

Superconducting Quantum Technologies

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Lecture 1

Course plan and Assessment

Lectures: Mondays, Tuesdays

Seminars/labs: Fridays (PhD students: Daria Kalacheva, Alexey Dmitriev, Gleb Fedorov)

During lectures, questions are very welcome

- Homework (20%)
Simulations or solving problems related to the course topic
- Presentation (20%)
25-30 minutes talk describing one of experimental work on quantum optics with superconducting quantum systems
- Exam (60%)
Solving problems

Lecture 1

- Introduction to the subject
- Cavity QED with natural atoms
- Two-level systems and qubits. Different qubit realizations
- Tools of quantum mechanics
- The Hamiltonians of a two-level system and the wavefunction. Bloch sphere
- Double well potential and the single electron charge qubit
- Eigenstates and eigenenergies of a two-level system.

Course specification

The goal of the course is to provide an overview of a new and rapidly developing field of physics – quantum optics with artificial atoms. The field is fundamentally interesting and promising in several applications: quantum computing; on-chip quantum electronics; sensing; metrology.

- Introduction in 1D quantum optics
- Superconducting quantum systems
- Quantum evolution
- On-chip Quantum Electrodynamics
- Real quantum systems. Decoherence
- Experimental realization of Quantum Optical Effects with artificial atoms
- Open quantum systems
- Two qubit gate

Detailed course specification

Quantum Optics in 1D

- Harmonic oscillators and non-linear oscillators.
- Photons and fields.
- Atoms interacting with quantized electromagnetic fields.
- Quantum mechanics of electrical circuits.
- Transmission lines and transmission line resonators.
- Traveling and standing waves. Solid-state quantum bits.

Artificial atoms. Control and manipulation of energies and quantum states

- Charge and flux quantization in superconducting circuits.
- Alternative superconducting quantum systems.
- Superconducting qubits and artificial atoms.

Artificial atoms interacting with electromagnetic fields

- Quantum optics with artificial atoms.
- Strong coupling of an atom to the fields.
- Fock states and coherent states.
- Scattering of propagating electromagnetic waves by a two-level system.
- Artificial quantum systems and natural atoms.
- Quantum-state control and manipulation in qubits.

Experimental realization of fundamental quantum phenomena on-chip

- Jaynes-Cummings Hamiltonian.
- Lamb shift and Stark effect.
- Generating of the Fock states with artificial atoms.
- Single-photon source.
- Spontaneous and stimulated emission.
- Lasing with single artificial atoms.
- Electromagnetically induced transparency.

What to read

M. Orszag, "Quantum Optics: Including Noise Reduction", Trapped Ions, Quantum Trajectories, and Decoherence, 2nd edition, Springer-Verlag, 2007

- *Atom – field interaction (semiclassical and quantum approaches)*
- *Fock states*

List of some useful articles:

1. Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai. "Coherent control of macroscopic quantum states in a single-Cooper-pair box", *Nature* 398, 786 (1999).
2. I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, J. E. Mooij. "Coherent Quantum Dynamics of a Superconducting Flux Qubit". *Science* 299, pp. 1869-1871 (2003).
3. A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R.- S. Huang, J. Majer, S. Kumar, S. M. Girvin, R. J. Schoelkopf. "Strong coupling of a single photon to a superconducting qubit using circuit quantum electrodynamics" *Nature (London)* 431, 162 (2004).
4. Max Hofheinz, E. M. Weig, M. Ansmann, Radoslaw C. Bialczak, Erik Lucero, M. Neeley, A. D. O'Connell, H. Wang, John M. Martinis, A. N. Cleland. "Generation of Fock states in a superconducting quantum circuit", *Nature* 454, 310-314 (2008).
5. O. Astafiev, A. M. Zagoskin, A. A. Abdumalikov Jr., Yu. A. Pashkin, T. Yamamoto, R. Inomata, Y. Nakamura, and J. S. Tsai. "Resonance fluorescence of a single artificial atom", *Science* 327, 840 (2010).

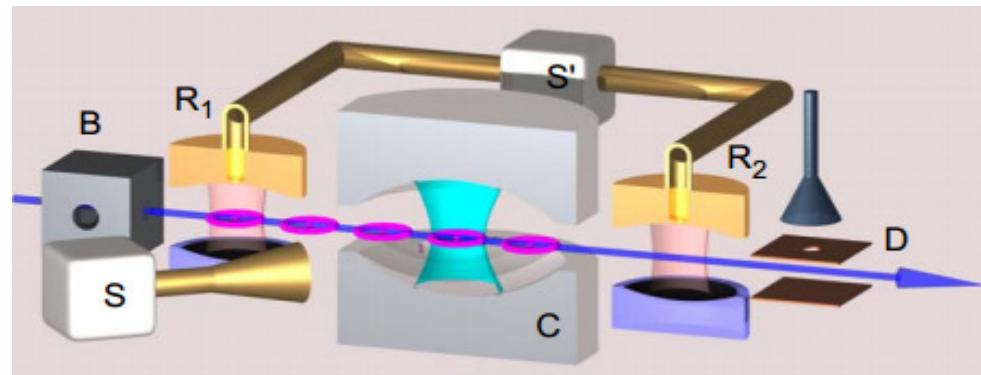
Cavity Quantum Electrodynamics

In 2012 Serge Haroche and David J. Wineland won the Physics Nobel Prize for “*ground breaking experimental methods that enable measuring and manipulation of individual quantum systems*” in Cavity QED



Cavity QED studies the interaction between light confined in a cavity and single atoms

Single atoms interact with a microwave cavity

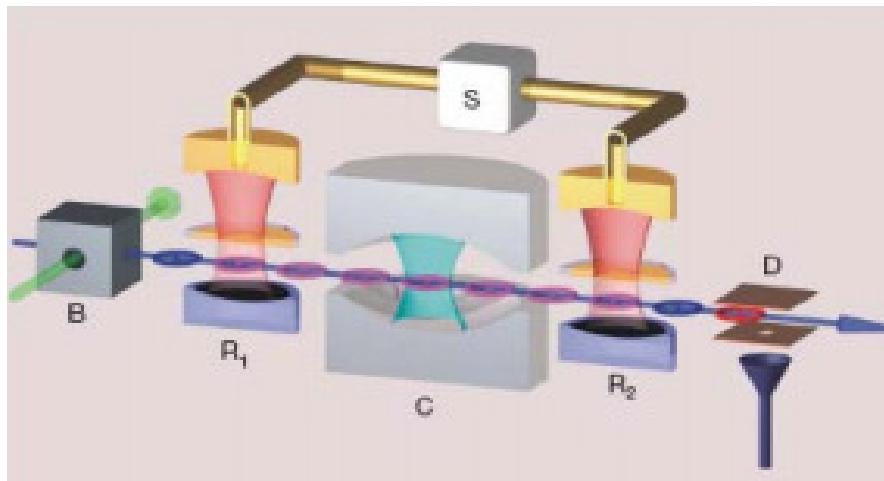


Circuit Quantum Electrodynamics

Similarity with Cavity QED

Cavity QED

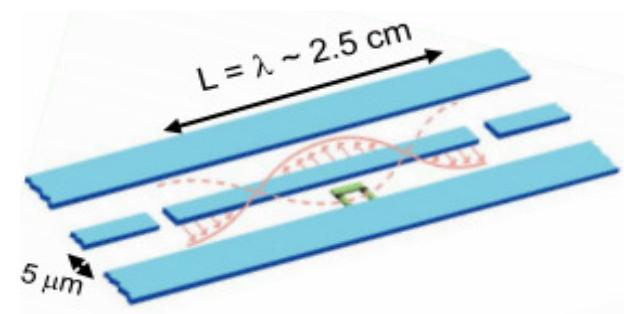
- 3D cavities
- Natural atoms



Circuit QED

- 1D transmission lines
- Artificial atoms (superconducting quantum systems)

Electrical circuits:
Fabricated with standard
nanotechnological processes



Two-level quantum system (quantum bit)

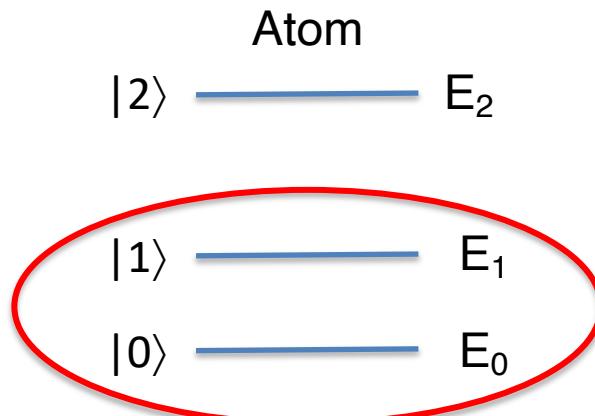
Two-level quantum systems – fundamental objects of Quantum mechanics

The two-level quantum systems can be used as quantum bits (qubits) to encode an information

Qubit is a unit of quantum information – the quantum analogue of a classical bit.

