Superconducting Quantum Technologies

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Lecture 8

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- Noise in quantum systems
- Relaxation and dephasing
- Physical origin of decoherence

Quantum dynamics of two-level systems with decoherence

Bloch Sphere for dissipative spin dynamics

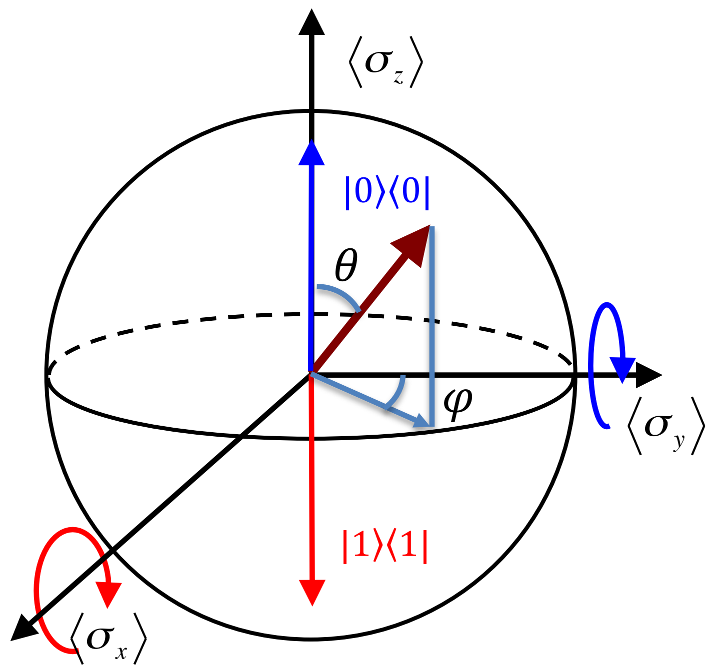
$$|\psi\rangle\langle\psi| = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} & e^{-i\varphi}\sin\frac{\theta}{2} \end{pmatrix} = \begin{pmatrix} \cos^2\frac{\theta}{2} & e^{-i\varphi}\cos\frac{\theta}{2}\sin\frac{\theta}{2} \\ e^{i\varphi}\cos\frac{\theta}{2}\sin\frac{\theta}{2} & \sin^2\frac{\theta}{2} \end{pmatrix} = \begin{pmatrix} \cos^2\frac{\theta}{2} & \frac{1}{2}e^{-i\varphi}\sin\theta \\ \frac{1}{2}e^{i\varphi}\sin\theta & \sin^2\frac{\theta}{2} \end{pmatrix}$$

$$\rho_{00} + \rho_{11} = 1$$

$$\langle\sigma_x\rangle = \rho_{01} + \rho_{10} = \frac{1}{2}e^{-i\varphi}\sin\theta + \frac{1}{2}e^{i\varphi}\sin\theta = \cos\varphi\sin\theta$$

$$\langle\sigma_y\rangle = i\rho_{01} - i\rho_{10} = \frac{i}{2}e^{-i\varphi}\sin\theta + \frac{i}{2}e^{i\varphi}\sin\theta = \sin\varphi\sin\theta$$

$$\langle\sigma_z\rangle = \rho_{00} - \rho_{11} = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = \cos\theta$$



Polar coordinates:

$$x = \cos\varphi\sin\theta$$

$$y = \sin\varphi\sin\theta$$

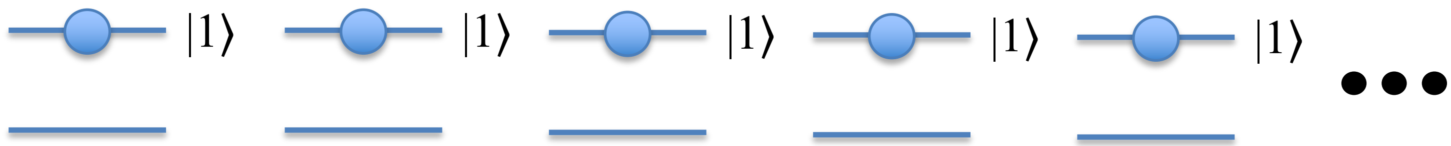
$$z = \cos\theta$$

More generally:

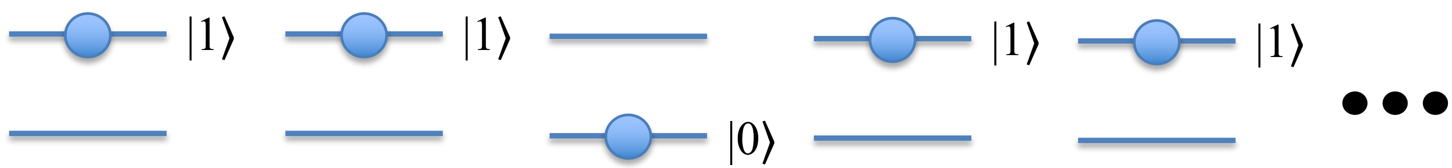
$$x \to \langle\sigma_x\rangle \quad y \to \langle\sigma_y\rangle \quad z \to \langle\sigma_z\rangle$$

Pure and Mixed states

Pure state: wavefunction is $|1\rangle$; Density matrix: $\rho = |1\rangle\langle 1|$



Mixed state: the state which can not be described by a wavefunction => probabilities
Density matrix: $\rho = 0.8|1\rangle\langle 1| + 0.2|0\rangle\langle 0|$



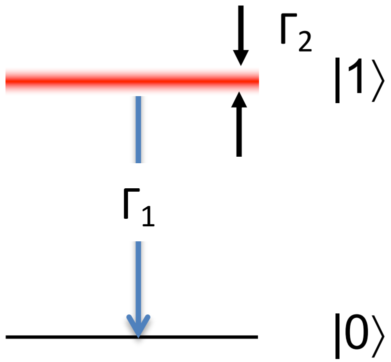
Phase fluctuations:

$$\left|\Psi_1\right\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad \rho = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad \left|\Psi_2\right\rangle = \frac{|0\rangle + e^{i\varphi}|1\rangle}{\sqrt{2}} \quad \rho = \begin{pmatrix} \frac{1}{2} & \frac{e^{i\varphi}}{2} \\ \frac{e^{-i\varphi}}{2} & \frac{1}{2} \end{pmatrix} \qquad \left|\Psi_3\right\rangle = \frac{|0\rangle + e^{-i\varphi}|1\rangle}{\sqrt{2}} \quad \rho = \begin{pmatrix} \frac{1}{2} & \frac{e^{-i\varphi}}{2} \\ \frac{e^{i\varphi}}{2} & \frac{1}{2} \end{pmatrix} \quad \bullet\bullet\bullet$$

$$\rho_{00} = \rho_{11} = \frac{1}{2} \qquad \rho_{01} = \frac{1}{N}\left(\frac{1}{2} + \frac{e^{i\varphi}}{2} + \frac{e^{-i\varphi}}{2} + \ldots\right) \approx \frac{1}{2}\frac{1}{N}\left(1 + \left(1 + i\varphi + \frac{(i\varphi)^2}{2!}\right) + \left(1 - i\varphi + \frac{(-i\varphi)^2}{2!}\right) + \ldots\right) = \frac{1 - \frac{\varphi^2}{3}}{2}$$

Mixed state: $\left|\rho_{01}\right| = \left|\rho_{10}\right| < \sqrt{\left|\rho_{00}\right|\left|\rho_{11}\right|} = \frac{1}{2}$

The Master Equation and the Lindblad term



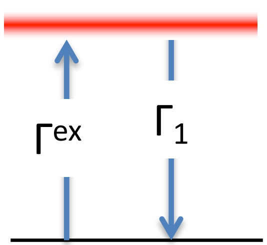
Relaxation and dephasing: Γ_1, Γ_2

$$\boxed{\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H,\rho] + L}$$

$$\begin{pmatrix} \dot{\rho}_{00} & \dot{\rho}_{01} \\ \dot{\rho}_{10} & \dot{\rho}_{11} \end{pmatrix}$$

The Lindblad term

$$\boxed{L = \begin{pmatrix} \Gamma_1 \rho_{11} & -\Gamma_2 \rho_{01} \\ -\Gamma_2 \rho_{10} & -\Gamma_1 \rho_{11} \end{pmatrix}}$$



Accounting excitation (e.g. T > 0)

$$\boxed{L = \begin{pmatrix} \Gamma_1 \rho_{11} - \Gamma^{ex} \rho_{00} & -\Gamma_2 \rho_{01} \\ -\Gamma_2 \rho_{10} & -\Gamma_1 \rho_{11} + \Gamma^{ex} \rho_{00} \end{pmatrix}}$$

Decay of the off-diagonal terms

$\rho_{01} = \rho_{10}^*$ therefore they decay with the same rate Γ_2

Γ_2 is the dephasing rate

First we assume only relaxation in our system

$$\rho_{10}|1\rangle\langle 0|\rho_{01}|0\rangle\langle 1| = \rho_{10}\rho_{01}|1\rangle\langle 1| = |\rho_{10}|^2|1\rangle\langle 1|$$

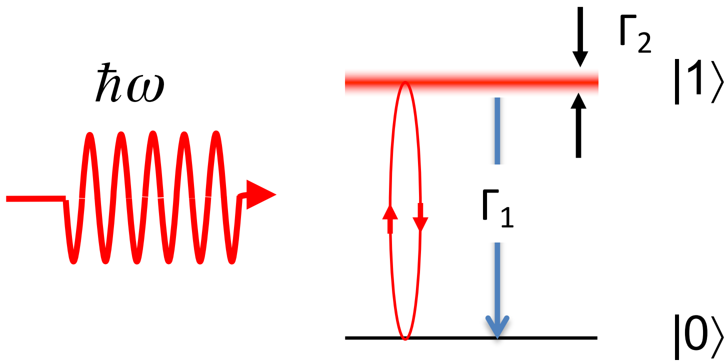
$$|\rho_{10}(t)| = |\rho_{10}(t)| = \sqrt{\rho_{11}(t)} = \sqrt{\rho_{11}(0)e^{-\Gamma_1 t}} = \sqrt{\rho_{11}(0)}e^{-\frac{\Gamma_1}{2}t}$$

$$\boxed{\Gamma_2 = \frac{\Gamma_1}{2}}$$

Dephasing rate can not be smaller than half of the relaxation rate

Driven two-level system with incoherent processes

The general form of the Hamiltonian driven by a wave with an arbitrary phase shift



Relaxation and dephasing: Γ_1, Γ_2

Dephasing: $\Gamma_2 = \dfrac{\Gamma_1}{2} + \Gamma_\varphi$

Pure dephasing

$$H = -\frac{\hbar\omega}{2}\sigma_z + \hbar\Omega\sigma_x\cos(\omega t - \varphi) \qquad U = e^{-i\frac{\omega t}{2}\sigma_z}$$

$$\boxed{H' = \frac{\hbar\Omega}{2}\left(\sigma_x\cos\varphi + \sigma_y\sin\varphi\right)}$$

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H',\rho] + L \qquad \rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \qquad L = \begin{pmatrix} \Gamma_1\rho_{11} & -\Gamma_2\rho_{01} \\ -\Gamma_2\rho_{10} & -\Gamma_1\rho_{11} \end{pmatrix}$$

$$L = \begin{pmatrix} \Gamma_1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} -\Gamma_1\rho_{00} & -\Gamma_2\rho_{01} \\ -\Gamma_2\rho_{10} & -\Gamma_1\rho_{11} \end{pmatrix}$$

Driven two-level system with incoherent processes

The RWA Hamiltonian:

$$\boxed{H=\frac{\hbar\Omega}{2}\left(\sigma_x\cos\varphi+\sigma_y\sin\varphi\right)}$$

The Master Equation:

$$\boxed{\frac{\partial\rho}{\partial t}=-\frac{i}{\hbar}\left[H,\rho\right]+L}$$

The Lindblad term:

$$L=\begin{pmatrix}\Gamma_1\rho_{11} & -\Gamma_2\rho_{01}\\ -\Gamma_2\rho_{10} & -\Gamma_1\rho_{11}\end{pmatrix}$$

