EE301 - HW3

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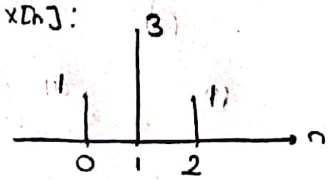
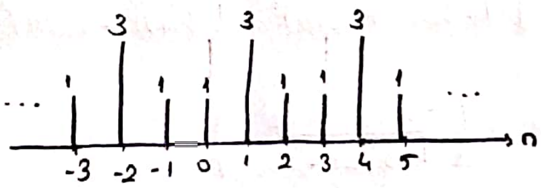
① · for $x[n] = \delta[n] + 3\delta[n-1] + \delta[n-2]$

a) $\tilde{x}_N[n] = \sum_{k=-\infty}^{\infty} x[n-kN]$ for $N=3$:





b) · DTFS coefficients of $\tilde{x}_3[n]$; $\tilde{X}_3[k]$

$\Rightarrow \tilde{x}_3[n] \longleftrightarrow \tilde{X}_3[k]$ such that $\tilde{X}_3[k] = \frac{1}{N} \cdot \sum_{n=\langle N\rangle} \tilde{x}_3[n] e^{-jk\frac{2\pi}{N}n}$ where $N=3$.

$$\Rightarrow \tilde{X}_3[k] = \frac{1}{3} \sum_{n=0}^{2} \tilde{x}_3[n] e^{-jk\frac{2\pi}{3}n}$$

$$= \frac{1}{3}\left(1 + 3e^{-jk\frac{2\pi}{3}} + e^{-jk\frac{4\pi}{3}}\right) = \frac{1}{3} + e^{-jk\frac{2\pi}{3}} + e^{-jk\frac{4\pi}{3}}$$

↳ periodic w/ 3.

$$\Rightarrow \tilde{x}_3[n] = \sum_{k=\langle N\rangle} \tilde{X}_3[k] e^{jk\frac{2\pi}{N}n} \quad \text{for } N=3.$$

c) · DTFS coefficients of $\tilde{x}_5[n]$: $\tilde{X}_5[k]$

$\Rightarrow \tilde{x}_5[n] \longleftrightarrow \tilde{X}_5[k]$ so that $\tilde{X}_5[k] = \frac{1}{N} \sum_{n=\langle N\rangle} \tilde{x}_5[n] e^{-jk\frac{2\pi}{N}n}$ where $N=5$.

$$\Rightarrow \tilde{X}_5[k] = \frac{1}{5} \sum_{n=0}^{4} \tilde{x}_5[n] e^{-jk\frac{2\pi}{5}n}$$

$$= \frac{1}{5}\left(1 + 3e^{-jk\frac{2\pi}{5}} + e^{-jk\frac{4\pi}{5}}\right) = \frac{1}{5} + \frac{3}{5}e^{-jk\frac{2\pi}{5}} + \frac{1}{5}e^{-jk\frac{4\pi}{5}}$$

↳ periodic w/ 5.

$$\Rightarrow \tilde{x}_5[n] = \sum_{k=\langle N\rangle} \tilde{X}_5[k] e^{jk\frac{2\pi}{N}n} \quad \text{for } N=5.$$

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d) · DTFT of $x[n]$: $X(e^{j\Omega})$

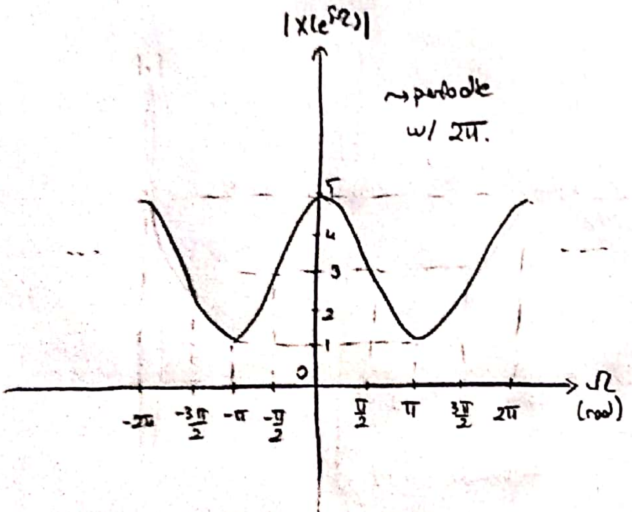
$\Rightarrow x[n] \overset{F}{\longleftrightarrow} X(e^{j\Omega})$