Physically, the qubit is a two-level quantum system

Natural qubits are: polarization of a single photon, electron spin, charge quantum, flux quantum



Artificial qubits: quantum dots, superconducting systems

Quantum bits (qubits)

The information is encoded by quantum states

Ground state:

0



E_1



E_0

Excited state:

1

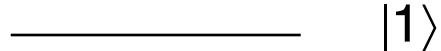


E_1

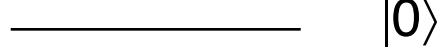


E_0

Two-level
quantum system

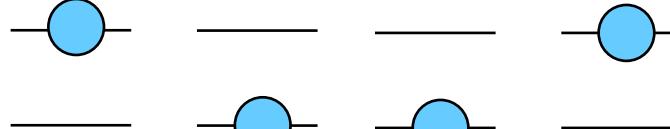


$|1\rangle$



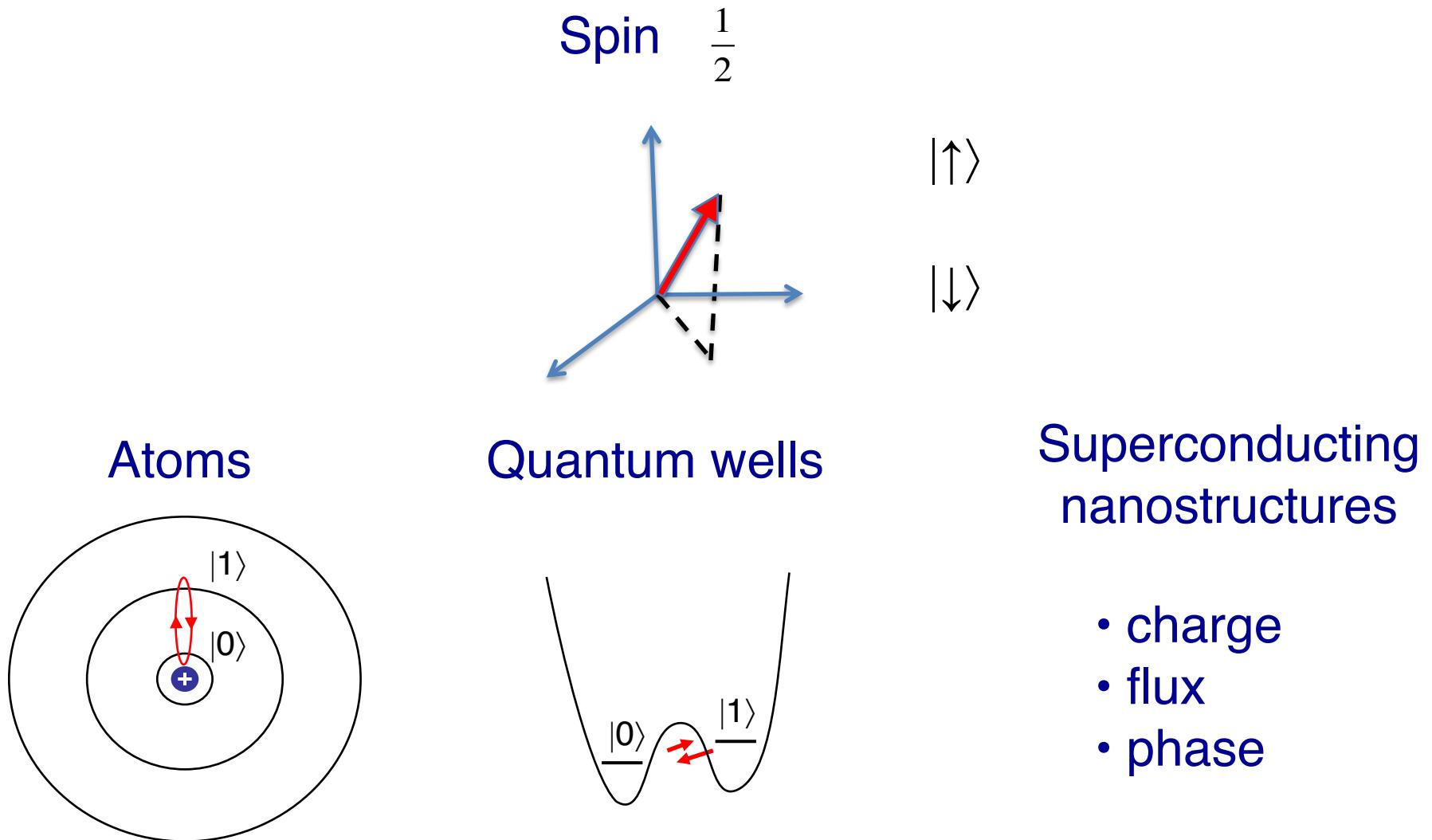
$|0\rangle$

Encoding numbers in quantum systems



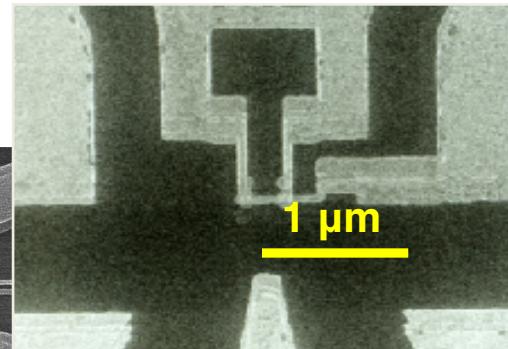
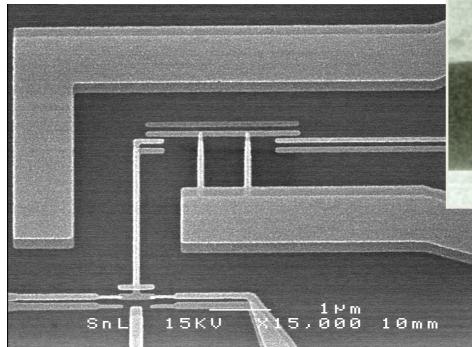
$|1001\rangle$

Different qubit realizations

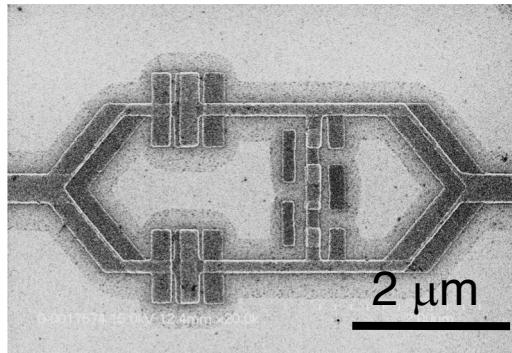
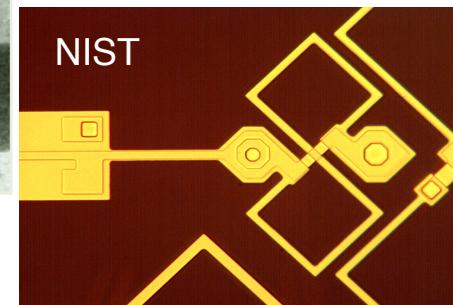


Realizations of superconducting quantum circuits

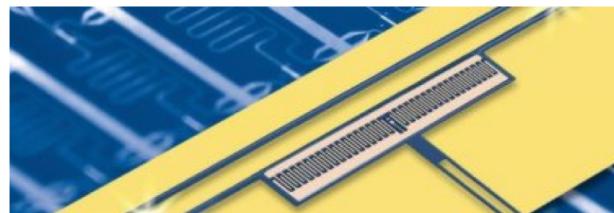
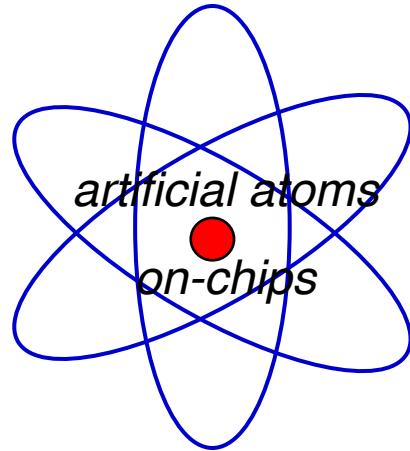
charge qubits