Three independent equations for $\rho_{00},\ \rho_{01},\ \rho_{10}$

$\rho_{00}+\rho_{11}=1$

$$\begin{pmatrix}\dot\rho_{00} & \dot\rho_{01}\\ \dot\rho_{10} & \dot\rho_{11}\end{pmatrix}=-\frac{i}{\hbar}\left(H\rho-\rho H\right)+L$$

$$\frac{\partial\langle\sigma_j\rangle}{\partial t}=\mathrm{tr}\left[\sigma_j\frac{\partial\rho}{\partial t}\right]$$

$$\frac{\partial\langle\sigma_j\rangle}{\partial t}=\mathrm{tr}\left[-\frac{i}{\hbar}\sigma_j\left(H\rho-\rho H\right)+\sigma_jL\right]=-\frac{i}{\hbar}\mathrm{tr}\left[\sigma_jH\rho-H\sigma_j\rho\right]+\mathrm{tr}\left[\sigma_jL\right]=\frac{i}{\hbar}\left\langle\left[H,\sigma_j\right]\right\rangle+\mathrm{tr}\left[\sigma_jL\right]$$

$$\frac{\partial\langle\sigma_x\rangle}{\partial t}=-i\frac{\Omega}{2}\left(\left\langle\left[\sigma_x,\sigma_x\right]\right\rangle\cos\varphi+\left\langle\left[\sigma_y,\sigma_x\right]\right\rangle\sin\varphi\right)+\mathrm{tr}\left[\sigma_xL\right]$$

$$\frac{\partial\langle\sigma_y\rangle}{\partial t}=-i\frac{\Omega}{2}\left(\left\langle\left[\sigma_x,\sigma_y\right]\right\rangle\cos\varphi+\left\langle\left[\sigma_y,\sigma_y\right]\right\rangle\sin\varphi\right)+\mathrm{tr}\left[\sigma_yL\right]$$

$$\frac{\partial\langle\sigma_z\rangle}{\partial t}=-i\frac{\Omega}{2}\left(\left\langle\left[\sigma_x,\sigma_z\right]\right\rangle\cos\varphi+\left\langle\left[\sigma_y,\sigma_z\right]\right\rangle\sin\varphi\right)+\mathrm{tr}\left[\sigma_zL\right]$$

$$\frac{\partial\langle\sigma_x\rangle}{\partial t}=-\Omega\langle\sigma_z\rangle\sin\varphi-\Gamma_2\langle\sigma_x\rangle$$

$$\frac{\partial\langle\sigma_y\rangle}{\partial t}=\Omega\langle\sigma_z\rangle\cos\varphi-\Gamma_2\langle\sigma_y\rangle$$

$$\frac{\partial\langle\sigma_z\rangle}{\partial t}=\Omega\left(-\langle\sigma_y\rangle\cos\varphi+\langle\sigma_x\rangle\sin\varphi\right)-\Gamma_1\langle\sigma_z\rangle+\Gamma_1$$

$$\underbrace{\frac{\partial}{\partial t}\begin{pmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{pmatrix}}_{\frac{\partial\vec\sigma}{\partial t}}=\underbrace{\begin{pmatrix}-\Gamma_2 & 0 & -\Omega\sin\varphi\\ 0 & -\Gamma_2 & \Omega\cos\varphi\\ \Omega\sin\varphi & -\Omega\cos\varphi & -\Gamma_1\end{pmatrix}}_{B}\underbrace{\begin{pmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{pmatrix}}_{\vec\sigma}+\underbrace{\begin{pmatrix}0\\ 0\\ \Gamma_1\end{pmatrix}}_{\vec b}$$

Dynamics of the two level system is described by:

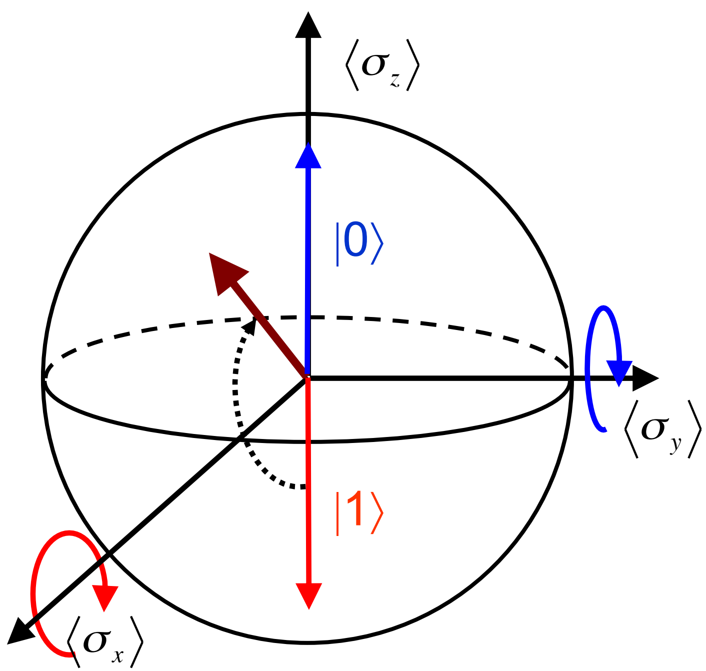
$$\boxed{\frac{d\vec\sigma}{dt}=B\vec\sigma+\vec b}$$

Bloch Sphere for dissipative spin dynamics

$$\frac{\partial}{\partial t}\begin{pmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{pmatrix}=\begin{pmatrix}-\Gamma_2 & 0 & -\Omega\sin\varphi\\ 0 & -\Gamma_2 & \Omega\cos\varphi\\ \Omega\sin\varphi & -\Omega\cos\varphi & -\Gamma_1\end{pmatrix}\begin{pmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{pmatrix}+\begin{pmatrix}0\\ 0\\ \Gamma_1\end{pmatrix}$$

$$\frac{d\vec{\sigma}}{dt}=B\vec{\sigma}+\vec{b}$$

The vector can be less than one
(alternative criteria of mixed states)

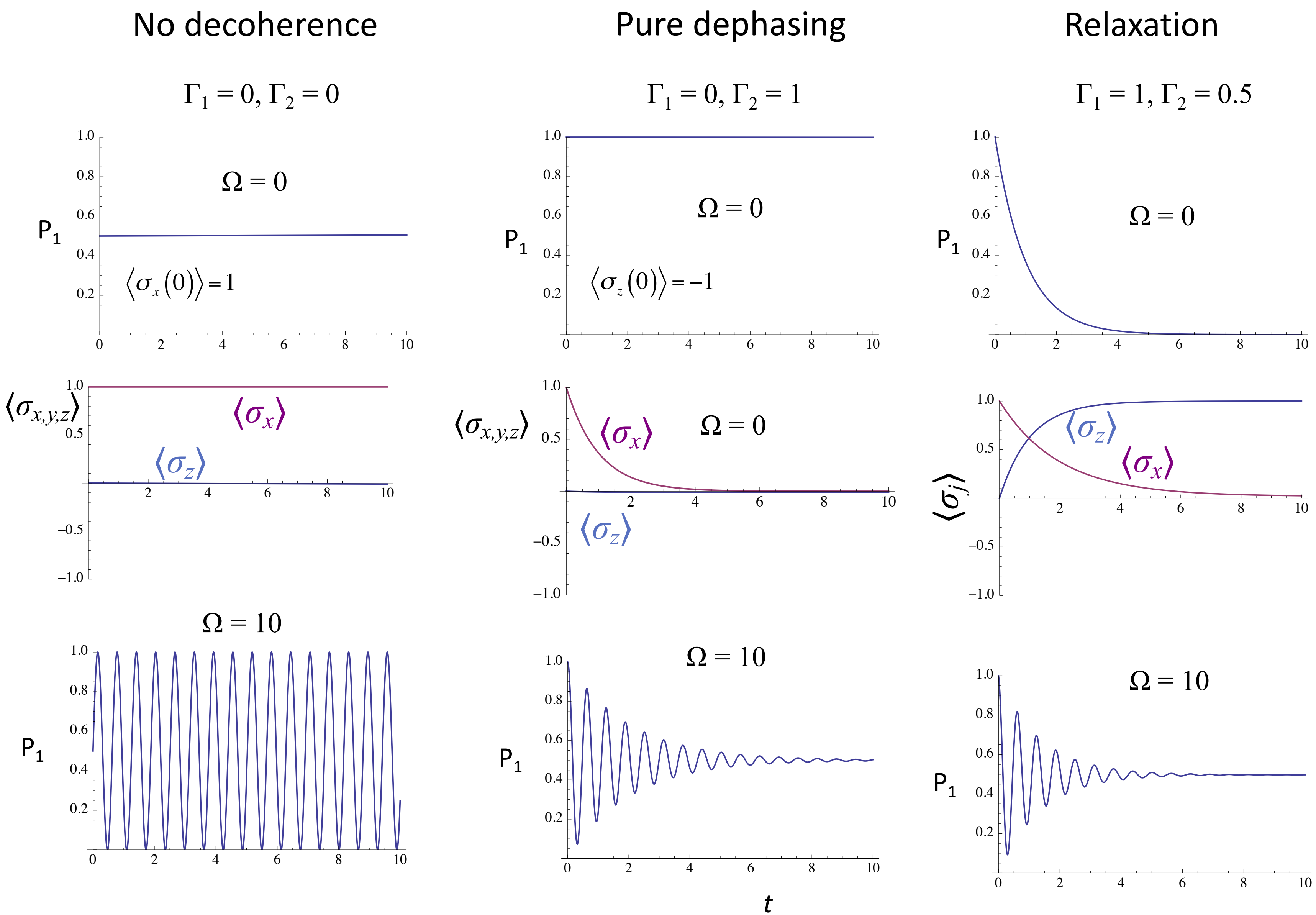


$$\vec{\sigma}(t)=e^{Bt}\vec{\sigma}(0)+B^{-1}\left(e^{Bt}-1\right)\vec{b}$$

$$\vec{\sigma}(t)=e^{Bt}\left[\vec{\sigma}(0)+B^{-1}\vec{b}\right]-B^{-1}\vec{b}$$

Dynamics of the two-level system is exactly same as the dynamics of spin 1/2

Rabi oscillations: Oscillations in a two-level system under external drive. Oscillations of atomic states.



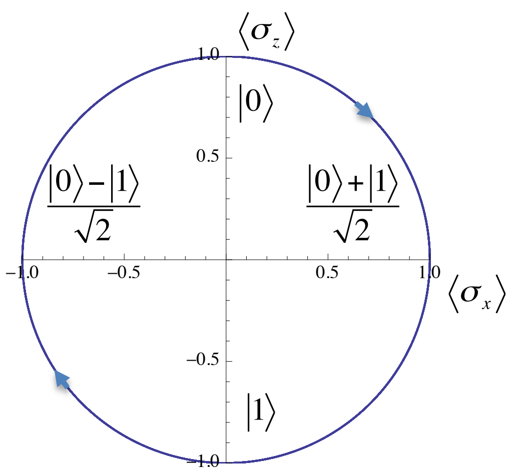
No decoherence $\Gamma_1 = 0, \Gamma_2 = 0$

$$\Psi = \cos\frac{\Omega t}{2}|0\rangle + \sin\frac{\Omega t}{2}|1\rangle$$

$$\rho_{00} = \sin^2\frac{\Omega t}{2} = \frac{1+\cos\Omega t}{2} \qquad \langle\sigma_x\rangle = \rho_{01}+\rho_{10} = \sin\Omega t$$

$$\rho_{01} = \cos\frac{\Omega t}{2}\sin\frac{\Omega t}{2} = \frac{\sin\Omega t}{2} \qquad \langle\sigma_y\rangle = i\rho_{01} - i\rho_{10} = 0$$

$$\rho_{10} = \cos\frac{\Omega t}{2}\sin\frac{\Omega t}{2} = \frac{\sin\Omega t}{2} \qquad \langle\sigma_z\rangle = 2\rho_{00} - 1 = \cos\Omega t$$



With decoherence

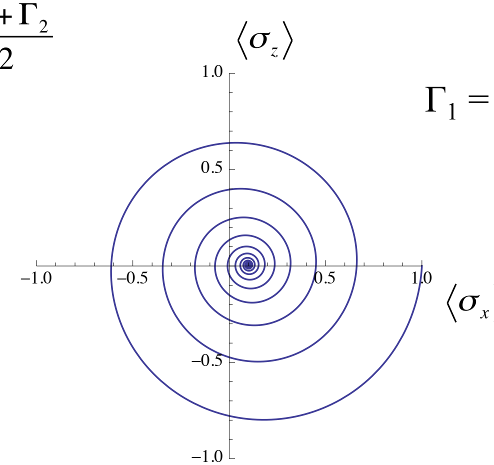
$$\rho_{00} = 1$$

$$\langle\sigma_x\rangle \approx e^{-\gamma t}\sin\Omega t \qquad \rho_{00} = \frac{\langle\sigma_z+1\rangle}{2} = \frac{1+e^{-\gamma t}\cos\Omega t}{2}$$

$$\langle\sigma_y\rangle = 0 \qquad \rho_{01} = \frac{e^{-\gamma t}\sin\Omega t}{2}$$

$$\langle\sigma_z\rangle \approx e^{-\gamma t}\cos\Omega t \qquad \rho_{10} \approx \frac{e^{-\gamma t}\sin\Omega t}{2}$$

$$\gamma = \frac{\Gamma_1+\Gamma_2}{2}$$



$\Gamma_1 = 1, \Gamma_2 = 0.5$

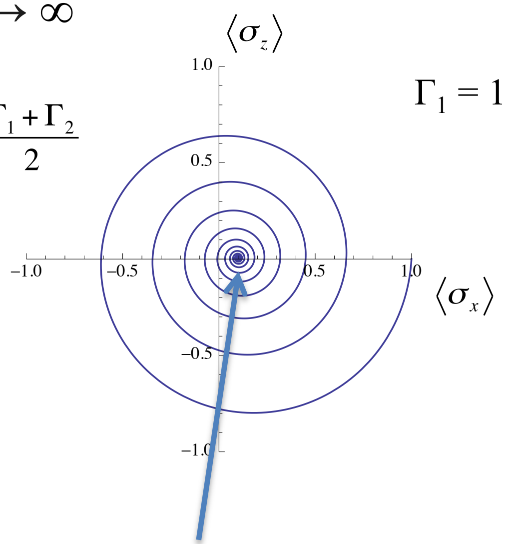
Stationary Master Equation

$$\frac{\partial \rho}{\partial t} = 0 \qquad -\frac{i}{\hbar}[H,\rho] + L = 0$$

$$\frac{\partial \vec{\sigma}}{\partial t} = 0 \qquad B\vec{\sigma} + \vec{b} = 0 \qquad \vec{\sigma} = -B^{-1}\vec{b}$$

