Extending Richardson's Arms Race Model: Grievance as a Variable

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Applied Math 115: Mathematical Modeling

Abstract

Arms race models are useful to understand how to diffuse or slow/stop a cold-war situation when pitted against a country with known reaction patterns. The standard Richardson Model predicts disarmament or armageddon, which fails to stand up to the evidence provided by the Cold War and modern nuclear standoffs. Our model leverages a carrying capacity for the nuclear arsenal combined with a more nuanced approach to handling grievances to decisively converge to stable points with minimized over/under reactions. This more accurately captures the rapid escalations and de-escalations found in real-world arms races.

1 Introduction

Lewis Richardson kick-started the field of arms race modeling when he began studying how nations develop and deploy arsenals after the horrific events of WWI and WWII. His model, detailed below, serves as the launch board for most modern studies and models of the subject.

$$\frac{dx}{dt} = ay - mx + r$$
$$\frac{dy}{dt} = bx - ny + s$$

(*a,b = aggression coefficients; m,n = fatigue coefficients; r,s = grievance coefficients*)

In this model, the aggression coefficients *a* and *b* represent the aggressiveness of a given country. A higher aggression coefficient leads to a more dramatic response to the competing country's arsenal size. The fatigue coefficients *m* and *n* represent the rate at which countries retire arms, either because of the cost of maintenance or because of a general desire to disarm. The grievance coefficients *r* and *s* represent external or internal pressures on a country which influence how they increase or decrease their arsenal independent of arsenal size (such as pressure from the UN to disarm, internal politics, or active embargos).

However his model has one major failing point: it always ends in disarmament or an exponential increase in arms (i.e. breakout of war). While this may have been a realistic model for the wars that Richardson experienced during his time and thus had available data for, it fails to stand up to the long-lasting nuclear standoffs we know today such as the Cold War between the United States and Soviet Union/Russia, as well as India and Pakistan.

The most common way to remedy this all-or-nothing result of the Richardson model is to amend it with a carrying capacity for each country's arsenal, thereby enabling the system to reach non-zero stable points. While this fixes the most glaring problem with the Richardson model, it still takes an overly simplistic approach to how countries react to arms stockpiling. Mainly, both the standard Richardson and Richardson with carrying capacity models only account for the *quantity* of arms; each neglects to account for the *rate of change* of arms stockpiles. This is an important issue to address because failure to do so leaves the system prone to slow responses, which may result in over or under reactions in the short term, and longer times to reach a resolution.

2 Description of Model

As stated in the introduction, the most common way to remedy the all-or-nothing approach of the Richardson model is to add a carrying capacity to the differential equations [1]. This produces the following equations:

$$\frac{dx}{dt} = (1 - \frac{x}{x_{max}})(ay - mx + r)$$
$$\frac{dy}{dt} = (1 - \frac{y}{y_{max}})(bx - ny + s)$$

However, this model still utilizes constants to represent the grievance. Our modification to the model treats grievance as a variable dependent on the rate of change of the opposing country's arsenal. Thus our model is as follows:

$$\frac{dx}{dt} = (1 - \frac{x}{x_{max}})(ay - mx + B\frac{dy}{dt} + r)$$
$$\frac{dy}{dt} = (1 - \frac{y}{y_{max}})(bx - ny + G * \frac{dx}{dt} + s)$$

In order to better understand how this Grievance as a Variable model (GAV) effects outcomes in comparison to the Grievance as a Constant model (GAC), we characterized three different archetype states to enter into an arms race against each other.

Country Type	Carrying Capacity (x_{max}, y_{max})	Aggression Coeffs (a, b)	Fatigue Coeffs (m, n)	Grievance Coeffs (B, G)	External Pressure Coeffs (r, s)	Initial Stock-pile (x_0, y_0)
Aggressive	10	2	.2	2	2	2
Defensive	8	1.5	2	1	2	5
Diplomatic	4	.5	1	.1	-.5	3

Aggressive countries are characterized by high arms acquisition responses to opposing country arms acquisition (high aggression), low "internal brakes" to further arms increases (low fatigue) [2], and high grievance levels (now a function of the opposing country's rate of arms acquisitions). We initialized the Aggressive country to have a high carrying capacity, given its willingness to invest a higher percentage of GDP towards arms acquisitions, and a lower initial arms level, since aggressive countries usually don't have the economic capacity to develop many nuclear warheads (e.g. North Korea, etc.), compared to more economically prosperous countries that tend more towards diplomatic or defensive strategies.