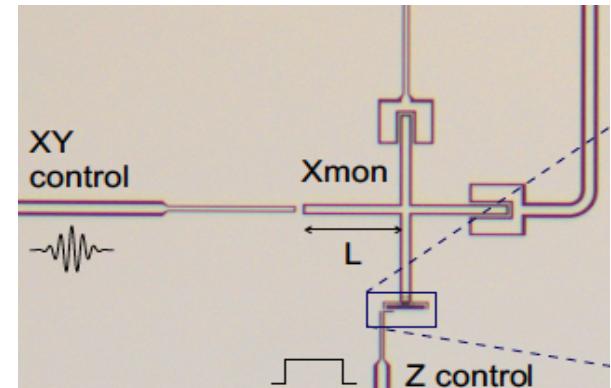
phase qubit



Flux qubits

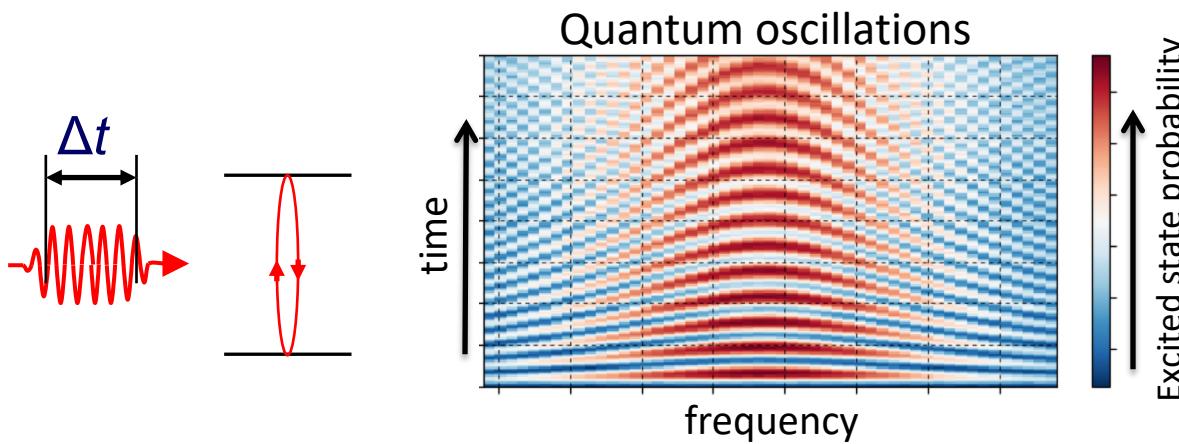
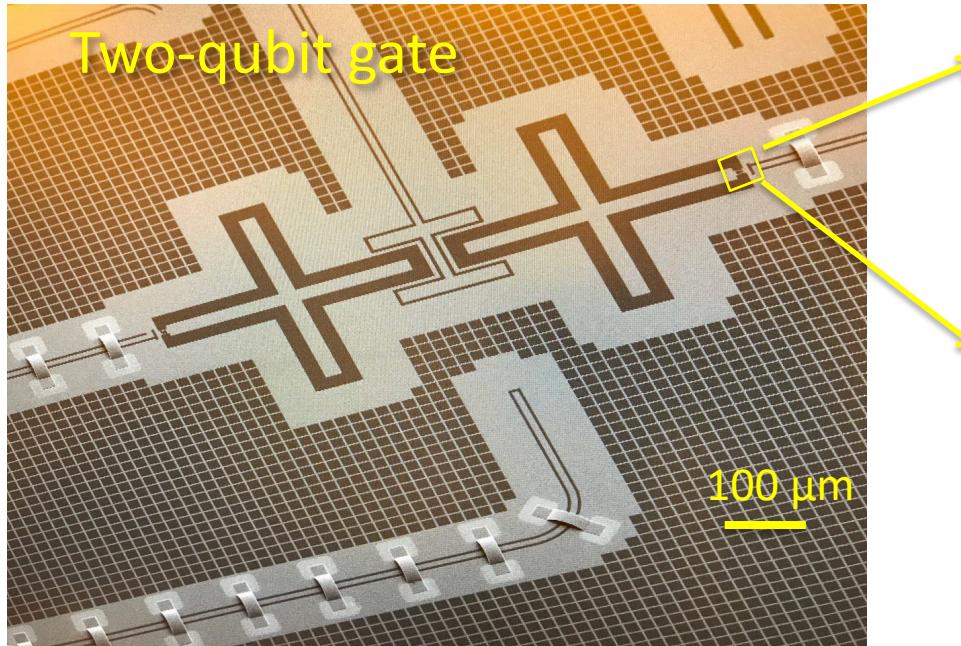


Transmon



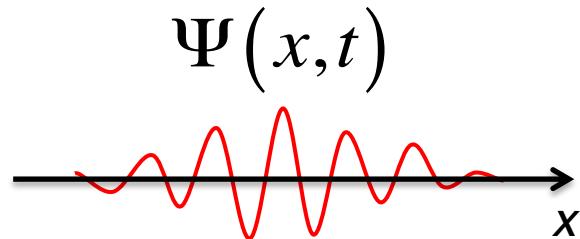
Xmon

Superconducting Quantum Systems



Superconducting quantum systems are at the core of quantum technology and fundamental physics

Wave function
Probability amplitude:



Measurement result: collapse of wavefunction

Probability:

$$P(x, t) = |\Psi(x, t)|^2 \equiv \Psi^*(x, t)\Psi(x, t) \quad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Psi^*(x, t)\Psi(x, t) dx dt = 1$$

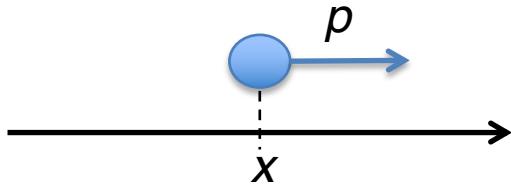
Operator:

$$\hat{O} \rightarrow \hat{O}\Psi(x, t) = \Psi^{(O)}(x, t) \quad \rightarrow \quad \langle \hat{O}(t) \rangle = \int_{-\infty}^{\infty} \Psi^*(x, t)\hat{O}\Psi(x, t) dx$$

Can be presented in various forms

Expectation value:

Quantum MECHANICS



Momentum operator:

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

Momentum:

$$\langle p \rangle = \int_{-\infty}^{\infty} \Psi^*(x) \left(-i\hbar \frac{\partial \Psi(x)}{\partial x} \right) dx$$

The Hamiltonian (total energy):

$$H = U(x) + \frac{p^2}{2m}$$

$$\hat{H} = U(x) + \frac{\hat{p}^2}{2m}$$

$$\hat{H} = U(x) - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}$$

The Schrödinger equation

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = \hat{H} \Psi(x,t)$$

The Schrödinger equation and time evolution

$$i\hbar \frac{\partial \psi(t)}{\partial t} = \hat{H}\psi(t) \quad \psi(t) = \exp\left(-\frac{i}{\hbar} \int_0^t H dt'\right) \psi(0)$$

The system evolution of the time-independent Hamiltonian

$$\psi(t) = \exp\left(-i \frac{H}{\hbar} t\right) \psi(0)$$

Eigenstates and eigenenergies

$$\hat{H}\psi_k(t) = E_k \psi_k(t)$$

$$\psi_k(t) = \psi_k(0) e^{-i\omega_k t}$$

Wavefunction representation

$$\hat{H}\psi_k(t) = E_k\psi_k(t) \quad \psi_k(t) = \psi_k(0)e^{-i\omega_k t}$$

Dirac's notation

Ket: $\psi_k = |k\rangle$

Bra: $\psi_k^* = \langle k|$

$|k\rangle = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$ *k-th element is non-zero*

$\langle k| = \begin{pmatrix} 0 & 0 & \dots & 1 & \dots & 0 \end{pmatrix}$

Inner product

$$\langle k|n\rangle = \begin{pmatrix} 0 & 0 & \dots & 1 & \dots & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

$$\langle k|n\rangle = 0$$

$$\langle n|n\rangle = 1$$

Outer product

$$|k\rangle\langle n| = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \dots & \dots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 \end{pmatrix}_k^n$$

