

# Quantum Electronics of Nanostructures

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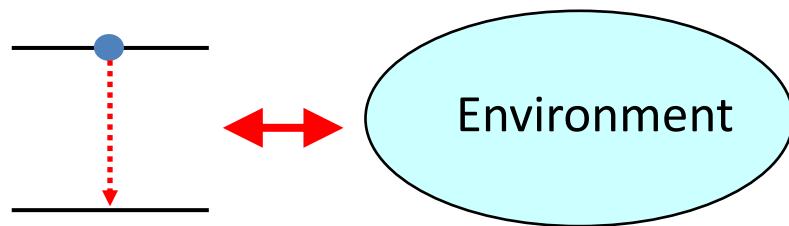
Lecture 9

# Lecture 9

- Noise in quantum systems
- Relaxation and dephasing
- Physical origin of decoherence
- Concept of strong coupling

# Noise in quantum systems

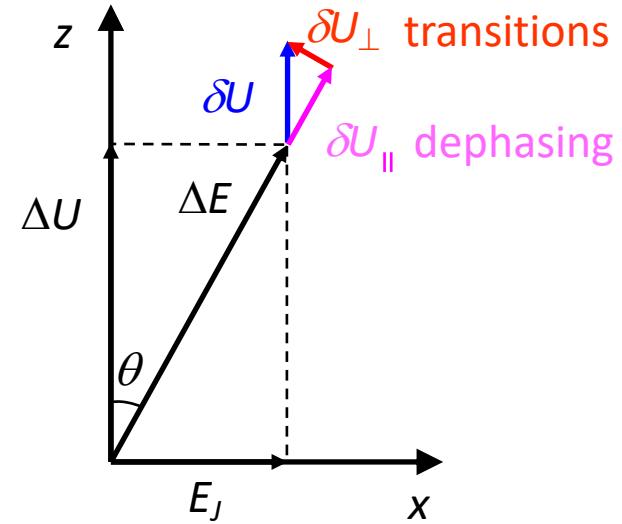
Two-level system



$$H = -\frac{\Delta E}{2} \sigma_z - \delta U(t) (\sigma_z \cos \theta + \sigma_x \sin \theta)$$

Energy fluctuation (dephasing)

Transitions (relaxation/excitation)

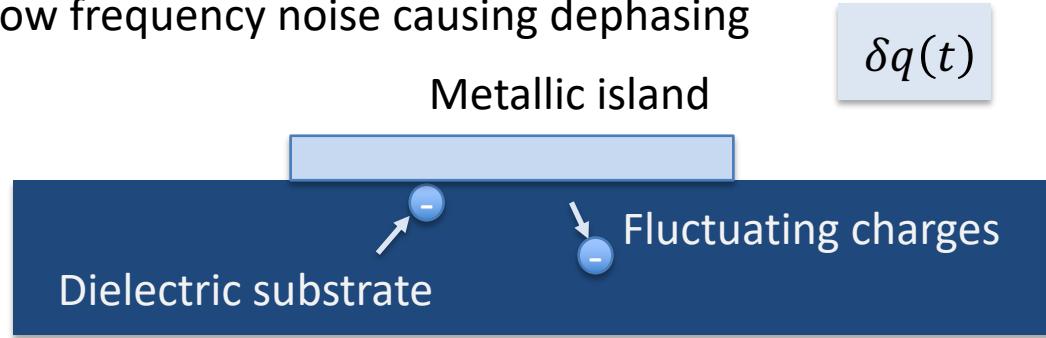


Low frequency noise (< measurement time) results in energy fluctuations

High frequency noise (at  $\omega \approx \omega_a$ ) results in relaxation/excitation

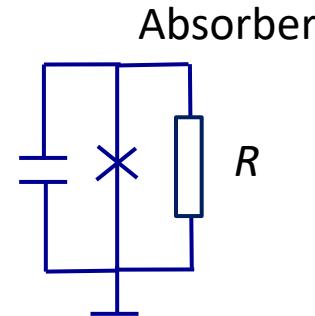
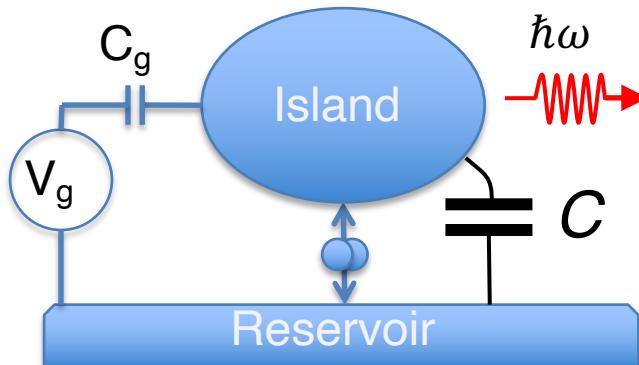
# Physical origin of the noise

Low frequency noise causing dephasing



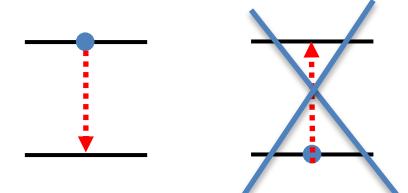
Energy of the two-level system:  $\Delta E(q + \delta q)$

High frequency noise causing relaxation:



Spontaneous emission

At T = 0:



At T = 0 noise is asymmetric (quantum)

# Noise spectral density

Spectral noise density of random variable  $x$ :  $S_x(\omega)$

Dispersion of  $x$  in the frequency range  $\omega_0 \leq \omega \leq \omega_1$ :  $\langle x^2 \rangle = \int_{\omega_0}^{\omega_1} S_x(\omega) d\omega$

Long integration time  $T \Rightarrow$  narrow frequency range  $\Delta\omega = \frac{2\pi}{T}$

$$\langle x^2 \rangle = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt \approx S_x(0) \Delta\omega$$

$$S_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle x(\tau)x^*(0) \rangle e^{i\omega\tau} d\tau$$

$$\langle x(\tau)x^*(0) \rangle = \int_{-\infty}^{\infty} S_x(\omega) e^{-i\omega\tau} d\omega$$

$$\langle x(\tau)x^*(0) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t+\tau)x^*(t) dt$$

For classical variable:  $x(t)$  - real  $\Rightarrow x(t) = x^*(t) \Rightarrow S_x(\omega) = S_x(-\omega)$

Noise spectral density for real values can be defined for  $\omega \geq 0$ :  $S'_x(\omega) = S_x(-\omega) + S_x(\omega)$

# High frequency noise

$$H_0 = -\frac{\hbar\omega_a}{2}\sigma_z \quad H_{int} = \hbar\Omega(t)\sigma_x$$

For charge qubit:

$$\hbar\Omega(t) = \mu_q V(t)$$

$$\mu = C_k \frac{\partial U}{\partial q}$$

$V(t)$  can be voltage fluctuations from lines

$$\psi(t) \approx \left( 1 - \frac{i}{\hbar} \int_0^t H_{int}(t') dt' \right) \psi(0)$$

$\mu_q = 2e \sin \theta$  is the dipole moment projected on  $\sigma_x$

$$\text{If } \psi(0) = |1\rangle \quad \psi(t) \approx -\frac{i}{\hbar} \int_0^t \langle 0 | H_{int}(t') | 1 \rangle e^{-i\omega_a t'} dt' = -\frac{i\sigma^-}{\hbar} \int_0^t \Omega(t') e^{-i\omega_a t'} dt'$$

$$p_0(t) \approx \frac{1}{\hbar^2} \int_0^t \int_0^t \Omega(t') \Omega(t'') e^{i\omega_a(t''-t')} dt'' dt' \rightarrow \frac{1}{\hbar^2} \int_0^t \int_0^t \langle \Omega(t') \Omega(t'') \rangle e^{i\omega_a(t''-t')} dt'' dt'$$

$$p_0(t) \approx \frac{t}{\hbar^2} \int_0^t \langle \Omega(\tau) \Omega(0) \rangle e^{i\omega_a \tau} d\tau$$

$$S_V(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle V(\tau) V(0) \rangle e^{i\omega\tau} d\tau$$

