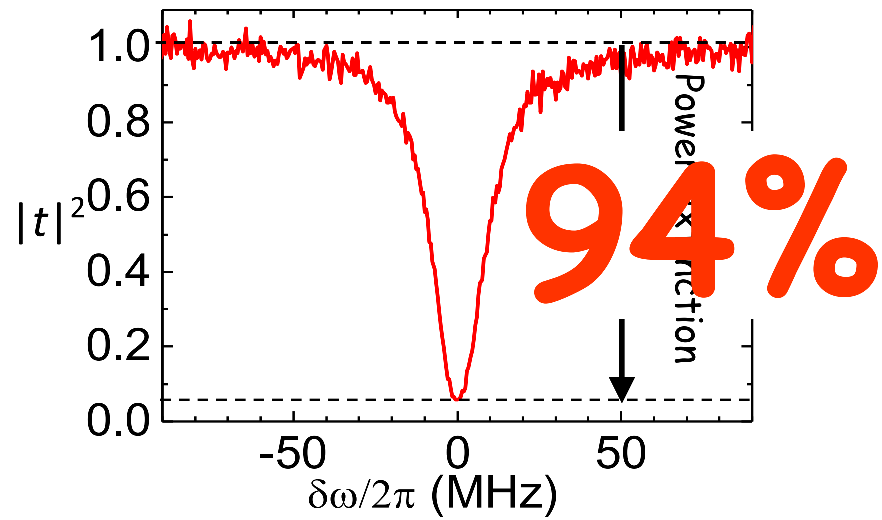


$$r = r_0 \frac{1 + i\delta\omega / \Gamma_2}{1 + \left(\delta\omega / \Gamma_2\right)^2 + \Omega^2 / \Gamma_1 \Gamma_2}$$

Resonance fluorescence: Extinction at the degeneracy point

Direct transmission spectroscopy





Previous record in optical systems was 12%

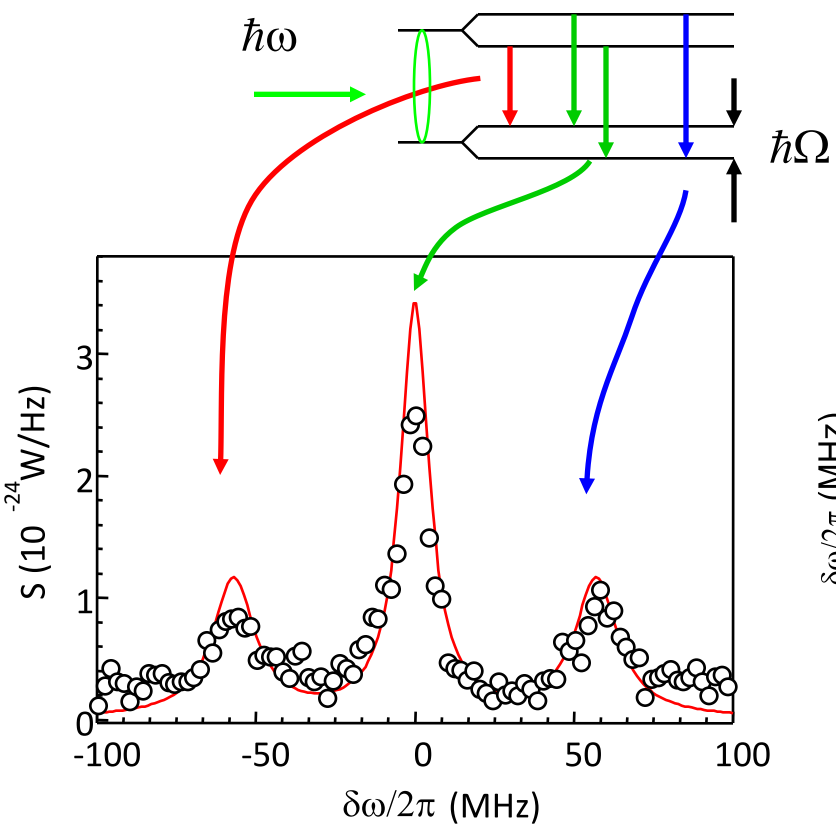
The artificial atom strongly interacts with modes of 1D open space

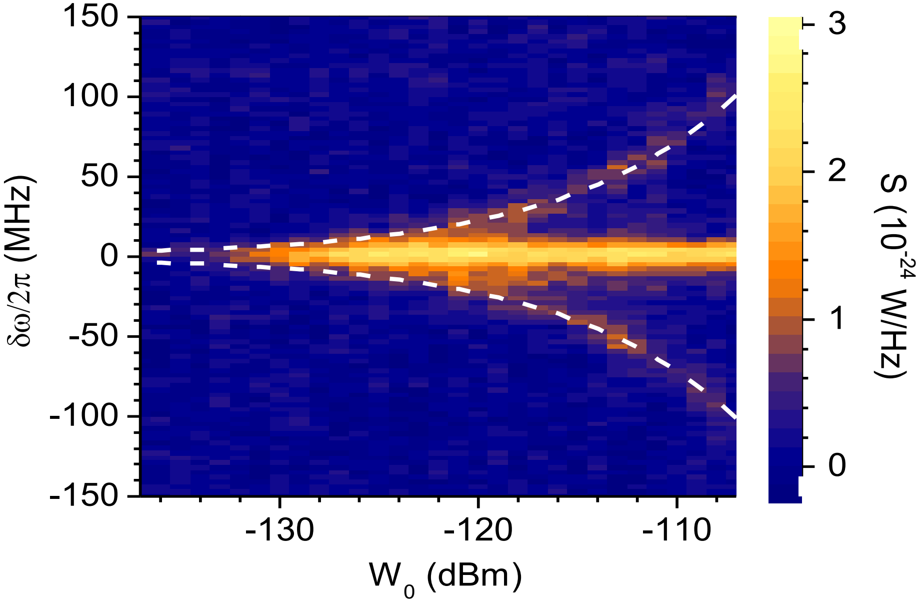
$\Downarrow$

Promising candidate for quantum information processing

O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov, Yu. A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J. S. Tsai. Resonance fluorescence of a single artificial atom. Science. 327 (2010).

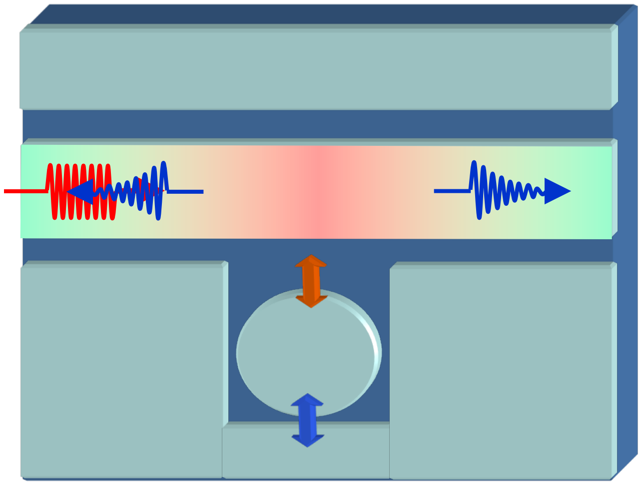
Resonance Fluorescence (inelastic scattering spectrum)



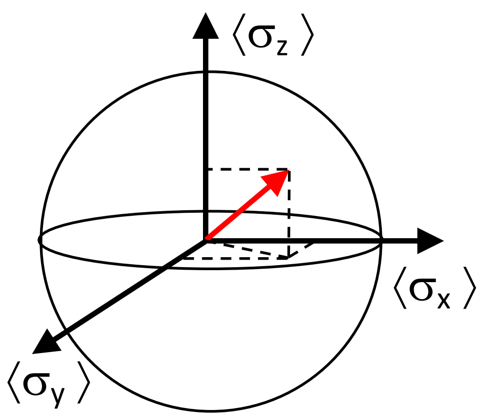


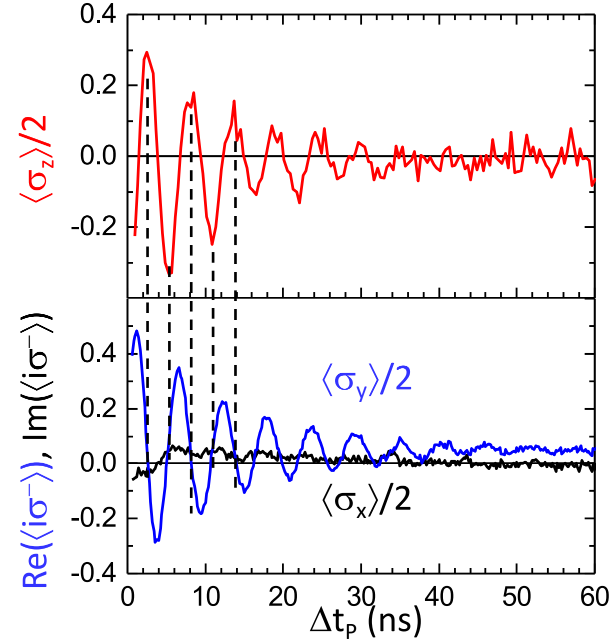
O. Astafiev, A. M. Zagoskin, A.A. Abdumalikov, Yu.A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J.S. Tsai.
Resonance fluorescence of a single artificial atom. Science. 327 (2010).