$t \longrightarrow \infty$

$$\gamma = \frac{\Gamma_1 + \Gamma_2}{2}$$



$\Gamma_1 = 1, \Gamma_2 = 0.5$

Stationary conditions

$$\langle \sigma_x \rangle = \frac{\Gamma_1 \Omega}{\Gamma_1 \Gamma_2 + \Omega^2} \xrightarrow[\Omega \to \infty]{} \frac{\Gamma_1}{\Omega}$$

$$\langle \sigma_y \rangle = 0$$

$$\langle \sigma_z \rangle = \frac{\Gamma_1 \Gamma_2}{\Gamma_1 \Gamma_2 + \Omega^2}$$

Driven two-level system with detuning

$$H = -\frac{\hbar\omega_0}{2}\sigma_z + \frac{\hbar\Omega}{2}\sigma_x\cos(\omega t + \varphi) \qquad U = e^{-i\frac{\omega t}{2}\sigma_z}$$

$$\boxed{H' = -\frac{\hbar\delta\omega}{2}\sigma_z + \frac{\hbar\Omega}{2}\left(\sigma_x\cos\varphi + \sigma_y\sin\varphi\right)}$$

$$\delta\omega = \omega_0 - \omega$$

$$\frac{\partial\langle\sigma_x\rangle}{\partial t} = -i\frac{\delta\omega}{2}\langle[\sigma_z,\sigma_x]\rangle + \dots$$

$$\frac{\partial\langle\sigma_y\rangle}{\partial t} = -i\frac{\delta\omega}{2}\langle[\sigma_z,\sigma_y]\rangle + \dots$$

$$\frac{\partial\langle\sigma_z\rangle}{\partial t} = -i\frac{\delta\omega}{2}\langle[\sigma_z,\sigma_z]\rangle + \dots$$

$$\frac{\partial\langle\sigma_x\rangle}{\partial t} = -\delta\omega\langle\sigma_y\rangle - \Omega\langle\sigma_z\rangle\sin\varphi - \Gamma_2\langle\sigma_x\rangle$$

$$\frac{\partial\langle\sigma_y\rangle}{\partial t} = \delta\omega\langle\sigma_x\rangle + \Omega\langle\sigma_z\rangle\cos\varphi - \Gamma_2\langle\sigma_y\rangle$$

$$\frac{\partial\langle\sigma_z\rangle}{\partial t} = \Omega\left(\langle\sigma_x\rangle\cos\varphi - \langle\sigma_y\rangle\sin\varphi\right) - \Gamma_1\langle\sigma_z\rangle + \Gamma_1$$

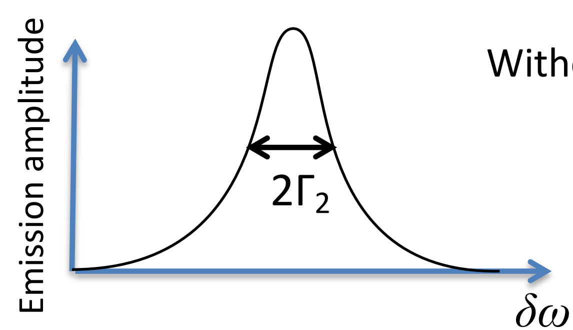
$$\frac{\partial}{\partial t}\begin{pmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{pmatrix} = \begin{pmatrix}-\Gamma_2 & -\delta\omega & -\Omega\sin\varphi\\ \delta\omega & -\Gamma_2 & \Omega\cos\varphi\\ \Omega\sin\varphi & -\Omega\cos\varphi & -\Gamma_1\end{pmatrix}\begin{pmatrix}\langle\sigma_x\rangle\\ \langle\sigma_y\rangle\\ \langle\sigma_z\rangle\end{pmatrix} + \begin{pmatrix}0\\ 0\\ \Gamma_1\end{pmatrix}$$

Stationary solution:

$$B\vec{\sigma} + \vec{b} = 0 \qquad \vec{\sigma} = -B^{-1}\vec{b}$$

In the limit of weak drive:

$$V_{emit} = V_0\left\langle\sigma^+\right\rangle \propto \frac{1}{\Gamma_2 + i\delta\omega}$$

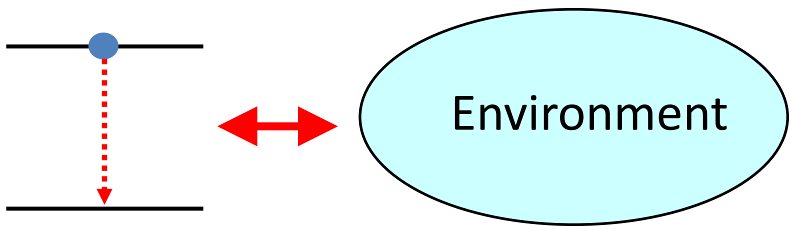


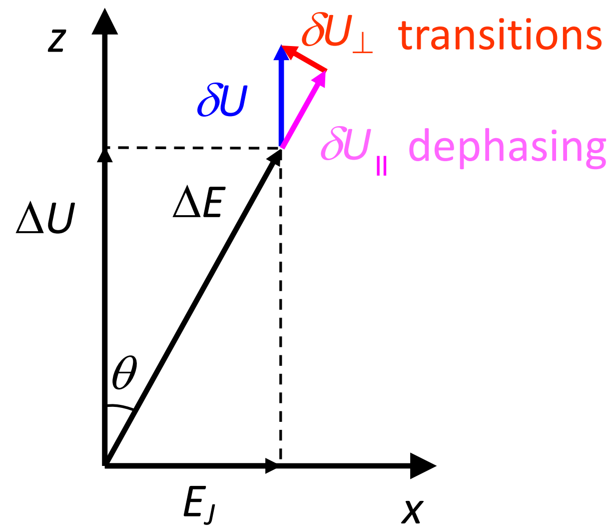
Without dephasing:

$$\Gamma_2 = \frac{\Gamma_1}{2}$$

Noise in quantum systems

Two-level system





$$H = -\frac{\Delta E}{2}\sigma_z - \delta U(t)(\sigma_z \cos\theta + \sigma_x \sin\theta)$$

Energy fluctuation (dephasing) Transitions (relaxation/excitation)

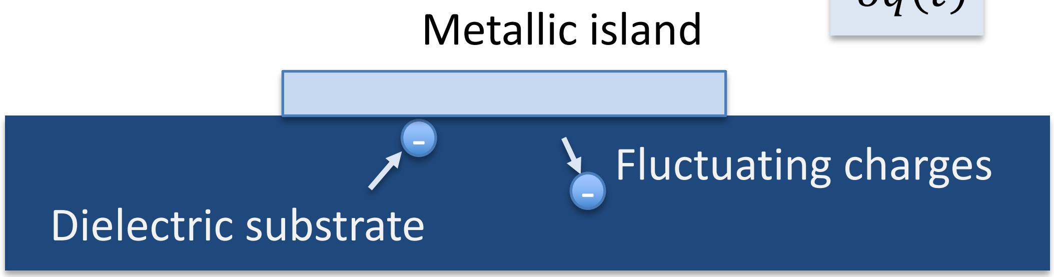
Low frequency noise (< measurement time) results in energy fluctuations

High frequency noise (at $\omega \approx \omega_a$) results in relaxation/excitation

Physical origin of the noise

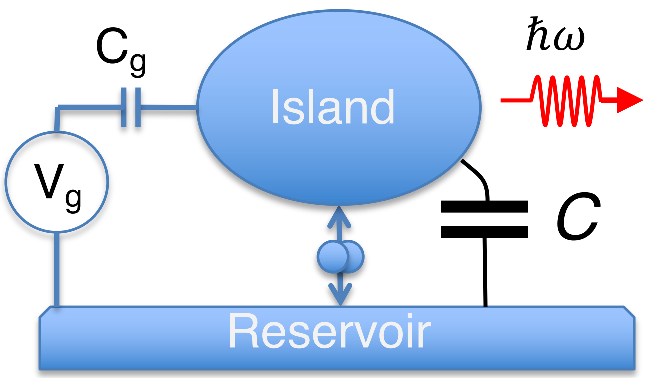
Low frequency noise causing dephasing

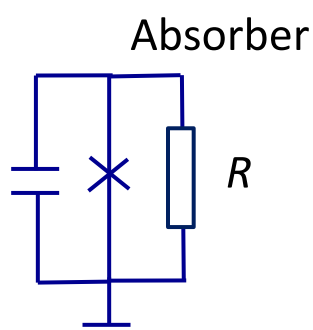
$\delta q(t)$



Energy of the two-level system: $\Delta E(q + \delta q)$

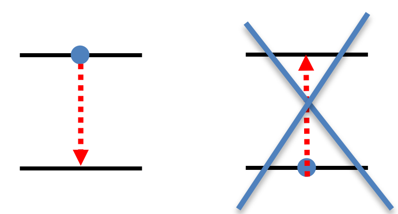
High frequency noise causing relaxation:





Spontaneous emission

At T =0:



At T = 0 noise is asymmetric (quantum)

Noise spectral density

Spectral noise density of random variable x: $S_x(\omega)$

Dispersion of x in the frequency range $\omega_0 \leq \omega \leq \omega_1$: $\langle x^2 \rangle = \int_{\omega_0}^{\omega_1} S_x(\omega) d\omega$

Long integration time $T \Longrightarrow$ narrow frequency range $\Delta\omega = \frac{2\pi}{T}$

$$\langle x^2 \rangle = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt \approx S_x(0) \Delta\omega$$

$$\boxed{S_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle x(\tau) x^*(0) \rangle e^{i\omega\tau}\, d\tau}$$

$$\langle x(\tau) x^*(0) \rangle = \int_{-\infty}^{\infty} S_x(\omega) e^{-i\omega\tau}\, d\omega$$

$$\langle x(\tau) x^*(0) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t+\tau) x^*(t) dt$$

For classical variable: $x(t)$ - real $\Rightarrow x(t) = x^*(t) \;\Rightarrow S_x(\omega) = S_x(-\omega)$

Noise spectral density for real values can be defined for $\omega \geq 0$: $S_x'(\omega) = S_x(-\omega) + S_x(\omega)$

High frequency noise

$$H_0 = -\frac{\hbar\omega_a}{2}\sigma_z \qquad H_{int} = \hbar\Omega(t)\sigma_x$$

For charge qubit:

$\hbar\Omega(t) = \mu_q V(t)$

$$\mu = C_k \frac{\partial U}{\partial q}$$

$V(t)$ can be voltage fluctuations from lines

$$|\psi(t)\rangle \approx \left(1 - \frac{i}{\hbar}\int_0^t H_{int}(t)dt\right)|\psi(0)\rangle$$

$\mu_q = 2e\sin\theta$ is the dipole moment projected on σ_x

If $|\psi(0)\rangle = |1\rangle$ $\quad \langle 0|\psi(t)\rangle \approx -\frac{i}{\hbar}\int_0^t \langle 0|H_{int}(t)|1\rangle e^{-i\omega_a t}dt = -\frac{i\sigma^-}{\hbar}\int_0^t \Omega(t')e^{-i\omega_a t'}dt'$

$$p_0(t) \approx \frac{1}{\hbar^2}\int_0^t\int_0^t \Omega(t')\Omega(t'')e^{i\omega_a(t''-t')}dt''\,dt' \rightarrow \frac{1}{\hbar^2}\int_0^t\int_0^t \langle\Omega(t')\Omega(t'')\rangle e^{i\omega_a(t''-t')}dt''\,dt'$$

$$p_0(t) \approx \frac{t}{\hbar^2}\int_0^t \langle\Omega(\tau)\Omega(0)\rangle e^{i\omega_a\tau}d\tau \qquad S_V(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\langle V(\tau)V(0)\rangle e^{i\omega\tau}d\tau$$