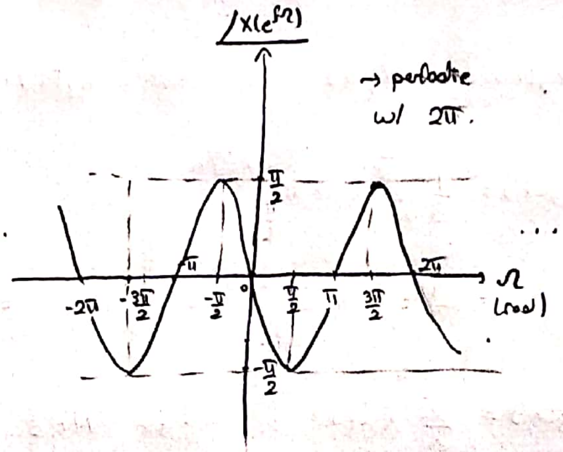
$\Rightarrow x[n] = \delta[n] + 3\delta[n-1] + \delta[n-2] \longleftrightarrow X(e^{j\Omega}) = 1 + 3e^{-j\Omega} + e^{-j2\Omega} \ldots$

$\ldots = 1 + 3\cos\Omega - j3\sin\Omega + \cos 2\Omega - j\sin 2\Omega$

$\Rightarrow |X(e^{j\Omega})| = \sqrt{(1+3\cos\Omega+\cos 2\Omega)^2 + (3\sin\Omega + \sin 2\Omega)^2}$

$\angle X(e^{j\Omega}) = \arctan\left(\dfrac{-3\sin\Omega - \sin 2\Omega}{1+3\cos\Omega + \cos 2\Omega}\right)$





e) · $\tilde{X}_3[k] \overset{?}{=} \frac{1}{3} X(e^{j\Omega})\Big|_{\Omega = k\frac{2\pi}{3}}$

$\Rightarrow \frac{1}{3} X(e^{j\Omega})\Big|_{\Omega = k\frac{2\pi}{3}} = \frac{1}{3} + e^{-jk\frac{2\pi}{3}} + \frac{1}{3} e^{-jk\frac{4\pi}{3}} = \tilde{X}_3[k]$ from parts b and d.

· $\tilde{X}_5[k] \overset{?}{=} \frac{1}{5} X(e^{j\Omega})\Big|_{\Omega = k\frac{2\pi}{5}}$

$\Rightarrow \frac{1}{5} X(e^{j\Omega})\Big|_{\Omega = k\frac{2\pi}{5}} = \frac{1}{5} + \frac{3}{5} e^{-jk\frac{2\pi}{5}} + \frac{1}{5} e^{-jk\frac{4\pi}{5}} = \tilde{X}_5[k]$ from parts b and d.

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② · For an LTI system with

$$y[n] - \frac{1}{2} y[n-1] = x[n] - x[n-1] + x[n-2]$$

a) Since the system is LTI, we have $y[n] = x[n] * h[n]$.

$$\Rightarrow \underbrace{Y(e^{j\Omega}) = X(e^{j\Omega}) H(e^{j\Omega})}_{\text{conv. property}} \quad \Rightarrow H(e^{j\Omega}) = Y(e^{j\Omega}) / X(e^{j\Omega})$$

$$\Rightarrow Y(e^{j\Omega})\left(1 - \frac{1}{2}e^{-j\Omega}\right) = X(e^{j\Omega})\left(1 - e^{-j\Omega} + e^{-j2\Omega}\right)$$

$$\Rightarrow H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \frac{1 - e^{-j\Omega} + e^{-j2\Omega}}{1 - \frac{1}{2}e^{-j\Omega}}$$

b) -At the end!-

c) If $x[n] = \cos\left(\frac{\pi}{3}n\right) + \sin\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$ is the input to the system, we have:

$$\cdot\ x[n] = \frac{e^{j\frac{\pi}{3}n} + e^{-j\frac{\pi}{3}n}}{2} + \frac{1}{j2}\left(e^{j\frac{\pi}{4}} \cdot e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{4}} e^{-j\frac{\pi}{2}n}\right)$$