Towards photonic quantum computers: quantum state control



Pseudo-spin on Bloch sphere



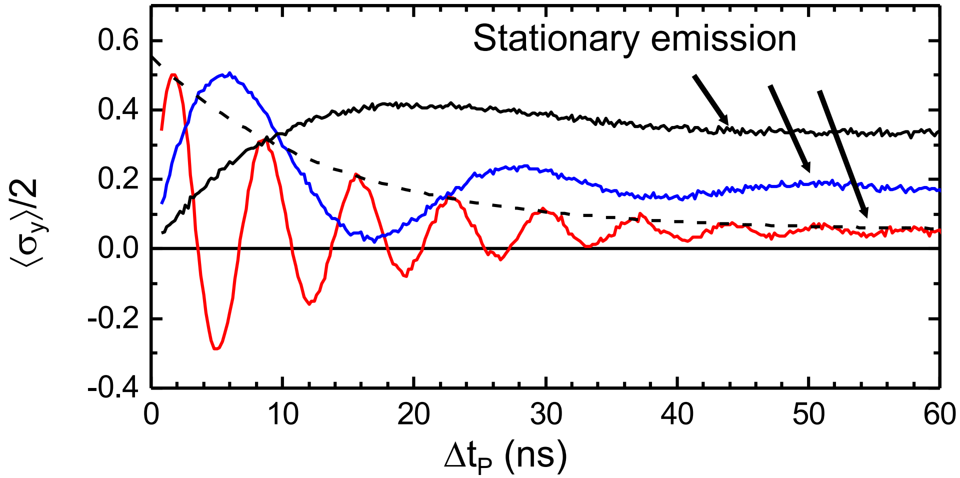


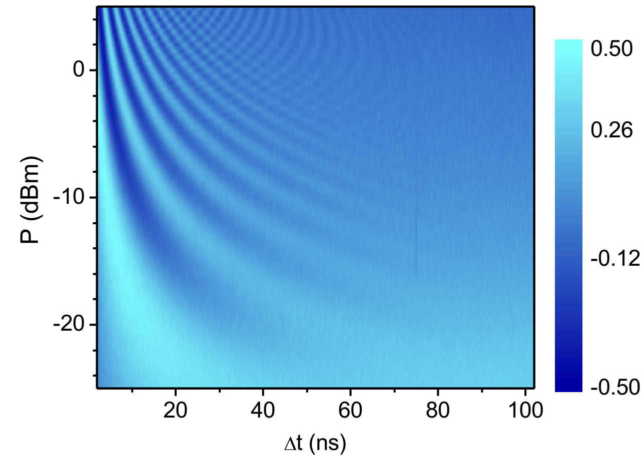
Strong coupling allows to measure simultaneously all three components of quantum states

A. A. Abdumalikov, Jr., O. V. Astafiev, Yu. A. Pashkin, Y. Nakamura, and J. S. Tsai. Dynamics of Coherent and Incoherent Emission from an Artificial Atom in a 1D Space. *Phys. Rev. Lett.* **107**, (2011).

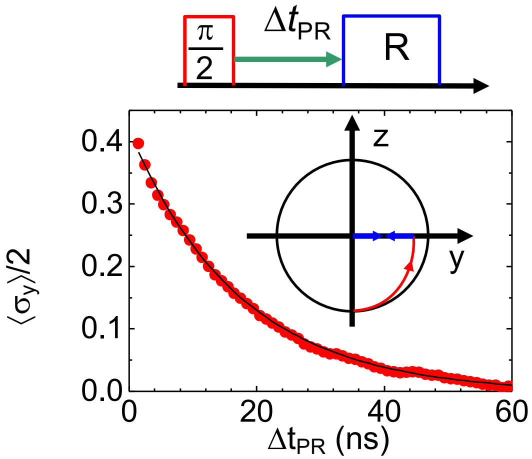
Dynamics of dipole emission

$$\langle \dot{\sigma}^- \rangle = -\gamma \langle \sigma^- \rangle - \frac{i\Omega}{2} \langle \sigma_z \rangle$$