Relaxation ( $|1\rangle \rightarrow |0\rangle$ ):

$$\Gamma_{10} = \frac{\mu^2}{\hbar^2} \int_{-\infty}^{\infty} \langle V(\tau) V(0) \rangle e^{i\omega_a \tau} d\tau = \frac{2\pi\mu^2}{\hbar^2} S_V(\omega)$$

Excitation ( $|0\rangle \rightarrow |1\rangle$ ):

$$\Gamma_{01} = \frac{\mu^2}{\hbar^2} \int_{-\infty}^{\infty} \langle V(\tau) V(0) \rangle e^{-i\omega_a \tau} d\tau = \frac{2\pi\mu^2}{\hbar^2} S_V(-\omega)$$

# Quantum noise

$$S_V(\omega) \neq S_V(-\omega)$$

Johnson Nyquist noise ( $hf \ll k_B T$ ):

$$\langle \delta V^2 \rangle = 4 k_B T R \Delta f$$

$$S_V(\pm\omega) = \frac{2k_B T R}{2\pi}$$

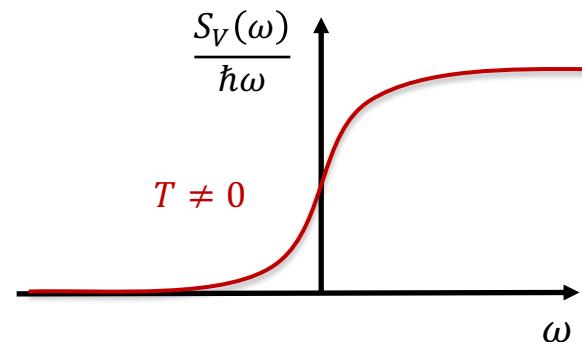
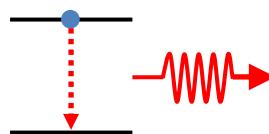
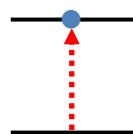
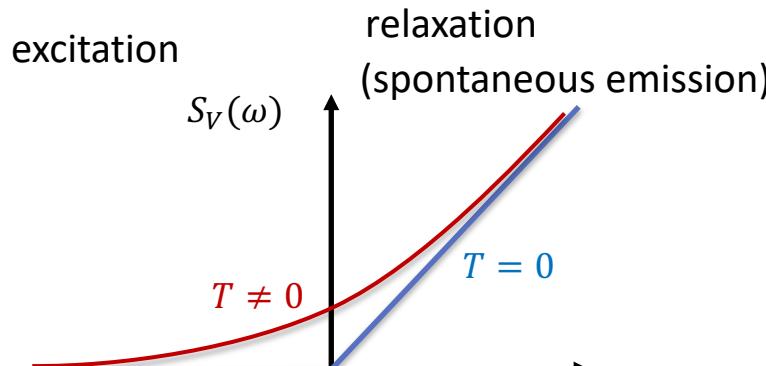
$$\langle \delta V^2 \rangle = S_V(-\omega)\Delta\omega + S_V(\omega)\Delta\omega$$

$$S_V(\pm\omega) = \frac{2k_B T R}{2\pi} \frac{\frac{\hbar\omega}{k_B T}}{e^{\frac{\hbar\omega}{k_B T}} - 1}$$

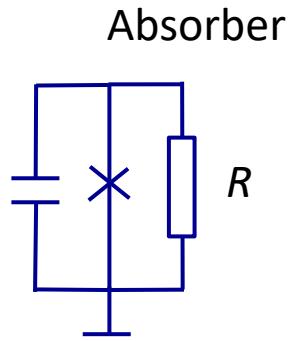
At  $T = 0$ :

$$S_V(\omega) = \frac{\hbar\omega R}{\pi}$$

$$S_V(-\omega) = 0$$



# Relaxation due to ohmic environment



$$S_V(\omega) = \frac{\hbar\omega R}{\pi}$$

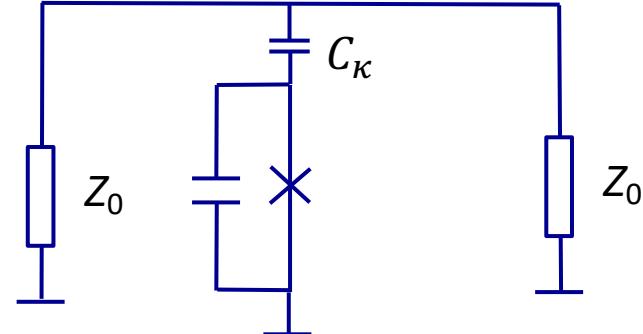
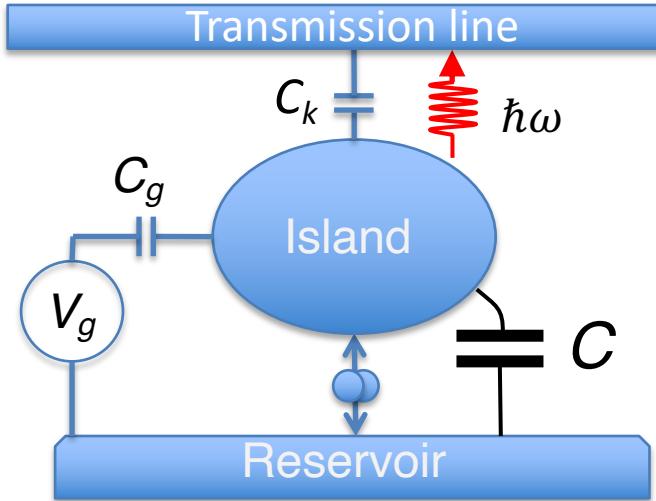
$$R \rightarrow \text{Re}[Z]$$

$$\mu_q = C_\kappa \frac{E_C}{2e} \sin \theta$$

$$\Gamma_{10} = \frac{2\pi\mu_q^2}{\hbar^2} S_V(\omega) = \frac{2\hbar\omega\mu_q^2 R}{\hbar^2}$$

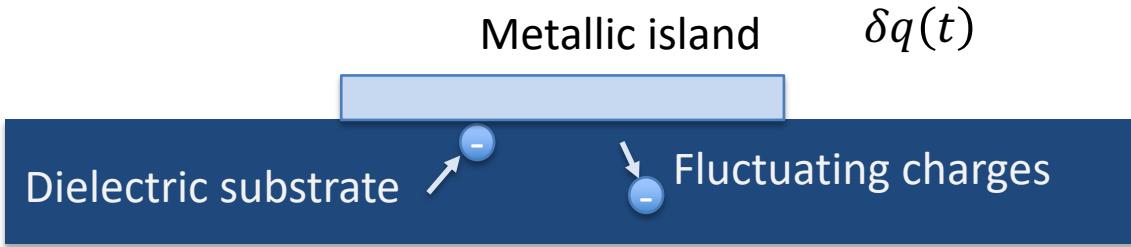
$$\Gamma_1 = \frac{2\mu^2\omega}{\hbar} \text{Re}[Z]$$

Charge qubit:  $\Gamma_1 = \frac{2\omega Z_0}{2\hbar} \left( \frac{2eC_\kappa}{C_q} \sin \theta \right)^2$



At  $T = 0$ , relaxation is a result of high frequency quantum noise

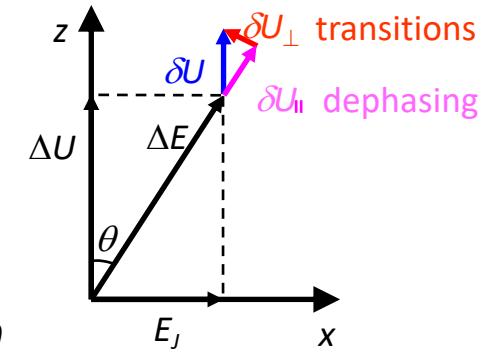
# Dephasing and low frequency noise



$$H_0 = -\frac{\hbar\omega_a}{2}\sigma_z$$

For charge qubit:

$$H_q = \mu_q \delta q(t) \sigma_z \quad \mu_q = \frac{\partial \Delta E}{\partial q} = \frac{2E_C}{2e} \cos \theta$$