⊛ As complex exponentials are eigenfunctions of the system:

$$⊛\ \underbrace{z^n}_{x[n]} \longrightarrow \underbrace{H(z)\,z^n}_{y[n]} \quad \text{where } z = e^{j\Omega}, \ H(e^{j\Omega}) \text{ is from part a.}$$

$$\Rightarrow y[n] = \frac{1}{2}\left(H(e^{j\frac{\pi}{3}})e^{j\frac{\pi}{3}n} + H(e^{-j\frac{\pi}{3}})e^{-j\frac{\pi}{3}n}\right) + \frac{1}{j2}\left(e^{j\frac{\pi}{4}} H(e^{j\frac{\pi}{2}}) e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{4}} H(e^{-j\frac{\pi}{2}}) e^{-j\frac{\pi}{2}n}\right)$$

$$\Rightarrow y[n] = \frac{1}{2}\left(\frac{e^{j\frac{\pi}{3}n} - e^{j\frac{\pi}{3}(n-1)} + e^{j\frac{\pi}{3}(n-2)}}{1 - \frac{1}{2}e^{-j\frac{\pi}{3}}}\right) + \frac{1}{2}\left(\frac{e^{-j\frac{\pi}{3}n} - e^{-j\frac{\pi}{3}(n-1)} + e^{-j\frac{\pi}{3}(n-2)}}{1 - \frac{1}{2}e^{j\frac{\pi}{3}}}\right) \ldots$$

$$\ldots + \frac{1}{j2}\left(e^{j\frac{\pi}{4}}\left(\frac{e^{j\frac{\pi}{2}n} - e^{j\frac{\pi}{2}(n-1)} + e^{j\frac{\pi}{2}(n-2)}}{1 - \frac{1}{2}e^{-j\frac{\pi}{2}}}\right)\right) - \frac{1}{j2}\left(e^{-j\frac{\pi}{4}}\left(\frac{e^{-j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}(n-1)} + e^{-j\frac{\pi}{2}(n-2)}}{1 - \frac{1}{2}e^{j\frac{\pi}{2}}}\right)\right)$$

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d) . Prove $H(e^{j\Omega}) = H^*(e^{j(2\pi-\Omega)})$

→ We know that DTFT must be periodic w/ 2π. Thus,

$$H^*(e^{j(2\pi-\Omega)}) = H^*(e^{-j\Omega}).$$

conjugate property

$\Rightarrow$ If $H(e^{j\Omega}) = H^*(e^{-j\Omega}) \longleftrightarrow h[n] = h^*[n]$ so that $h[n]$ is a real signal!

$\Rightarrow$ We have: $$H(e^{j\Omega}) = \frac{1}{1-\frac{1}{2}e^{-j\Omega}} - \frac{e^{-j\Omega}}{1-\frac{1}{2}e^{-j\Omega}} + \frac{e^{-j2\Omega}}{1-\frac{1}{2}e^{-j\Omega}}$$

IFT:

$$\Rightarrow h[n] = \left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{2}\right)^{n-1} u[n-1] + \left(\frac{1}{2}\right)^{n-2} u[n-2]$$