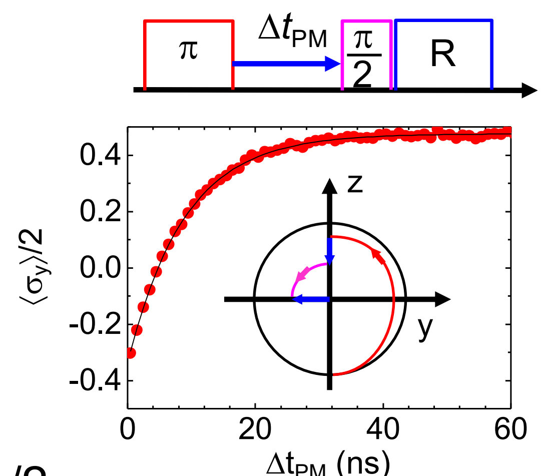




Phase relaxation



Energy relaxation

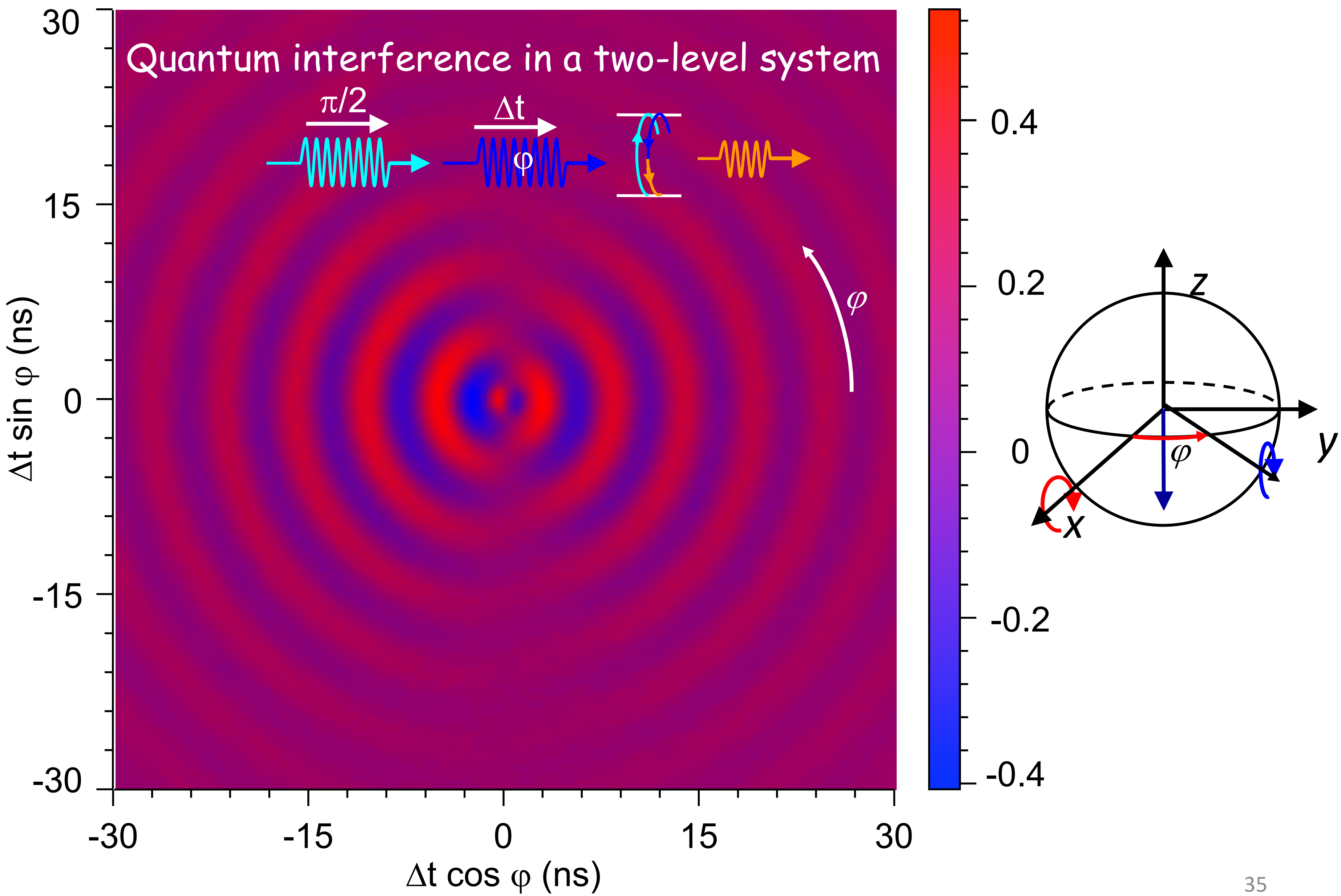


$\Gamma_2/2\pi = 9.1$ MHz

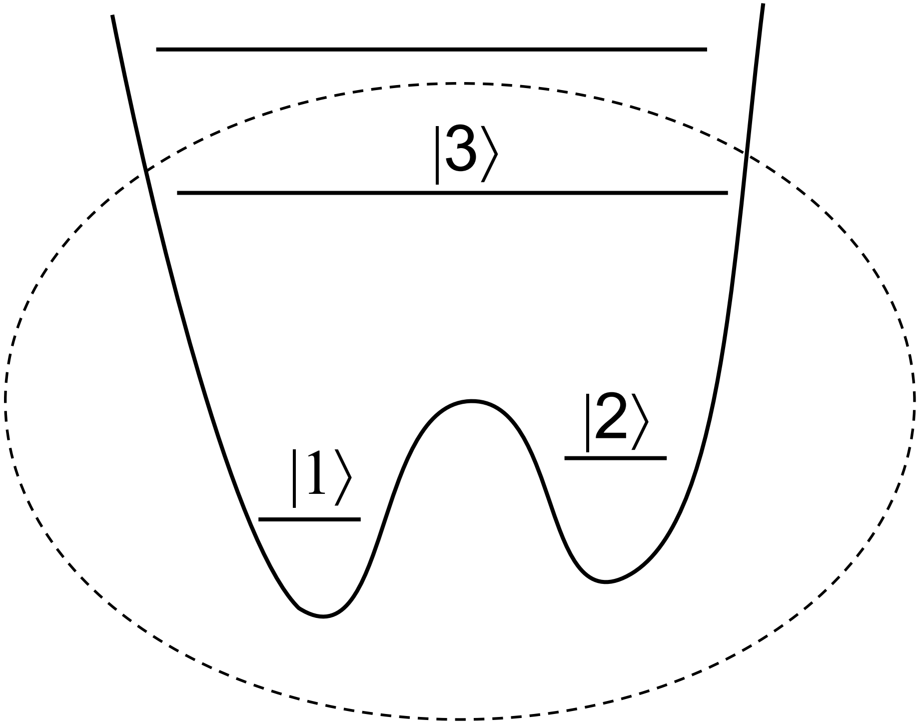
$\Gamma_2 = \Gamma_1/2$

$\Gamma_1/2\pi = 18$ MHz

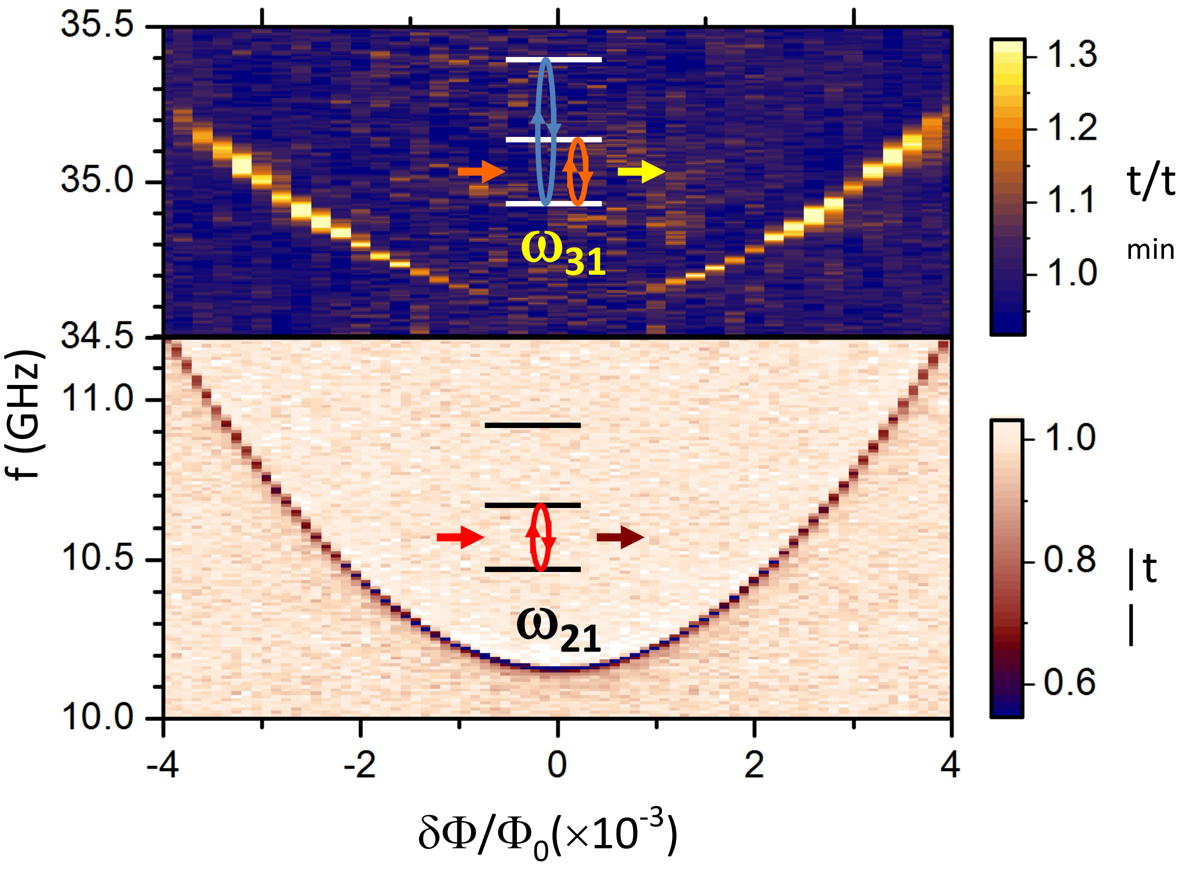
Quantum interference in a two-level system

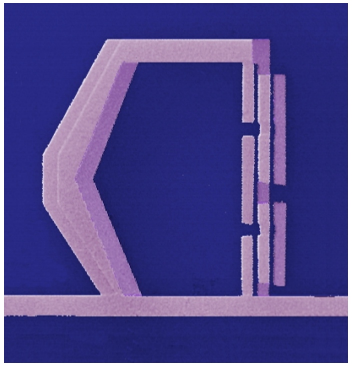


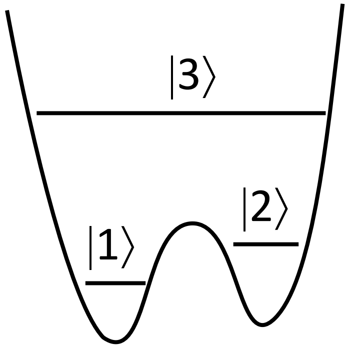
Three-level quantum system



Spectroscopy of three-level atom







Quantum optics on a single atom:

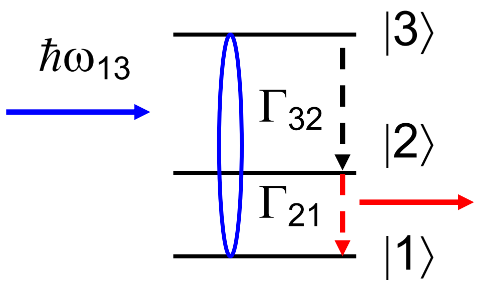
- Spontaneous emission
- Stimulated emission
- Quantum amplifier
- Electromagnetically induced transparency

Ultimate on-chip quantum amplifier

- Spontaneous emission
- Stimulated emission

O. Astafiev, A.A. Abdumalikov, A. M. Zagoskin, Yu.A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J.S. Tsai. Ultimate on-chip quantum amplifier. *Phys. Rev. Lett* 104, 183603 (2010).

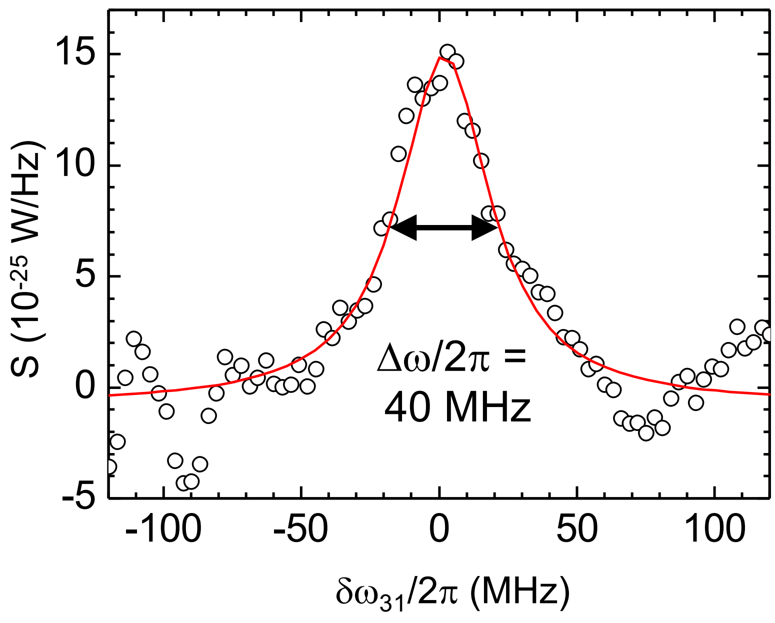
Spontaneous emission

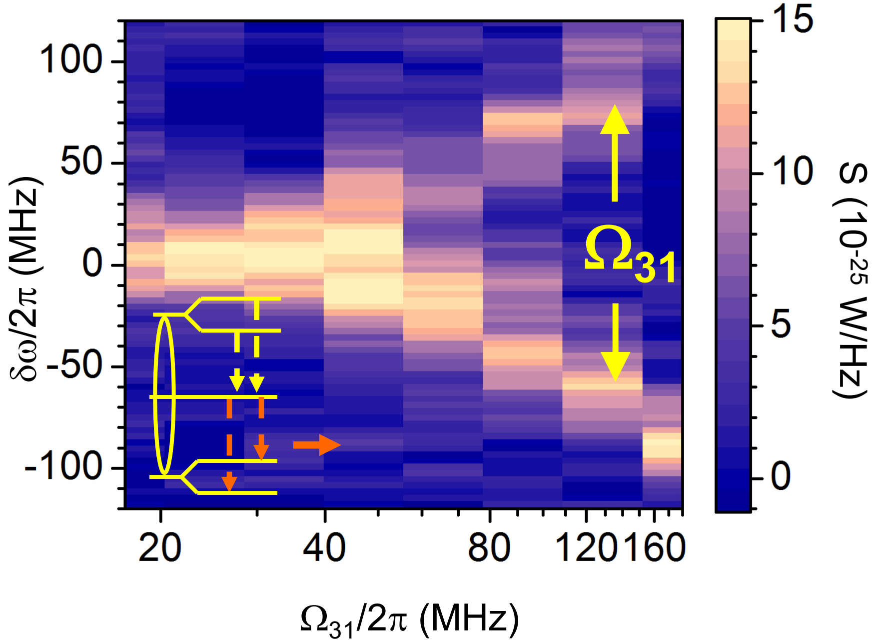


Noise spectral density (*weak driving limit*)

$$S(f) = \frac{\hbar\omega\Gamma_{21}}{2}\frac{\gamma_{21}}{\gamma_{21}^2 + \delta\omega^2}$$

