

Homework 1

1. (a) Consider a discrete-time LTI system whose impulse response is given by:

$$h[n] = \delta[n] + \delta[n - 5]$$

- Describe in words the effect of this system on the input signal.
- Write down the input-output relationship of this system (i.e., the relation between the output $y[n]$ and the input $x[n]$).
- Find the system output $y[n]$ for the input

$$x[n] = \sin\left(\frac{4\pi n}{5}\right) (n^3 - \sqrt{|n|}).$$

- (b) Now consider a discrete-time LTI system whose impulse response is given by a pulse of duration L as follows:

$$h[n] = u[n] - u[n - L]$$

You can consider the pulse duration $L \geq 2$ to be a variable number. The system is given the following input:

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

- Find the system output $y[n]$ for an arbitrary pulse duration $L \geq 2$.
 - Sketch the input $x[n]$ and the output $y[n]$ for $L = 3$.
2. (a) Suppose we have an unknown black-box system, which is known to be a continuous-time LTI system. We would like to completely identify the behavior of this system, i.e., we would like to gain full knowledge of what the system output would be for any input. For this purpose, we do an experiment where we stimulate the system by giving certain input signals $x(t)$ and observing the output $y(t)$.
- The system is given the input $x(t) = u(t)$ and the output is observed to be $y(t) = u(t - 5)$. Discuss whether this experiment is sufficient to fully identify the system. If yes, find the input-output relationship of the system (i.e., the general relationship between an input $x(t)$ and its corresponding output $y(t)$.)
 - Propose an/another input signal $x(t)$ so that the system behavior can be fully identified by observing the corresponding system output.
 - Discuss whether the following statement is correct: “Any arbitrary nonzero input signal $x(t)$ would be suitable for fully identifying the system behavior, by observing the corresponding output $y(t)$ ”. If you agree, prove the statement. If you disagree, give a counterexample.
- (b) Now consider a continuous-time LTI system whose impulse response is given as

$$h(t) = e^{-t}[u(t) - u(t - 1)].$$

The following input is applied to this system:

$$x(t) = e^{-t}[u(t - 1) - u(t - 3)]$$

- Plot the signals $x(t)$ and $h(t)$.
- Find and plot the system output $y(t)$.

- (c) Now consider that the following input is applied to the system in part (b):

$$v(t) = e^{-(t+1)}[u(t) - u(t-2)].$$

Plot the system output. (**Hint:** Do not make any calculations. Think about how the input signal $v(t)$ is related to the previous input signal $x(t)$, and recall how LTI systems behave when the input is modified in this way.)

3. Consider the following LTI systems with input/output relationships given below:

System 1:

$$y_1[n] = \sum_{k=-\infty}^{n+2} x_1[k]$$

System 2:

$$y_2[n] = \sum_{k=-\infty}^{n-2} x_2[k]$$

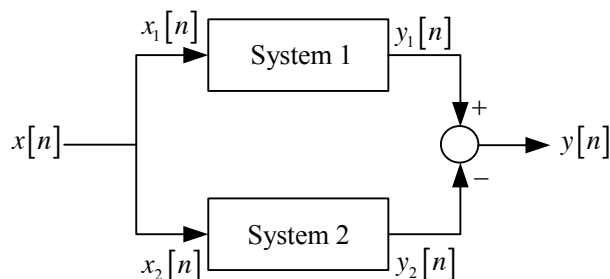
- (a) Find the impulse responses of these systems.

- (b) Are these systems

- i. Memoryless?
- ii. Causal?
- iii. Stable?

Justify your answers.

- (c) These two systems are connected in parallel to form the system given below:



- i. Find the impulse response of this system.

- ii. Is this system

- a) Memoryless?
- b) Causal?
- c) Stable?

Justify your answers.

4. In this question, we will use MATLAB to study the discrete-time convolution operation.

- (a) Remember the system with impulse response $h[n] = u[n] - u[n-L]$ and the input signal $x[n] = (1/2)^n u[n]$ studied in Question 1(b). In Question 1(b), you found the output signal $y[n]$ by theoretically calculating the convolution of $x[n]$ and $h[n]$. Let's first verify this result by convolving these two signals in MATLAB.