Relaxation ($|1\rangle \rightarrow |0\rangle$):

$$\Gamma_{10} = \frac{\mu^2}{\hbar^2}\int_{-\infty}^{\infty}\langle V(\tau)V(0)\rangle e^{i\omega_a\tau}d\tau = \frac{2\pi\mu^2}{\hbar^2}S_V(\omega)$$

Excitation ($|0\rangle \rightarrow |1\rangle$):

$$\Gamma_{01} = \frac{\mu^2}{\hbar^2}\int_{-\infty}^{\infty}\langle V(\tau)V(0)\rangle e^{-i\omega_a\tau}d\tau = \frac{2\pi\mu^2}{\hbar^2}S_V(-\omega)$$

Quantum noise

$$S_V(\omega) \neq S_V(-\omega)$$

Johnson Nyquist noise ($hf \ll k_B T$):

$$\langle \delta V^2 \rangle = 4\, k_B T R \Delta f$$

$$\langle \delta V^2 \rangle = S_V(-\omega)\Delta\omega + S_V(\omega)\Delta\omega$$

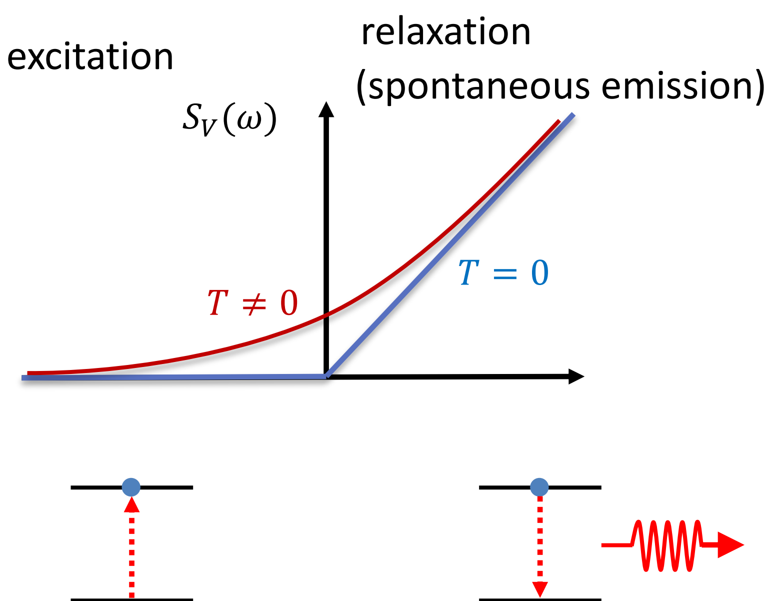
$$S_V(\pm\omega) = \frac{2k_B T R}{2\pi}$$

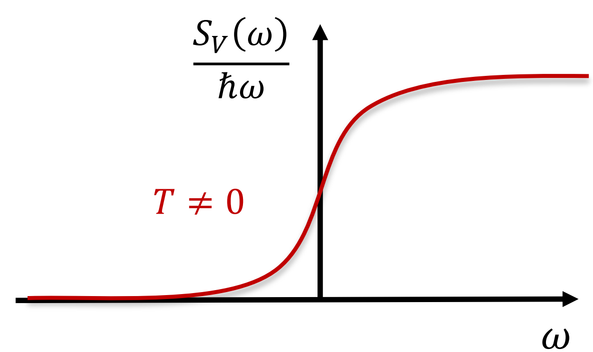
$$S_V(\pm\omega) = \frac{2k_B T R}{2\pi} \frac{\frac{\hbar\omega}{k_B T}}{e^{\frac{\hbar\omega}{k_B T}} - 1}$$

At $T = 0$:

$$S_V(\omega) = \frac{\hbar\omega R}{\pi}$$

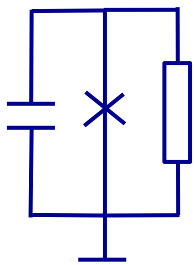
$$S_V(-\omega) = 0$$





Relaxation due to ohmic environment

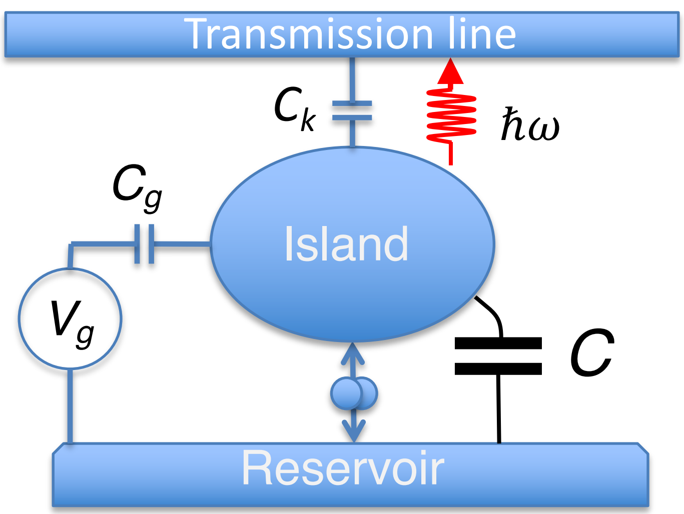
Absorber

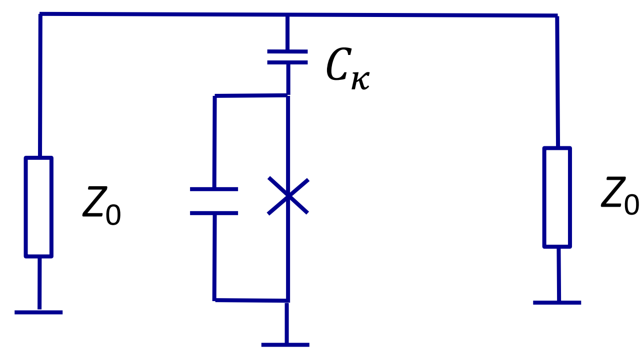


$$S_V(\omega) = \frac{\hbar\omega R}{\pi} \qquad R \to \mathrm{Re}[Z] \qquad \mu_q = C_\kappa \frac{E_C}{2e} \sin\theta$$

$$\Gamma_{10} = \frac{2\pi\mu_q^2}{\hbar^2} S_V(\omega) = \frac{2\hbar\omega\mu_q^2 R}{\hbar^2} \qquad \Gamma_1 = \frac{2\mu^2\omega}{\hbar} \mathrm{Re}[Z]$$

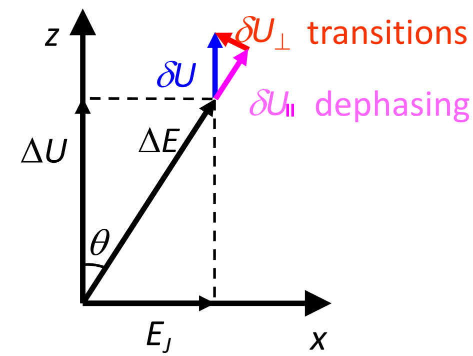
Charge qubit: $\Gamma_1 = \dfrac{2\omega Z_0}{2\hbar} \left(\dfrac{2eC_\kappa}{C_q} \sin\theta \right)^2$





At T = 0, relaxation is a result of high frequency quantum noise

Dephasing and low frequency noise



For charge qubit:

$$H_0 = -\frac{\hbar\omega_a}{2}\sigma_z \qquad H_q = \mu_q \delta q(t) \sigma_z \qquad \mu_q = \frac{\partial \Delta E}{\partial q} = \frac{2E_C}{2e}\cos\theta$$

Energy fluctuations due to charge fluctuations : $\langle \delta E^2 \rangle = \mu_q^2 \langle \delta q^2 \rangle$

Superposed state: $\psi = \frac{|0\rangle e^{\frac{i\omega t}{2}} + e^{-\frac{i\omega t}{2}}|1\rangle}{\sqrt{2}} \Rightarrow \rho_{01} = \frac{1}{2}e^{i\omega t} \rightarrow \frac{1}{2}e^{i\omega t + i\delta\varphi(t)} = \frac{1}{2}e^{i\omega t}e^{i\delta\varphi(t)}$

Phase fluctuations: $\left\langle e^{i\delta\varphi} \right\rangle \approx 1 + i\langle \delta\varphi \rangle - \frac{\langle \delta\varphi^2 \rangle}{2} \qquad \langle \delta\varphi^2 \rangle = \frac{\langle \delta E^2 \rangle T_2^2}{\hbar^2}$

$$\frac{\mu^2 S_q(0) \Gamma_2 T_2^2}{2\pi\hbar^2} = 1 \qquad \Gamma_2 = \frac{\mu^2 S_q(0)}{2\pi\hbar^2}$$

However it is not a realistic case because $S(0) = \infty$ due to 1/f noise

Dephasing and low frequency noise

Charge noise

$$S_q(f) = \frac{\alpha_q^2}{f}$$

Flux noise:

$$S_\Phi(f) = \frac{\alpha_\Phi^2}{f}$$

$$\langle \delta E^2 \rangle = \mu^2 \int_{1/T_{meas}}^{\omega_{cut}} \frac{\alpha_q^2}{\omega}\frac{d\omega}{2\pi} = \frac{\mu_q^2 \alpha_q^2}{2\pi} \ln(\omega_{cut} T_{meas})$$

Typical $(\omega_{cut} T_{meas}) \sim 10^3 - 10^4$ and $\ln(\omega_{cut} T_{meas}) \sim 7 - 9$

$$\langle \delta E^2 \rangle = \mu^2 \int_{1/T_{meas}}^{\omega_{cut}} \frac{\alpha_q^2}{\omega}\frac{d\omega}{2\pi} = \frac{\mu_q^2 \alpha_q^2}{2\pi} \ln(\omega_{cut} T_{meas}) \qquad \sim 1$$

$$\langle \delta \varphi^2 \rangle = \frac{\langle \delta E^2 \rangle T_2^2}{\hbar^2} = 1$$

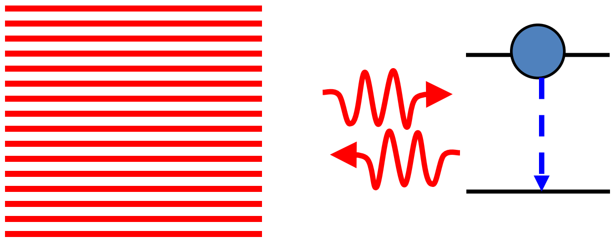
$$\Gamma_2 = \frac{\sqrt{\langle \delta E^2 \rangle}}{\hbar} = \frac{\mu_q \alpha_q}{\hbar} \sqrt{\frac{\ln(\omega_{cut} T_{meas})}{2\pi}} \approx \frac{\mu_q \alpha_q}{\hbar}$$

$$\mu_q = \frac{\partial \Delta E}{\partial q} = \frac{E_C}{2e} \cos\theta$$

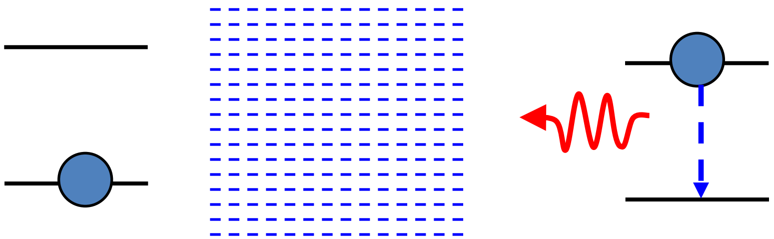
The degeneracy point ($\theta = \pi$) is the magic point with suppressed decohernce

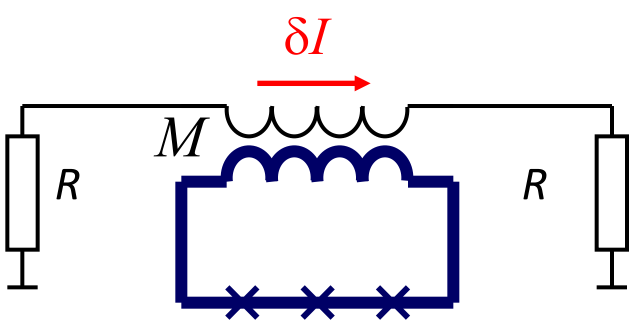
The artificial atom as a sensor of the quantum noise

Hot environment (T >> $\hbar\omega$)
(classical noise)