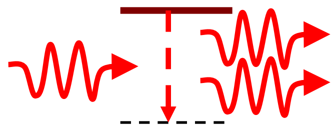
$\Omega_{31}/2\pi$ = 24 MHz

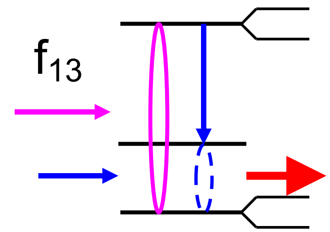


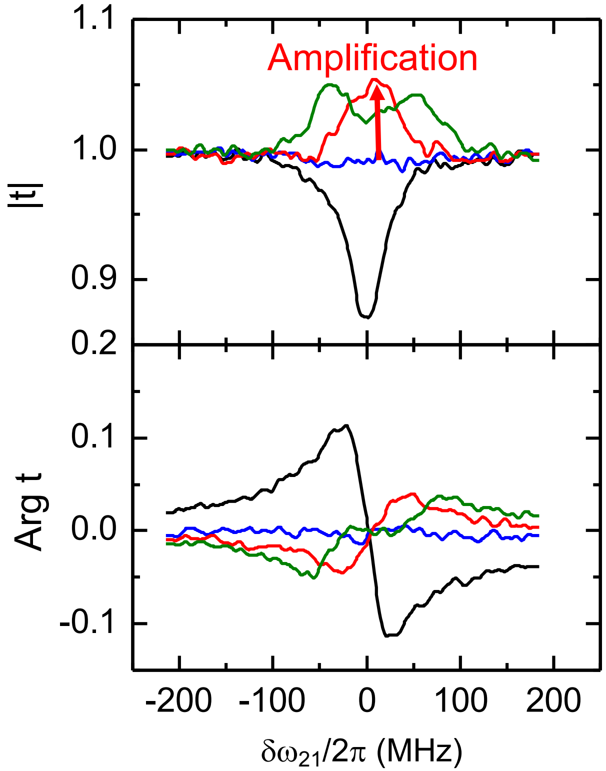


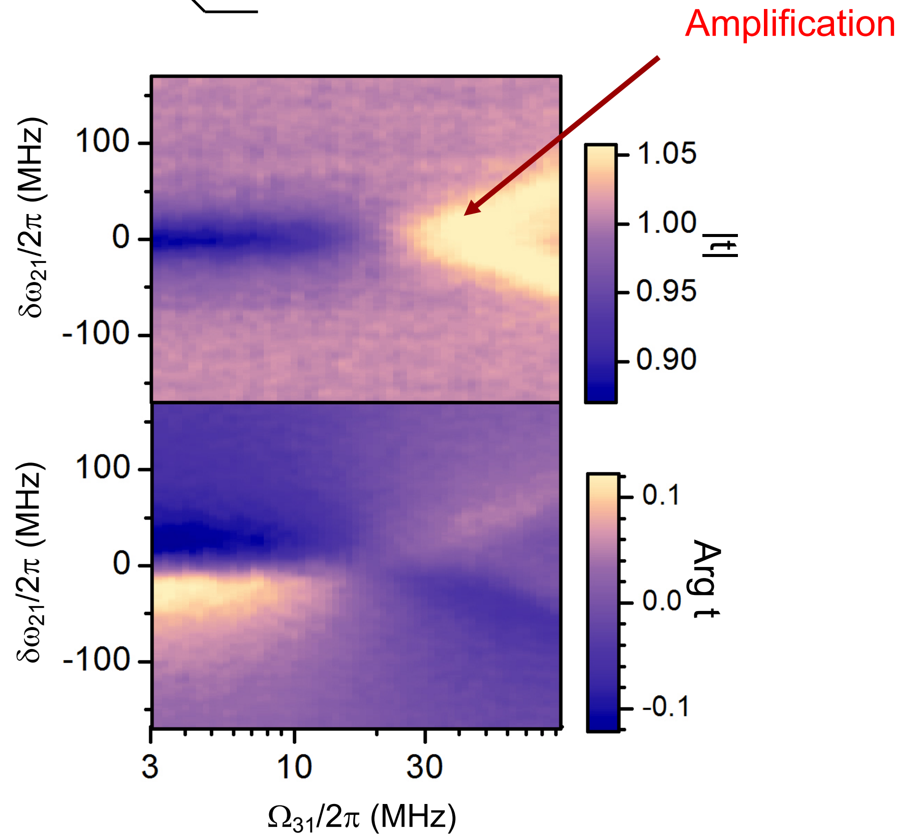
Noise level of the 4K amplifier is 10^{-22} W/Hz!

Stimulated emission

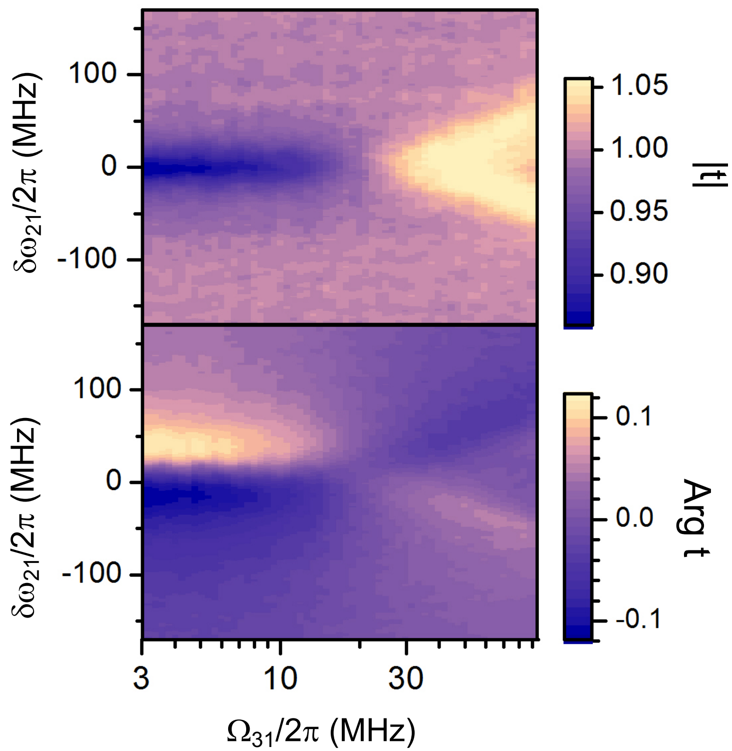




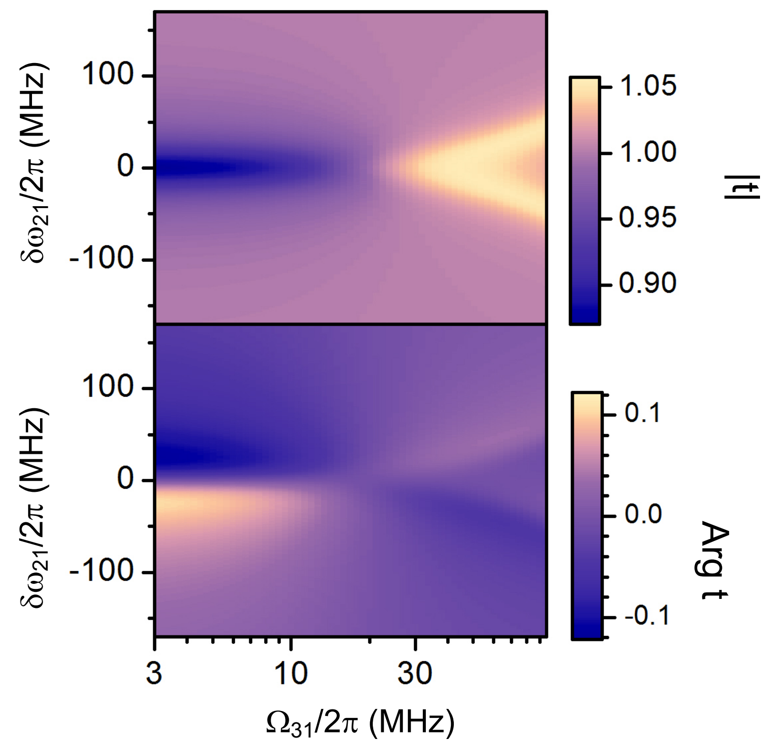




Measurements

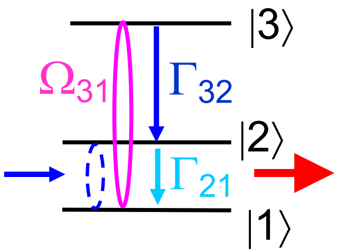


Calculations



$\Gamma_{21}/2\pi$ = 11 MHz $\qquad$ $\Gamma_{32}/2\pi$ = 35 MHz

$\Gamma_{32} >> \Gamma_{21}$



Transmission at resonance without pure dephasing

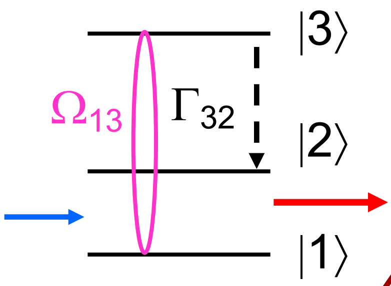
$$t \approx 1 + \frac{\rho_{22} - \rho_{11}}{1 + \Omega_{31}^2 / \left(\Gamma_{32}\Gamma_{21}\right)}$$

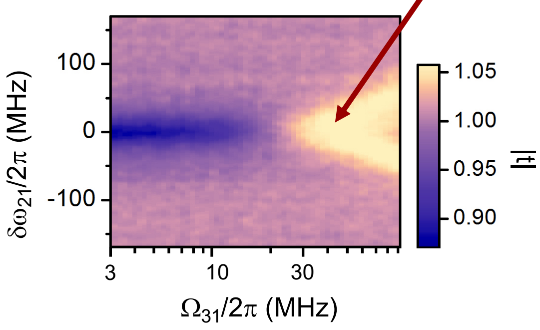
Maximum transmission

$$t_{\max} = \boxed{1\frac{1}{8}} \quad \text{at} \quad \Omega_{31}^2 = 3\Gamma_{32}\Gamma_{21}$$