Operators in Dirac's notation

$$\hat{o} = \sum_{k,j} c_{kj} |k\rangle\langle j|$$

Quantized energy levels (e.g. natural atoms)

Eigenstates, eigenenergies

$$\hat{H}\psi_k(t) = E_k\psi_k(t)$$

$$|k\rangle \xrightarrow{\quad} E_k$$

■
■
■

$$|2\rangle \xrightarrow{\quad} E_2$$

$$|1\rangle \xrightarrow{\quad} E_1$$

$$|0\rangle \xrightarrow{\quad} E_0$$

Quantum bit is a two-level quantum system

$$|1\rangle \xrightarrow{\quad} E_1$$

$$|0\rangle \xrightarrow{\quad} E_0$$

Two-level atom Hamiltonian in the eigenbasis

Ground state

$$E_1 \text{ --- } |1\rangle$$

$$E_0 \text{ --- } |0\rangle$$

$$H|0\rangle = E_0|0\rangle$$

Excited state

$$E_1 \text{ --- } |1\rangle$$

$$E_0 \text{ --- } |0\rangle$$

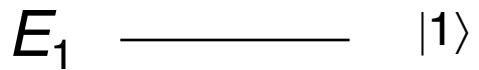
$$H|1\rangle = E_1|1\rangle$$

$$H = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

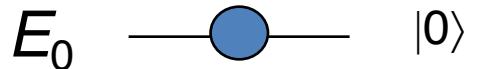
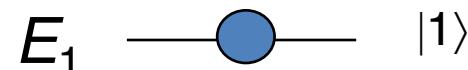
$$H = \begin{pmatrix} E_0 & 0 \\ 0 & E_1 \end{pmatrix}$$

Two-level atom

Ground state: $|0\rangle$



Excited state: $|1\rangle$



Arbitrary state:

$$\Psi = \alpha|0\rangle + \beta|1\rangle$$

$$\begin{aligned} \Psi^* \Psi &= (\alpha^* \langle 0 | + \beta^* \langle 1 |)(\alpha |0\rangle + \beta |1\rangle) = \alpha^* \alpha \langle 0 | 0 \rangle + \alpha^* \beta \langle 0 | 1 \rangle + \alpha \beta^* \langle 1 | 0 \rangle + \beta^* \beta \langle 1 | 1 \rangle = |\alpha|^2 + |\beta|^2 \\ &\quad |\alpha|^2 + |\beta|^2 = 1 \end{aligned}$$

$$e^{i\delta} \cos \frac{\theta}{2} |0\rangle + e^{i\vartheta} \sin \frac{\theta}{2} |1\rangle = e^{i\delta} \left(\cos \frac{\theta}{2} |0\rangle + e^{i(\vartheta-\delta)} \sin \frac{\theta}{2} |1\rangle \right)$$

$$\Psi = \cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle$$

Wavefunction of a two-level system on the Bloch sphere

$$\Psi = \cos \frac{\theta}{2} |0\rangle + e^{i\varphi} \sin \frac{\theta}{2} |1\rangle$$

θ and φ define position
of a unit vector on the sphere:

$$x = \sin \theta \cos \varphi$$

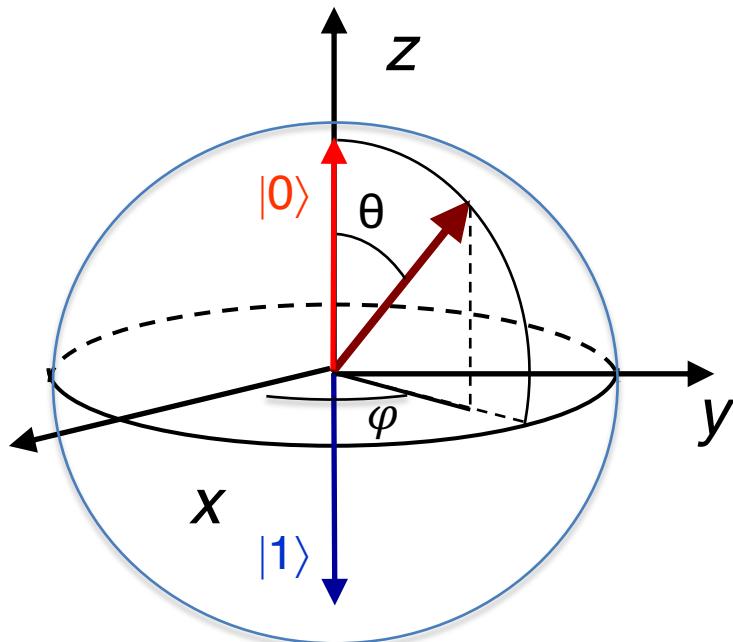
$$y = \sin \theta \sin \varphi$$

$$z = \cos \theta$$

$$\Psi = \cos \frac{\theta}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + e^{i\varphi} \sin \frac{\theta}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\varphi} \sin \frac{\theta}{2} \end{pmatrix}$$

$$0 \leq \theta \leq \pi$$

$$0 \leq \varphi \leq 2\pi$$



Mapping of an arbitrary state on the Bloch sphere

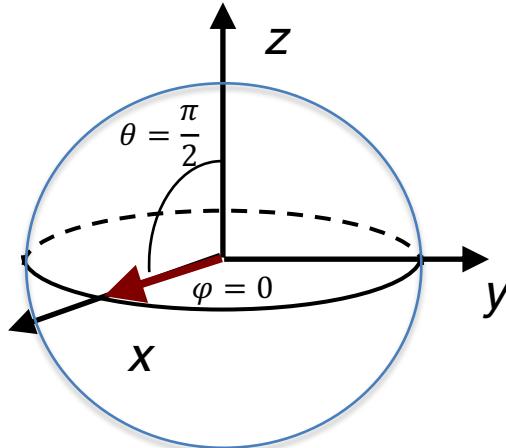
$$\Psi = \alpha|0\rangle + \beta|1\rangle$$

$$\Psi = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi} \sin\frac{\theta}{2}|1\rangle$$

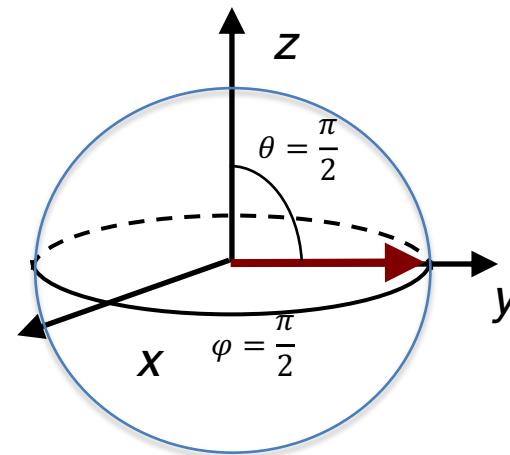
$$\theta = 2 \operatorname{atan} \left| \frac{\beta}{\alpha} \right| \quad \varphi = -i \ln \left(\frac{\beta}{\alpha} \cot \frac{\theta}{2} \right)$$

Examples of states on the Bloch sphere

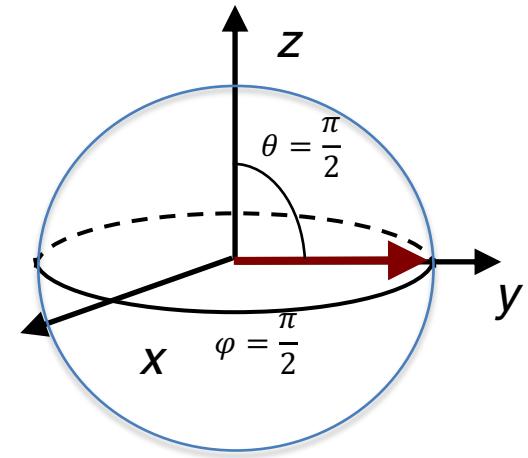
$$\Psi = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \cos \frac{\pi/2}{2} |0\rangle + \sin \frac{\pi/2}{2} |0\rangle$$



$$\begin{aligned}\Psi &= \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \cos \frac{\pi/2}{2} |0\rangle - \sin \frac{\pi/2}{2} |0\rangle = \\ &\quad \cos \frac{\pi/2}{2} |0\rangle + e^{i\pi} \sin \frac{\pi/2}{2} |0\rangle\end{aligned}$$