Energy fluctuations due to charge fluctuations :  $\langle \delta E^2 \rangle = \mu_q^2 \langle \delta q^2 \rangle$

Superposed state:  $\psi = \frac{|0\rangle e^{\frac{i\omega t}{2}} + |1\rangle e^{-\frac{i\omega t}{2}}}{\sqrt{2}}$   $\Rightarrow \rho_{01} = \frac{1}{2} e^{i\omega t} \rightarrow \frac{1}{2} e^{i\omega t + i\delta\varphi(t)} = \frac{1}{2} e^{i\omega t} e^{i\delta\varphi(t)}$

Phase fluctuations:  $\langle e^{i\delta\varphi} \rangle \approx 1 + i\langle \delta\varphi \rangle - \frac{\langle \delta\varphi^2 \rangle}{2}$        $\langle \delta\varphi^2 \rangle = \frac{\langle \delta E^2 \rangle T_2^2}{\hbar^2}$

$$\frac{\mu^2 S_q(0) \Gamma_2 T_2^2}{2\pi\hbar^2} = 1 \quad \Gamma_2 = \frac{\mu^2 S_q(0)}{2\pi\hbar^2}$$

However it is not a realistic case because  $S(0) = \infty$  due to 1/f noise

# Dephasing and low frequency noise

Charge noise

$$S_q(f) = \frac{\alpha_q^2}{f}$$

Flux noise:

$$S_\Phi(f) = \frac{\alpha_\Phi^2}{f}$$

$$\langle \delta E^2 \rangle = \mu^2 \int_{1/T_{meas}}^{\omega_{cut}} \frac{\alpha_q^2}{\omega} \frac{d\omega}{2\pi} = \frac{\mu_q^2 \alpha_q^2}{2\pi} \ln(\omega_{cut} T_{meas})$$

Typical  $(\omega_{cut} T_{meas}) \sim 10^3 - 10^4$  and  $\ln(\omega_{cut} T_{meas}) \sim 7 - 9$

$$\langle \delta E^2 \rangle = \mu^2 \int_{1/T_{meas}}^{\omega_{cut}} \frac{\alpha_q^2}{\omega} \frac{d\omega}{2\pi} = \frac{\mu_q^2 \alpha_q^2}{2\pi} \ln(\omega_{cut} T_{meas}) \quad \nearrow \sim 1$$

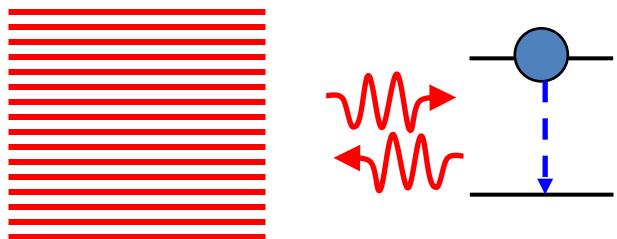
$$\langle \delta \varphi^2 \rangle = \frac{\langle \delta E^2 \rangle T_2^2}{\hbar^2} = 1 \quad \Gamma_2 = \frac{\sqrt{\langle \delta E^2 \rangle}}{\hbar} = \frac{\mu_q \alpha_q}{\hbar} \sqrt{\frac{\ln(\omega_{cut} T_{meas})}{2\pi}} \approx \frac{\mu_q \alpha_q}{\hbar}$$

$$\mu_q = \frac{\partial \Delta E}{\partial q} = \frac{E_C}{2e} \cos \theta$$

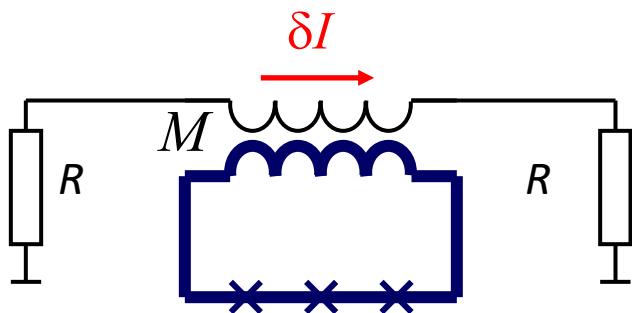
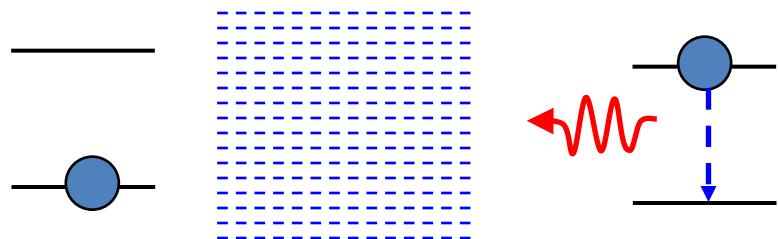
The degeneracy point ( $\theta = \pi$ ) is the magic point with suppressed decoherence

# The artificial atom as a sensor of the quantum noise

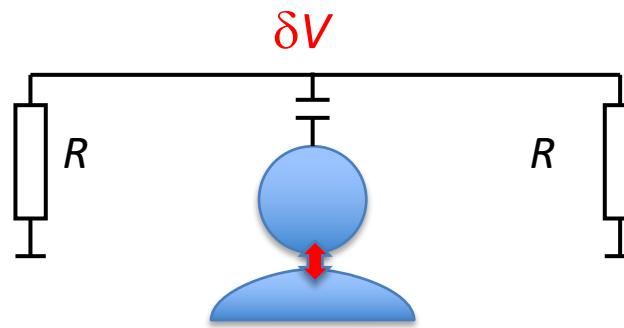
Hot environment ( $T \gg \hbar\omega$ )  
(classical noise)



Cold environment ( $T \ll \hbar\omega$ )  
(quantum noise)



$$\hbar\Omega = \phi_p \delta I$$



$$\hbar\Omega = \mu \delta V$$

The resistors can absorb energy

# Atom-resonator Strong coupling

# Dissipation in resonators (photon decay)

$$H_{JC} = -\frac{\Delta E}{2}\sigma_z + \hbar\omega_r a^\dagger a + g_0(a\sigma^+ + a^\dagger\sigma^-)$$

$$\frac{\partial\rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] + L$$

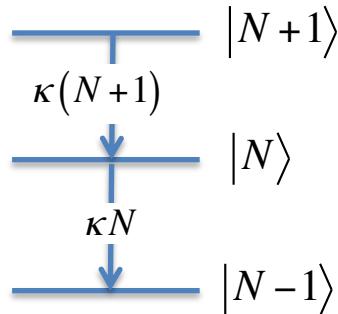
No pure dephasing:  $\Gamma_2 = \frac{\Gamma_1}{2}$

$$L = \begin{pmatrix} \Gamma_1 \rho_{11} & -\frac{\Gamma_1}{2} \rho_{01} \\ -\frac{\Gamma_1}{2} \rho_{10} & -\Gamma_1 \rho_{11} \end{pmatrix}$$

$$L = \frac{\Gamma_1}{2}(2\sigma^- \rho \sigma^+ - \sigma^+ \sigma^- \rho - \rho \sigma^+ \sigma^-)$$

Density matrix of a harmonic oscillator:

$$\rho^{(r)} = \sum_{M=0}^{\infty} \sum_{N=0}^{\infty} \rho_{NM}^{(r)} |N\rangle \langle M|$$



Lindblad operator of a harmonic oscillator:

$$L^{(r)} = \frac{\kappa}{2}(2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a)$$