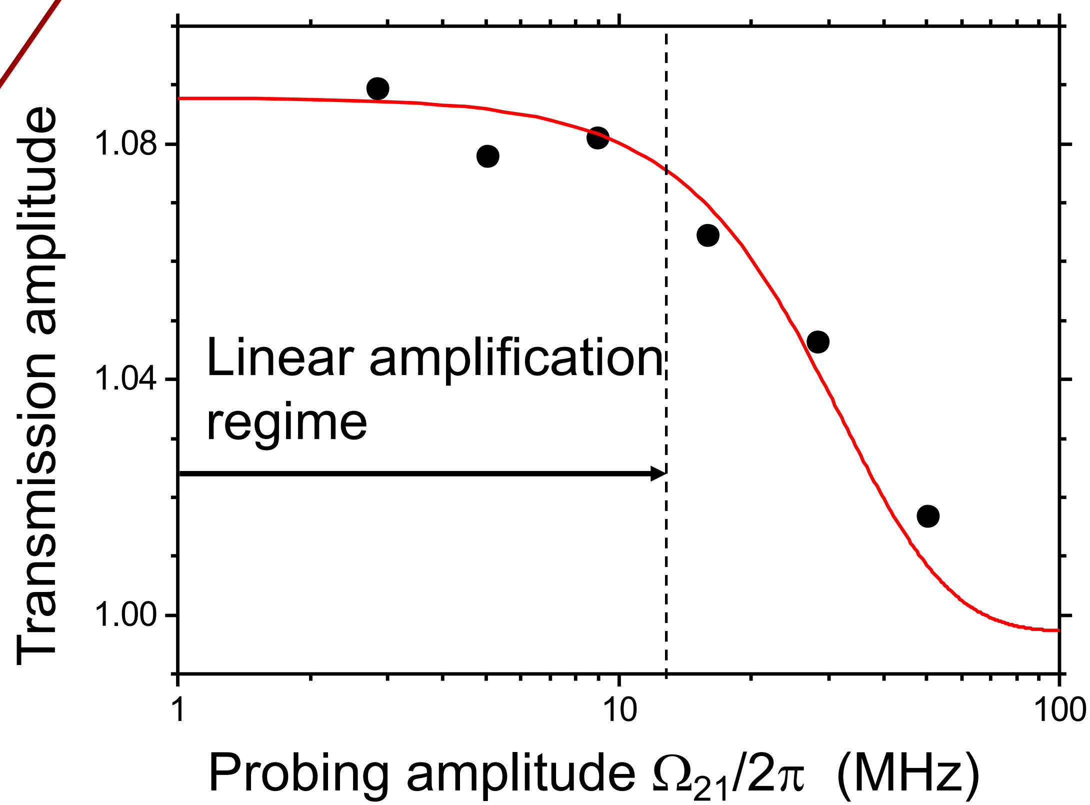
$$\rho_{11} = \frac{1}{4} \qquad \rho_{22} = \frac{3}{4}$$

