EE 301 Signals & Systems
Homework 3

Due: *January* 16, 2022, 23:55 via `odtuclass.metu.edu.tr`

1) **Problem 1:** Let $x[n] = \delta[n] + 3\delta[n-1] + \delta[n-2]$.

a) Plot $x[n]$ and its periodic extension, $\tilde{x}[n]$, for $N = 3$.

$$\tilde{x}_N[n] = \sum_{k=-\infty}^{\infty} x[n-kN]$$

b) Find the Discrete-time Fourier Series (DTFS) coefficients, $\tilde{X}_3[k]$, of $\tilde{x}_3[n]$. Write the DTFS representation of $\tilde{x}_3[n]$.

c) Find the Discrete-time Fourier Series (DFS) coefficients for N=5, $\tilde{X}_5[k]$, of $\tilde{x}_5[n]$. Write the DTFS representation of $\tilde{x}_5[n]$.

d) Find the DTFT, $X(e^{j\Omega})$, of $x[n]$. Plot its magnitude and phase.

e) Verify that $\tilde{X}_3[k] = X(e^{j\Omega})\big|_{\Omega=k\frac{2\pi}{3}}$ and $\tilde{X}_5[k] = X(e^{j\Omega})\big|_{\Omega=k\frac{2\pi}{5}}$, i.e., uniformly spaced samples of DTFT of $x[n]$. Show these samples on the magnitude and phase plot of $X(e^{j\Omega})$.

Problem 2: The following difference equation for a LTI system is given,

$$y[n] - \frac{1}{2}y[n-1] = x[n] - x[n-1] + x[n-2]$$

a) Find the frequency response, $H(e^{j\Omega})$.

b) Plot the magnitude, $|H(e^{j\Omega})|$ and phase response, $\angle H(e^{j\Omega})$, for $H(e^{j\Omega})$ in MATLAB using `freqz` command.

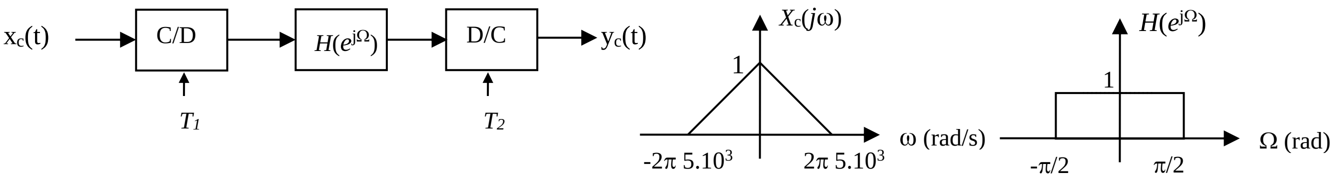
c) Let $x[n] = \cos\left(\frac{\pi}{3}n\right) + \sin\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$ be the system input. Find the output $y[n]$.

d) Prove that $H(e^{j\Omega}) = H^*(e^{j(2\pi - \Omega)})$.

Problem 3:

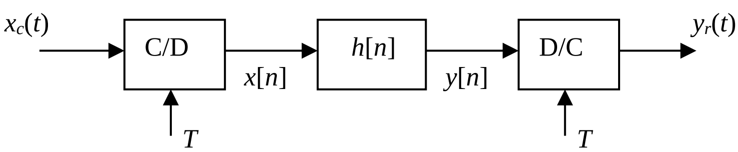
For the given system, $X_c(j\omega)$ and $H(e^{j\Omega})$, sketch and label the Fourier Transform of $y_c(t)$ for the following cases

a) $1/T_1 = 1/T_2 = 10^4$ **b)** $1/T_1 = 1/T_2 = 2.10^4$ **c)** $1/T_1 = 2.10^4$, $1/T_2 = 10^4$ **d)** $1/T_1 = 10^4$, $1/T_2 = 2.10^4$



Problem 4:

Assume the following system is provided:



$$Y_c(j\omega) = \begin{cases} |\omega| X_c(j\omega) & 1000\pi < |\omega| < 2000\pi \\ 0 & otherwise \end{cases}$$

Determine the maximum value for T and appropriate $H(e^{j\Omega})$ of the DT system, such that $Y_r(j\omega) = Y_c(j\omega)$.