1) a) DTFS coefficients are periodic with signal period
$N=10$.
$\boxed{\Rightarrow \text{True}}$

b) If the signal were real and even $\Rightarrow a_k = a_{-k}$

However the signal is only real, not even

$\Rightarrow a_k = a^*_{-k}$

$\boxed{\Rightarrow \text{False}}$

c) $x[n+2] \longleftrightarrow a_k e^{\frac{j2\pi k}{5}}$

Since $x[n+2]$ is even $\Rightarrow a_k e^{j\frac{2\pi k}{5}}$ is real. $\boxed{\Rightarrow \text{True}}$

d) $a_0 = \frac{1}{10} \sum_{n=\langle N \rangle,\ N=10} x[n] = 0$ $\boxed{\Rightarrow \text{True}}$

e) $x[n] - x[n-\frac{N}{2}] \longleftrightarrow a_k - a_k e^{-j\frac{2\pi}{N}\cdot\frac{N}{2}\cdot k} = b_k$

$= a_k(1 - e^{-j\pi k})$

$= \begin{cases} 2a_k, & k \text{ is odd} \\ 0, & k \text{ is even} \end{cases}$

$\boxed{\Rightarrow \text{False}}$

2)(a) $x(t) = \cos\left(\frac{\pi t}{2}\right) \cdot \mathrm{rect}\left(t/2\right)$

$\downarrow \mathcal{F}\{\,\}$

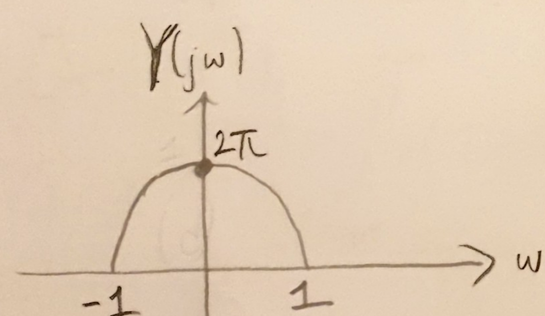
$$X(j\omega) = \frac{1}{2\pi}\left[\pi\delta\left(\omega - \frac{\pi}{2}\right) + \pi\delta\left(\omega + \frac{\pi}{2}\right)\right] \underset{\uparrow \text{convolution}}{*} \frac{2\sin(\omega)}{\omega}$$

$$= \left[\overbrace{\frac{\sin\left(\omega - \frac{\pi}{2}\right)}{\omega - \frac{\pi}{2}}}^{-\cos(\omega)} + \overbrace{\frac{\sin\left(\omega + \frac{\pi}{2}\right)}{\omega + \frac{\pi}{2}}}^{+\cos(\omega)}\right]$$

$$= \frac{\pi\cos(\omega)}{\left(\frac{\pi^2}{4} - \omega\right)} = \frac{4\pi\cos(\omega)}{(\pi^2 - 4\omega^2)}$$

(b) $C_1 = 2\pi x(t)\Big|_{t=0} = 2\pi$, $C_2 = X(j\omega)\Big|_{\omega=0} = \frac{4}{\pi}$

(c) $\mathcal{F}\{y(t)\} = 2\pi\, x(-\omega) = \begin{cases} 2\pi\cos\left(\frac{\pi\omega}{2}\right) & \text{if } |\omega| < 1 \\ 0 & \text{otherwise} \end{cases}$



as $\cos\left(-\frac{\pi\omega}{2}\right) = \cos\left(\frac{\pi\omega}{2}\right)$

and $|-\omega| = |\omega|$.

(d) $z(t) = \frac{\pi}{2} \cdot \underbrace{\frac{\sin(2t)}{\pi t}}_{\triangleq z_1(t)} \cdot \underbrace{\frac{\sin(8t)}{\pi t}}_{\triangleq z_2(t)} \cdot e^{j10t}$