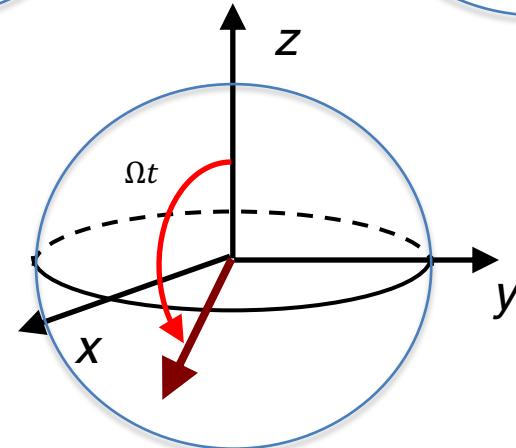


$$\begin{aligned}\Psi &= \frac{|0\rangle + i|1\rangle}{\sqrt{2}} = \cos \frac{\pi/2}{2} |0\rangle + i \sin \frac{\pi/2}{2} |0\rangle \\ &= \cos \frac{\pi/2}{2} |0\rangle + e^{i\pi/2} \sin \frac{\pi/2}{2} |0\rangle\end{aligned}$$



Evolution: $\Psi = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \cos \frac{\Omega t}{2} |0\rangle + \sin \frac{\Omega t}{2} |0\rangle$

Rotation around y-axis

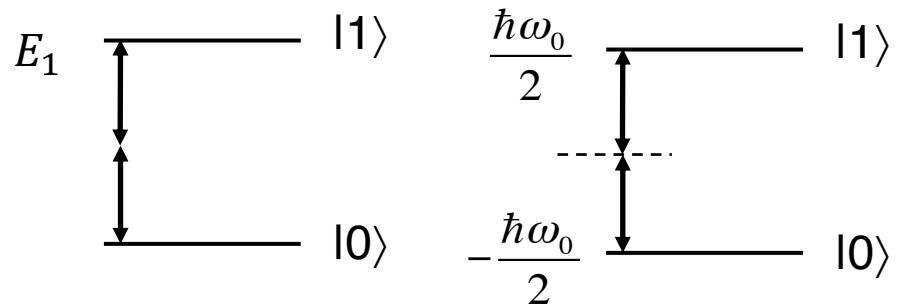


Two-level atom representation

$$H = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

$$\hbar\omega_0 = E_1 - E_0$$

Symmetrized diagonal Hamiltonian:



$$H = \begin{pmatrix} -\frac{\hbar\omega_0}{2} & 0 \\ 0 & \frac{\hbar\omega_0}{2} \end{pmatrix}$$

$$H = \frac{\hbar\omega_0}{2}(-|0\rangle\langle 0| + |1\rangle\langle 1|)$$

$$H = \frac{\hbar\omega_0}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = -\frac{\hbar\omega_0}{2} \sigma_z$$

Pauli matrices:

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

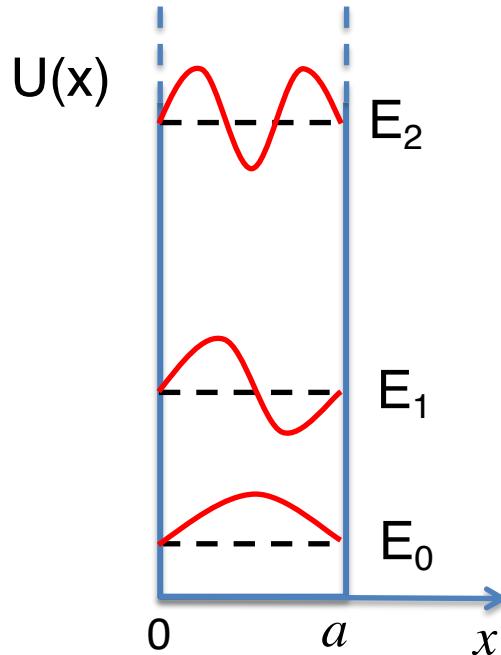
$$\sigma_x \sigma_y = -i \sigma_z$$

$$\sigma_y \sigma_z = -i \sigma_x$$

$$\sigma_z \sigma_x = -i \sigma_y$$

$$\sigma_k \sigma_k = I$$

Possible realization of a solid-state charge qubit



Schrodinger Equation: $\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U \right) \psi = E \psi$

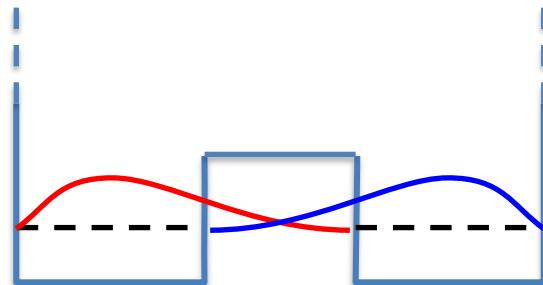
Wavefunction: $\psi = A e^{\kappa x} + B e^{-\kappa x}$ $\kappa = \frac{1}{\hbar} \sqrt{2m(U - E)}$

$$E_n = \frac{\pi^2 \hbar^2}{2ma^2} (n+1)^2$$

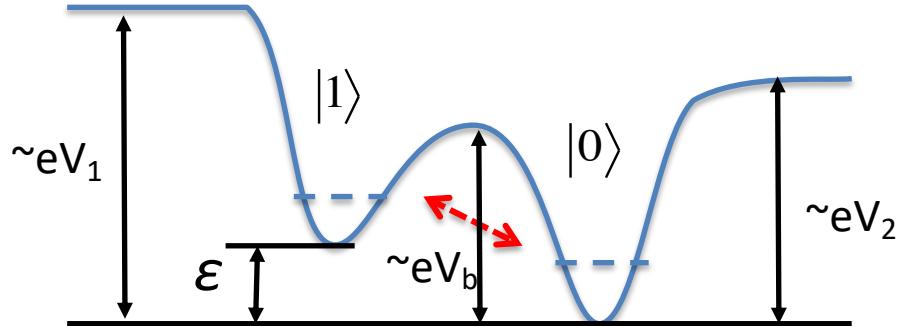
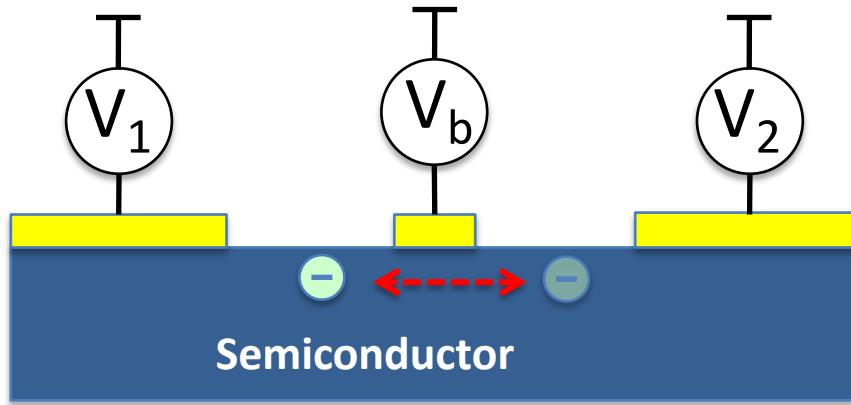
$$\psi_n = \sqrt{\frac{2}{a}} \sin \left[\frac{\pi(n+1)x}{a} \right]$$

$$H = E_0 |0\rangle\langle 0| + E_1 |1\rangle\langle 1| + E_2 |2\rangle\langle 2| + \dots$$

Wavefunctions are overlapped \Rightarrow tunneling



A charge qubit with a single electron



Potential energy difference
due to external gate voltages

$$H = -E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

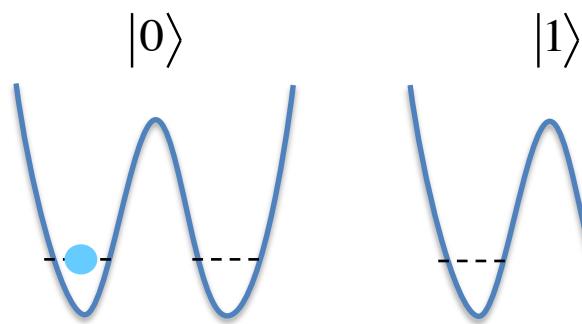
$$H = \begin{pmatrix} E_0 & -\frac{\Delta}{2} \\ -\frac{\Delta}{2} & E_1 \end{pmatrix}$$