Defensive countries are characterized by relatively high arms acquisition responses, moderately high fatigue levels, in addition to high grievance levels. We also initialize the Defensive country to have a relatively high carrying capacity, given its characterization as a nation that is able to sufficiently respond to opposing threats, as well as a high initial arms level, given a "defensive" country's preparedness for any potential threat.

Diplomatic countries are pacifists, and so are characterized by low arms acquisition responses (in hopes of lowering the prospect of war), moderate fatigue levels, as well as extremely low grievance levels. The diplomatic country is initialized with a low carrying capacity for arms, representing the nation's unwillingness to engage in militant activity (i.e. investing in arms), and a moderately low initial arms level, since more diplomatic countries generally have greater economic capabilities, and thus are generally more likely to have a more developed defense level (despite their more diplomatic tendencies).

3 Analysis

With grievance as a function of the change of arms acquisition in the opposing country, we consider each of the six possible combinations in the three different types of countries. In the figures below, we overlay the amended Richardson model with grievance as a constant, alongside our version of the amended Richardson model with grievance as a variable, in order to directly illustrate the differences in behavior

between the two models. The units of time are measured in *months* given the average timeline for an arms race to escalate (or devolve), according to historical measures [3].

The results shown in Figure 1 suggest that in an arms race between two aggressive countries, arms acquisitions escalate quickly to their respective carrying capacities, in roughly 1 to 2 months time. With grievance as a variable (in red), the time to escalation is evidently faster than the model with grievance as a constant, suggesting that the arms acquisition by the opposing country is a significant function of how fast the country builds their arms stockpile. This behavior was evident in the 1962 Cuban Missile Crisis, when photos by an American U2 spy plane on October 14th triggered a retaliatory naval blockade and imminent threat of war, ending the standoff on October 28th that same year [4]. Despite the particular nuances of the situation, both countries, which have particularly aggressive characteristics, escalate their rhetoric and arms strategy in about 14 days given the exponential threat of growing arms from the opposing country--similar to the behavior of our model. Note that the two Aggressive countries (Nation A and B) overlap each other due to their identical characteristics.

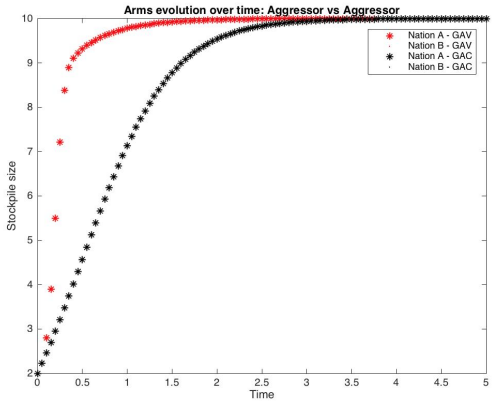

Figure 1

In Figure 2, we see that the original amended Richardson model (in black) counterintuitively suggests that a Defensive country (Nation B) would first decrease its arms capacity in response to rapid acquisition of arms from an Aggressive country (Nation A). This is due to the low initial stockpile of Nation A and the unchanging nature of grievance despite the rapid rate of change of Nation A's armaments. Because our model (in red) accounts for this, we see an immediate gradual increase in arms acquisition, stabilizing after roughly 4 months, for the Defensive country.

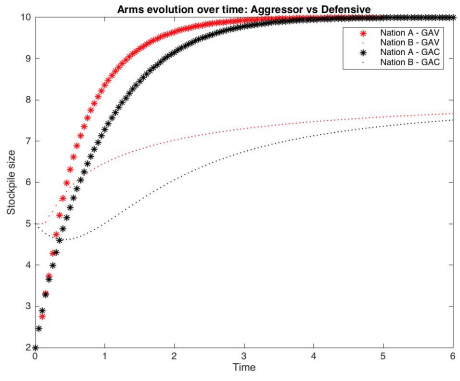

Figure 2

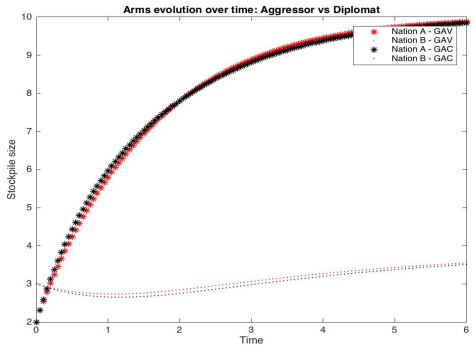

Figure 3

Figure 3 reveals that even with a Diplomatic country (Nation B), an aggressive country (Nation A) will still ultimately increase arms, albeit at a much slower pace, thus causing the Diplomatic country to very slowly raise their arms stockpile in response. The two models, given the Diplomatic country's reluctance to respond militantly, yield roughly the same behavior.