N-th element of the resonator Lindblad operator:

$$\dot{\rho}_{NN}^{(r)} \rightarrow \kappa(N+1)\rho_{N+1,N+1}^{(r)} - \kappa N \rho_{NN}^{(r)}$$

$\kappa$  is the photon decay rate

# Linewidth of the resonator (quantum mechanical approach)

$$H = \hbar\omega_r a^\dagger a + \hbar\Omega(a + a^\dagger) \cos \omega t$$

$$H' = U H U^\dagger - i\hbar U \dot{U}^\dagger$$

$$U = e^{i\omega t a^\dagger a}$$

$$H' = \hbar(\omega_r - \omega)a^\dagger a + \hbar\Omega e^{i\omega t a^\dagger a}(a + a^\dagger)e^{-i\omega t a^\dagger a} \cos \omega t = \hbar\delta\omega a^\dagger a + \hbar\Omega(e^{-i\omega t}a + e^{i\omega t}a^\dagger)\frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

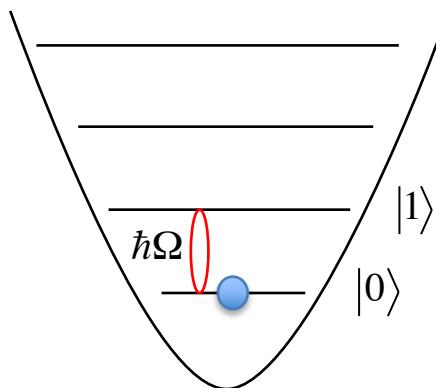
$$H' = -\hbar\delta\omega a^\dagger a + \frac{\hbar\Omega}{2}(a + a^\dagger)$$

$$\delta\omega = \omega - \omega_r$$

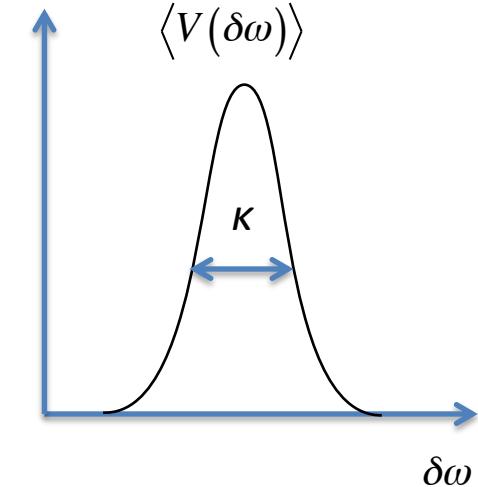
In a weak driving regime, when population of the  $|1\rangle$  is low ( $\rho_{11} \ll 1$ ) we can truncate the photon space by 0, 1

$$H\rho - \rho H + L = 0$$

$$L = \begin{pmatrix} \kappa\rho_{11} & -\frac{\kappa}{2}\rho_{01} \\ -\frac{\kappa}{2}\rho_{10} & -\kappa\rho_{11} \end{pmatrix}$$



$$\langle a + a^\dagger \rangle = \text{tr}[(a + a^\dagger)\rho] \approx \frac{2\Omega}{\kappa + 2\delta\omega}$$



$$H_{JC} = -\frac{\Delta E}{2}\sigma_z + \hbar\omega_r a^\dagger a + g_0(a\sigma^+ + a^\dagger\sigma^-)$$

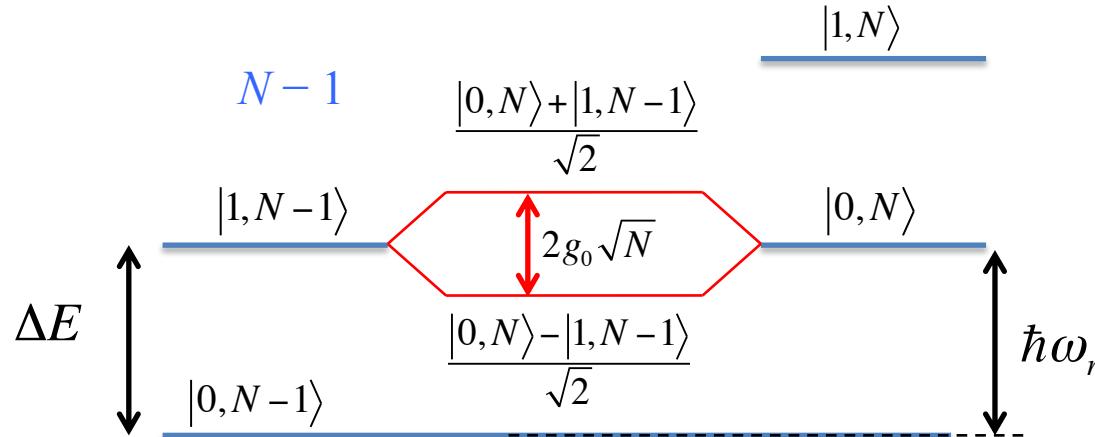
$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[H, \rho] + L^{(r)} + L^{(a)}$$

$$L^{(r)} = \frac{\kappa}{2}(2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a)$$

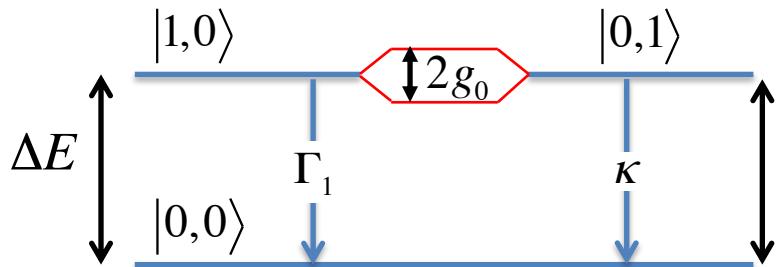
$$L^{(a)} = \frac{\Gamma_1}{2}(2\sigma^-\rho\sigma^+ - \sigma^+\sigma^-\rho - \rho\sigma^+\sigma^-)$$

$$\rho^{(ra)} = \sum_{N,M=0}^{\infty} \sum_{n,m=0}^1 \rho_{Nn,Mm} |Nn\rangle\langle Mm|$$

*N*



# Single-photon interaction



$$\rho = \begin{pmatrix} \rho_{01,01} & \rho_{01,10} \\ \rho_{10,01} & \rho_{10,10} \end{pmatrix}$$

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H, \rho] + L$$

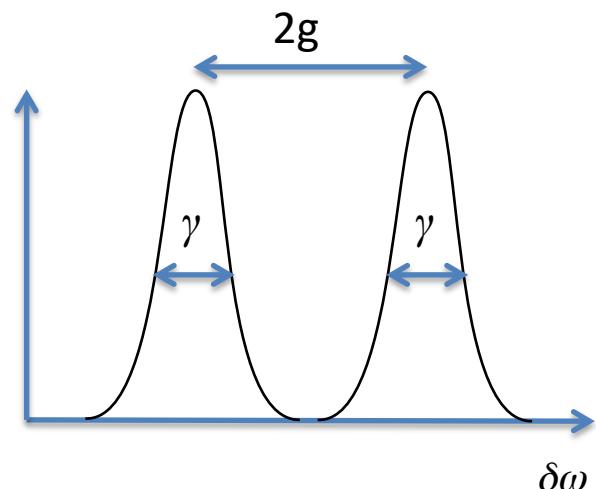
The states decay with rate:  $\gamma = \frac{\kappa + \Gamma_1}{2}$