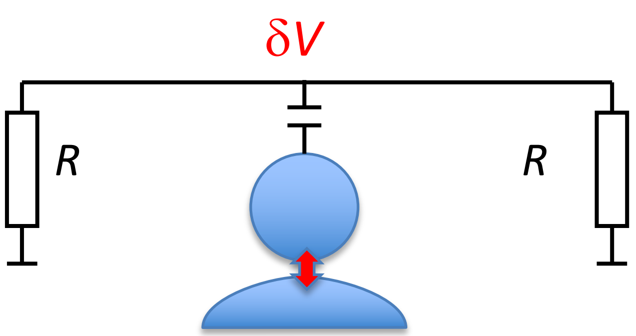


Cold environment (T << $\hbar\omega$)
(quantum noise)





$$\hbar\Omega = \phi_p \delta I$$



$$\hbar\Omega = \mu\delta V$$

The resistors can absorb energy

Atom-resonator
Strong coupling

Dissipation in resonators (photon decay)

$$\boxed{H_{JC} = -\frac{\Delta E}{2}\sigma_z + \hbar\omega_r a^+ a + g_0\left(a\sigma^+ + a^+\sigma^-\right)}$$

$$\boxed{\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] + L}$$

No pure dephasing: $\Gamma_2 = \frac{\Gamma_1}{2}$

$$L = \begin{pmatrix} \Gamma_1\rho_{11} & -\frac{\Gamma_1}{2}\rho_{01} \\ -\frac{\Gamma_1}{2}\rho_{10} & -\Gamma_1\rho_{11} \end{pmatrix}$$

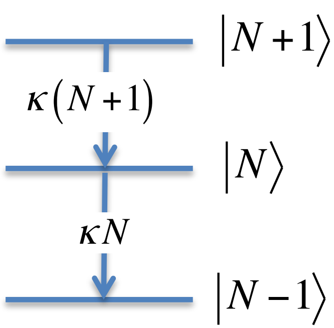
$$L = \frac{\Gamma_1}{2}\left(2\sigma^-\rho\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-\right)$$

Density matrix of a harmonic oscillator:

$$\rho^{(r)} = \sum_{M=0}^{\infty}\sum_{N=0}^{\infty}\rho_{NM}^{(r)}|N\rangle\langle M|$$

Lindblad operator of a harmonic oscillator:

$$L^{(r)} = \frac{\kappa}{2}\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$



N-th element of the resonator Lindblad operator:

$$\dot{\rho}_{NN}^{(r)} \to \kappa(N+1)\rho_{N+1,N+1}^{(r)} - \kappa N\rho_{NN}^{(r)}$$

κ is the photon decay rate

Linewidth of the resonator (quantum mechanical approach)

$$H = \hbar\omega_r a^\dagger a + \hbar\Omega\left(a + a^\dagger\right)\cos\omega t$$

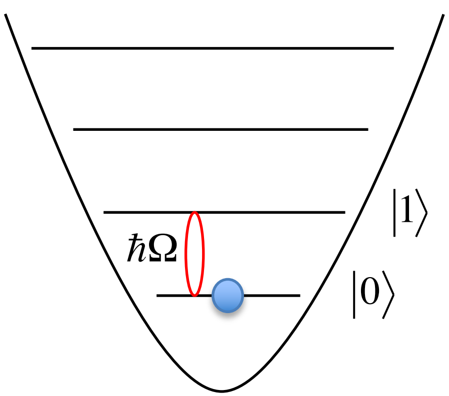
$$H' = UHU^\dagger - i\hbar U\dot{U}^\dagger \qquad U = e^{i\omega t a^\dagger a}$$

$$H' = \hbar\left(\omega_r - \omega\right)a^\dagger a + \hbar\Omega e^{i\omega t a^\dagger a}\left(a + a^\dagger\right)e^{-i\omega t a^\dagger a}\cos\omega t = \hbar\delta\omega a^\dagger a + \hbar\Omega\left(e^{-i\omega t}a + e^{i\omega t}a^\dagger\right)\frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

$$H' = -\hbar\delta\omega a^\dagger a + \frac{\hbar\Omega}{2}\left(a + a^\dagger\right) \qquad \delta\omega = \omega - \omega_r$$

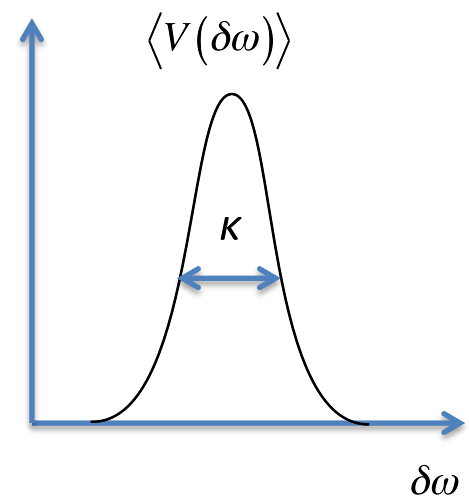
In a weak driving regime, when population of the $|1\rangle$ is low ($\rho_{11} << 1$) we can truncate the photon space by 0, 1

$$H\rho - \rho H + L = 0$$



$$L = \begin{pmatrix} \kappa\rho_{11} & -\frac{\kappa}{2}\rho_{01} \\ -\frac{\kappa}{2}\rho_{10} & -\kappa\rho_{11} \end{pmatrix}$$

$$\left\langle a + a^\dagger \right\rangle = \mathrm{tr}\left[\left(a + a^\dagger\right)\rho\right] \approx \frac{2\Omega}{\kappa + 2\delta\omega}$$



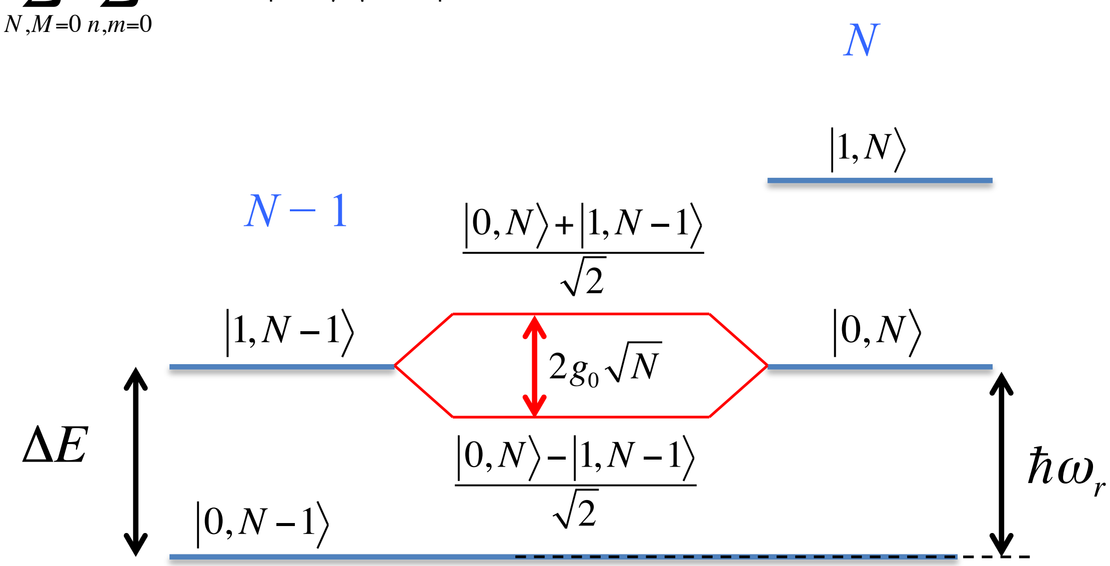
$$H_{JC} = -\frac{\Delta E}{2}\sigma_z + \hbar\omega_r a^+ a + g_0\left(a\sigma^+ + a^+\sigma^-\right)$$

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}\left[H,\rho\right] + L^{(r)} + L^{(a)}$$

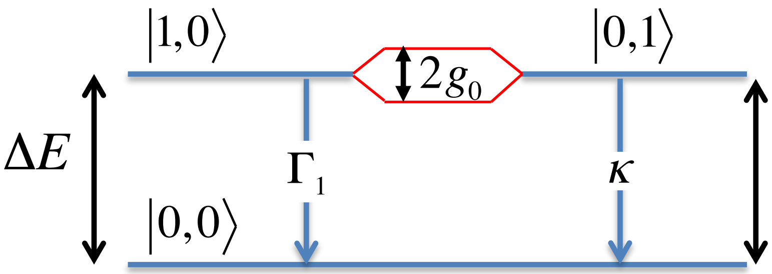
$$L^{(r)} = \frac{\kappa}{2}\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$

$$L^{(a)} = \frac{\Gamma_1}{2}\left(2\sigma^-\rho\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-\right)$$

$$\rho^{(ra)} = \sum_{N,M=0}^{\infty}\sum_{n,m=0}^{1}\rho_{Nn,Mm}\left|Nn\right\rangle\left\langle Mm\right|$$



Single-photon interaction



$$H' = g\left(|01\rangle\langle 10| + |10\rangle\langle 01|\right)$$

$$\rho = \begin{pmatrix} \rho_{01,01} & \rho_{01,10} \\ \rho_{10,01} & \rho_{10,10} \end{pmatrix}$$

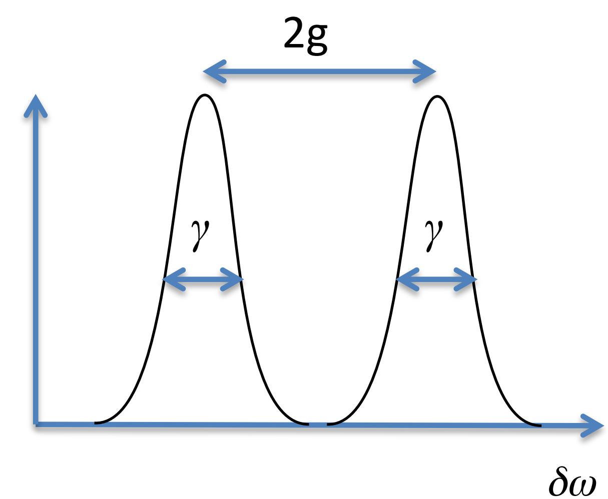
$$L = \begin{pmatrix} \kappa\rho_{01,01} + \Gamma_1\rho_{10,10} & -\frac{\kappa}{2}\rho_{00,01} & -\frac{\Gamma_1}{2}\rho_{00,10} \\ -\frac{\kappa}{2}\rho_{01,00} & -\kappa\rho_{01,01} & -\frac{\Gamma_1+\kappa}{2}\rho_{01,10} \\ -\frac{\Gamma_1}{2}\rho_{10,00} & -\frac{\Gamma_1+\kappa}{2}\rho_{10,01} & -\Gamma_1\rho_{10,10} \end{pmatrix}$$

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H,\rho] + L$$

The states decay with rate: $\gamma = \dfrac{\kappa + \Gamma_1}{2}$

To exchange energy between the qubit and the resonator the following condition should be fulfilled:

$$\boxed{(\Gamma_1,\ \Gamma_2,\ \kappa) << \frac{g}{\hbar}}$$



Matlab Quantum optics toolbox: qotoolbox

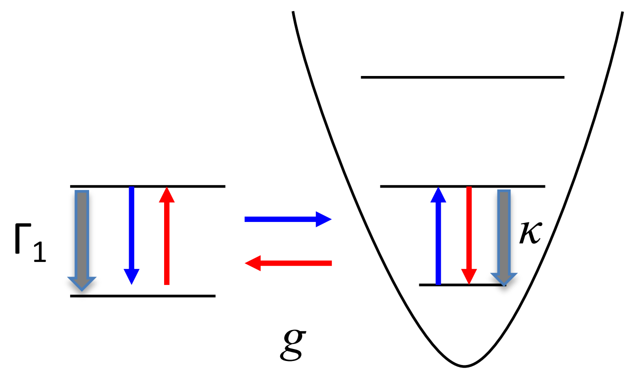
Strong coupling

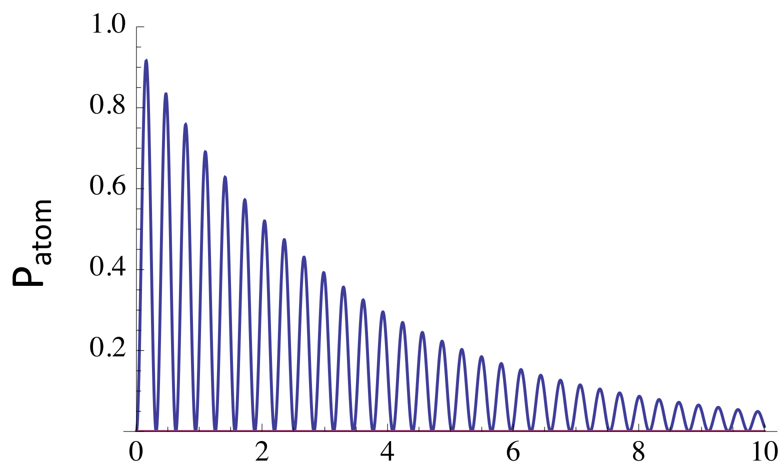
Energy exchange. Strong coupling.