Figures 4 (Defensive vs Defensive), 5 (Diplomatic vs Defensive), and 6 (Diplomatic vs Diplomatic), both reveal similar patterns of behavior where the less aggressive nature of each country induces a decline in arms acquisitions. Whereas convergence of arms stockpile differs for each scenario, the difference between the original amended Richardson model (with grievance as a constant) and our model (with grievance as a variable), is that the rate of arms decline for each country is expedited. With grievance as a variable, each country's aggression towards the other is highly dependent on the arms acquisition rate of the other country; therefore, we would expect that for two countries less inclined towards aggressive behavior, de-escalation of militant activity is accelerated.

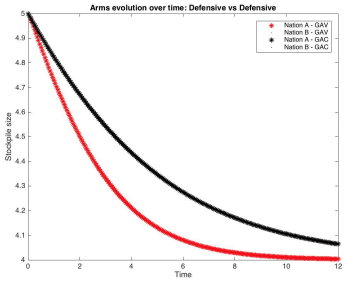

Figure 4

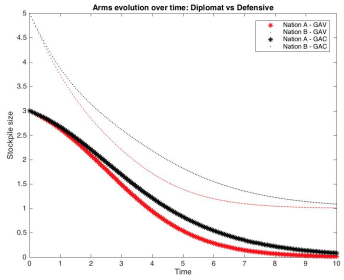

Figure 5

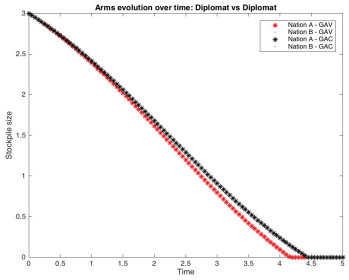

Figure 6

As an illustrative, when Ukraine became a nuclear-free state in June 1996, Belarus followed a mere 5 months afterwards on November 1996 when it removed its last nuclear missile [3]. Despite the unique context during that time period, this example demonstrates how issues of armament or disarmament are time compressed events given the imminent threat (or lack thereof) of war--something our model is more apt at emphasizing.

3.1 Sensitivity Analysis

Now, we observe the sensitivity of the model to three parameters: carrying capacity (x_max, y_max), and internal/external pressures (r, s), when an Aggressive country and Diplomatic country are in opposition, and the sensitivity of the model to initial conditions when an Aggressive country and a Defensive country are in opposition and when an Aggressive country vs Aggressive Country are in opposition. Varying carrying capacities reflect the evolution of the model when countries are able to expand their capacity for investing in arms acquisitions, and varying internal/external pressures reflect the evolution of the model when the country faces different internal/external conditions that enable them or disable them from pursuing more arms. We show the behavior of the systems in the first six days of the race in order to examine the granular effect, and eventual trajectories of these parameters. Furthermore, varying the initial conditions of the system we are able to show the different effects starting points in an arms race will have for any country.

In Figures 8 and 9 below, we see that for a Diplomatic Country, the carrying capacity can determine the initial trajectory of arms acquisitions (where past CC = 13, the trajectory trends upward); whereas for the Aggressive Country, the carrying capacity determines the rate of increasing arms acquisitions, since all carrying capacities result in an upward trending behavior. For both countries, we see that larger carrying capacity values have a nonlinear effect on the trajectory of arms acquisitions, perhaps indicating that for a large enough value of x_max or y_max, the countries are more prone to acquiring arms.

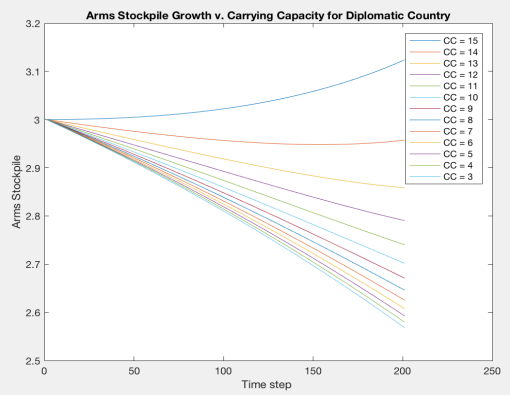

Figure 8

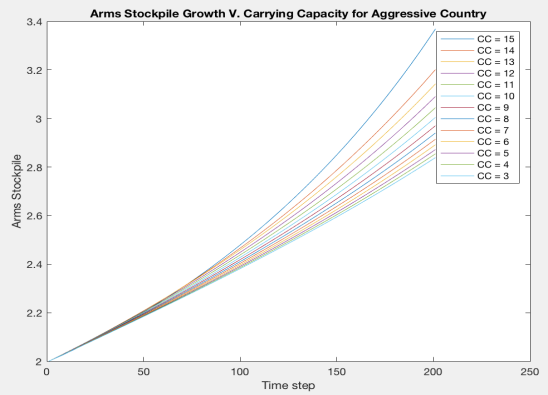

Figure 9

(Note: dt = .001, so 250 timesteps is equivalent to roughly 6 days)

