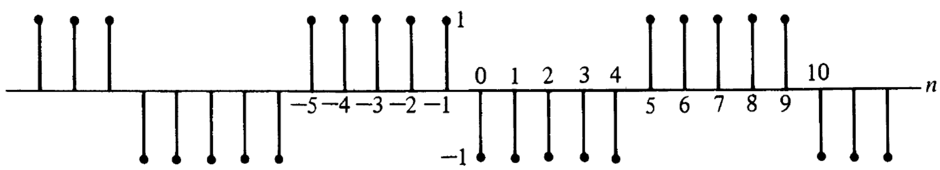
Middle East Technical University
Department of Electrical and Electronics Engineering
EE301: Signals and Systems

Homework 2

1. Consider a real periodic signal $x[n]$ shown in the figure given below. Let N denote the fundamental period of $x[n]$. Using the properties of the Fourier Series and without explicitly evaluating the Fourier Series coefficients a_k, determine whether the following are true or false. Please clearly explain your reasoning.



   (a) $a_k = a_{k+10}$ for all k.
   (b) $a_k = a_{-k}$.
   (c) $a_k e^{jk\frac{2\pi}{5}}$ is real.
   (d) a_0 is zero.
   (e) $x[n] - x[n - \frac{N}{2}]$ has Fourier series coefficients $b_k = \begin{cases} 2a_k & \text{k is even,} \\ 0 & \text{k is odd.} \end{cases}$

2. Please answer the following questions related to continuous time Fourier Transform and its properties.

   (a) Show that the Fourier Transform of $x(t)$ given below
   $$x(t) = \begin{cases} cos(\frac{\pi t}{2}) & \text{if } |t| < 1 \\ 0 & \text{otherwise.} \end{cases}$$
   is equal to $X(j\omega) = \frac{4\pi cos(\omega)}{\pi^2 - 4\omega^2}$.
   (b) Determine the values for $C_1 = \int_{-\infty}^{\infty} X(\omega) d\omega$ and $C_2 = \int_{-\infty}^{\infty} x(t) dt$.
   (c) Determine and plot the Fourier Transform $Y(j\omega)$ of $y(t) = \frac{4\pi cos(t)}{\pi^2 - 4t^2}$. (Hint: Use duality)
   (d) Determine and plot the Fourier Transform $Z(jw)$ of $z(t)$ given below.
   $$z(t) = \left(\frac{sin(2t) sin(8t)}{2\pi t^2} \right) e^{j10t}$$
   (e) Find the energy of $z(t)$, given by $E_z = \int_{-\infty}^{+\infty} |z(t)|^2 dt$?

3. An In-phase and quadrature *(I-Q) modulated* continuous time signal $x(t)$ is given as
   $$x(t) = m_1(t) cos(5t) + m_2(t) sin(5t),$$
   where $m_1(t)$ and $m_2(t)$ are the message signals whose Fourier Transforms are as follows
   $$M_1(j\omega) = \begin{cases} 1 + \omega & -1 \le \omega < 0 \\ 1 - \omega & 0 \le \omega < 1 \\ 0 & \text{elsewhere,} \end{cases}$$
   and
   $$M_2(j\omega) = \begin{cases} 0.5 & -1 \le \omega < 1 \\ 0 & \text{elsewhere,} \end{cases}$$

(a) Is x(t) a real signal? Justify.

(b) Find the Fourier Transform of x(t), namely $X(j\omega)$. Plot $Re\{X(j\omega)\}$ and $Im\{X(j\omega)\}$ separately.

(c) The *receiver*, namely the I-Q demodulator, can be represented as the following input/output relation:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)cos(5\tau)h(t-\tau)d\tau$$

where $h(t)$ is the impulse response of a low-pass filter with $H(j\omega) = \begin{cases} 2 & -5 \leq \omega \leq 5 \\ 0 & \text{elsewhere.} \end{cases}$

i. State whether the *receiver* is a linear time-invariant (LTI) system or not. Justify.

ii. When *I-Q modulated* signal $x(t)$ is given as input to the *receiver*, find and plot the Fourier Transform of the output $y(t)$, namely $Y(j\omega)$.

4. Let $x(t)$ be a periodic CT signal with fundamental period $T = 4$. Its Fourier Series (FS) coefficient is denoted by a_k. Over one period, this signal is expressed as

$$x(t) = \begin{cases} t-1 & \text{if } 0 \leq t < 2 \\ 3-t & \text{if } 2 \leq t < 4. \end{cases}$$

First, write your Matlab scripts

(a) To generate and sketch periodic *triangular* x(t) covering at least 5 periods. (Note: While plotting the curves, you can take 100 samples over one period.)

(b) Reconstruct an approximated time function using a given number of its Fourier Series coefficients (for instance, a 7-term reconstruction only uses the FS coefficients a_{-3}, $a_{-2}, \ldots$, a_0, $\ldots$, a_3 in the synthesis equation). You should provide the computed expressions in Matlab for the FS coefficients of these periodic signals as the input to your reconstruction script.

Part 1: Using the FS terms of the triangular pulse train $x(t)$, plot the reconstructed signal with 7, 9, 11, 21 and 51 FS terms on the same figure. Do you expect a significant improvement in the approximation if we further increase the number of terms in the synthesis equation? Why? (Comment on this while keeping Parseval's relation in mind.)

Part 2: Write a Matlab code that multiplies the Fourier Series coefficients of a waveform by appropriate numbers such that the new FS terms are the coefficients of the "delayed and differentiated waveform". Set delay=1, and plot the original and delayed and differentiated waveforms for the original triangular pulse train $x(t)$ above, with 11 and 21 and 51 terms of the synthesis equation. Do you observe convergence to the delayed and differentiated waveform at the edges after reconstruction? Zoom to the edges and notice the Gibbs effect. You can read about "Gibbs phenomenon" from `http://en.wikipedia.org/wiki/Gibbs_phenomenon`.

Do not forget to include the code that you have written.

5. Recommended problems from the textbook (for self study only, not to be collected): 4.9, 4.21, 4.29, 4.32-4.37, 4.43-4.45