Quantum amplifier gain

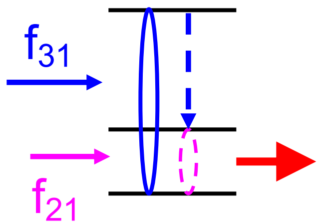


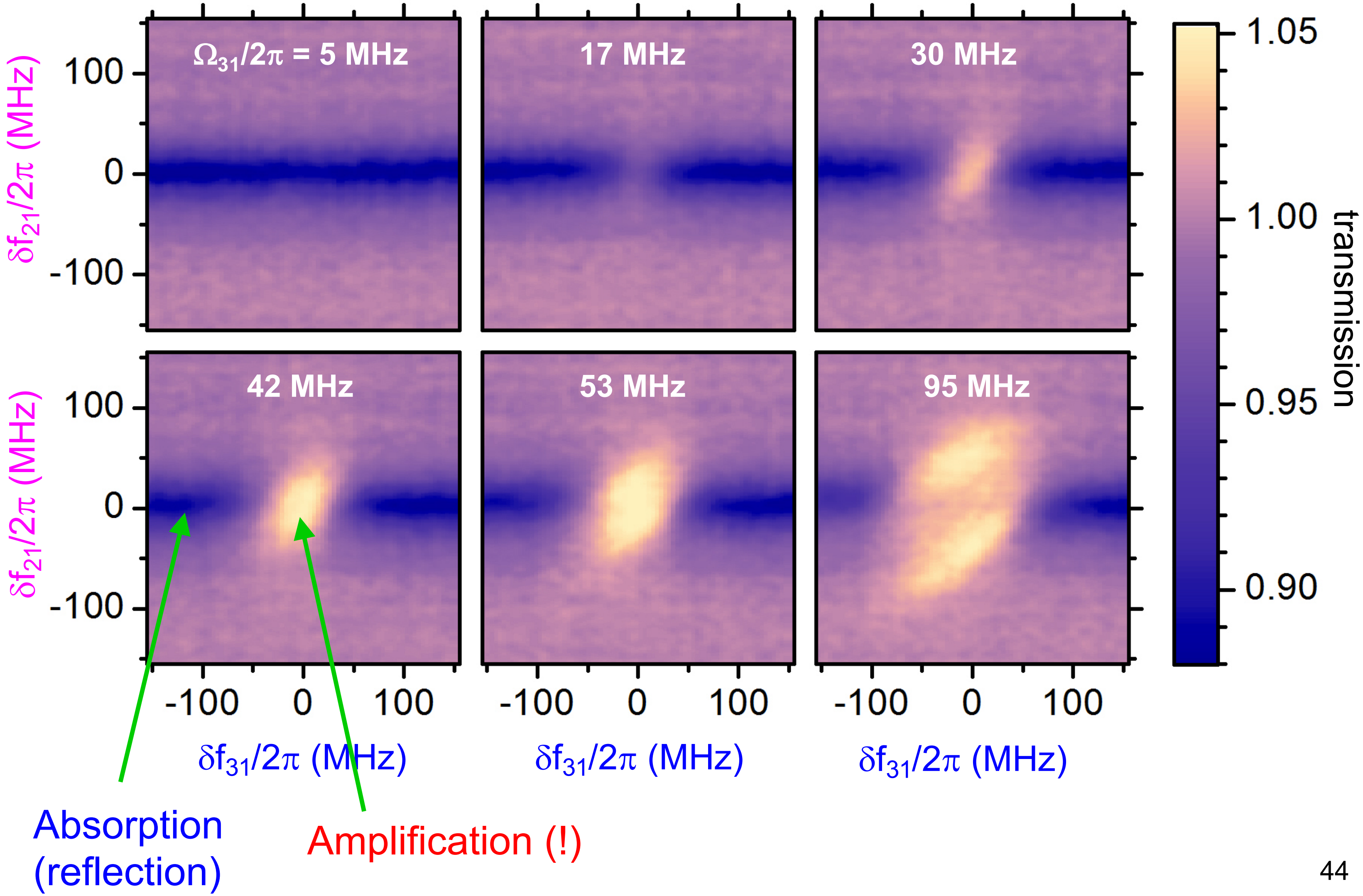


$\Omega_{31}/2\pi$ = 40 MHz

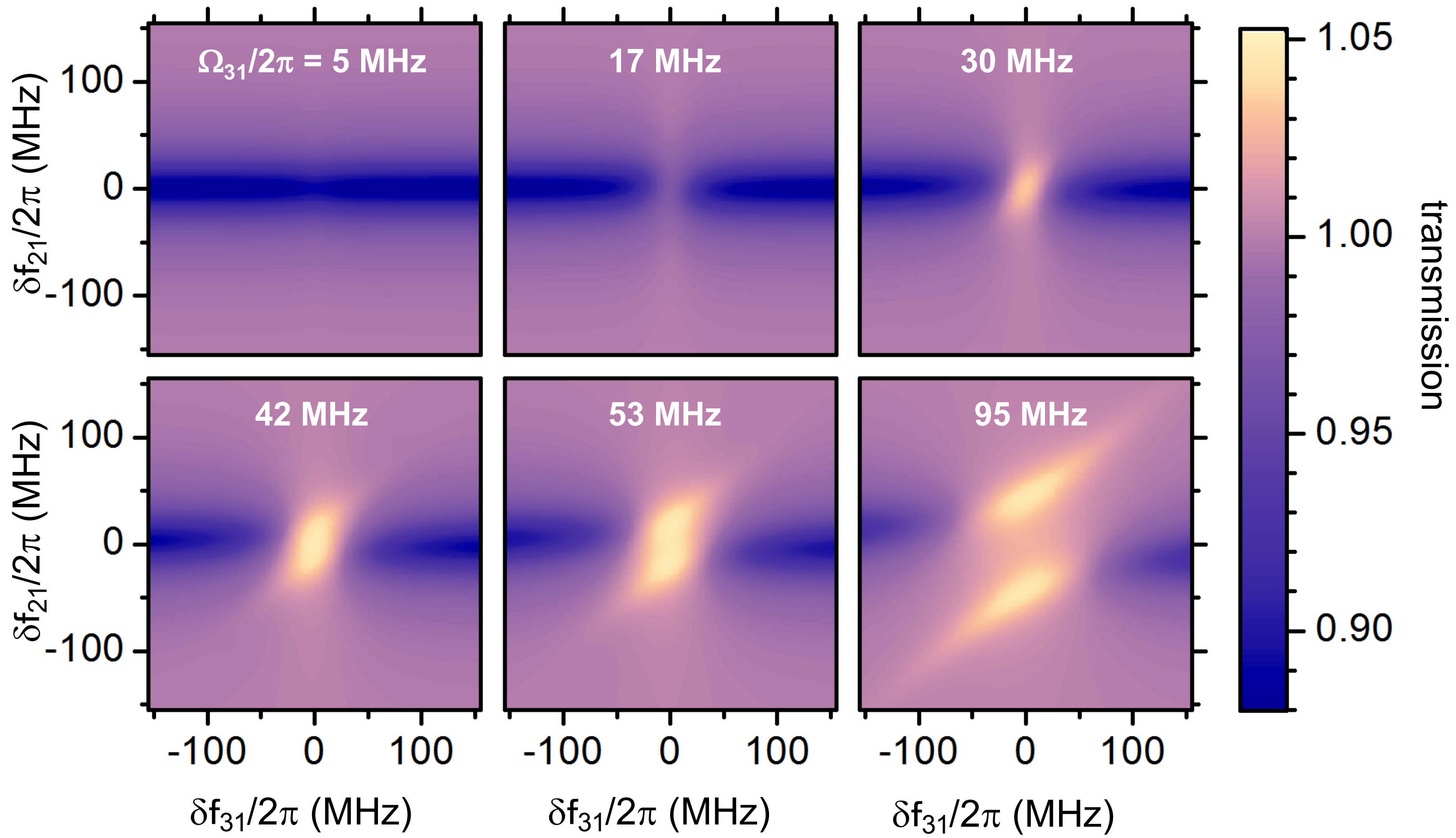


Stimulated emission

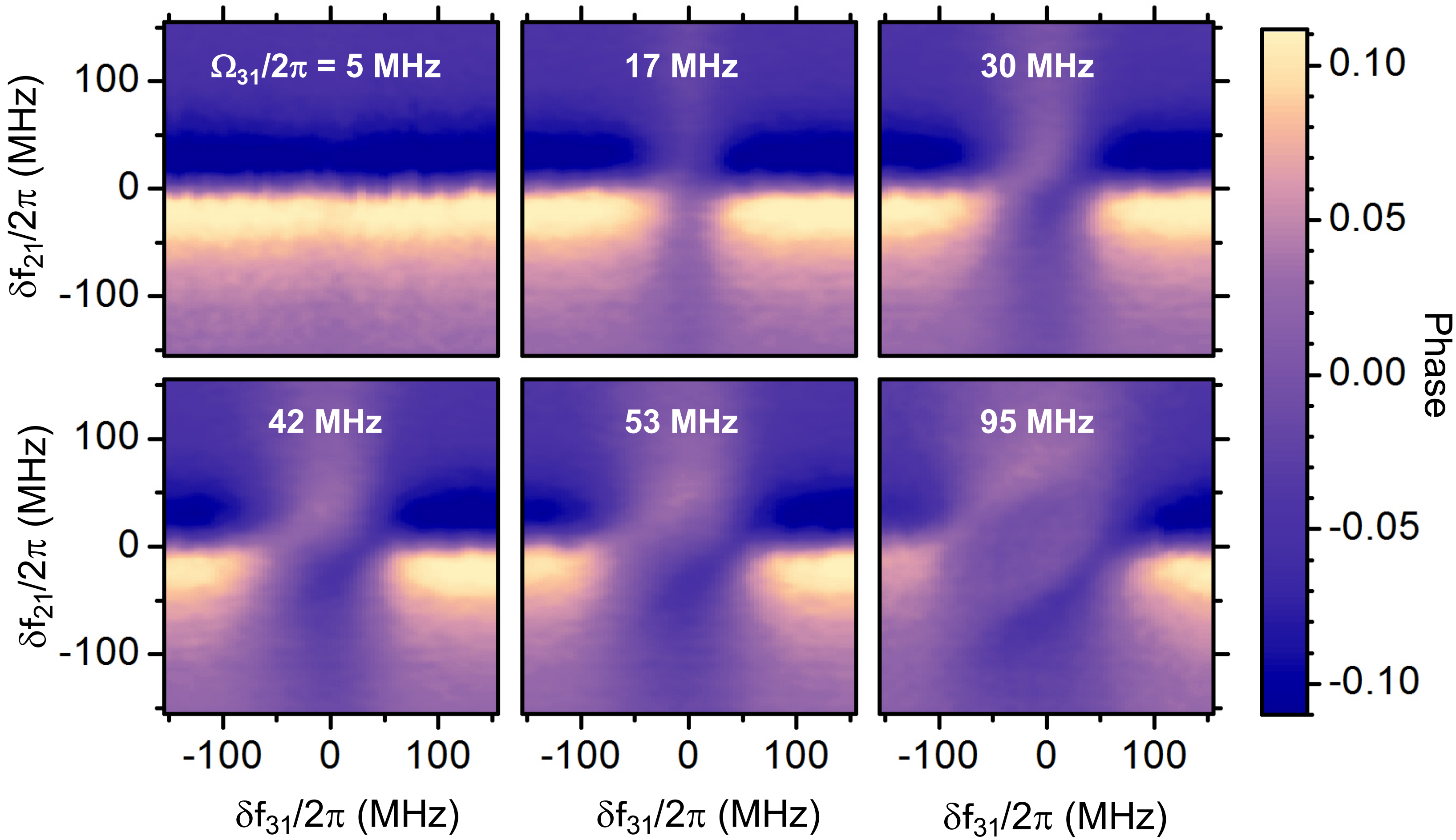




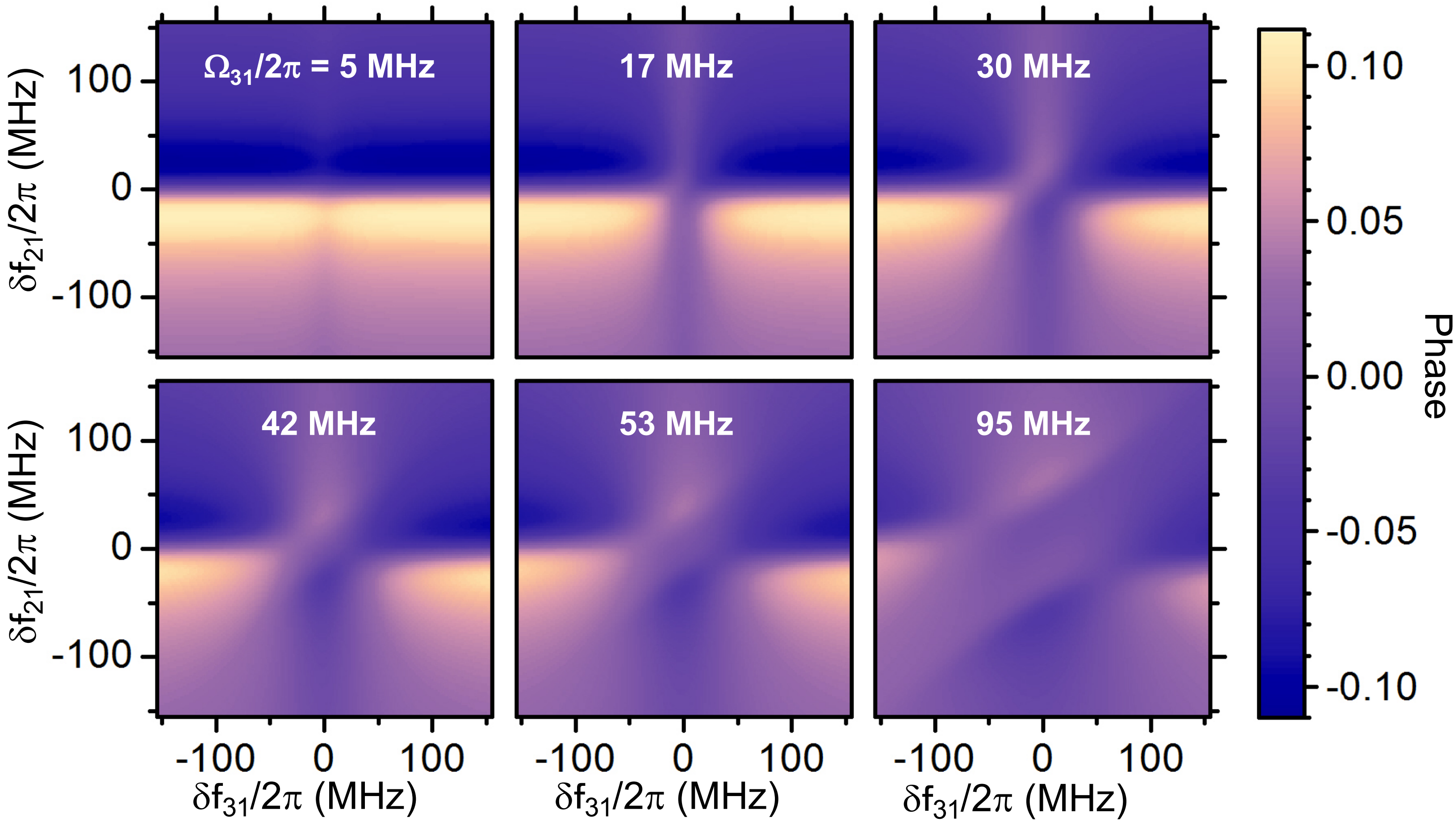
Stimulated emission
(calculations)



Phase in stimulated emission



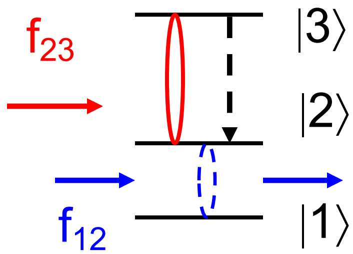
Phase in stimulated emission (calculations)



Electromagnetically induced transparency on the single atom

A. A. Abdumalikov, **O. V. Astafiev** A. M. Zagoskin, Yu.A. Pashkin, T. Yamamoto, K. Inomata, Y. Nakamura, and J.S. Tsai. Electromagnetically Induced Transparency on a Single Artificial Atom. *Phys. Rev. Lett* 104, 193601 (2010).

