An extension of an ILP-Based Rollout Policy for Distributed Supplier Production Planning (***incomplete***)

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Abstract

The previous work has developed the optimization policy for a supply chain where each local supplier has one downstream (parent) supplier and multiple upstream (children) suppliers. These upstream suppliers supply distinct types of materials to their parent supplier. In this extension of the production planning, we make the supply chain more robust by adding some flexibiliy to it. In this newly developed structure of supply chain, each supplier may have multiple upstream and downstream suppliers. Upstream suppliers may supply the same type material to their downstream supplier(s). In the end, we formulate the new optimization problem to fit with the new supply chain structure.

Notation and Problem

We are using the same notation as the original write-up to refer to the variables unless they have not been introduced earlier or need to be modified. Similar to the original work, we assume that we are on t-th day and Joe is the local supplier that we are considering. Let $\mathcal{N}^+_{Joe}$ and $\mathcal{N}^-_{Joe}$ be the set containing labels of Joe's parent and children suppliers, respectively. Among the suppliers in $\mathcal{N}^-_{Joe}$, some suppliers supply similar product parts to Joe. We define $\mathcal{D}_{Joe}$ to be the set containing different product parts. Regarding time horizon, each supplier has 2 types of horizon. The first type is time horizon h_p respect to each parent p. We collect them in a dictionary whose key is each parent's label, and value is the corresponding time horizon. (Note that in this case, there is not only one p.) $H^{Dict} = \{p : h_p\}$; $p \in \mathcal{N}^+_{Joe}$ Each h_p can be written as $H_p - \mathcal{T}_p$ where H_p refers to the second type or the overall horizon of parent p (we will proceed to this type of horizon right after this) and $\mathcal{T}_p$ refers to the travel time from Joe to parent supplier p. The second type of time horizon is $H_{\text{Joe}} = \max_{p \in \mathcal{N}^+_{Joe}} h_p$, which is basically the maximum of the first-type time horizon respect to all parent suppliers. In this document, we will simply refer to H_{Joe} as H.

Decision Variables

Production Plan

The production plan of Joe for the next H days is

$$\widetilde{\boldsymbol{x}}_t \triangleq [\widetilde{x}_{t,t}\ \widetilde{x}_{t,t+1} \ldots \widetilde{x}_{t,t+H-1}]$$

where $\widetilde{x}_{t,t'}$ is the number of parts to be produced on the $(t'-t)$-th day by Joe.

Input Inventory

$$\widetilde{\boldsymbol{R}}^I_t \triangleq [\widetilde{\boldsymbol{r}}^I_{t,t+1}\ \widetilde{\boldsymbol{r}}^I_{t,t+2} \ldots \widetilde{\boldsymbol{r}}^I_{t,t+H}]$$

Note that the input inventory is divided into $|\mathcal{D}_{Joe}|$ segments, and each segment stores input material for each distinct part. Therefore, at the beginning of day t', we have

$$\widetilde{\boldsymbol{r}}^{I}_{t,t'} \triangleq [\widetilde{r}^{I}_{d_1,t,t'}\, \widetilde{r}^{I}_{d_2,t,t'} \ldots \widetilde{r}^{I}_{d_{|\mathcal{D}_{Joe}|},t,t'}], \;\; \forall t' \in \mathbb{N}^{t+1}_{t+H}$$

$\widetilde{R}_{d_i,t,t'}$ denotes the amount of part $d_i \in \mathcal{D}_{Joe}$ (or $i \in \mathbb{N}^{1}_{|\mathcal{D}_{Joe}|}$) stored in the input inventory at the beginning of day t'.

Output Inventory

Output inventory is basically similar to input inventory except that it only has one segment since Joe only supplies one type of material, stored in the output, ready to be shipped to parent suppliers.

$$\widetilde{\boldsymbol{R}}^{O}_{t} \triangleq [\widetilde{r}^{O}_{t,t+1}\, \widetilde{r}^{O}_{t,t+2} \ldots \widetilde{r}^{O}_{t,t+H}]$$

Unmet Demand

Unmet demand refers to the projected undersupply of the supplies from Joe to each of his parent suppliers.

$$\widetilde{\boldsymbol{X}}^{UD}_{t} \triangleq [\widetilde{\boldsymbol{x}}^{UD}_{t,t}\, \widetilde{\boldsymbol{x}}^{UD}_{t,t+1} \ldots \widetilde{\boldsymbol{x}}^{UD}_{t,t+H-1}]$$