↳ This signal is real ; thus, $h[n] = h^*[n]$. ▨

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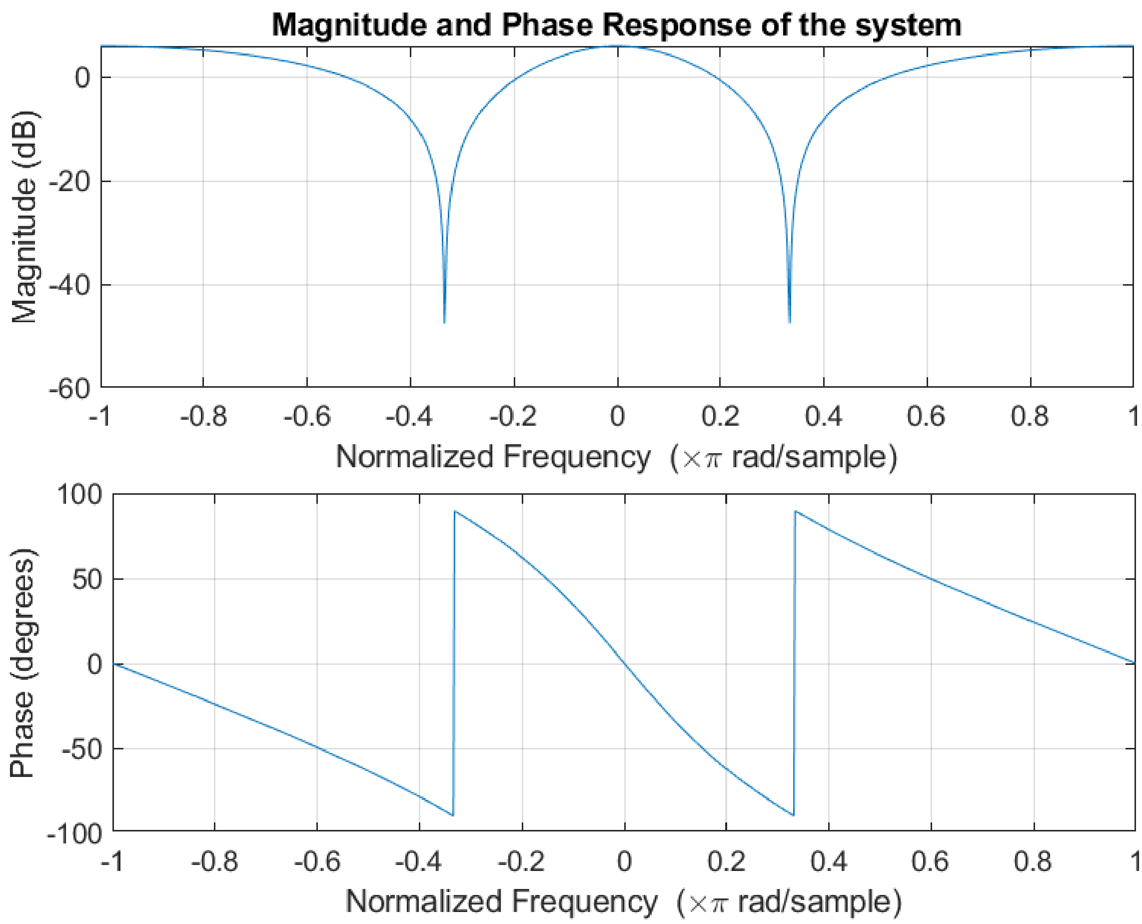
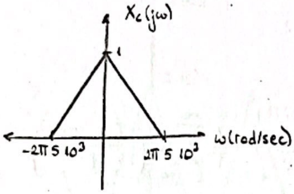
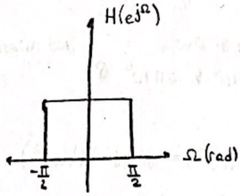
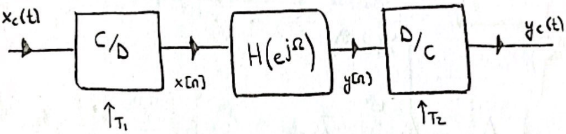


Figure 1. Magnitude and Phase Response plots of the system.

Question 3







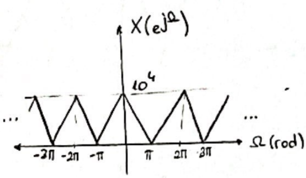
(a) $\frac{1}{T_1} = \frac{1}{T_2} = 10^4$ $\qquad \omega_m = 2\pi\, 5\, 10^3 = \pi\, 10^4$ rad/sec $\qquad$ if $\omega_s > 2\omega_m$, there is no aliasing.

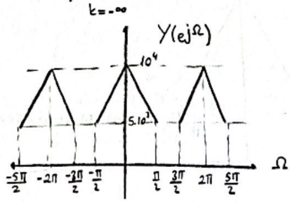
$$\omega_s = \frac{2\pi}{T_1} = 2\pi \times 10^4 \qquad \left(\begin{matrix} 2\pi \times 10^4 \geq 2\pi \times 10^4 \\ \omega_s \geq 2\omega_m \end{matrix}\right) \Rightarrow \text{No aliasing}$$