In Figures 9 and 10 below, we see that for a Diplomatic country, changes in external/internal pressures yield a relatively constant impact on the trajectory of arms acquisitions; whereas for the Aggressive country, there is a slight nonlinear impact of arms acquisition trajectory (as the line curves downward when r = -3) when there are associated negative internal/external pressures. The figures also reveal that internal and external pressures are pretty influential in determining the initial trajectories of arms acquisition rates, in contrast to the varying carrying capacity model, where trajectories stay the same (for the Aggressive country) unless a certain threshold is reached (for the Diplomatic country).

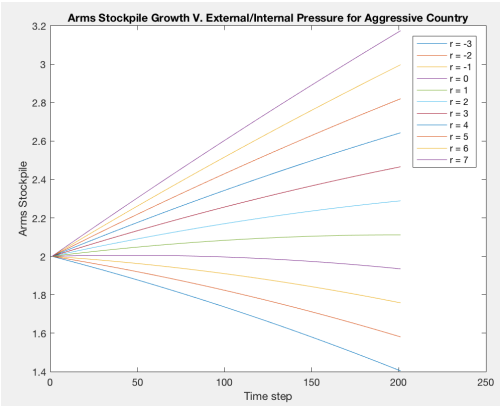

Figure 9

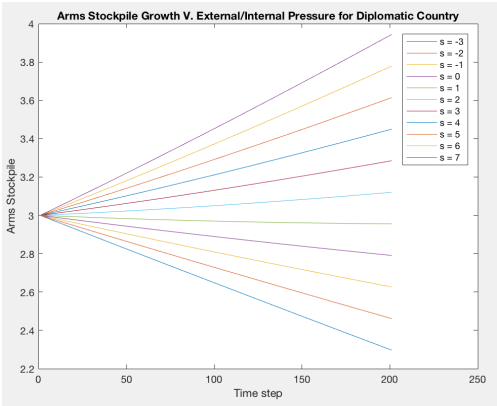

Figure 10

Furthermore, from Figures 10 and 11 below, we see that the model predicts that above a certain threshold in initial conditions, countries will go to their own respective carrying capacity, and below the threshold, converge to disarmament. If an Aggressive country opposes a Defensive country, we see that both countries will tend to go towards their carrying capacities of 10 and 8, respectively; however, once both countries' initial conditions are at relatively low values, both countries will disarm and go to zero. This analysis applies for when an Aggressive country opposes an Aggressive country; however, the initial conditions can be higher than the previously mentioned case and still go to disarmament, instead of to their respective armament carrying capacities. Lastly it must be noted that once the initial condition is above the carrying capacity, the defensive country will tend towards an infinite value while the aggressive country will go towards its carrying capacity. We consider these initial conditions to be impossible for a country since it would require a country to naturally have started above its carrying capacity and therefore consider such results irrelevant to our calculations.

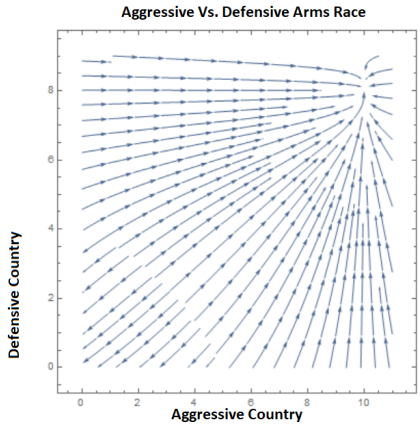

Figure 11

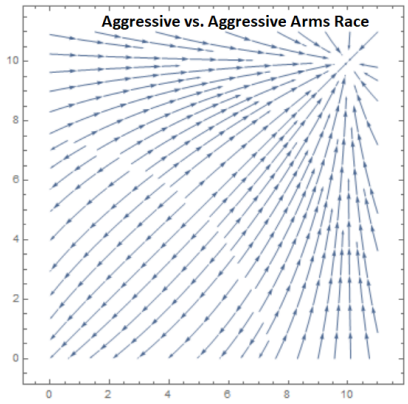

Figure 12

4 Discussion

Analysis of our Grievance as a Variable model demonstrates the rapid swings in armaments suggested by real world scenarios such as the Cuban Missile Crisis and the Ukraine/Belarus disarmaments. This has noticeable benefits over the amended Richardson model, particularly in scenarios like that shown in Figure 2 where the amended model allows an under-reaction to an Aggressor's stockpile, thus leaving the Defensive country extremely vulnerable for a short period of time. While both models reach the same end conclusions, we believe a model that accurately characterizes the rapid movements found in real world arms races is worth the additional complexity and computational cost.