Strong coupling regime is achieved, when the characteristic energy is higher than incoherent processes

For an atom coupled to a resonator the coupling energy g

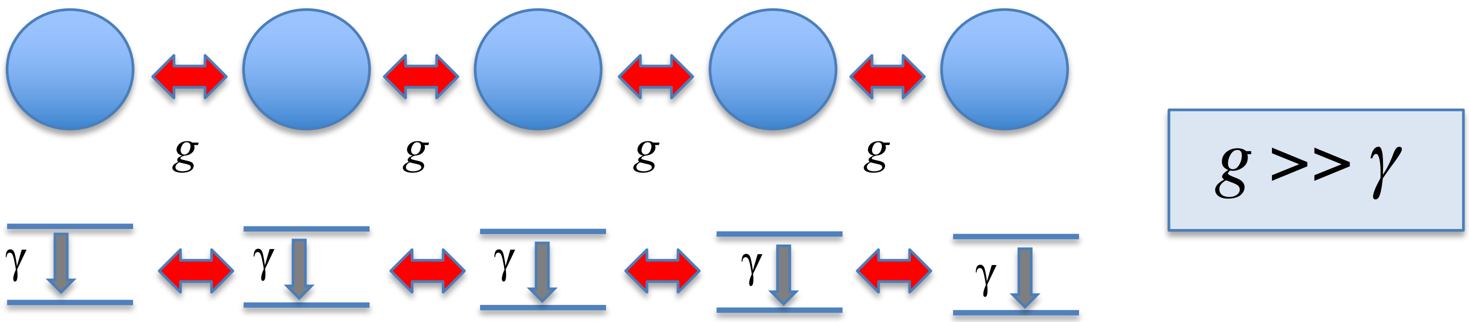
$$\boxed{\left(\hbar\Gamma_1,\ \hbar\Gamma_2,\ \hbar\kappa\right) << g}$$



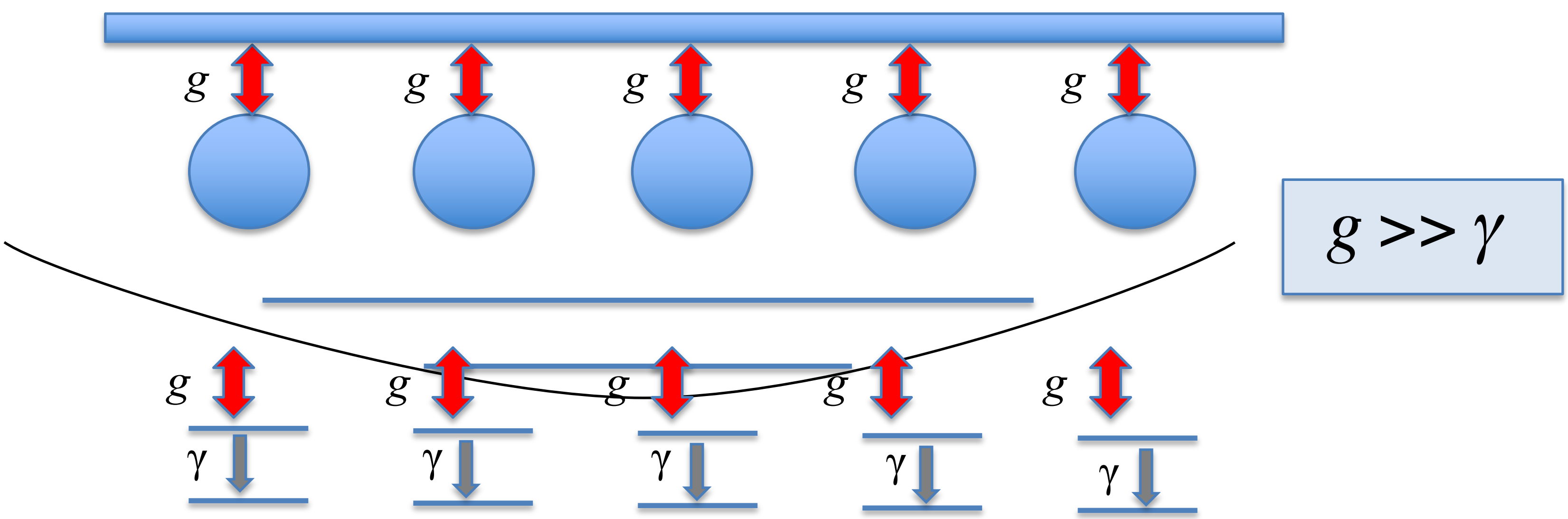


The strong coupling is necessary condition to manipulate with quantum states; to exchange information between different systems.

Possible geometries of quantum systems



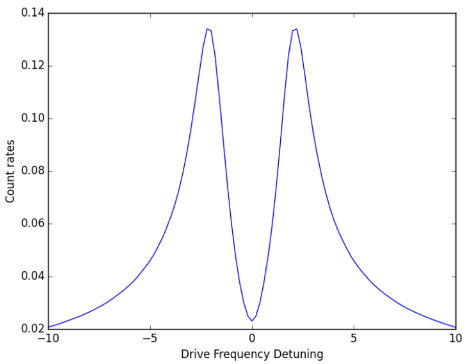
Qubits coupled to quantum baths

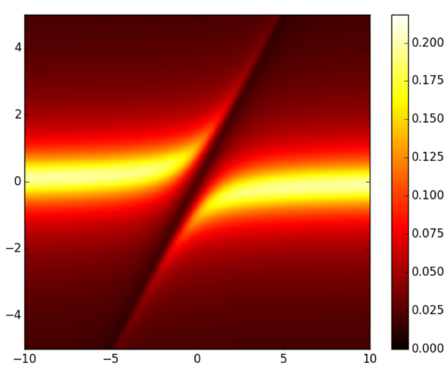


Two-level atom in a resonator

Strong coupling:

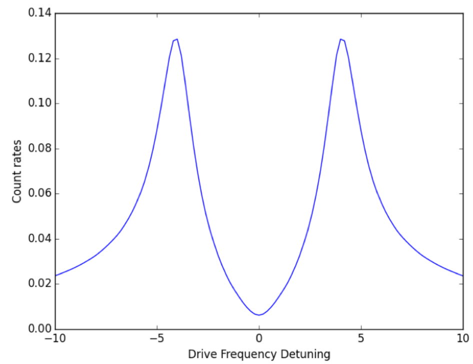
$\kappa = 0.5,\ \Gamma_1 = 0.5,\ g = 1$

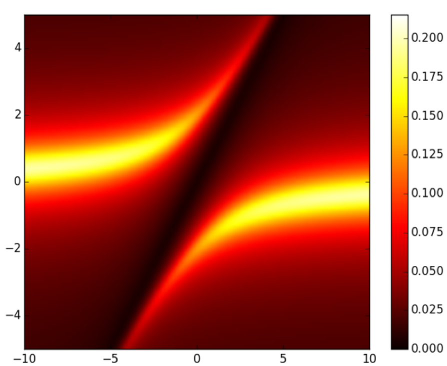




Strong coupling:

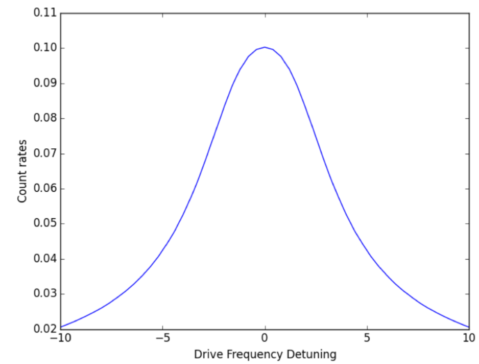
$\kappa = 0.5,\ \Gamma_1 = 0.5,\ g = 2$

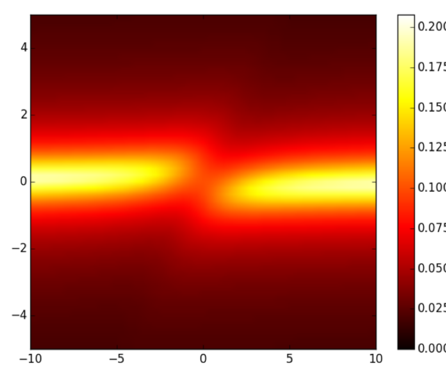




Weak coupling:

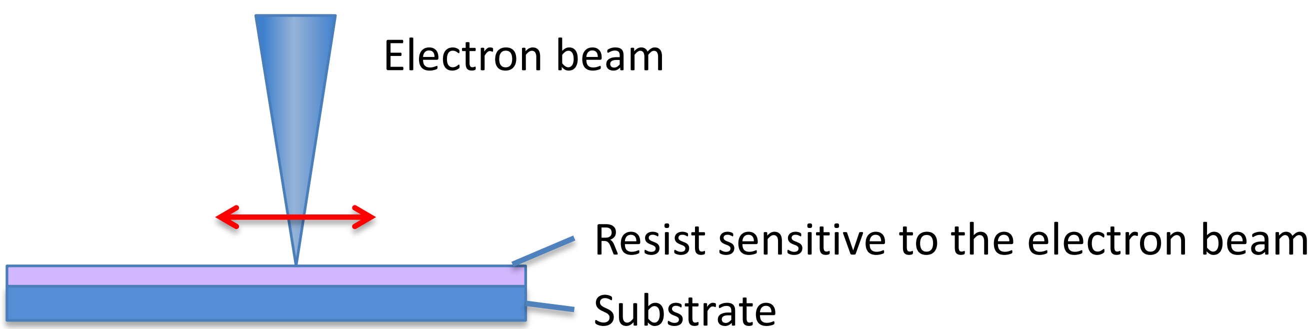
$\kappa = 0.5,\ \Gamma_1 = 4,\ g = 1$



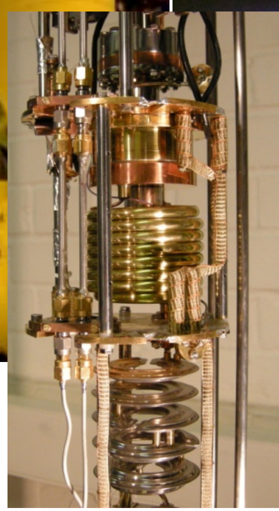


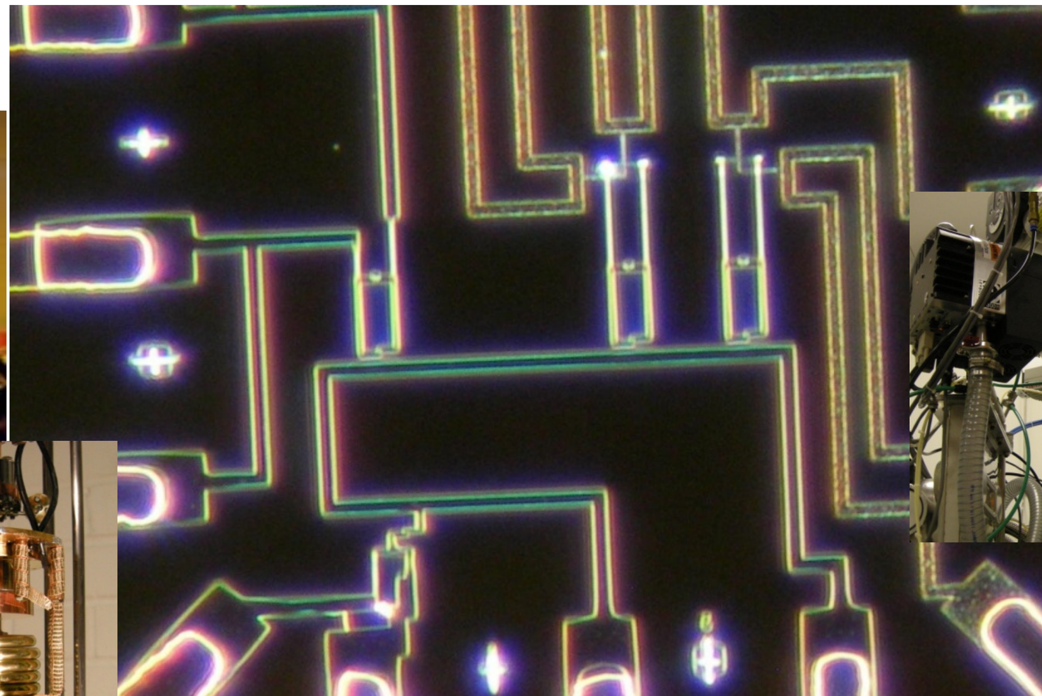
Sample Fabrication

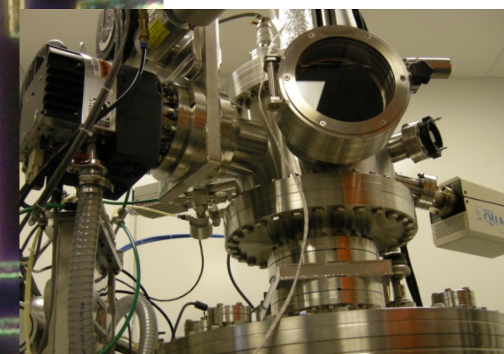
Electron-beam lithography systems (EBL)