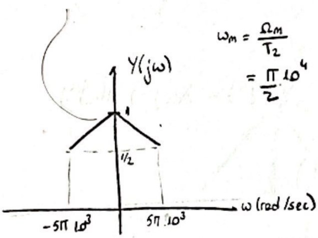
$$\Omega_m = \omega_m T_1 = \pi\, 10^4 \frac{1}{10^4} = \pi$$

$$Y(e^{j\Omega}) = X(e^{j\Omega})\, H(e^{j\Omega})$$

$$X(e^{j\Omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} x_c\left(j\frac{(\Omega - 2\pi k)}{T}\right)$$







$$\omega_m = \frac{\Omega_m}{T_2} = \frac{\pi}{2} 10^4$$

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(b) $\frac{1}{T_1} = \frac{1}{T_2} = 2.10^4$ $\qquad \omega_m = \pi\, 10^4$ rad/sec

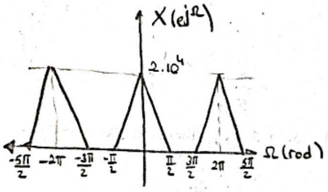
$\omega_s = \frac{2\pi}{T_1} = 4\pi\, 10^4$ $\qquad \omega_s \geq 2\omega_m$ $\qquad$ No Aliasing

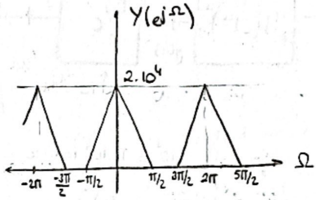
$4\pi\, 10^4 \geq 2\pi 10^4$ ✓

$\Omega_m = \omega_m T_1$

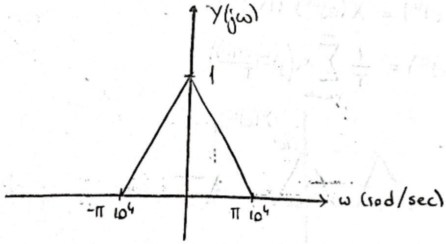
$= \pi 10^4 \frac{1}{2\,10^4} = \frac{\pi}{2}$

$Y(e^{j\Omega}) = X(e^{j\Omega}) H(e^{j\Omega}) \qquad X(e^{j\Omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} x_c\left(j\, \frac{(\Omega - 2\pi k)}{T}\right)$





$\omega_m = \frac{\Omega_m}{T_2} = \frac{\pi}{2}\, 2.10^4 = \pi . 10^4$ (rad/sec)



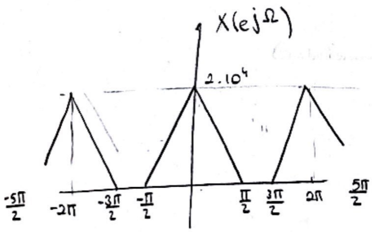
(c) $\frac{1}{T_1} = 2.10^4 \quad \frac{1}{T_2} = 10^4$ $\qquad \omega_m = \pi\, 10^4$ (rad/sec)

$\omega_s = \frac{2\pi}{T_1} = 4\pi\, 10^4$ $\qquad \omega_s \geq 2\omega_m$ $\qquad$ No Aliasing $\qquad Y(e^{j\Omega}) = X(e^{j\Omega}) H(e^{j\Omega})$

$4\pi\, 10^4 \geq 2\pi\, 10^4$ ✓

$\Omega_m = \omega_m T_1$

$= \pi\, 10^4 \frac{1}{2.10^4} = \frac{\pi}{2}$



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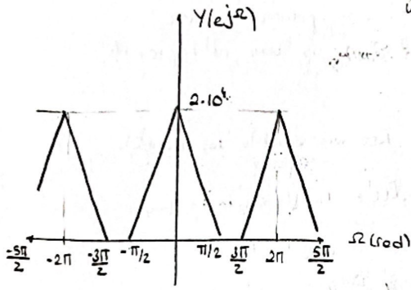
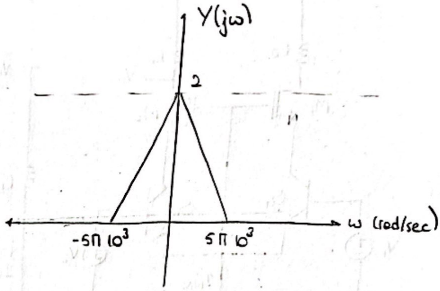