Tunneling energy

Possible realization of a solid-state charge qubit

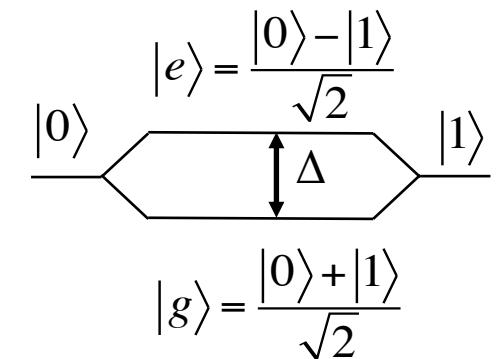
$$H = E_0 |0\rangle\langle 0| + E_0 |1\rangle\langle 1|$$

$$H = \begin{pmatrix} E_0 & 0 \\ 0 & E_0 \end{pmatrix}$$



$$H = E_0 |0\rangle\langle 0| + E_0 |1\rangle\langle 1| - \frac{\Delta}{2}(|0\rangle\langle 1| + |1\rangle\langle 0|)$$

$$H = \begin{pmatrix} E_0 & -\frac{\Delta}{2} \\ -\frac{\Delta}{2} & E_0 \end{pmatrix}$$

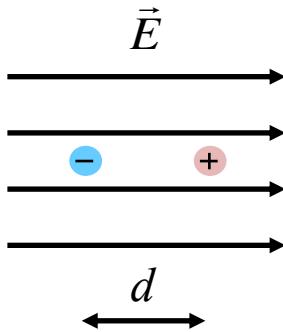


$$E_0 \rightarrow 0: \quad H = -\frac{\Delta}{2}(|0\rangle\langle 1| + |1\rangle\langle 0|)$$

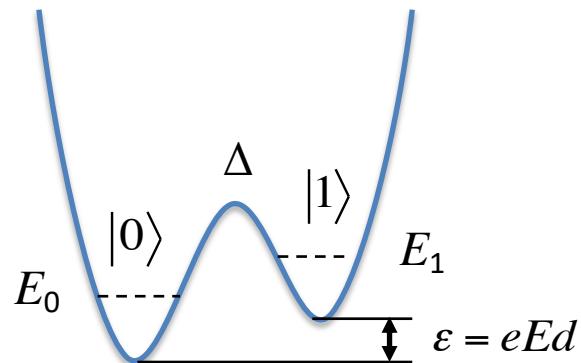
$$H|g\rangle = -\frac{\Delta}{2}(|0\rangle\langle 1| + |1\rangle\langle 0|) \frac{|0\rangle + |1\rangle}{\sqrt{2}} = -\frac{\Delta}{2} \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$H|e\rangle = -\frac{\Delta}{2}(|0\rangle\langle 1| + |1\rangle\langle 0|) \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \frac{\Delta}{2} \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

The electron in the double well potential biased by an electric field



$$H = E_0 |0\rangle\langle 0| + E_1 |1\rangle\langle 1| - \frac{\Delta}{2}(|0\rangle\langle 1| + |1\rangle\langle 0|)$$



$$E_1 - E_0 = \epsilon$$

$$H = \begin{pmatrix} E_0 & -\frac{\Delta}{2} \\ -\frac{\Delta}{2} & E_1 \end{pmatrix}$$

$$H = \begin{pmatrix} -\frac{\epsilon}{2} & -\frac{\Delta}{2} \\ -\frac{\Delta}{2} & \frac{\epsilon}{2} \end{pmatrix}$$

$$H = -\frac{\epsilon}{2}\sigma_z - \frac{\Delta}{2}\sigma_x$$

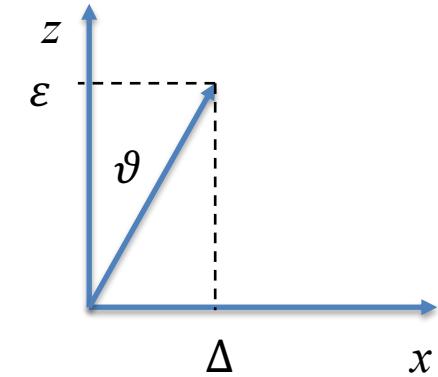
Eigenstates and eigenenergies of a two-level system

$$H = \begin{pmatrix} -\frac{\varepsilon}{2} & -\frac{\Delta}{2} \\ -\frac{\Delta}{2} & \frac{\varepsilon}{2} \end{pmatrix} \quad H = -\frac{\varepsilon}{2}\sigma_z - \frac{\Delta}{2}\sigma_x$$

$$\tan \vartheta = \frac{\Delta}{\varepsilon} \quad \sin \vartheta = \frac{\Delta}{\sqrt{\varepsilon^2 + \Delta^2}} \quad \cos \vartheta = \frac{\varepsilon}{\sqrt{\varepsilon^2 + \Delta^2}}$$

$$H = -\frac{\sqrt{\varepsilon^2 + \Delta^2}}{2} (\sigma_z \cos \vartheta + \sigma_x \sin \vartheta)$$

$$H\Psi_k = E_k \Psi_k$$



$$\begin{pmatrix} -\cos \vartheta & -\sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix} \Psi_k = \lambda_k \Psi_k \quad \begin{pmatrix} -\cos \vartheta - \lambda_k & -\sin \vartheta \\ -\sin \vartheta & (\cos \vartheta - \lambda_k) \end{pmatrix} \Psi_k = 0 \quad \lambda_k^2 - \cos^2 \vartheta - \sin^2 \vartheta = 0$$

$$\lambda_0 = -1 \quad E_0 = -\frac{\sqrt{\varepsilon^2 + \Delta^2}}{2}$$

$$\lambda_1 = 1 \quad E_1 = \frac{\sqrt{\varepsilon^2 + \Delta^2}}{2}$$

Ground state

$$H\Psi_0 = -\Delta E \Psi_0 \quad \lambda = -1$$

$$\begin{pmatrix} -\cos\vartheta+1 & -\sin\vartheta \\ -\sin\vartheta & \cos\vartheta+1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0$$

$$\alpha(1-\cos\vartheta) - \beta \sin\vartheta = 0$$

$$2\alpha \sin^2 \frac{\vartheta}{2} - 2\beta \sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2} = 0$$

$$\alpha \sin \frac{\vartheta}{2} = \beta \cos \frac{\vartheta}{2}$$

$$\alpha = \cos \frac{\vartheta}{2} \quad \beta = \sin \frac{\vartheta}{2}$$

$$\Psi_0 = \begin{pmatrix} \cos \frac{\vartheta}{2} \\ \sin \frac{\vartheta}{2} \end{pmatrix}$$

Excited state

$$H\Psi_1 = \Delta E \Psi_1 \quad \lambda = 1$$

$$\begin{pmatrix} -\cos\vartheta-1 & -\sin\vartheta \\ -\sin\vartheta & \cos\vartheta-1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = 0$$

$$-\alpha(\cos\vartheta+1) - \beta \sin\vartheta = 0$$

$$-2\alpha \cos^2 \frac{\vartheta}{2} - 2\beta \sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2} = 0$$

$$\alpha \cos \frac{\vartheta}{2} = -\beta \sin \frac{\vartheta}{2}$$

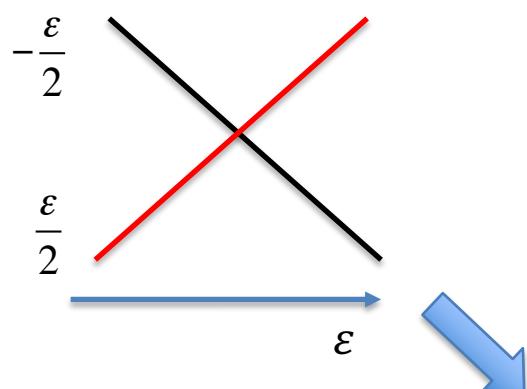
$$\alpha = \sin \frac{\vartheta}{2} \quad \beta = \cos \frac{\vartheta}{2}$$

$$\Psi_1 = \begin{pmatrix} \sin \frac{\vartheta}{2} \\ -\cos \frac{\vartheta}{2} \end{pmatrix}$$