To exchange energy between the qubit and the resonator the following condition should be fulfilled:

$$(\Gamma_1, \Gamma_2, \kappa) \ll \frac{g}{\hbar}$$

$$H' = g(|01\rangle\langle 10| + |10\rangle\langle 01|)$$

$$L = \begin{pmatrix} \kappa\rho_{01,01} + \Gamma_1\rho_{10,10} & -\frac{\kappa}{2}\rho_{00,01} & -\frac{\Gamma_1}{2}\rho_{00,10} \\ -\frac{\kappa}{2}\rho_{01,00} & -\kappa\rho_{01,01} & -\frac{\Gamma_1 + \kappa}{2}\rho_{01,10} \\ -\frac{\Gamma_1}{2}\rho_{10,00} & -\frac{\Gamma_1 + \kappa}{2}\rho_{10,01} & -\Gamma_1\rho_{10,10} \end{pmatrix}$$



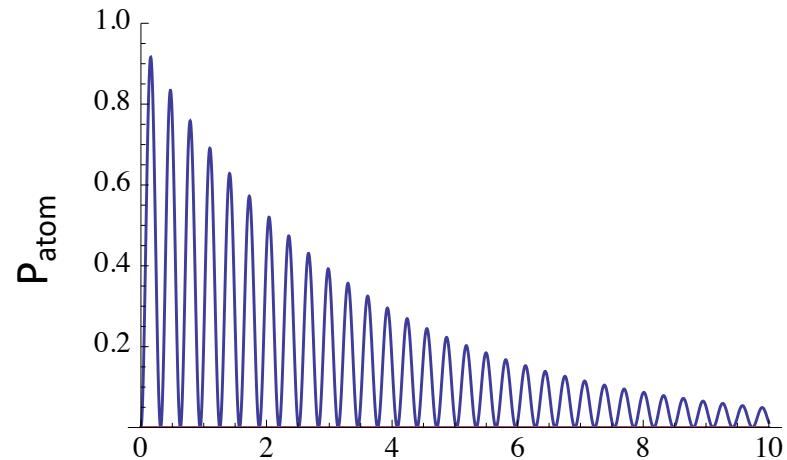
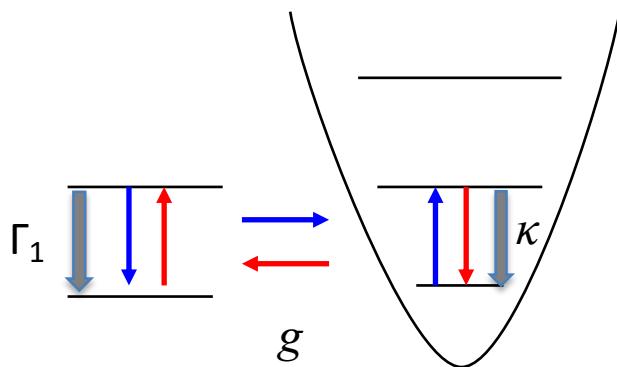
# Strong coupling

# Energy exchange. Strong coupling.

Strong coupling regime is achieved, when the characteristic energy is higher than incoherent processes

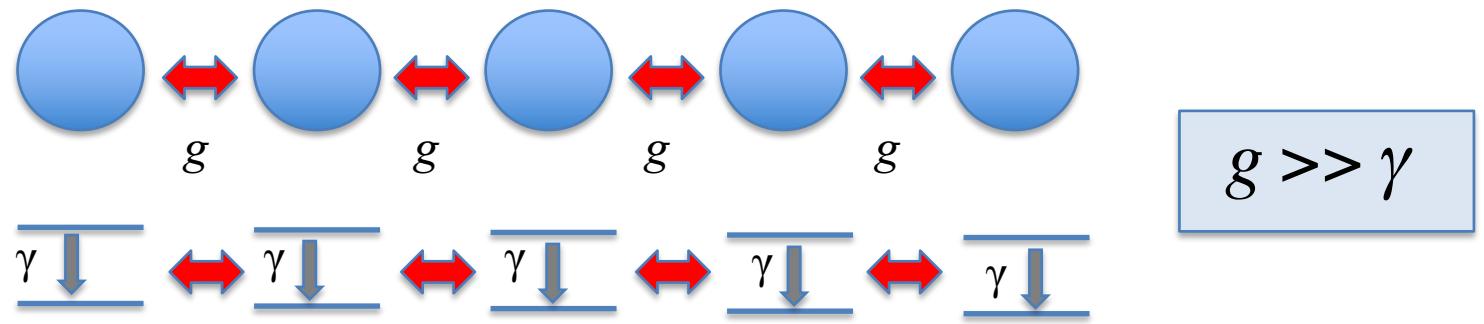
For an atom coupled to a resonator the coupling energy  $g$

$$(\hbar\Gamma_1, \hbar\Gamma_2, \hbar\kappa) \ll g$$

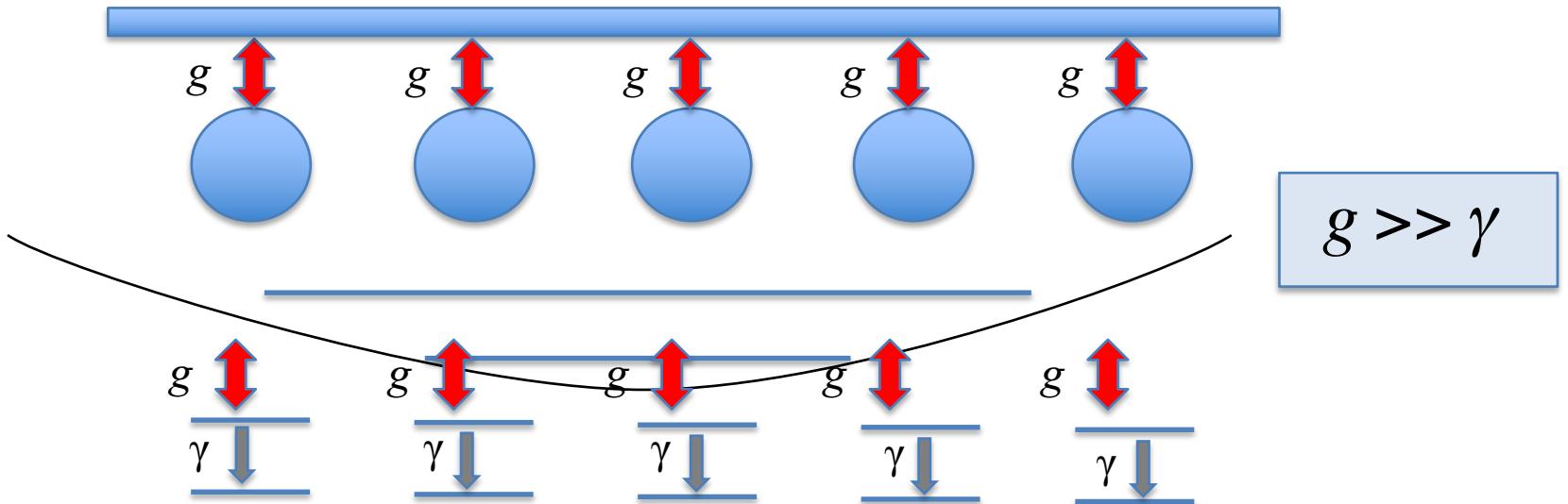


The strong coupling is necessary condition to manipulate with quantum states; to exchange information between different systems.