$Z_1(j\omega) = \mathrm{rect}(\omega/4)$

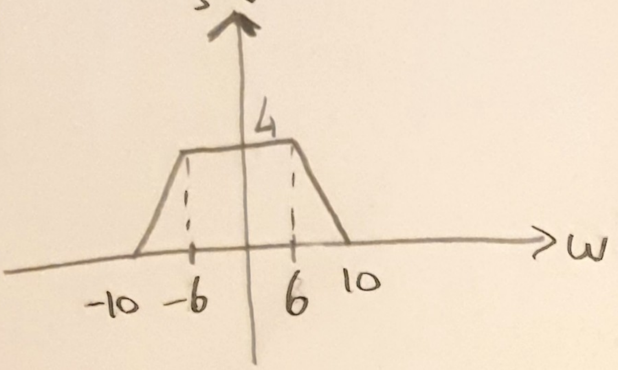
$Z_2(j\omega) = \mathrm{rect}(\omega/16)$

where $\mathrm{rect}(\omega/T) = \begin{cases} 1, & |\omega| < \frac{T}{2} \\ 0, & |\omega| > \frac{T}{2} \end{cases}$

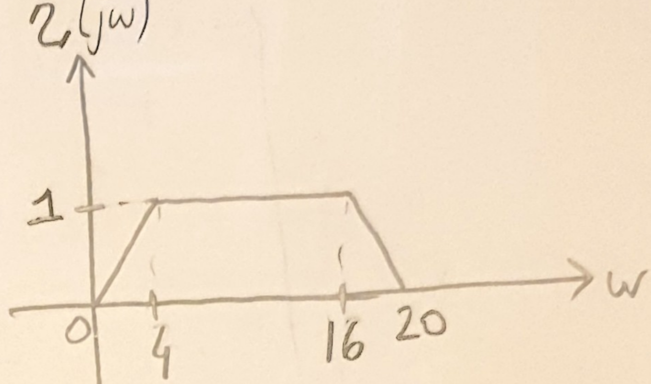
Q-2)(d) (Cont'd) $Z(j\omega) = \frac{\pi}{2}\frac{1}{(2\pi)^2}\underbrace{\left(Z_1(j\omega) * Z_2(j\omega)\right)}_{\triangleq Z_3(j\omega)} * 2\pi\delta(\omega-10)$

$$= \frac{1}{4} Z_3(j(\omega-10))$$

$Z_3(j\omega) = Z_1(j\omega) * Z_2(j\omega)$



$\Longrightarrow$



(e) $E_Z = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|Z(j\omega)|^2 d\omega = \underbrace{\int_0^4 \frac{1}{16}\omega^2 d\omega}_{I_1} + \underbrace{\int_4^{16} 1\, d\omega}_{I_2} + \underbrace{\int_{16}^{20}|Z(j\omega)|^2 d\omega}_{I_3}$

$\Longrightarrow \boxed{I_2 = 12}$ $\quad I_1 = I_3$

$I_1 = \frac{1}{48}\omega^3\Big|_0^4$

$\boxed{= 4/3} = I_3$

$\Longrightarrow E_Z = \frac{1}{2\pi}\left[12 + \frac{4}{3}\cdot 2\right]$

$$\boxed{= \frac{22}{3\pi}}$$

3) (a) $m_1(t)$ and $m_2(t)$ are all real since

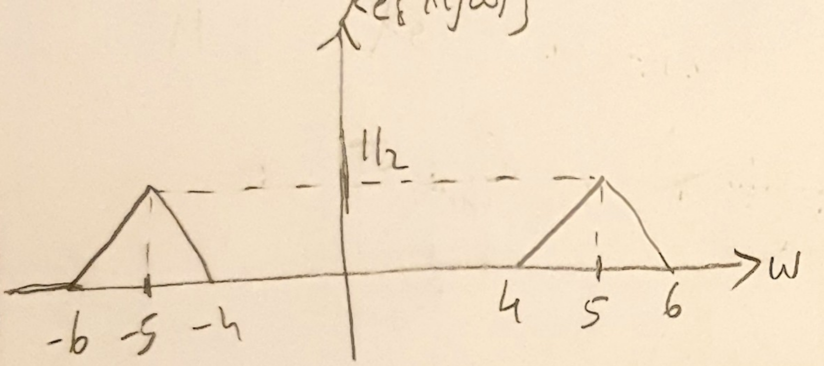
$M_1(jw) = M_1^*(-jw)$ and $M_2(jw) = M_2^*(-jw)$.