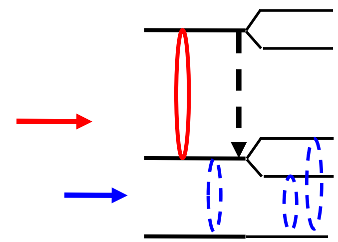
Electromagnetically induced transparency



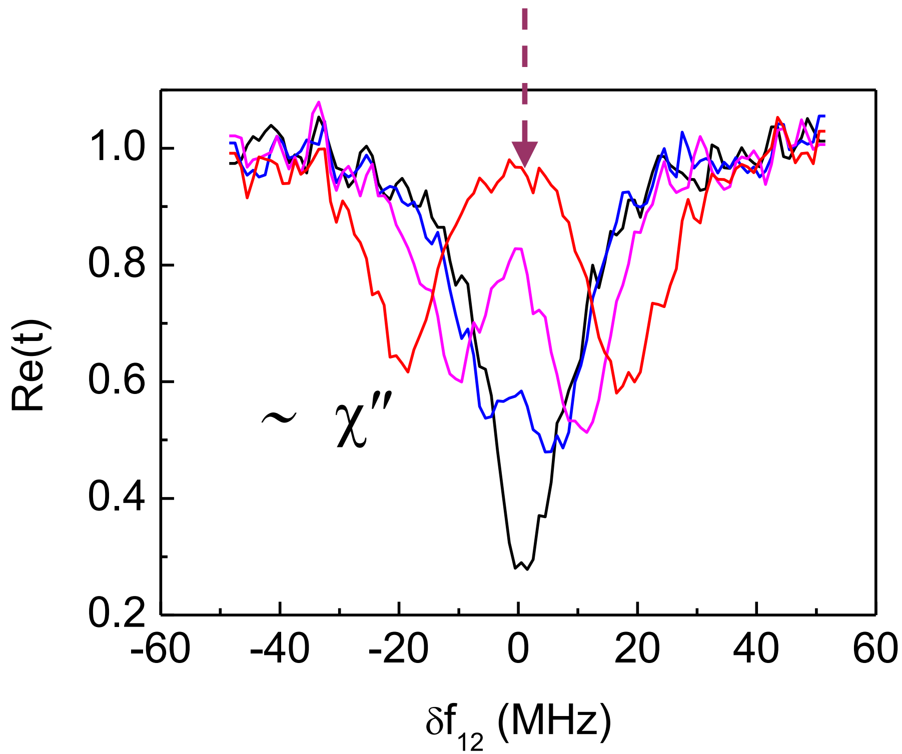
$$\chi = \chi' + i\chi''$$

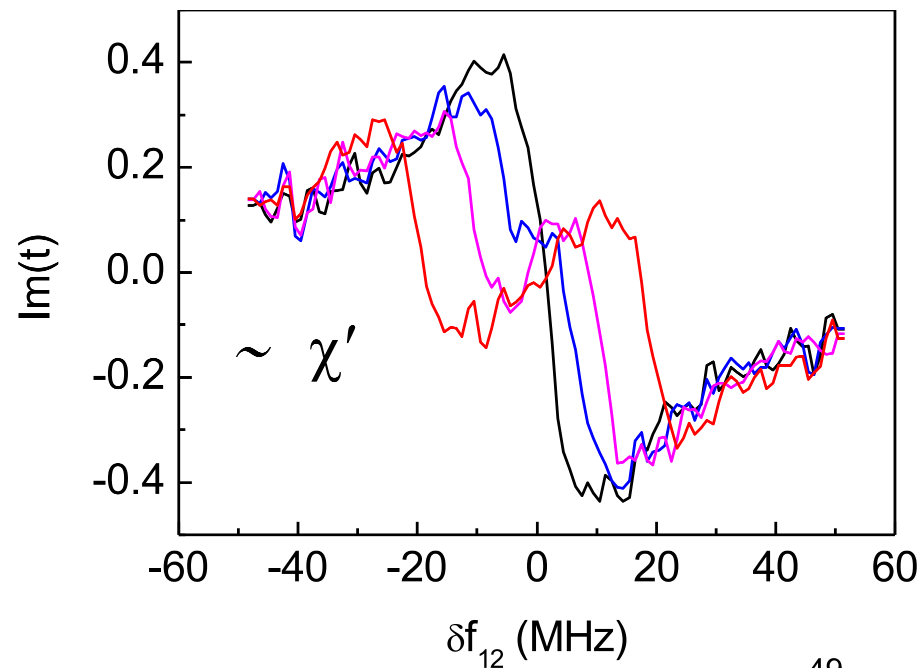
$\alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle$

Dark state (no absorption) $\beta = 0$

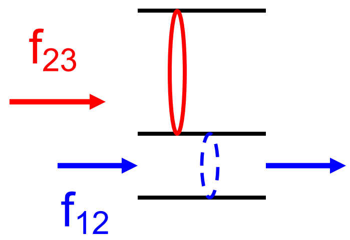


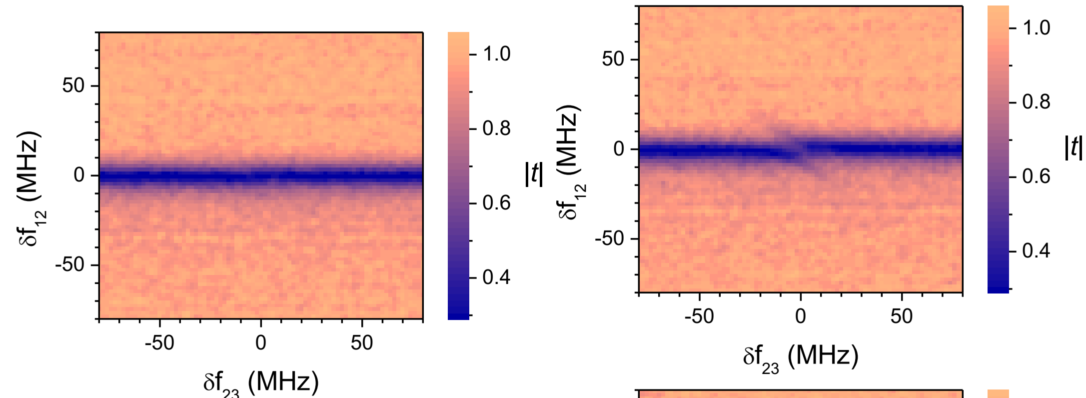
Induced transparency

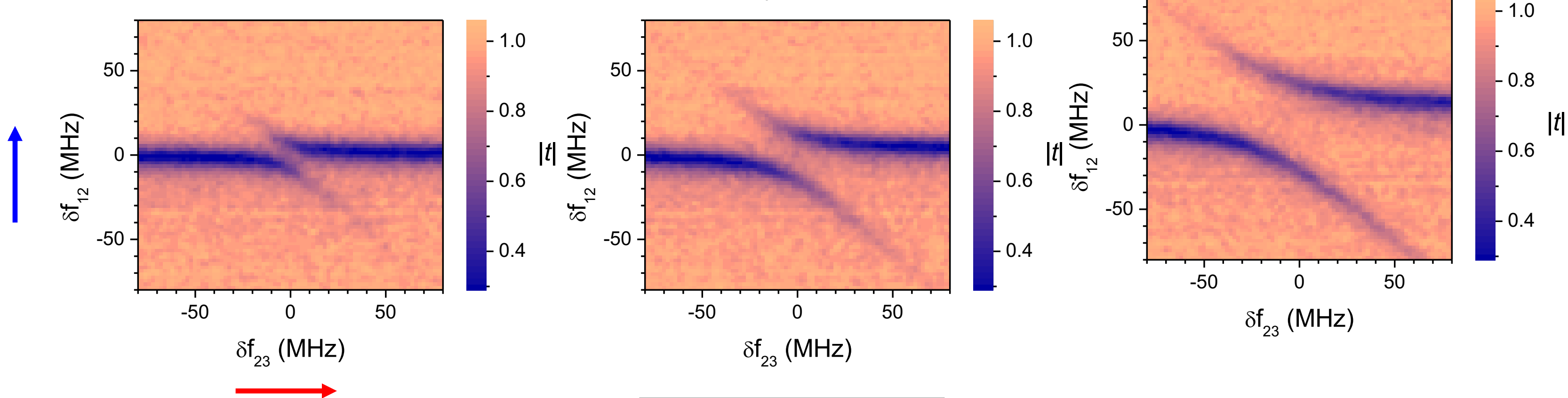


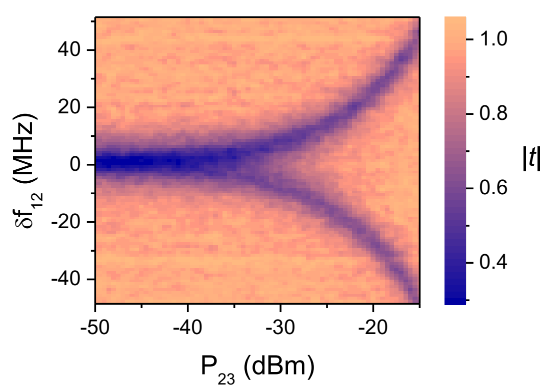


Electromagnetically induced transparency









Power dependence

$\alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle$

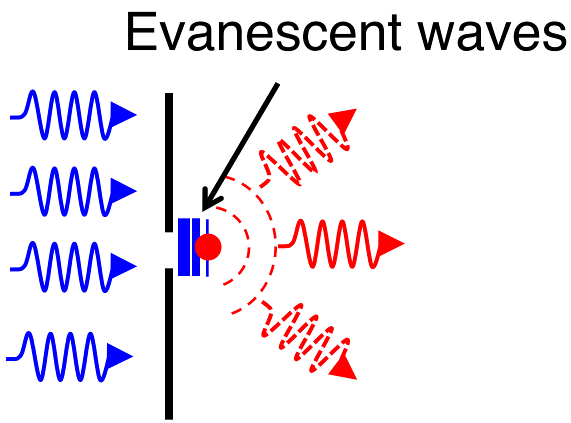
Dark state (no absorption)