$\omega_m = \frac{\Omega_m}{T_2} = \frac{\pi}{2}\, 10^4$ (rod/sec)



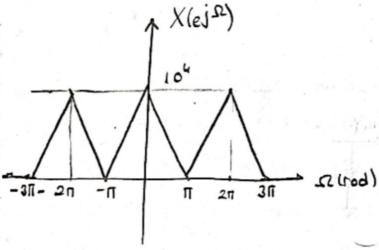
(d) $\frac{1}{T_1} = 10^4$ $\quad \frac{1}{T_2} = 2 \cdot 10^4$ $\quad \omega_m = \pi\, 10^4$ (rod/sec)

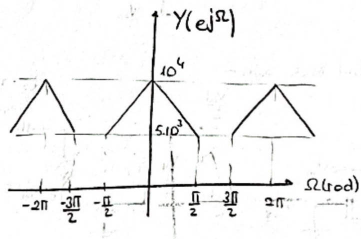
$\omega_s = \frac{2\pi}{T_1} = 2\pi\, 10^4$ $\quad \omega_s \geq 2\omega_m$ $\quad$ (No Aliasing)

$2\pi\, 10^4 \geq 2\pi\, 10^4$

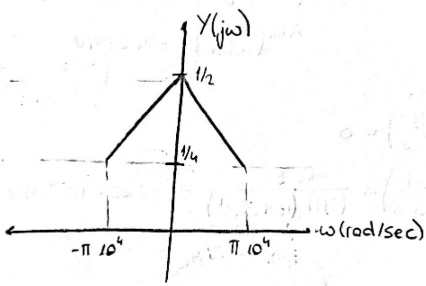
$\Omega_m = \omega_m T_1 = \pi$

$Y(e^{j\Omega}) = X(e^{j\Omega})\, H(e^{j\Omega})$





$\omega_m = \frac{\Omega_m}{T_2} = \frac{2\pi}{2}\, 10^4$ (rod/sec)



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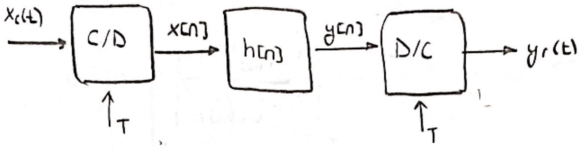
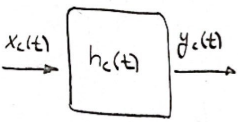
Problem 4

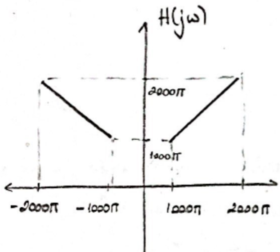


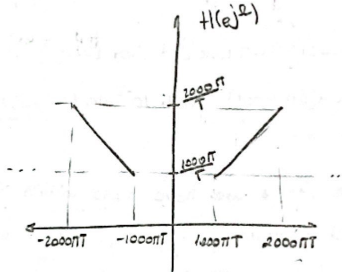
$$Y_c(j\omega) = \begin{cases} |\omega|\, X_c(j\omega) & 1000\pi < |\omega| < 2000\pi \\ 0 & \text{otherwise} \end{cases}$$



$$Y_c(j\omega) = |\omega|\, X_c(j\omega) = H(j\omega)\, X_c(j\omega)$$

$$H(j\omega) = |\omega| \;, \text{ if } \; 1000\pi < |\omega| < 2000\pi \,.$$

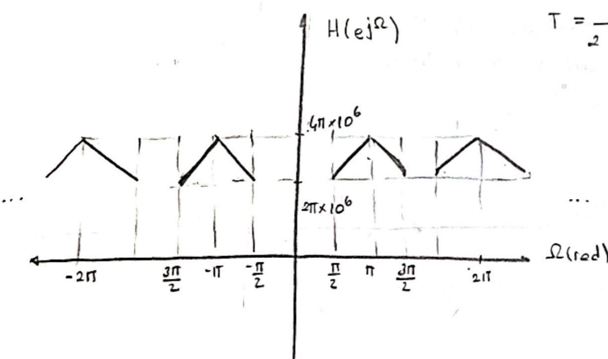




$$\Omega_m = \omega_m T$$

$$2000\pi T = \pi$$

$$T = \frac{1}{2\, 10^3}$$



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