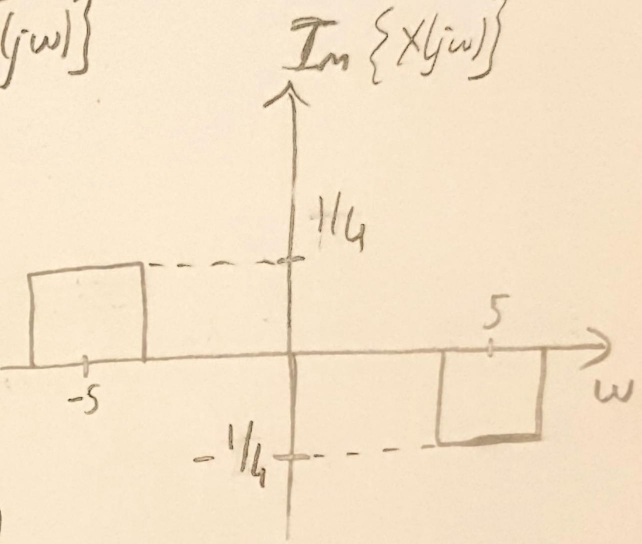
(Fourier transforms are all even symmetrical).

$\Rightarrow x(t)$ is real.

(b) $$X(jw) = \underbrace{\frac{\pi}{2\pi}\left[M_1(j(w-5)) + M_1(j(w+5))\right]}_{Re\{X(jw)\}}$$

$$\underbrace{j\frac{(-\pi)}{2\pi}\left[M_2(j(w-5)) - M_2(j(w+5))\right]}_{Im\{X(jw)\}}$$





(c) i) $x(t) \longrightarrow$ Receiver $\longrightarrow y_1(t)$

$\underline{x'(t)} = x(t-t_d) \longrightarrow$ Receiver $\longrightarrow y_2(t)$

$$y_2(t) = \int_{-\infty}^{+\infty} \underbrace{x(\tau - t_d)\cos(5\tau)}\, h(t-\tau)\, d\tau$$

If this were

$x(\tau - t_d)\cos(5(\tau - t_d)) \Rightarrow y_2(t) = y_1(t-t_d)$

However, it is not

$\Rightarrow$ System is not time invariant!!

2.3

c) ii) First find $x(\tau)\cos(5\tau) \stackrel{\Delta}{=} x_2(\tau)$

$$x_2(\tau) = x(\tau)\cos(5\tau) = m_1(\tau)\cos^2(5\tau) + m_2(\tau)\sin(5\tau)\cos(5\tau)$$

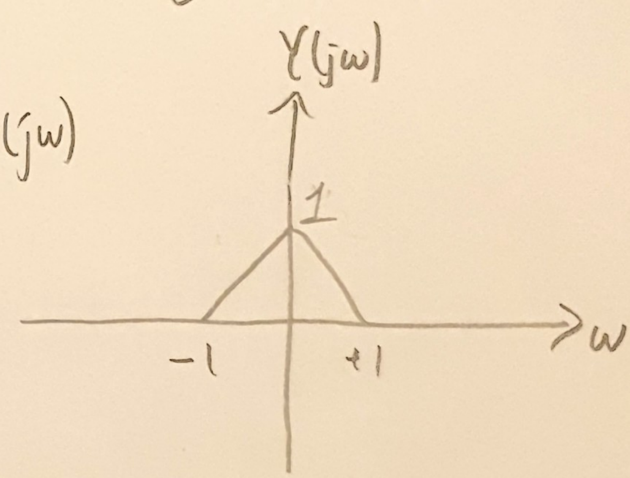
$$= \frac{m_1(\tau)}{2}\left[1+\cos(10\tau)\right] + \frac{1}{2}m_2(\tau)\sin(10\tau)$$

Find $X_2(j\omega) = \frac{M_1(j\omega)}{2} + \frac{\pi\left[M_1(j(\omega-10)) + M_1(j(\omega+10))\right]}{2}$

$$+ \frac{\pi\left[M_1(j(\omega-10)) - M_1(j(\omega+10))\right]}{2j}$$

Since the last two terms are non-zero only when $9 < |\omega| < 11$, they will be suppressed by the low-pass filter.

$$\Rightarrow Y(j\omega) = \frac{M_1(j\omega)}{2} H(j\omega) = M_1(j\omega)$$



Q4) The Fourier Series coefficients can be calculated for the given triangular wave (after several steps involving integration by parts) as

$$a_k = -\frac{2}{\pi^2 k^2} + \frac{2(-1)^k}{\pi^2 k^2} \;\; for\; k \neq 0$$

$$a_k = 0\; for\; k = 0.$$

Part 1)