# Possible geometries of quantum systems



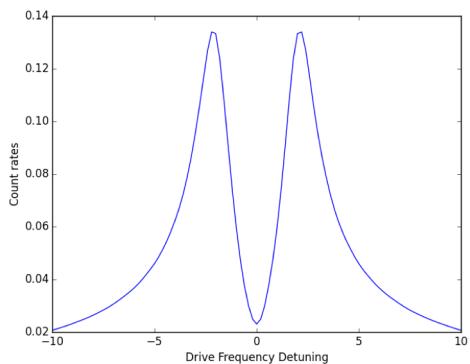
## Qubits coupled to quantum baths



# Two-level atom in a resonator

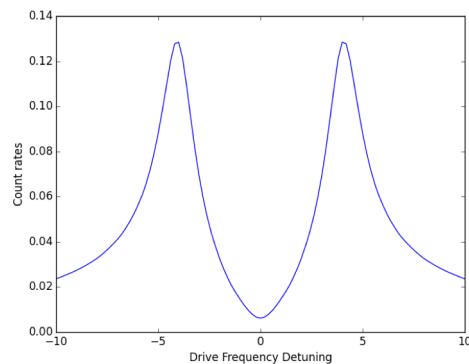
Strong coupling:

$$\kappa = 0.5, \Gamma_1 = 0.5, g = 1$$



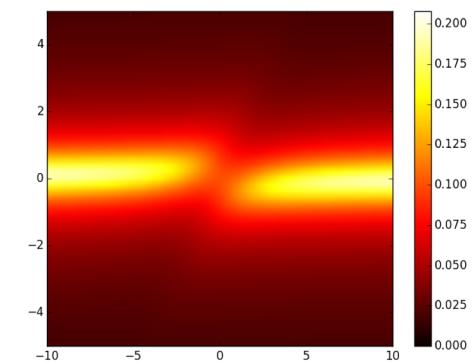
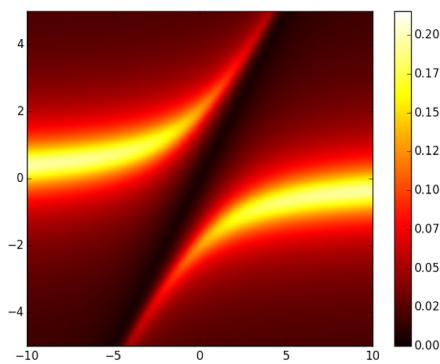
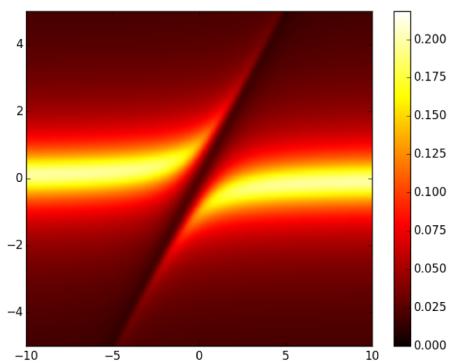
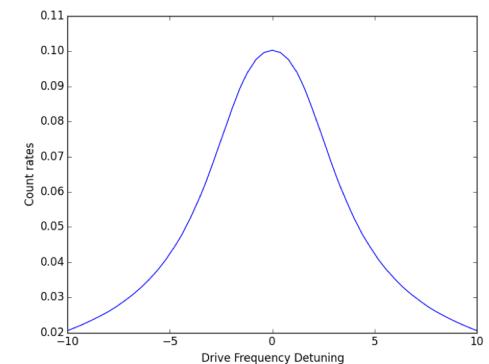
Strong coupling:

$$\kappa = 0.5, \Gamma_1 = 0.5, g = 2$$



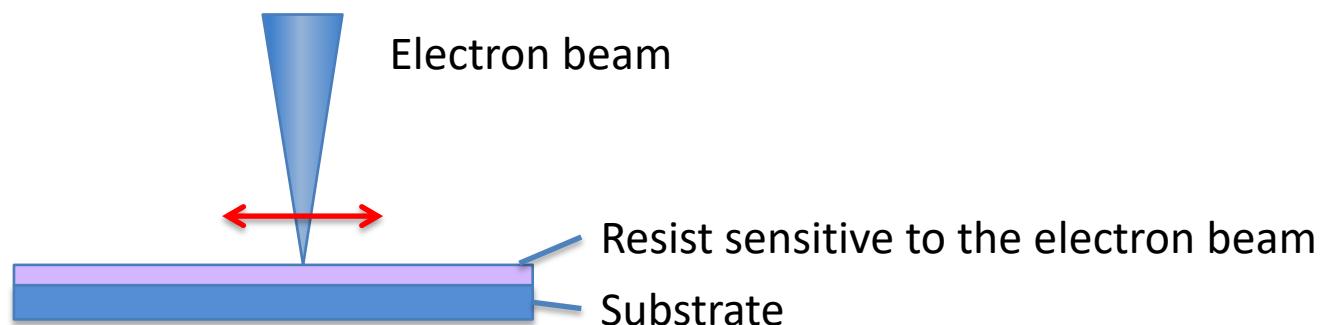
Weak coupling:

$$\kappa = 0.5, \Gamma_1 = 4, g = 1$$

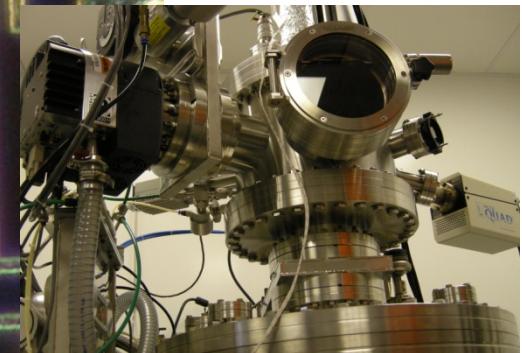
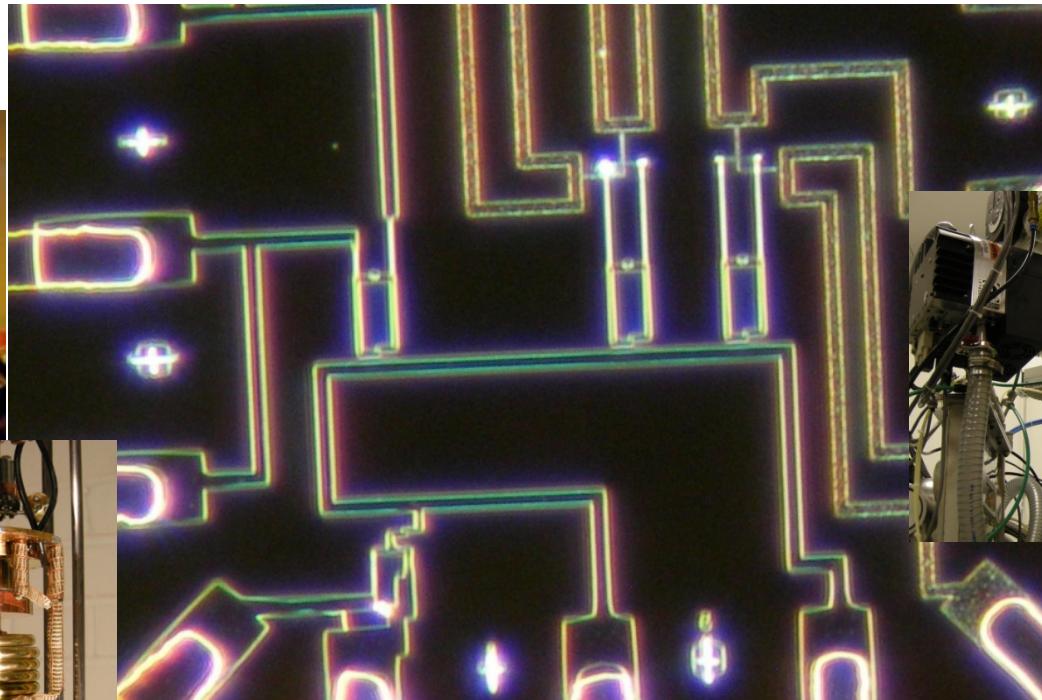