The charge qubit states and energies

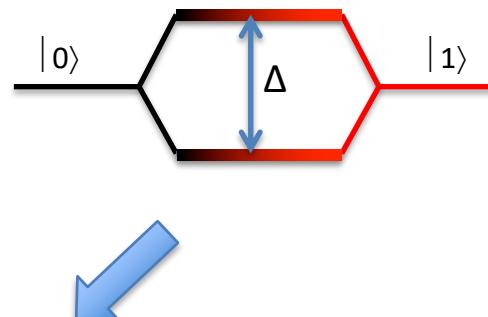
Potential energy:

$$U = \begin{pmatrix} -\varepsilon/2 & 0 \\ 0 & \varepsilon/2 \end{pmatrix} = -\frac{\varepsilon}{2} \sigma_z$$

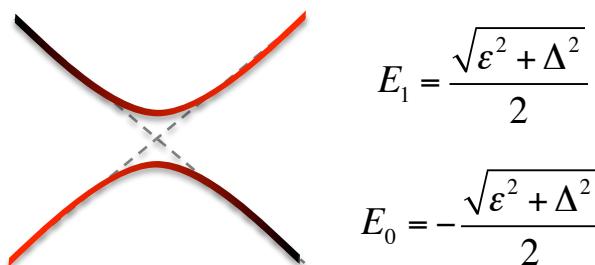


Kinetic energy:

$$T = \begin{pmatrix} 0 & -\Delta/2 \\ -\Delta/2 & 0 \end{pmatrix} = -\frac{\Delta}{2} \sigma_x$$

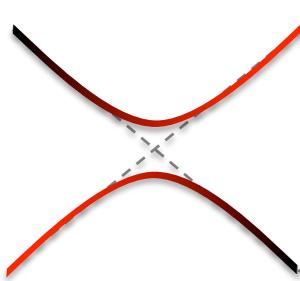


$$H = \frac{1}{2} \begin{pmatrix} -\varepsilon & -\Delta \\ -\Delta & \varepsilon \end{pmatrix} = -\frac{\varepsilon}{2} \sigma_z - \frac{\Delta}{2} \sigma_x$$



The charge qubit Hamiltonian (charge basis)

$$H = \frac{1}{2} \begin{pmatrix} -\varepsilon & -\Delta \\ -\Delta & \varepsilon \end{pmatrix} = -\frac{\varepsilon}{2} \sigma_z - \frac{\Delta}{2} \sigma_x$$



$$E_1 = \frac{\sqrt{\varepsilon^2 + \Delta^2}}{2}$$

$$\Psi_1 = \sin \frac{\vartheta}{2} |0\rangle - \cos \frac{\vartheta}{2} |1\rangle$$

$$E_0 = -\frac{\sqrt{\varepsilon^2 + \Delta^2}}{2}$$

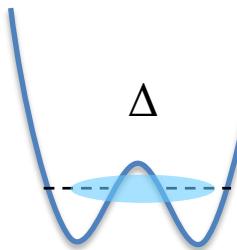
$$\Psi_0 = \cos \frac{\vartheta}{2} |0\rangle + \sin \frac{\vartheta}{2} |1\rangle$$

$$\varepsilon = 0 : \quad \vartheta = \frac{\pi}{2} \quad E_1 = \frac{\Delta}{2} \quad E_0 = -\frac{\Delta}{2}$$

$$\Psi_1 = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$\Psi_0 = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$H = -\frac{\Delta}{2} \sigma_x$$



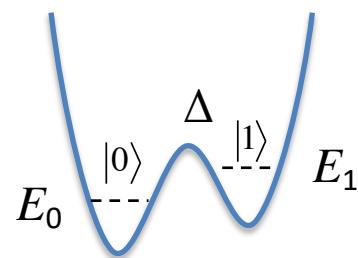
Delocalized charges

$$\varepsilon \gg \Delta : \quad \vartheta \approx 0 \quad E_1 = \frac{\varepsilon}{2} \quad E_0 = -\frac{\varepsilon}{2}$$

$$\Psi_1 \approx -|1\rangle$$

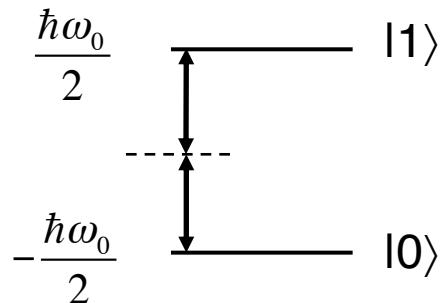
$$\Psi_0 \approx |0\rangle$$

$$H \approx -\frac{\varepsilon}{2} \sigma_z$$



Localized charges

Evolution of arbitrary states



$$H = \begin{pmatrix} -\frac{\hbar\omega_0}{2} & 0 \\ 0 & \frac{\hbar\omega_0}{2} \end{pmatrix}$$

$$\psi(t) = \exp\left(-i\frac{H}{\hbar}t\right)\psi(0)$$

$$\psi(t) = \exp\left[i\frac{\omega_0 t}{2}\sigma_z\right]\psi(0)$$

$U(t)$

The eigenstate evolution operator

$$U(t) = \begin{pmatrix} e^{\frac{i\omega t}{2}} & 0 \\ 0 & e^{-\frac{i\omega t}{2}} \end{pmatrix}$$

$$U(t) = e^{\frac{i\omega t}{2}}|0\rangle\langle 0| + e^{-\frac{i\omega t}{2}}|1\rangle\langle 1|$$

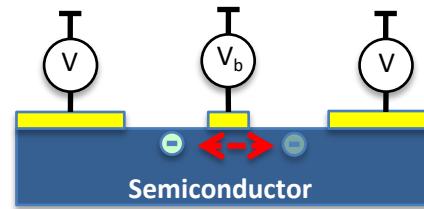
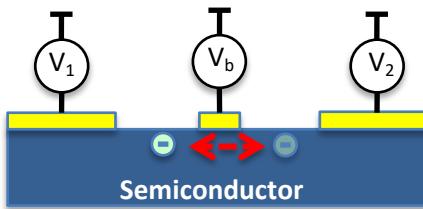
$$\left(e^{\frac{i\omega t}{2}}|0\rangle\langle 0| + e^{-\frac{i\omega t}{2}}|1\rangle\langle 1|\right)|0\rangle = e^{\frac{i\omega t}{2}}|0\rangle$$

$$\left(e^{\frac{i\omega t}{2}}|0\rangle\langle 0| + e^{-\frac{i\omega t}{2}}|1\rangle\langle 1|\right)|1\rangle = e^{-\frac{i\omega t}{2}}|1\rangle$$

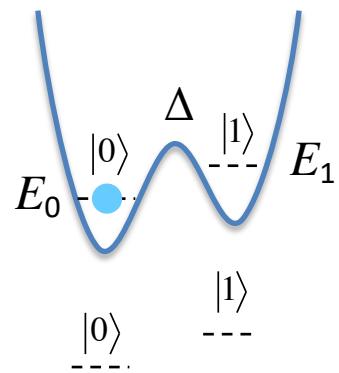
$$P(0) = e^{-\frac{i\omega t}{2}}\langle 0|e^{\frac{i\omega t}{2}}|0\rangle = 1$$

$$P(1) = e^{\frac{i\omega t}{2}}\langle 1|e^{-\frac{i\omega t}{2}}|1\rangle = 1$$

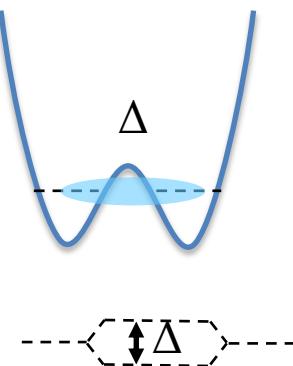
Control of quantum states



$$eV_1 < eV_2$$



$$eV_1 = eV_2$$



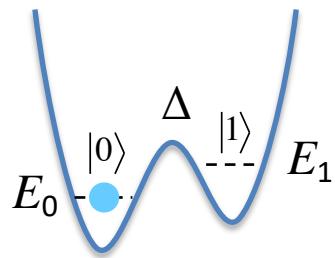
$$H \approx -\frac{\epsilon}{2} \sigma_z \quad \Psi_0 \approx |0\rangle$$

$$H = -\frac{\Delta}{2} \sigma_x \quad \Psi_0 \approx \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$$

Evolution of the two-level system

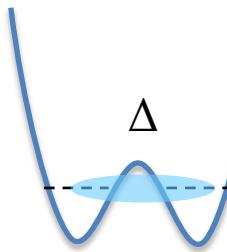
Assume that the electric field is suddenly switched off for time Δt

$$t = 0$$



$$H \approx -\frac{\epsilon}{2} \sigma_z \quad \Psi_0 \approx |0\rangle$$

$$0 < t < \Delta t$$



$$H = -\frac{\Delta}{2} \sigma_x$$

$$H = U(t) \Psi_0 \quad U(t) = \exp\left(i \frac{\Delta}{2\hbar} t \sigma_x\right)$$