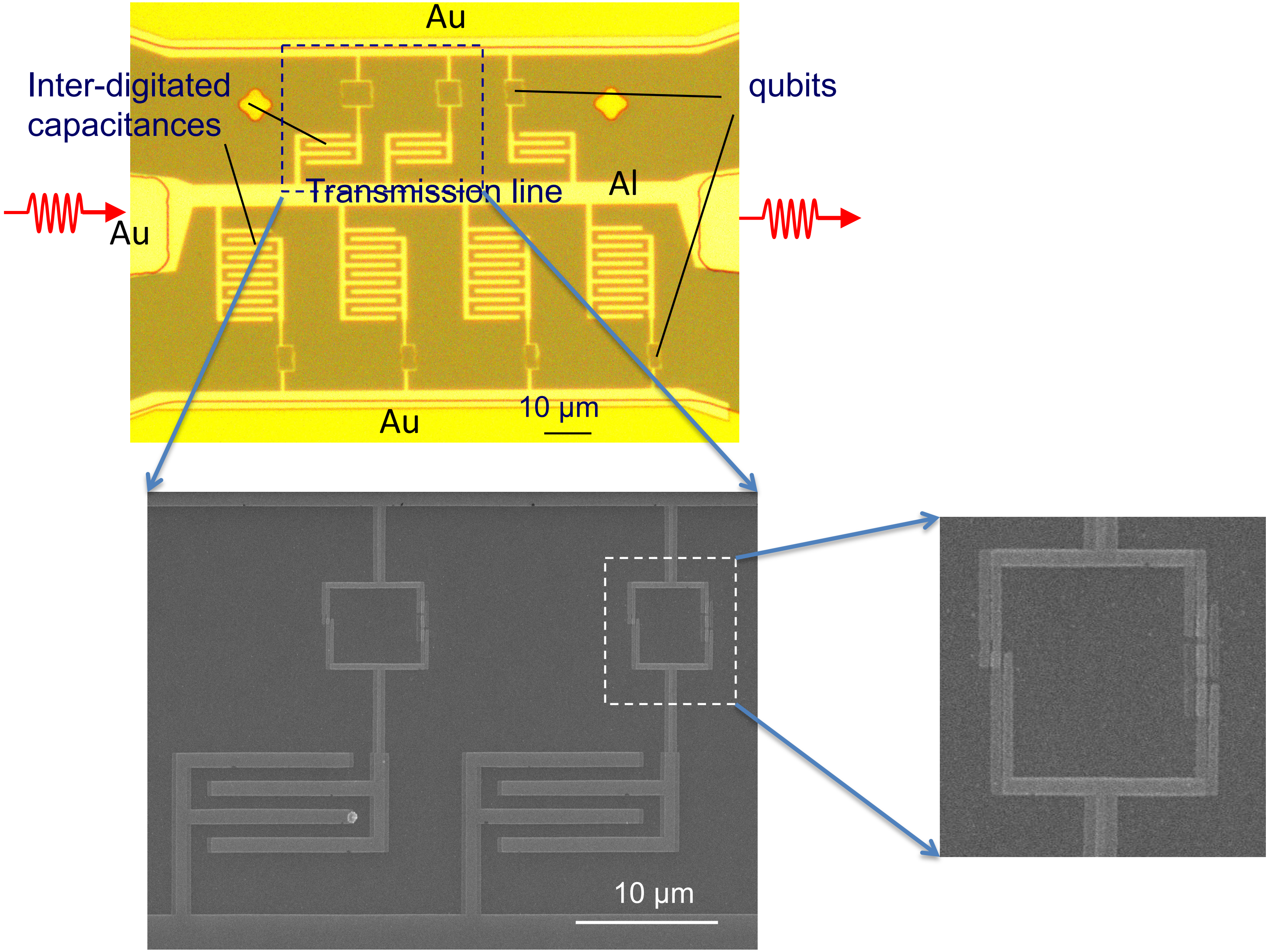


Hybrid nanostructures fabrication









Au
Inter-digitated
capacitances
qubits
Transmission line
Al
Au
Au
10 µm
10 µm

Lithography

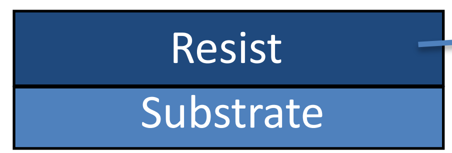
Resists are organic materials, which change their properties under deep UV or electron beams

Positive resists: Exposed areas are washed away during development processes

Negative resists: Unexposed areas are washed away during development processes

Positive resist processing

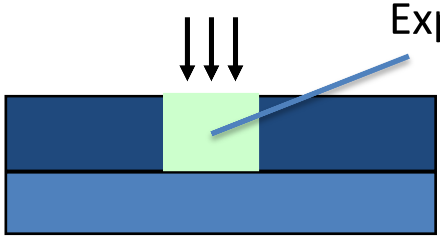
Spin coating



Thin film of resist (0.05 – 1 µm)

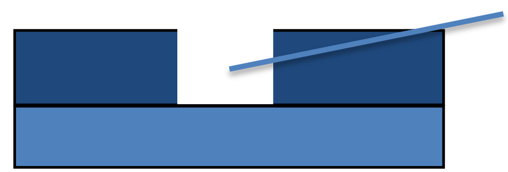
Exposure

Exposed resist



Development in liquids (developers)

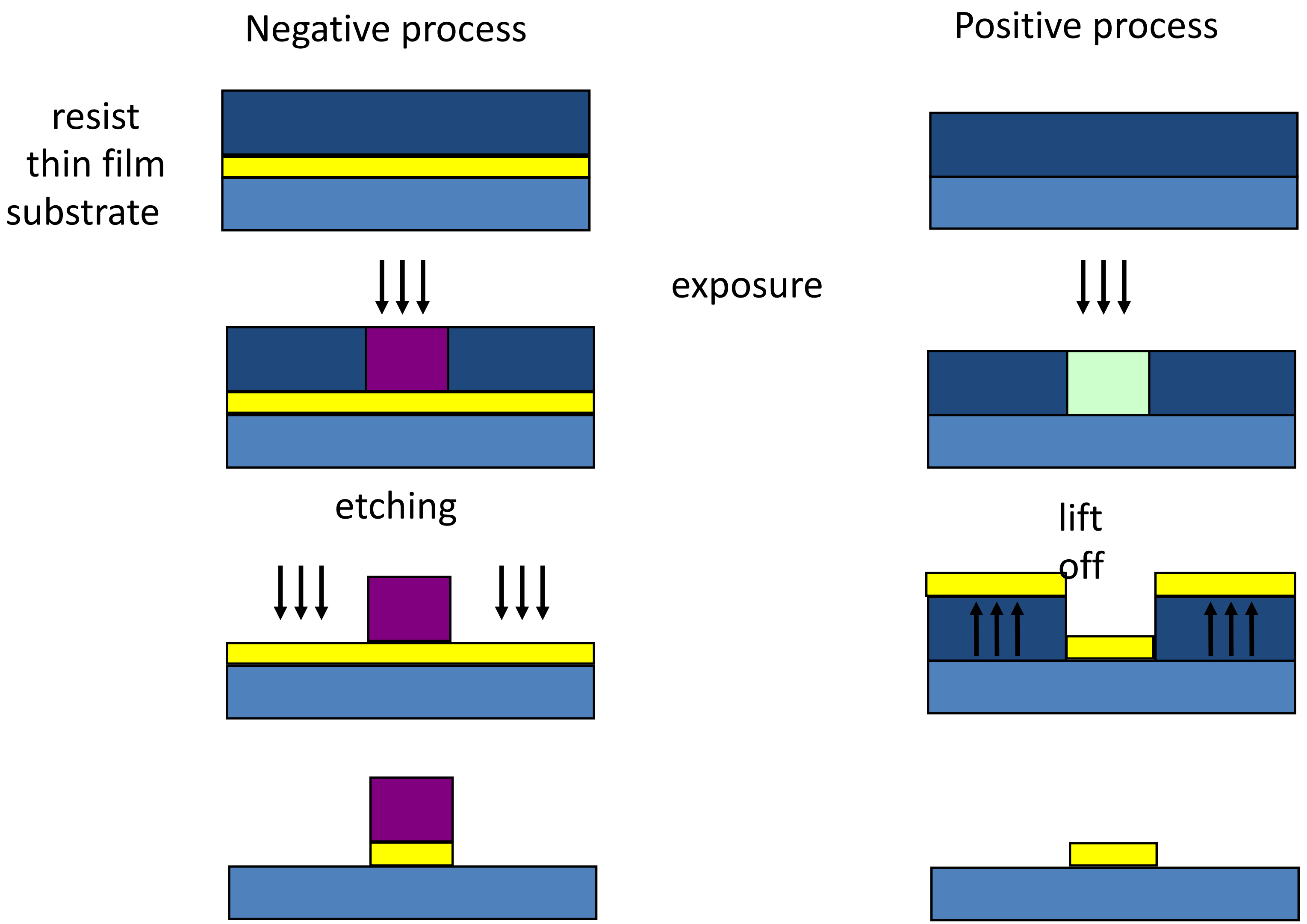
Developed resist is washed away after development



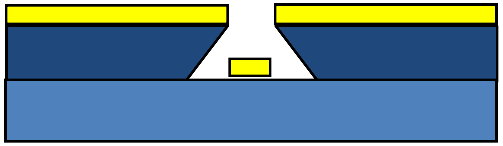
Resolution:
Photo-lithography: 0.5 µm
EB-lithography: 10 nm

The wafer is ready for metal deposition or etching

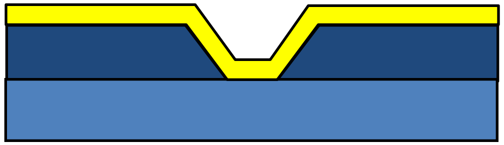
Lithography steps



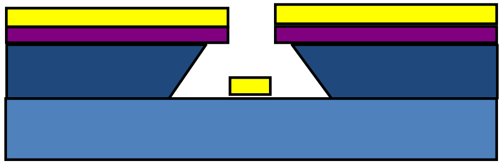
undercut



overcut



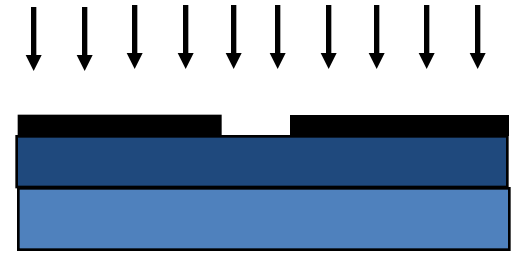
double layers resist



PMMA / Copolimer e-beam resists
S1813 / LOR5B photoresists

Bottom layer “softer’’ than top layer
Selective developers

Photolithography



UV light changes the properties of the

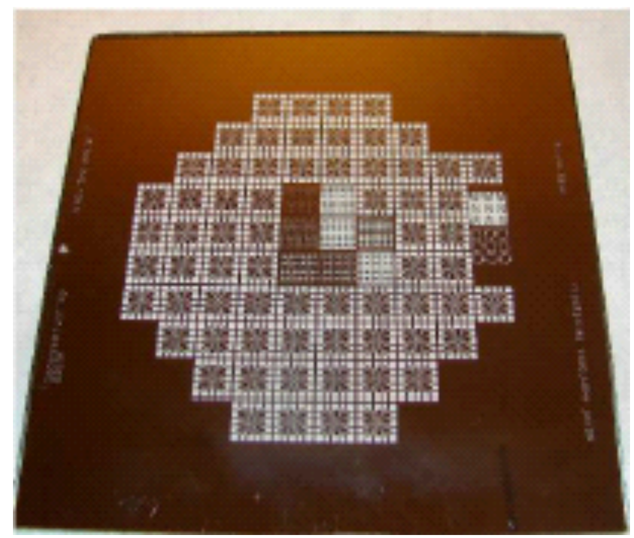
resist in the openings of the mask

Resist mask formed after development.