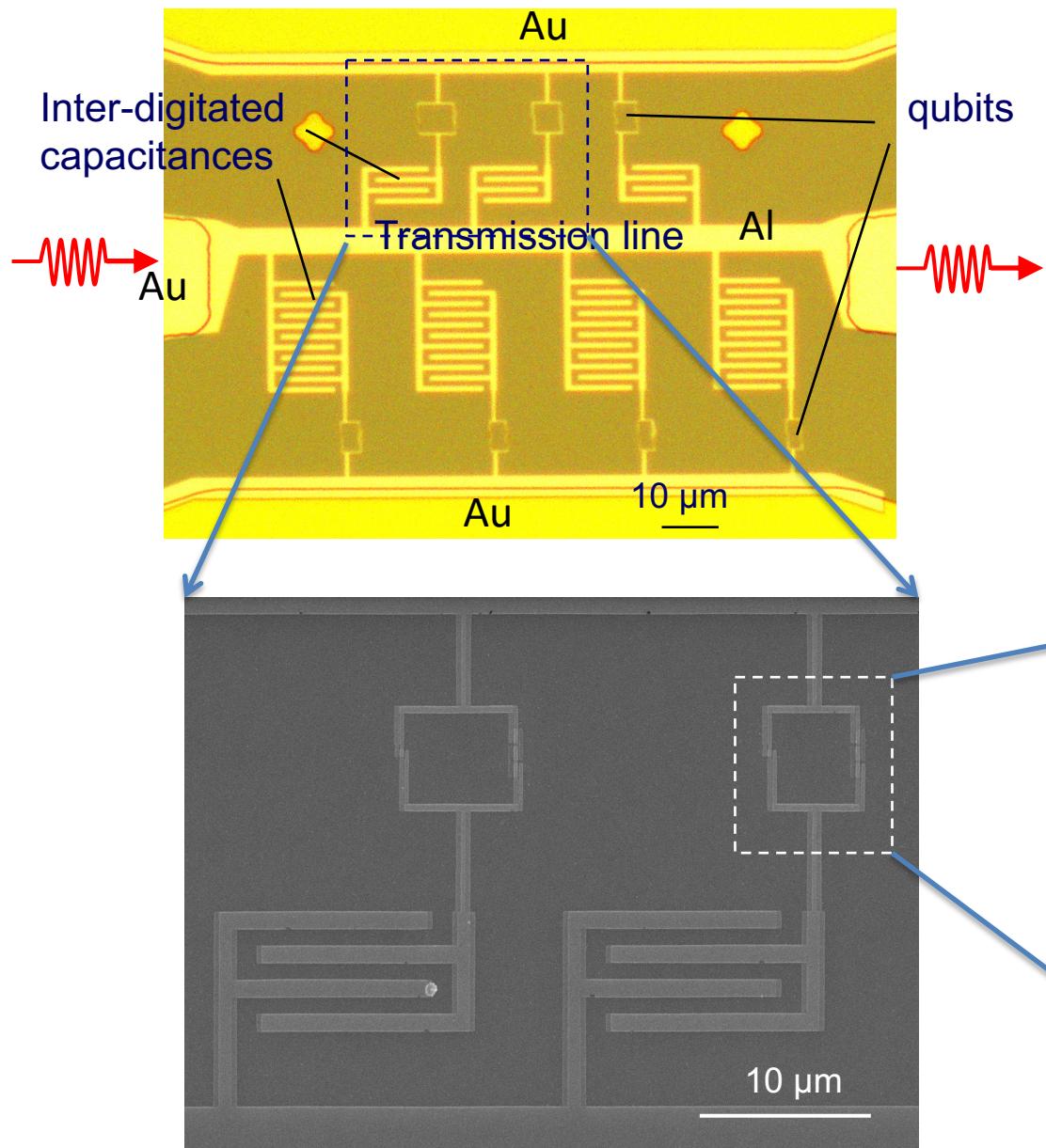
# Sample Fabrication

# Electron-beam lithography systems (EBL)



# Hybrid nanostructures fabrication





# Lithography

Resists are organic materials, which change their properties under deep UV or electron beams

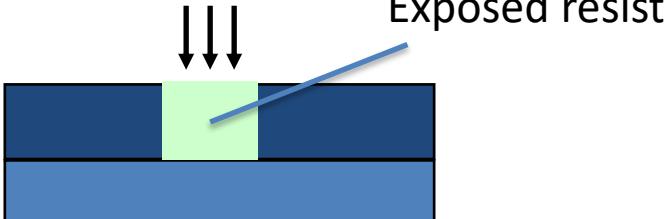
Positive resists: Exposed areas are washed away during development processes

Negative resists: Unexposed areas are washed away during development processes

## Positive resist processing

Substrate (e.g. Si)

### Exposure



## Spin coating



Thin film of resist  
(0.05 – 1 µm)

### Development in liquids (developers)



Developed resist is  
washed away  
after development

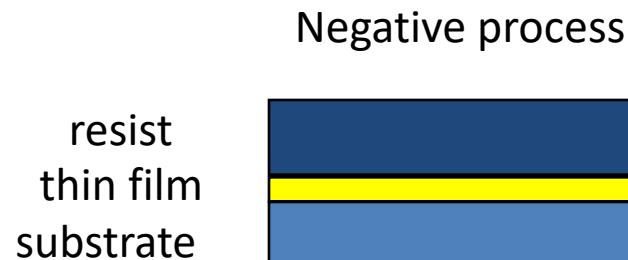
### Resolution:

Photo-lithography: 0.5 µm

EB-lithography: 10 nm

The wafer is ready for metal deposition or etching

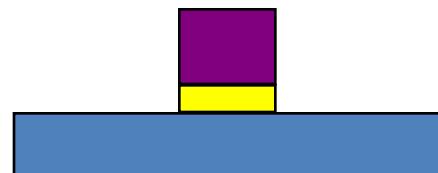
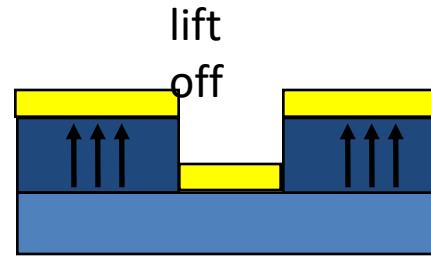
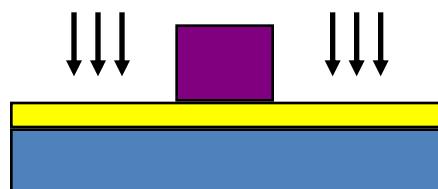
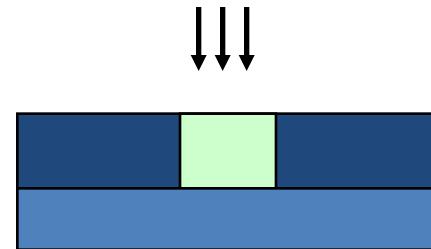
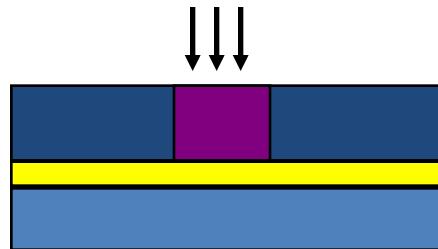
# Lithography steps



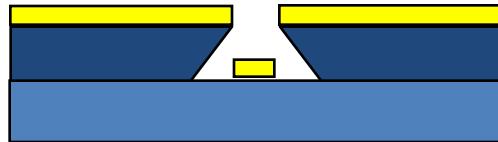
Positive process



exposure



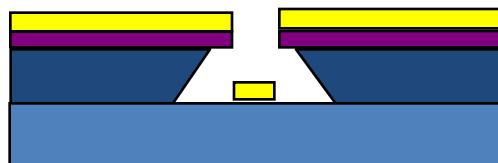
undercut



overcut



double layers resist



PMMA / Copolimer e-beam resists  
S1813 / LOR5B photoresists

Bottom layer “softer” than top layer  
Selective developers

## *Photolithography*

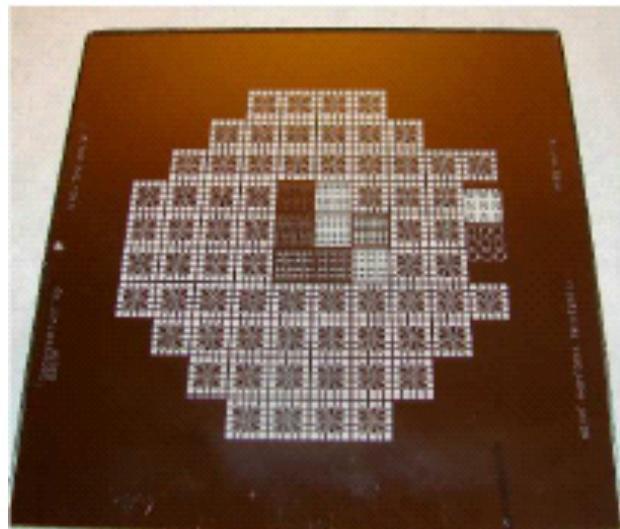
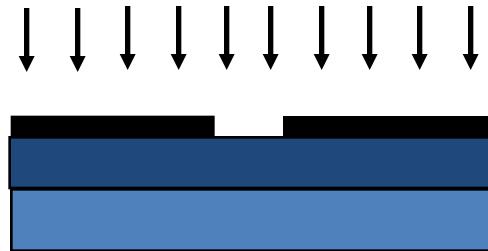
UV light changes the properties of the resist in the openings of the mask

Resist mask formed after development.