Note that this is the projected unmet demand. Therefore, with index t', it refers to the projected unmet demand that we supply on day t', not the unmet demand of the supply that arrives at the parent suppliers on day t'. On day t', the unmet demand is

$$\widetilde{\boldsymbol{x}}^{UD}_{t,t'} \triangleq [\widetilde{x}^{UD}_{p_1,t,t'}\, \widetilde{x}^{UD}_{p_2,t,t'} \ldots \widetilde{x}^{UD}_{p_{|\mathcal{N}^{+}_{Joe}|},t,t'}]$$

where $\widetilde{x}^{UD}_{p_i,t,t'}$ is the unmet demand (supply) of the parent $p_i \in \mathcal{N}^{+}_{Joe}$ (from Joe).
It is interesting to note that there are times, later in the horizon H of Joe, when Joe will

not be able to supply products to some of his parents in time if the travel times are large. Therefore, we set some of $\widetilde{x}^{UD}_{p_i,t,t'}$ to be 0, and here is the rule. If we supply on day t', then the product will arrive at parent p on day $t' + \mathcal{T}_p$ and takes another day to move to the production plant, which will be on day $t' + \mathcal{T}_p + 1$. The latest this could be is on day $t + H$. Therefore, t' has to be less than or equal to $t + (H - \mathcal{T}_p - 1)$ or $t + (h_p - 1)$. Hence, for any parent p of Joe, $\widetilde{x}^{UD}_{p,t,t'} = 0$ for $t' = t + h_p$, ..., $t + H - 1$

Upstream Demand

Joe's upstream demand on day t is

$$\widetilde{\boldsymbol{X}}^{UpD}_t \triangleq [\widetilde{\boldsymbol{x}}^{UpD}_{t,t}\ \widetilde{\boldsymbol{x}}^{UpD}_{t,t+1} \ldots \widetilde{\boldsymbol{x}}^{UpD}_{t,t+H-1}]$$

and his upstream demand on each day is distributed to each child c_i, $\forall i \in \mathbb{N}^1_{|\mathcal{N}^-_{Joe}|}$

$$\widetilde{\boldsymbol{x}}^{UpD}_{t,t'} \triangleq [\widetilde{x}^{UpD}_{c_1,t,t'}\ \widetilde{x}^{UpD}_{c_2,t,t'} \ldots \widetilde{x}^{UpD}_{c_{|\mathcal{N}^-_{Joe}|},t,t'}]$$

where $\widetilde{x}^{UpD}_{c_i,t,t'}$ refers to the amount of products that Joe demands from the child supplier c_i and wants to have them ready at the production site on day t'. Joe can request his children suppliers today (day t) and ask them to supply him the amount that he may specify. Then, tomorrow or day $t+1$, his children suppliers will start supplying the product parts to him. Parts from the child c_i will take $\mathcal{T}_{c_i}$ days to arrive at Joe's, which would be on day $t+1+\mathcal{T}_{c_i}$. With another day from shipment to input inventory, finally, the product parts will be ready for Joe's production on day $t + 2 + \mathcal{T}_{c_i}$

Therefore, for c_i, $\widetilde{x}^{UpD}_{c_i,t,t'} = 0, \forall t' \in \mathbb{N}^t_{t+1+\mathcal{T}_{c_i}}$

Parameters and Input Data

Parameters

$\theta_{p,t,t'}$ is the cost of unmet demand, or the penalty for not meeting the demand of parent p of Joe at time t'.

$K^O_{t'}$ and $K^I_{t'}$ are the costs for storing output and input products in the output and input inventory, respectively, at time t'.

K^{Pro} is the cost of production at Joe's production plant.

K^{Pur}_c is the purchasing cost from the child c.

Input Data

$\widetilde{x}^D_{p,t,t'}$ is the upstream demand expected to arrive at parent p on day t', ordered to Joe on day t (so Joe receive and uses this information on day $t+1$, which is why we use $\widetilde{x}^D_{p,t-1,t'}$ on day t).

$\widetilde{x}^s_{c,t,t'}$ is the projected shipment from child c to Joe, shipped on day t', ordered from Joe to child c on day t.

Q_d is the demand quantity of part d needed for a unit of output product of Joe.

F^d_c is the fraction of supply of part d that the child c has to contribute.