- i. Let's first generate the signal $x[n]$. Remember that we can generate the signal $x[n]$ only in a finite interval $0 \leq n \leq N-1$ in MATLAB. Set the signal length N to a suitable value (e.g. around 10-20) and plot $x[n]$ as follows.

```

n=0:1:N-1;
x=(1/2).^n;
figure;
stem(n,x);
xlabel('n');
ylabel('x[n]');

```

- ii. We can now similarly generate and plot the impulse response $h[n]$, which is a pulse of duration L and amplitude 1. Set the pulse duration L to a suitable value (typically smaller than the input signal length N). Plot $h[n]$ as follows.

```

h=ones(1,L); % Pulse of length L
figure;
stem(0:L-1, h);
xlabel('n');
ylabel('h[n]');

```

- iii. Let's now implement the convolution operation. First obtain the time-reversed impulse response $h[-k]$ as follows:

```

h_reversed=flip(h); % Take time reverse of h

```

h_reversed is in fact the same as **h** in this example, since the shape of our pulse is flat.

Remembering the convolution sum

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

our next job is to slide the shifted and time-reversed impulse response $h[n-k]$ through $x[k]$ and take their scalar product to obtain $y[n]$ for each n value. However, since we generated $x[k]$ only in the finite interval $0 \leq k \leq N-1$ in our code, in order to make the computation of $y[n]$ possible for the first few and the last few n values we are dealing with, we need to add 0's to the beginning and to the end of **x**. This is called zero-padding. Zero-pad the input signal as follows:

```

x_padded = [zeros(1,L-1) x zeros(1,L-1)]; % Apply zero-padding to input signal

```

We can now execute the convolution sum. Observe that the length of the output signal should be $N + L - 1$ when the convolved signals have lengths N and L .

```

y = zeros(1,N+L-1); % Allocate memory for y
for i=1:N+L-1
    y(i) = x_padded(i:i+L-1) * h_reversed'; % Take scalar product of x and time-reversed h
end

```

In the above lines, we compute $y[n]$ in the interval $0 \leq n < N+L-1$ by “sliding” **h_reversed** through **x_padded**, where the variable **i** adjusts how far we slide.

Now, observe that, when zero-padding the input signal, the zeros added before **x** do not lead to a mistake in the computation of $y[n]$ for the first few n values, since the input signal $x[n] = (1/2)^n u[n]$ is actually equal to 0 for $n < 0$. However, the zeros added at the end of **x** will make the computed output **y** deviate from the true output $y[n]$ for the last few n values, since the signal $x[n]$ is never exactly equal to 0 for $n > 0$. Noticing that our computation of $y[n]$ is valid only in the interval $0 \leq n \leq N-1$, let's extract only this part of the signal and plot it.

```

y=y(1:N);
figure;
stem(n,y);
xlabel('n');
ylabel('y[n]');

```

- iv. Compare the signal $y[n]$ to the theoretical result you obtained in Question 1(b) and comment.
- v. In fact, MATLAB also has a built-in function `conv` to perform convolution. Use this function to convolve the input signal and the impulse response as follows:

```

y=conv(x,h)
figure;
stem(0:N+L-2, y);
xlabel('n');
ylabel('y[n]');

```

Compare this result to the one you found in part (iii) and comment.

- (b) We will now study the effect of the pulse duration L on the output signal. In order to see this more clearly, let's test our system on a different input signal. Generate and plot a “random” input signal $x[n]$ of length $N = 40$:

```

N=40; %Signal length
n=0:1:N-1;
x=randn(1,N); % You must remember the meaning of this command your EE230 class ;)
figure;
stem(n,x);
xlabel('n');
ylabel('x[n]');

```

Pass this input signal through our LTI system by convolving it with the impulse response $h[n] = u[n] - u[n - L]$ (You can simply use the built-in MATLAB function `conv` for this). Plot the output signal $y[n]$.

Then repeat the same by changing the pulse duration L . Try especially small and large values (such as $L = 2$ and $L = 15$) and comment on the effect of the pulse duration on the output signal. Such systems with an impulse response consisting of a pulse of constant amplitude are also called “averaging filters”. Think about how the length L of the averaging filter influences the characteristics of the output signal. Does the output signal $y[n]$ look smoother for smaller or larger L values?