Our simulations make the following assumptions:

- In the very first time step of the simulation, both countries know the other country's arsenal size but have no knowledge of their rate of change. Thus the first movement of each simulation assumes an initial derivative of 0 for each country's arsenal size.
- Carrying capacity of a country is constant. Even as war progresses, they will not increase their willingness to spend more and raise their carrying capacity
- We do not account for the wealth of a nation when determining carrying capacity, rather we treat it more as a tolerance for how much arms they will hold, independent of whether they can afford it; this simplifies the model at the cost of generalizing the characteristics of aggressive, defensive, and diplomatic countries
- We use a continuous model even though you cannot have fractions of armaments
- All countries have the ability to immediately mobilize reactions to changes in an opponent's arsenal
- We assume that the rate of change of the opposition's arsenal does not affect the aggression terms (i.e. there is no y*dy/dt term in dx/dt and vice versa)

While the first of these is fairly benign as it only affects the very first time step, the latter assumptions could affect the outcome and/or behavior of an arms race. In our initial investigatory phase we proposed a second, more complicated model for grievance that would capture the last assumption. That model is as follows:

$$\frac{dx}{dt} = (1 - \frac{x}{x_{max}})(ay * B\frac{dy}{dt} - mx + r)$$
$$\frac{dy}{dt} = (1 - \frac{y}{y_{max}})(bx * G\frac{dx}{dt} - ny + s)$$

However, we chose to focus on the simpler model to allow us to focus on deeper analysis rather than grappling with the complexities of this model.

5 Conclusion

Especially with recent rhetoric by Russian officials about their nuclear capacity towards the United States and other countries [5], understanding arms races and the specific implications that different parameters have on their evolution is increasingly imperative for policy makers to understand. As our model suggests, the nature of grievance in response to another country is particularly important in regards to the timing of the arms race. Our model also suggests that carrying capacity (influenced by economic and technical capability by country), influences the rate at which these arms races progress, informing the speed at which a policymaker should respond to threats (both immanent and potential), given the respective capacities of their opposition. Empirical studies of capacity parameters (e.g. which economic conditions lead to restrictions in arms investment capacity), in addition to characterizations and parameterizations of arms races in artificial intelligence [6] remain as possible areas of extension for our model.

References

[1] Lane, Lehman, and John McEwen. “Modifying the Richardson Arms Race Model with a Carrying Capacity”. Retrieved February 27, 2018, from http://plaza.ufl.edu/blane116/modifying_the_richardson_arms_race_model.pdf

[2] Olinick, M. “A Modeling Application: Modeling An Arms Race”. Retrieved February 25, 2018, from https://www.cengage.com/math/book_content/0495108243_zill/projects_archive/debv4e/AModelingApplication5.pdf

[3] Nuclear weapons timeline. (n.d.). Retrieved March 01, 2018, from http://www.icanw.org/the-facts/the-nuclear-age/

[4] (n.d.). Retrieved March 01, 2018, from http://www.cbc.ca/news2/interactives/tl-cuban-missile-crisis/timeline/timeline.html

[5] Roth, A. (2018, March 01). Putin threatens US arms race with new missiles declaration. Retrieved March 03, 2018, from https://www.theguardian.com/world/2018/mar/01/vladimir-putin-threatens-arms-race-with-new-missiles-announcement

[6] Barnes, J. E., & Chin, J. (2018, March 02). The New Arms Race in AI. Retrieved March 03, 2018, from https://www.wsj.com/articles/the-new-arms-race-in-ai-1520009261

Contributions:

Jordan Rogers: Arms stockpile evolution simulation code, abstract, introduction, description of model, discussion, same parts in PowerPoint, Model Development

Daniel Kang: Sensitivity Analysis code (Carrying Capacity, Internal External Pressure), Description of Model, Analysis, Sensitivity Analysis (Carrying Capacity, Internal/External Pressure), Conclusion, same parts in PowerPoint, Model Development

Alan Estrada: Sensitivity analysis code (initial Conditions), Description of sensitivity of analysis code, Model development, and various parts of write up.