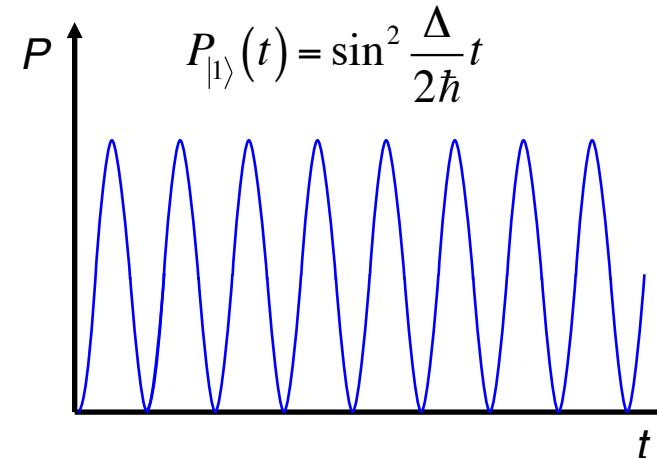
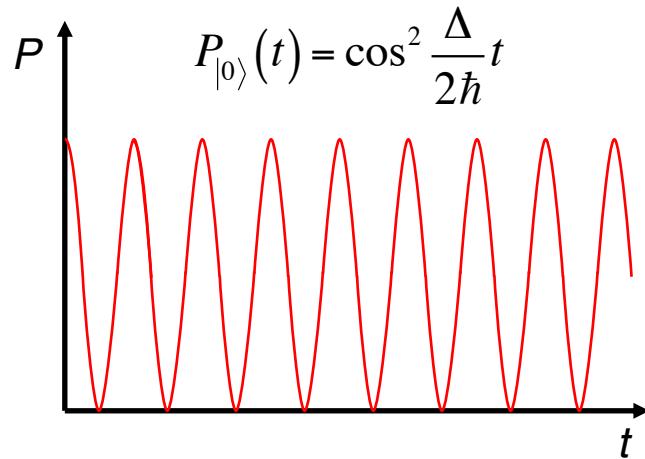
$$U(t) = \exp\left(i \frac{\Delta}{2\hbar} t \sigma_x\right) = I \cos \frac{\theta}{2} + i \sigma_x \sin \frac{\theta}{2} \quad \theta = \frac{\Delta}{\hbar} t$$

Quantum state control:

$$U(t)|0\rangle = \cos \frac{\theta}{2}|0\rangle + i \sin \frac{\theta}{2}|1\rangle$$

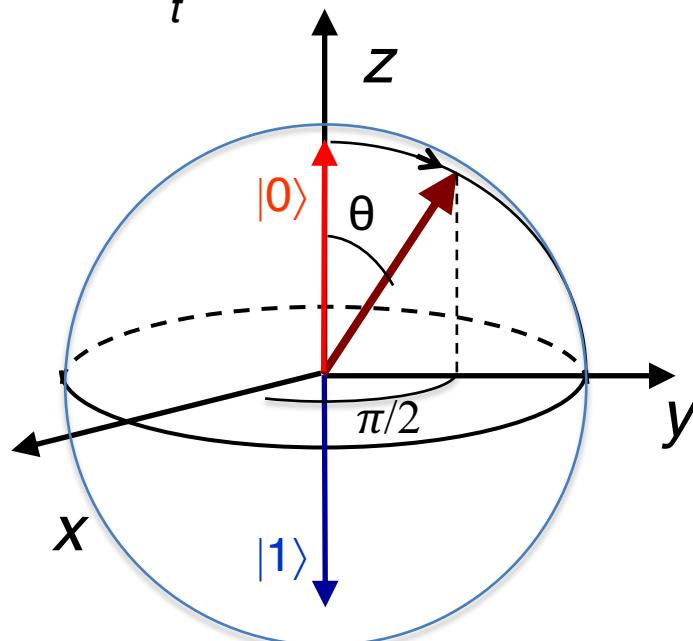
Probability oscillations

$$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + i\sin\frac{\theta}{2}|1\rangle \quad \theta = \frac{\Delta}{\hbar}t$$



$$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\frac{\pi}{2}} \sin\frac{\theta}{2}|1\rangle$$

$$\theta = \frac{\Delta}{\hbar}t \quad \varphi = \frac{\pi}{2}$$

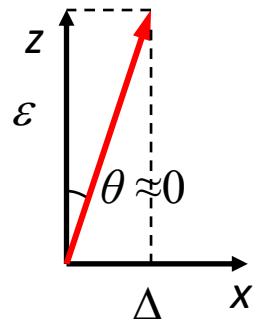
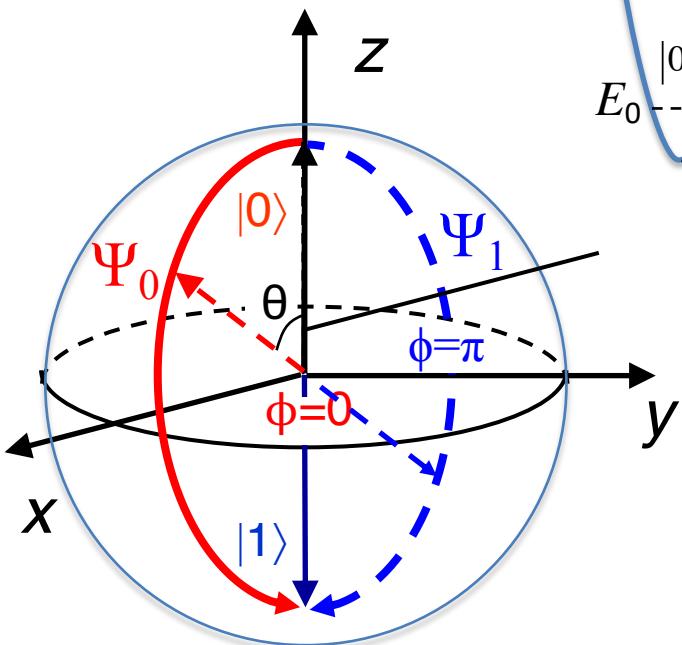


Stationary quantum state control (Week 1)

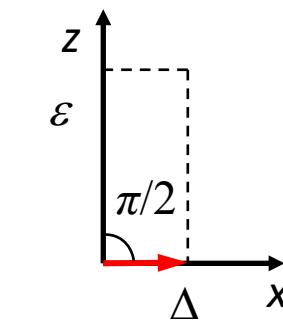
$$\Psi_0 = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} |1\rangle$$

$$\Psi_1 = \sin \frac{\theta}{2} |0\rangle - \cos \frac{\theta}{2} |1\rangle$$

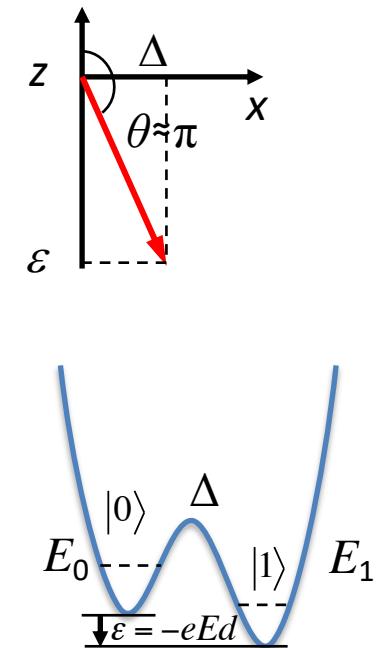
$$\Psi_1 = \cos\left(\frac{\pi-\theta}{2}\right) |0\rangle - \sin\left(\frac{\pi-\theta}{2}\right) |1\rangle$$



$\theta \approx 0 :$
 $\Psi_0 \approx |0\rangle$
 $\Psi_1 \approx -|1\rangle$



$\theta \approx \frac{\pi}{2} :$
 $\Psi_0 \approx \frac{|0\rangle + |1\rangle}{\sqrt{2}}$,
 $\Psi_1 \approx \frac{|0\rangle - |1\rangle}{\sqrt{2}}$



$\theta \approx \pi :$
 $\Psi_0 \approx |1\rangle$
 $\Psi_1 \approx |0\rangle$