Optimization Problem

Objective function

On day t, Joe would want to plan his production, supply and demand orders for the next H days so that he minimizes the following objective function.

$$\sum_{t' \in \mathbb{N}_{t+H-1}^{t}} ((\sum_{p \in \mathcal{N}_{Joe}^{+}} \theta_{p,t,t'} \widetilde{x}_{p,t,t'}^{UD}) + K_{t'+1}^{O} \widetilde{r}_{t,t'+1}^{O} + (\sum_{d \in \mathcal{D}_{Joe}} K_{d,t'+1}^{I} \widetilde{r}_{d,t,t'+1}^{I}) + K^{Pro} \widetilde{x}_{t,t'} + (\sum_{c \in \mathcal{N}_{Joe}^{-}} K_{c}^{Pur} \widetilde{x}_{c,t,t'}^{UpD}))$$

Constraints

First of all, the difference in output inventory between tomorrow and today is attributed to the amount that we produce today and the amount that we ship out to parent suppliers. This can be formulated as

$$\widetilde{r}_{t,t'+1}^{O} = \widetilde{r}_{t,t'}^{O} + \widetilde{x}_{t,t'} - \sum_{p \in \mathcal{N}_{Joe}^{+}} (\widetilde{x}_{p,t-1,t'}^{D} - \widetilde{x}_{p,t,t'}^{UD}) \tag{1}$$

for all $t' \in \mathbb{N}_{t+H-1}^{t}$

Second, the difference in input inventory between tomorrow and today is attributed to the supplies from children suppliers and the amount that we transfer from the input inventory to the production site. This can be formulated as

$$\widetilde{r}_{d,t,t'+1}^{I} = \widetilde{r}_{d,t,t'}^{I} + \sum_{i \in \mathbb{N}_{|d|}^{1}} (\widetilde{x}_{c_i^d,t-1,t'}^{s} - \widetilde{x}_{c_i^d,t,t'}^{UpD}) - Q_d \widetilde{x}_{t,t'} \tag{2}$$

for all $t' \in \mathbb{N}_{t+H-1}^{t}$ and distinct input part $d \in \mathcal{D}_{Joe}$, where $|d|$ refers to the number of children suppliers who supply the part d to Joe. c_i^d is the i-th child in the group of children who supply part d to Joe. Q_d refers to the quantity of part d that Joe needs in order to produce a unit of his product.

Moreover, we have to include some constraints on variables' nonnegativity. Considering each of Joe's parents, each parent's unmet demand cannot go below zero since negative unmet demand would mean that Joe supplies more than he is asked to. Unmet demand also needs to stay below demand.

$$0 \leq \widetilde{x}^{UD}_{p,t,t'} \leq \widetilde{x}^{D}_{p,t,t'} \tag{3}$$

All the other variables also need to be nonnegative. This includes input inventory, output inventory, product decision and upstream demand.

$$\widetilde{r}^{I}_{d,t,t'}, \widetilde{r}^{O}_{t,t'}, \widetilde{x}_{t,t'}, \widetilde{x}^{UpD}_{c^d_i,t,t'} \geq 0 \tag{4}$$

Finally, we need a constraint on the ratio of supplies among Joe's upstream suppliers who supply the same product/part to Joe.

$$\frac{\widetilde{x}^{UpD}_{c^d_i,t,t'} + \widetilde{x}^{S}_{c^d_i,t-1,t'}}{\sum_{j\in\mathbb{N}^1_{|d|}} (\widetilde{x}^{UpD}_{c^d_j,t,t'} + \widetilde{x}^{S}_{c^d_j,t-1,t'})} = \frac{F_{c^d_i}}{\sum_{j\in\mathbb{N}^1_{|d|}} F_{c^d_j}} \tag{5}$$

We include this equation for all (d, i, t') such that $\mathcal{T}_{c^d_i} \leq t' - t - 2$ because for c^d_i, on day t' such that $\mathcal{T}_{c^d_i} > t' - t - 2$, it will not be supplying anymore i.e. $\widetilde{x}^{UpD}_{c^d_i,t,t'} = \widetilde{x}^{S}_{c^d_i,t-1,t'} = 0$.

Further Extension

The optimization problem above could be done in both linear programming, where variables can take any real numbers (real nonnegative numbers in this case), and integer linear programming, where all variables have to be integers. In the real world, it would make more sense to produce products in full units i.e. 2 pieces of bathroom faucets rather than 1.25 piece. However, with the problem being integer programming, it can take much longer time to solve compare to when the problem is simply a linear programming. It is possible that we can modify this problem in some sense to obtain integer final results but maintain the optimization problem as a linear programming.

References

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Dionysios S. Kalogerias, Warren B. Powell, *An ILP-Based Rollout Policy for Distributed Supplier Production Planning*, April 2018, Department of Operations Research and Financial Engineering, Princeton University, Princeton, NJ 08544, USA