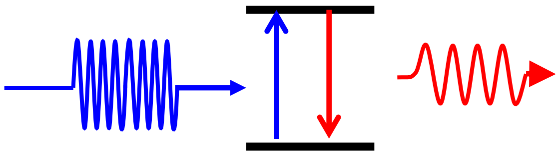
$\beta = 0$

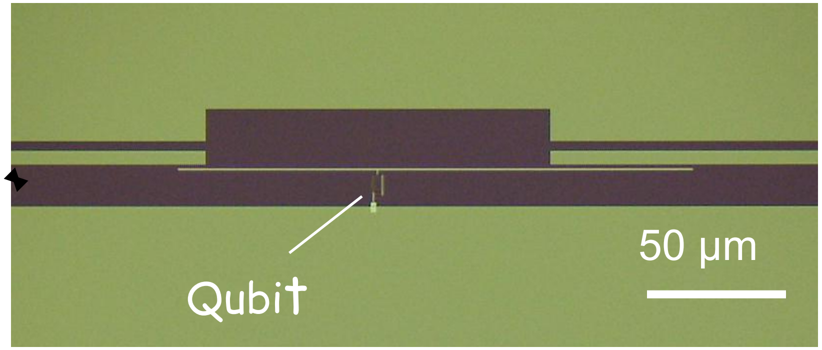
On-demand single-photon source

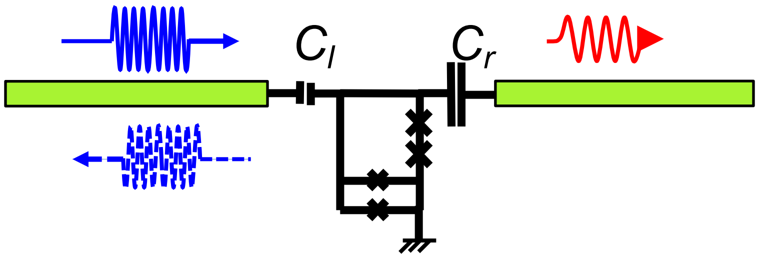
Tuneable on-demand single-photon source in the microwave range
Z. H. Peng, S. E. de Graaf, J. S. Tsai & O. V. Astafiev,
Nature Communications 7, Article number: 12588 (2016)

On-demand microwave photon source









$C_l \approx 0.5$ fF
$C_r \approx 5$ fF

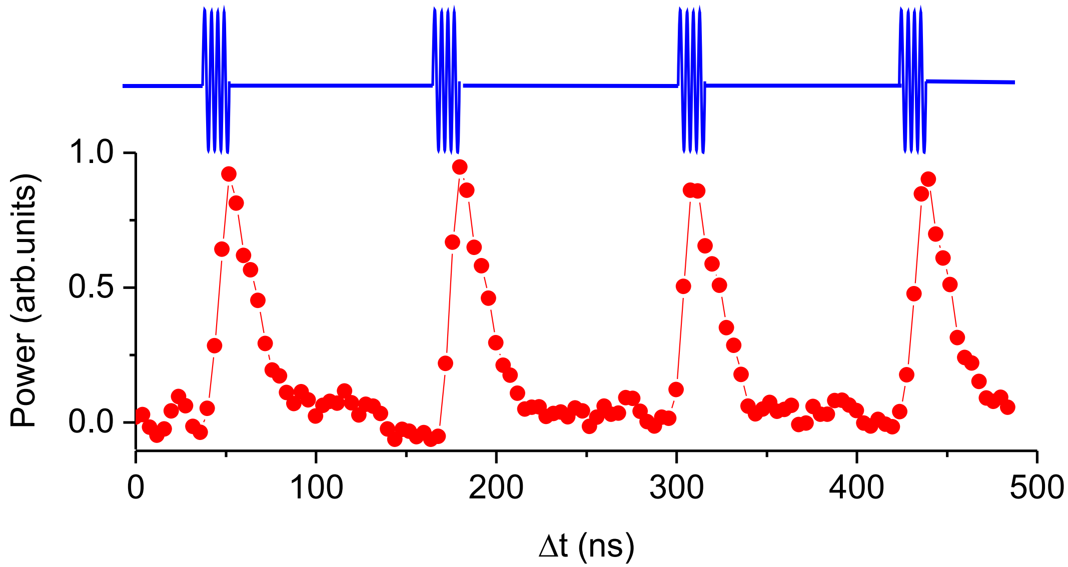
Transmitted power at 10 GHz $(\omega C Z_0)^2 \approx 10^{-6}$

Photon emission efficiency ~ $1-(C_l/C_r)^2 = 0.99$

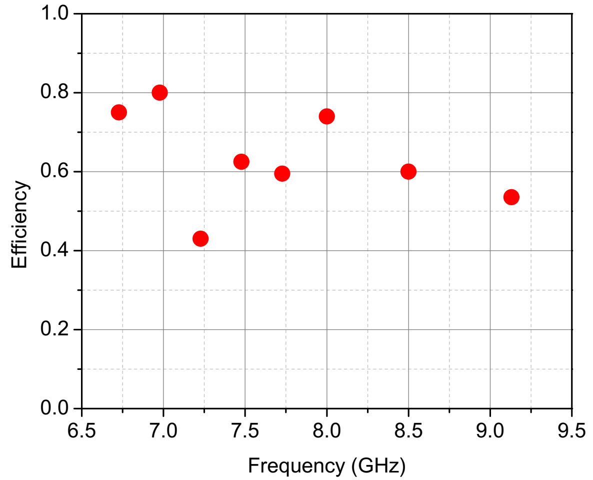
Time traces and efficiency

Excitation pulses

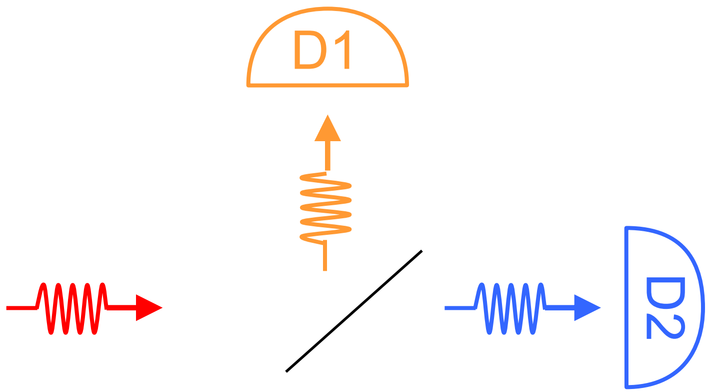
Emissions averaged photon shapes: $\langle \sigma_z \rangle$

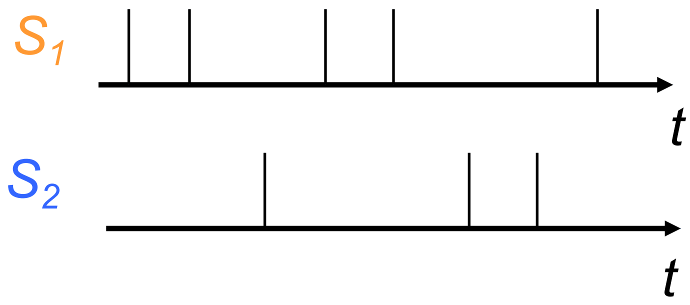


Efficiency over frequency wide range

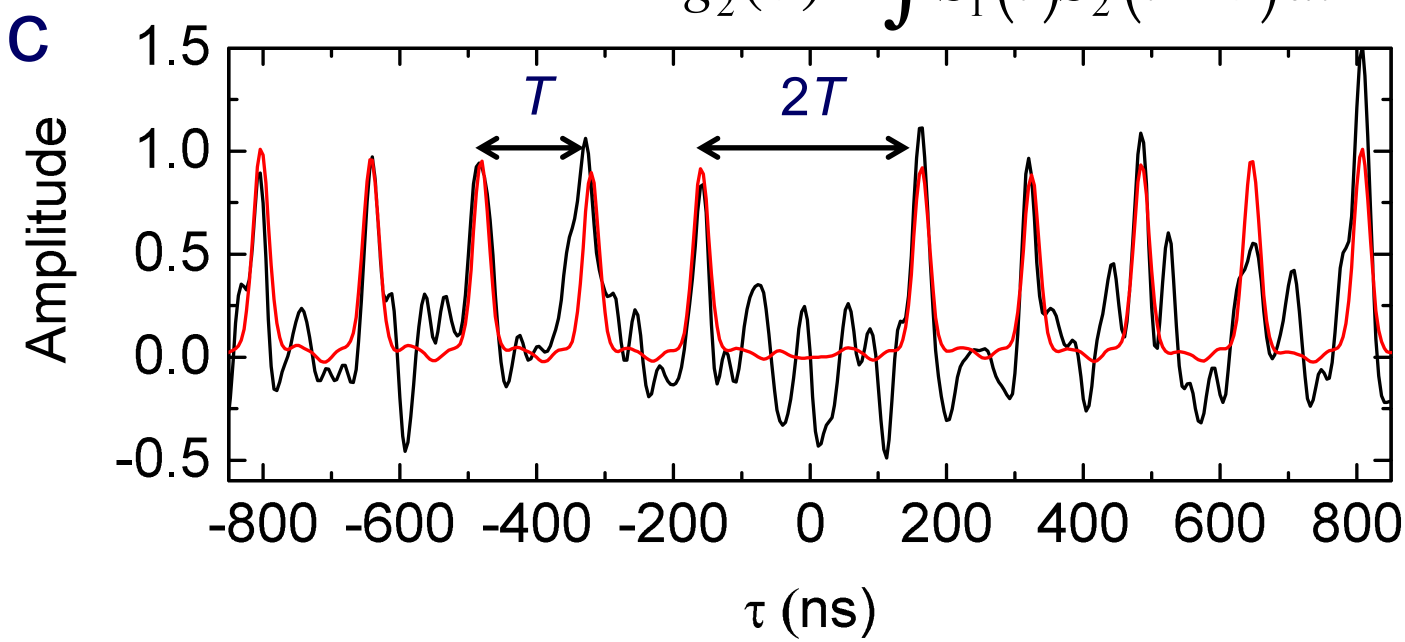


Second order correlation function





$$g_2(\tau) = \int S_1(t) S_2(t+\tau)\, dt$$



10^{10} traces: Took a month of averaging!