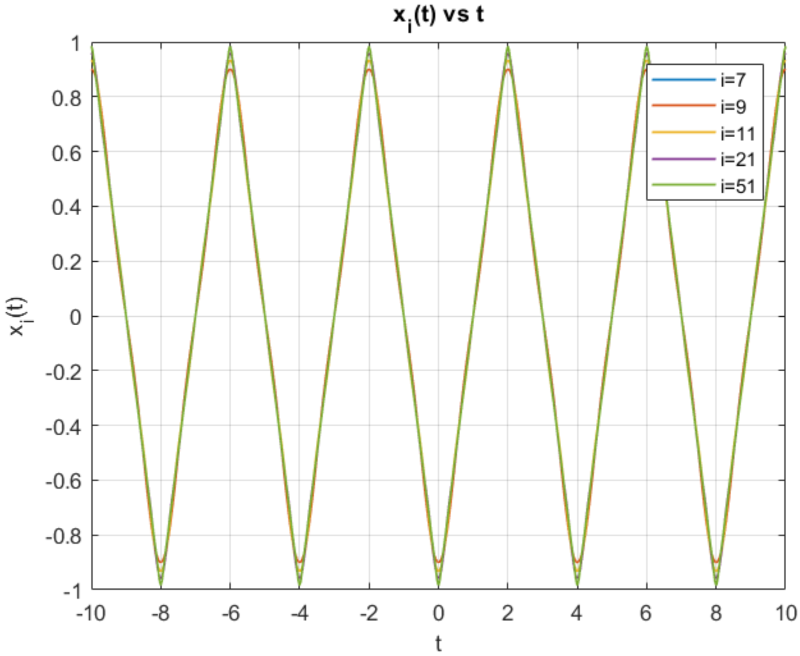


Fig.1: Reconstructed signal for various number of FS terms (i on the legend indicates the number of FS terms)

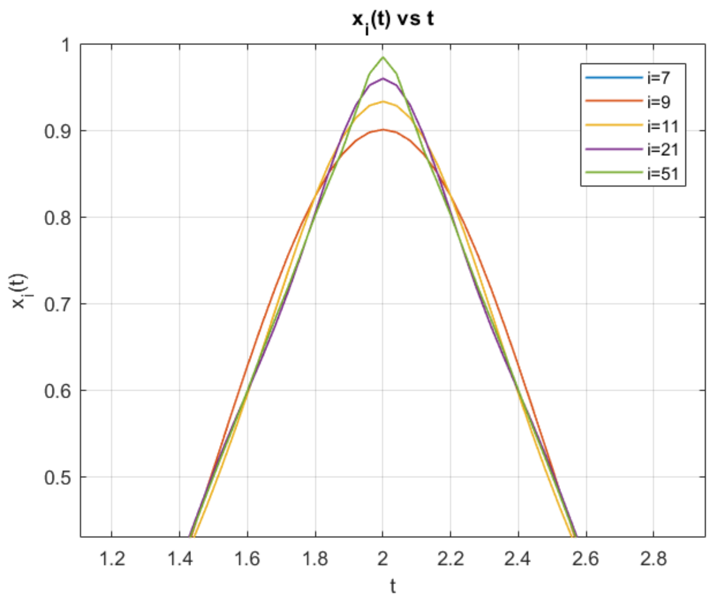


Fig.2: Reconstructed signal for various number of FS terms (zoomed in at one of the peak locations)

As can be noted in Fig.1, the reconstructed signal converges to the original triangular waveform as the number of FS terms increase. We do not expect a significant improvement when the number of FS terms increase as we can see that the signal is close to the original triangular waveform, its energy over time domain will be close to the energy of the original triangular waveform, thus the conntribution of the remaining FS terms in frequency domain will be negligible according to the Parseval's relation.

Part II) Original waveform:

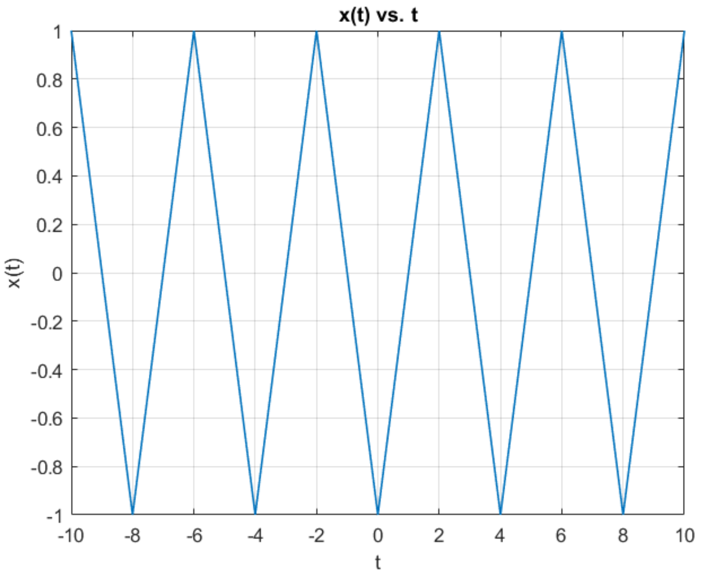


Fig. 3: Original Waveform (reconstructed versions can be found in Fig.1)

Delayed waveform:

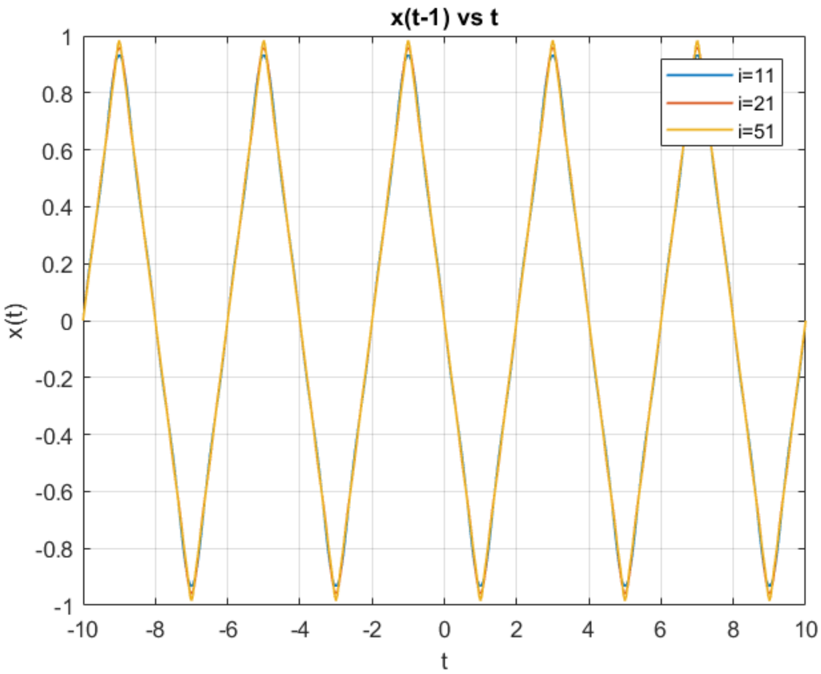


Fig. 4: Delayed Waveform

Delayed and differentiated waveform:

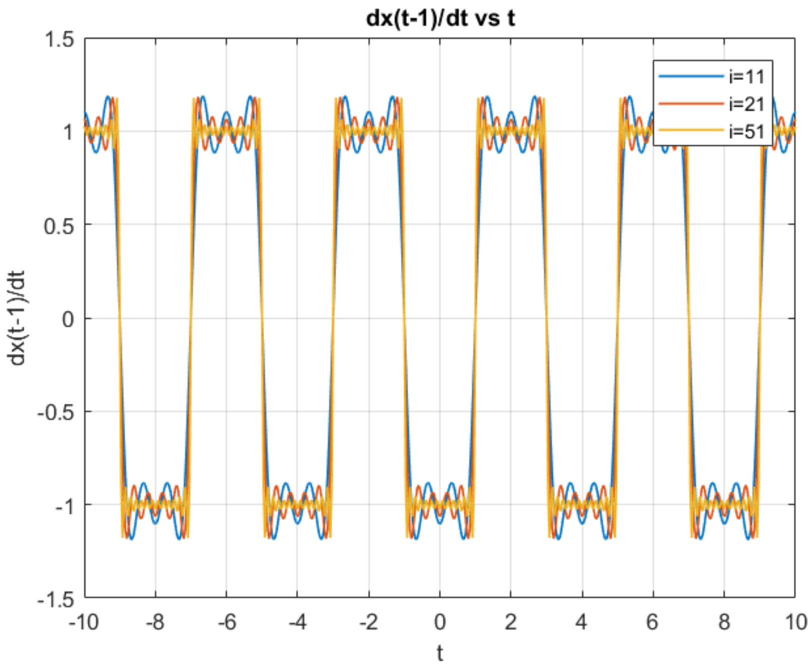


Fig. 5: Delayed and Differentiated Waveform

We observe convergence to original waveform except at the edges. When we look at the edges (it is most obvious for 51 term reconstruction in Fig.5) we do not observe convergence to the original waveform. This is the result of Gibbs effect. Although the energy of the reconstructed signal gets close to the original waveform as more FS terms are included, we cannot observe pointwise convergence at the edges, as single points at the edges do not have significant contribution to the integration made to find the signal energy.

```
clear all
close all
%% Part a
T=4;
no_of_terms=[7 9 11 21 51];
t_p=-T/2:T/100:T/2;
t=-2.5*T:T/100:2.5*T;
x=2*(2/T*abs(t_p)-0.5);
y=[x x(2:end) x(2:end) x(2:end) x(2:end)];
figure;
plot(t,y,'linewidth',1)
grid on
xlabel('t')
ylabel('x(t)')
title('x(t) vs. t')
%% Part b and Part 1
k=[-(no_of_terms(end)-1)/2:-1 1:(no_of_terms(end)-1)/2];
a_k=-1./(pi^2*k.^2)+(-1).^k./(pi^2*k.^2);    % Calculating
A_k for nonzero k
a_k=a_k*2;
a_k=[a_k(1:(no_of_terms(end)-1)/2) 0
a_k((no_of_terms(end)-1)/2+1:end)];                  %
Inserting a_0
k=[-(no_of_terms(end)-1)/2:-1 0 1:(no_of_terms(end)-
1)/2];
figure; stem(k,a_k); grid on; xlabel('k'); ylabel('a_k');
title('a_k vs k');

iter=0;
for c=(no_of_terms-1)/2  % c is the number up to which
harmonic components are included in F.S.
iter=iter+1;                  % For example when c=4,
reconstruction has harmonics up to
                   % 4th harmonics

for d=-c:c
x_d(d+c+1,:)=a_k(d+(no_of_terms(end)-
1)/2+1)*exp(1i*2*pi/T*d*t);
end
x_i(iter,:)=sum(x_d,1);
if(iter==1)
figure;
end
plot(t,x_i(iter,:),'linewidth',1)
hold on
    end
legend('i=7','i=9','i=11','i=21','i=51');
```

```
xlabel('t')
ylabel('x_i(t)')
title('x_i(t) vs t');
grid on;
%% Part 2
a_k_delayed=a_k.*exp(-1i*2*pi/T*k); % Effect of delay on
the FS coefficients.
a_k_delayed_diffrntiated=a_k_delayed.*(1i*k*2*pi/T);
no_of_terms=[11 21 51];
iter=0;

for c=(no_of_terms-1)/2  % c is the number up to which
harmonic components are included in F.S.
iter=iter+1;                    % For example when c=4,
reconstruction has harmonics up to
                              % 4th harmonics

for d=-c:c
x_d_2(d+c+1,:)=a_k_delayed(d+(no_of_terms(end)-
1)/2+1)*exp(1i*2*pi/T*d*t); % When d=0, a_k(21) gets the
0th harmonic
end
x_i_new(iter,:)=sum(x_d_2,1);
if(iter==1)
figure;
end
plot(t,x_i_new(iter,:),'linewidth',1)
hold on
end
legend('i=11','i=21','i=51','i=21','i=51');
xlabel('t')
ylabel('x_i(t)')
title('x_i(t) vs t');
grid on;

iter=0;
for c=(no_of_terms-1)/2  % c is the number up to which
harmonic components are included in F.S.
iter=iter+1;                    % For example when c=4,
reconstruction has harmonics up to
                              % 4th harmonics

for d=-c:c
x_d_3(d+c+1,:)=a_k_delayed_diffrntiated(d+(no_of_terms(en
d)-1)/2+1)*exp(1i*2*pi/T*d*t); % When d=0, a_k(21) gets
the 0th harmonic
end
```

```
x_i_new2(iter,:)=sum(x_d_3,1);
if(iter==1)
figure;
end
plot(t,x_i_new2(iter,:),'linewidth',1)
hold on
end
legend('i=11','i=21','i=51','i=21','i=51');
xlabel('t')
ylabel('x_i(t)')
title('x_i(t) vs t');
grid on;
```