This allows the substrate to be selectively exposed to etching or deposition

Wavelength < 1μm for Hg lamps;
180-280nm for deep UV lamps

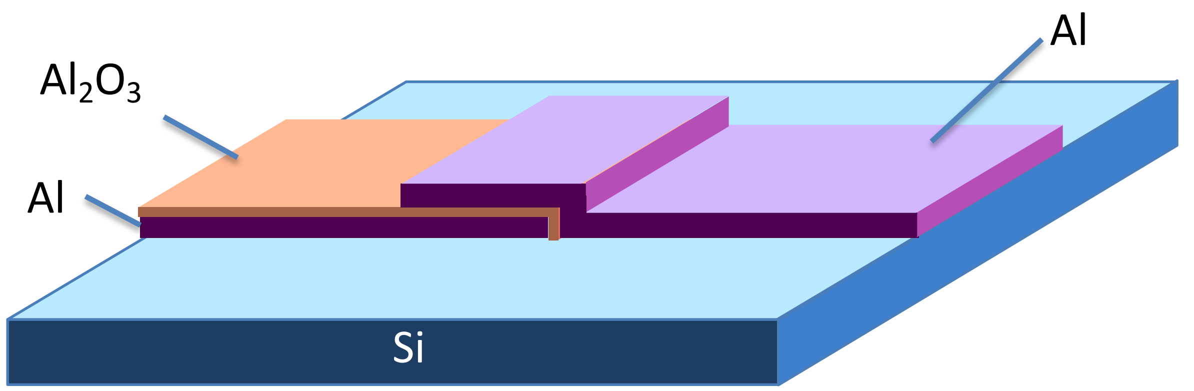
Direct printing resolution $w=(k \lambda g)^{1/2}$
k constant depending on resist
λ wavelength
g gap between resist and mask



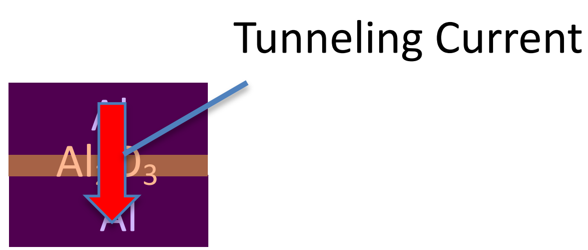
Photomask for 3 inch wafer

Fabrication of Josephson junctions

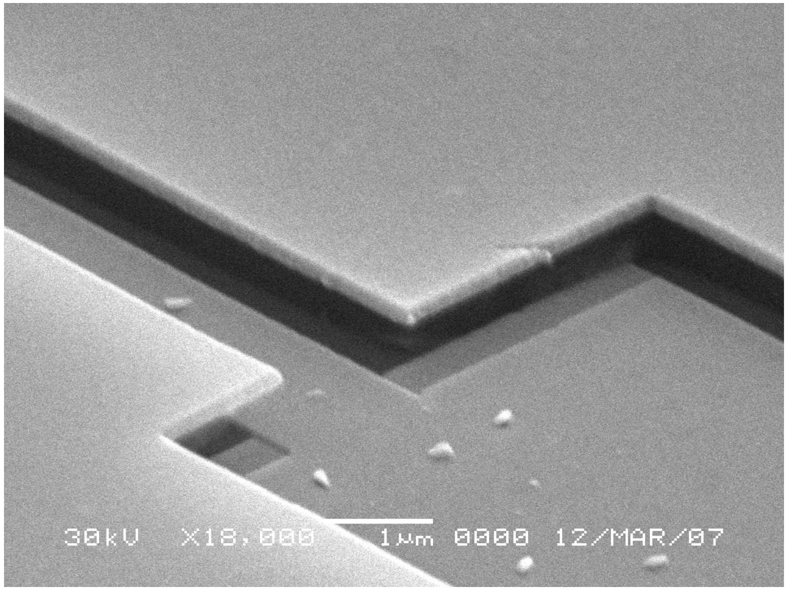
1. Bare silicon substrate
2. Aluminium deposition
3. Aluminium oxidation
4. Aluminium deposition (layer 2)

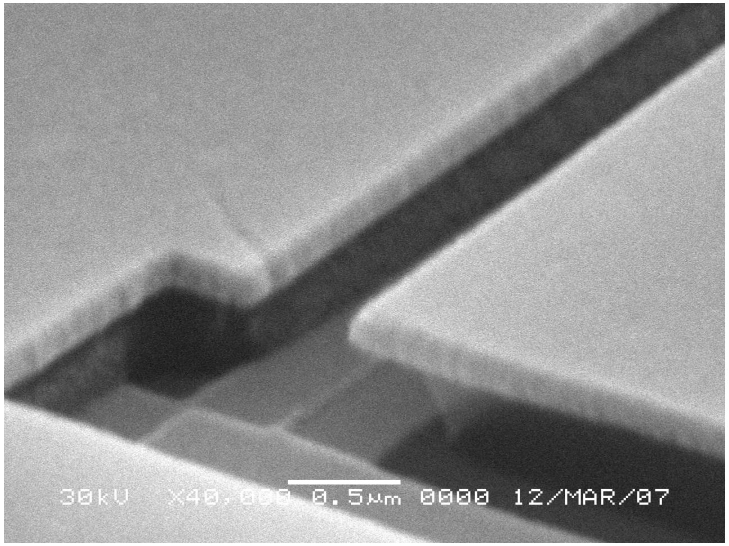


Crossection of the
Josephson junction

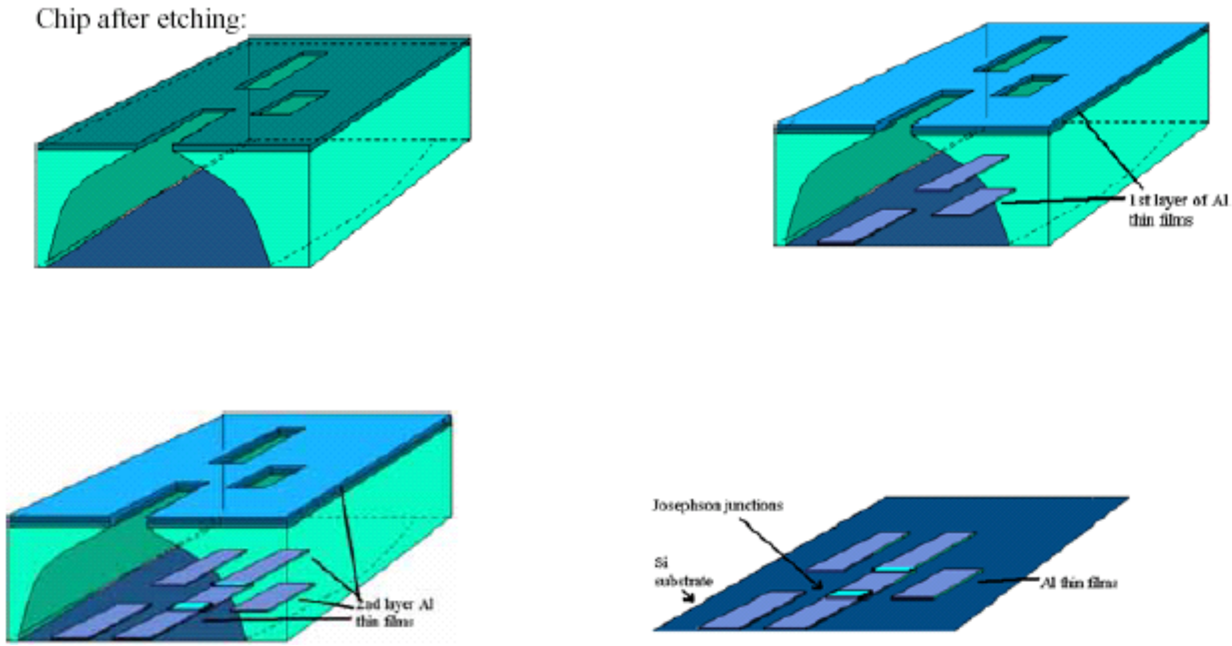


Au film pictured after evaporation on double layer resist



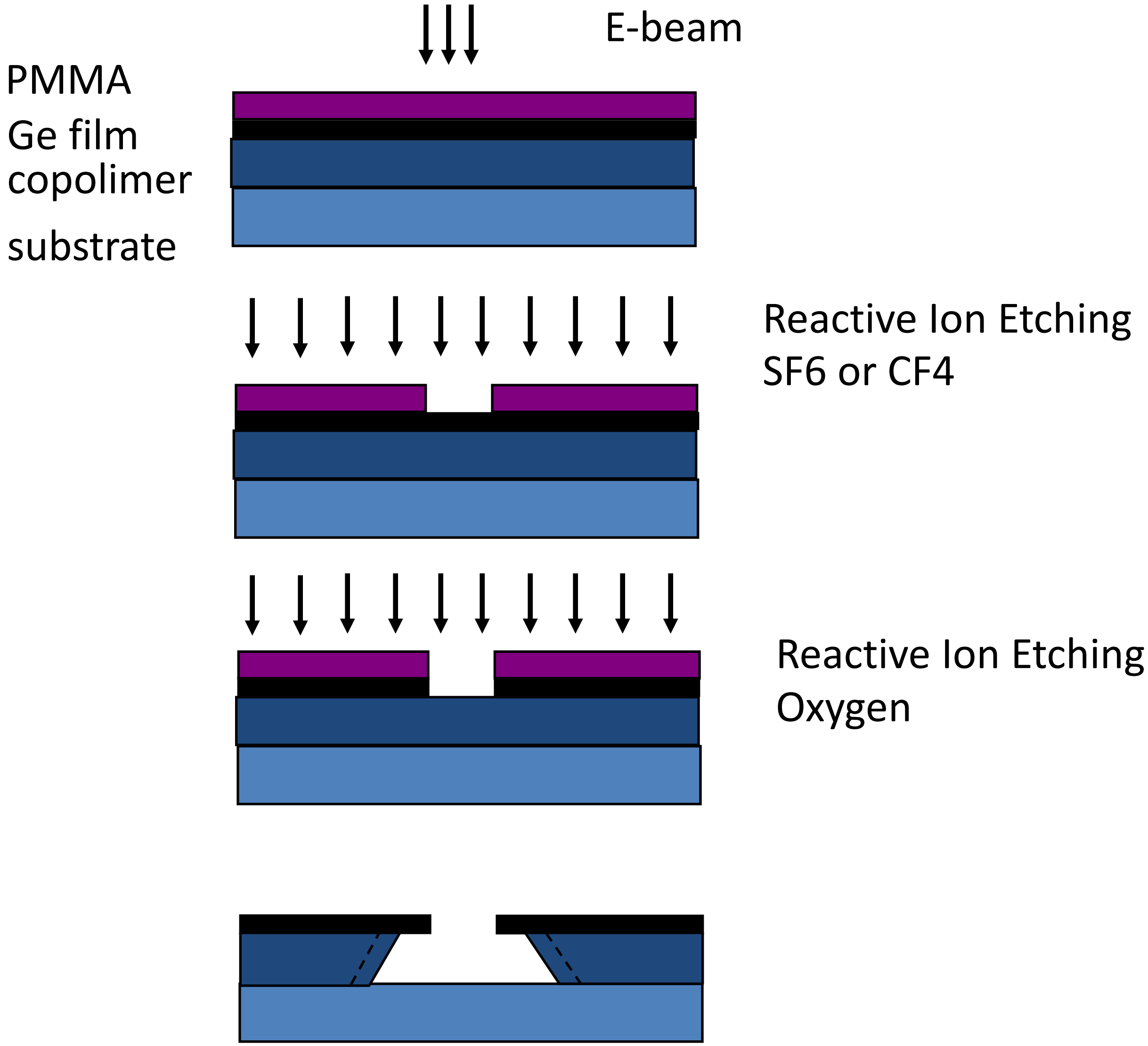


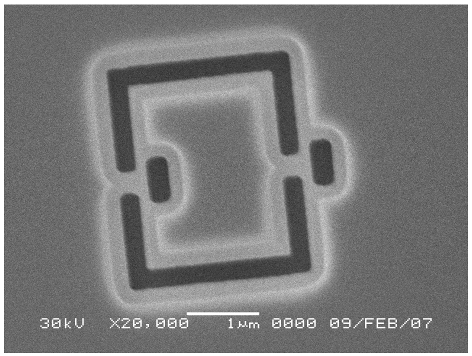
Angular evaporation + oxidation

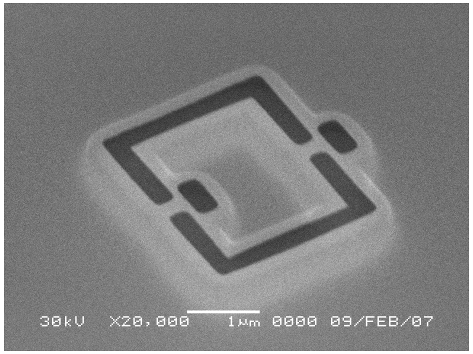


Soft mask
PMMA / Copolimer double layer resist
ZEP / Copolimer resist layer

Hard mask


E-beam
PMMA
Ge film
copolimer
substrate
Reactive Ion Etching
SF6 or CF4
Reactive Ion Etching
Oxygen


30kV X20,000 1μm 0000 09/FEB/07


30kV X20,000 1μm 0000 09/FEB/07