This allows the substrate to be selectively exposed to etching or deposition

Wavelength < 1 $\mu$ m for Hg lamps;  
180-280nm for deep UV lamps

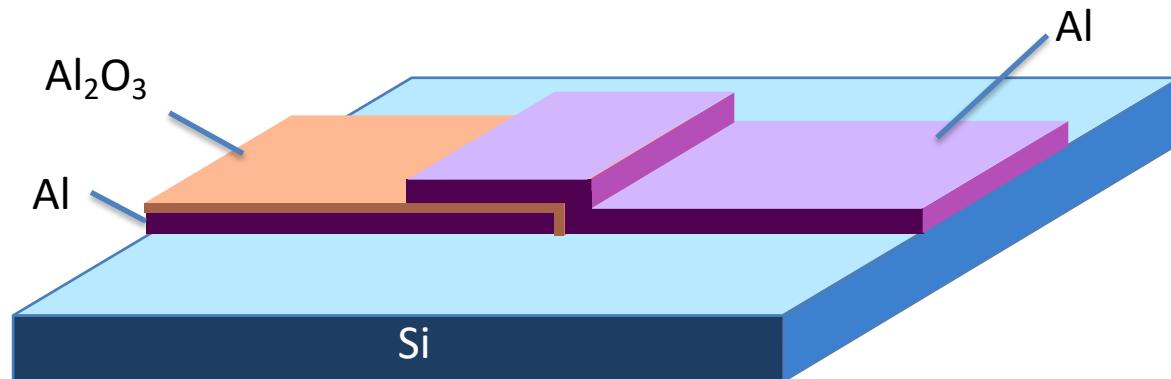
Direct printing resolution  $w=(k \lambda g)^{1/2}$   
k constant depending on resist  
 $\lambda$  wavelength  
g gap between resist and mask



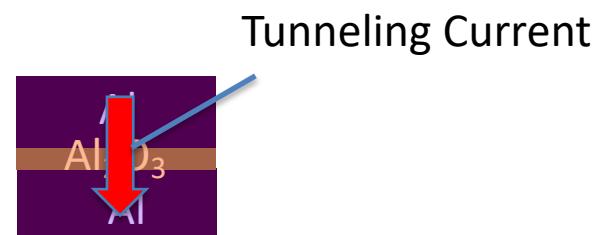
Photomask for 3 inch wafer

# Fabrication of Josephson junctions

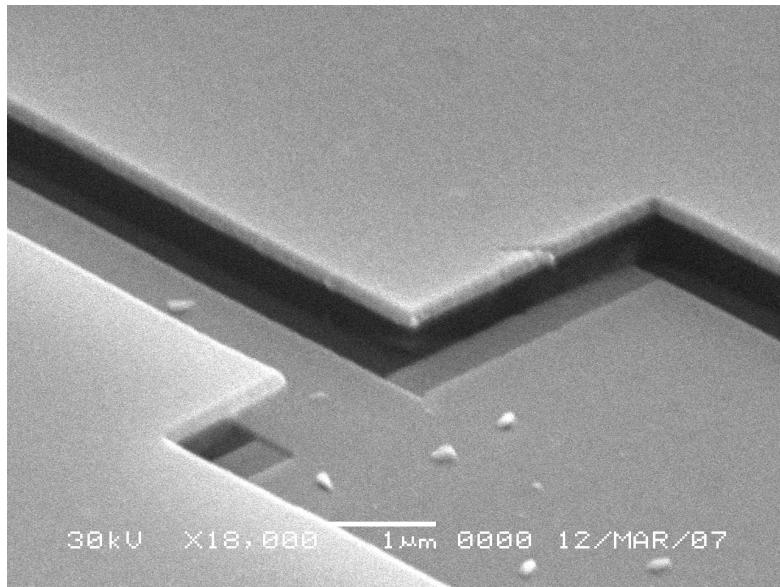
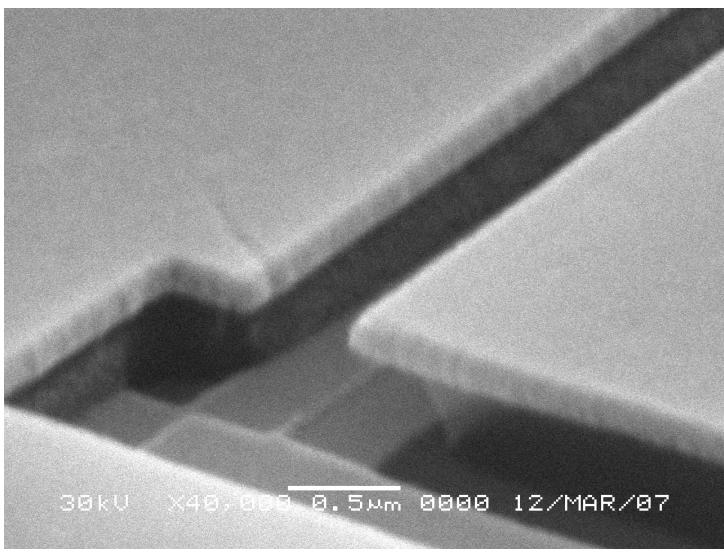
1. Bare silicon substrate
2. Aluminium deposition
3. Aluminium oxidation
4. Aluminium deposition (layer 2)



Crosssection of the  
Josephson junction

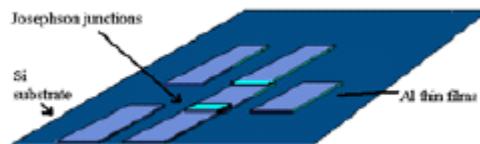
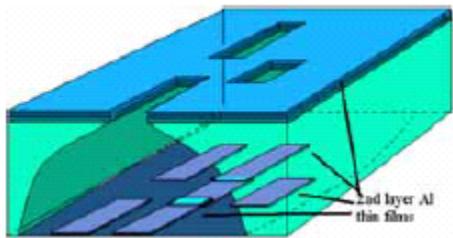
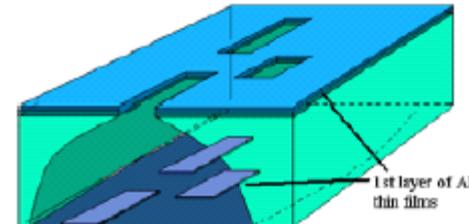
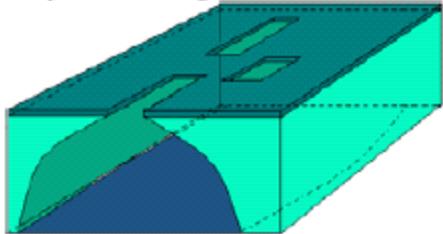


Au film pictured after evaporation  
on double layer resist



## *Angular evaporation + oxidation*

Chip after etching:



Soft mask

PMMA / Copolimer double layer resist

ZEP / Copolimer resist layer

