EE-301

Fall-2021

HW#1

Solutions

1) (a) i) It gives the summation of the input and 5-sample delayed version of input as the output.

(ii) $y[n] = x[n] + x[n-5]$

(iii) $$y[n] = \sin\left(\frac{4\pi n}{5}\right)\left(n^3 - \sqrt{|n|}\right) + \underbrace{\sin\left(\frac{4\pi(n-5)}{5}\right)}_{\sin\left(\frac{4\pi n}{5}\right)}\left((n-5)^3 - \sqrt{|n-5|}\right)$$

$$= \sin\left(\frac{4\pi n}{5}\right)\left[2n^3 - 15n^2 + 75n - 125 - \sqrt{|n|} - \sqrt{|n-5|}\right]$$

(b) i. Since $x[n]=0$ for $n<0$, $y[n]=0$ for $n<0$.

For $n \geq L \Rightarrow y[n] = \sum_{k=0}^{n} x[n-k] - \sum_{k=0}^{n-L} x[n-L-k]$

$$= \sum_{k=0}^{n} x[n-k] - \sum_{k'=L}^{n} x[n-k']$$

$$y[n] = \sum_{k=0}^{L-1} x[n-k], \quad n \geq 0, \; n \geq L.$$

$$y[n] = \sum_{k=0}^{n} x[n-k], \quad n \geq 0, \; n < L.$$

Q1) b)

i. (Continued) For $n \geq 0$, $n \geq L$,

$$y[n] = \sum_{k=0}^{L-1} \left(\frac{1}{2}\right)^{n-k} \underbrace{u[n-k]}_{\substack{=1 \text{ since} \\ k<L\leq n}}$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{L-1} 2^k = \left(\frac{1}{2}\right)^n \frac{2^0 - 2^L}{1-2} = \boxed{\left(\frac{1}{2}\right)^n (2^L - 1), \quad n \geq 0, n \geq L} \quad 1$$

For $n \geq 0$, $n < L$,

$$y[n] = \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{-k} = \left(\frac{1}{2}\right)^n \frac{2^0 - 2^{n+1}}{1-2} = \left(\frac{1}{2}\right)^n (2^{n+1} - 1)$$

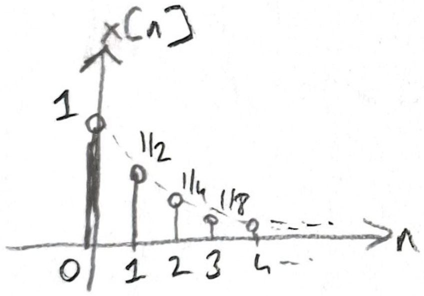
$$\boxed{= 2 - 2^{-n}, \quad n \geq 0, n < L} \quad 2$$

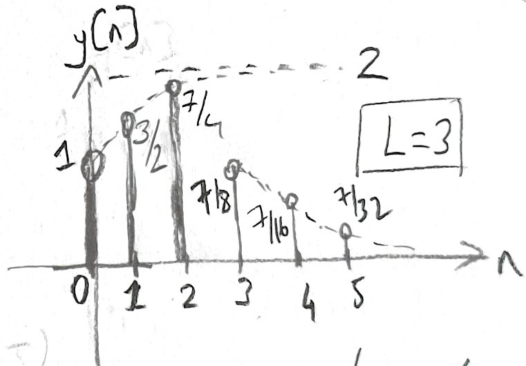
For $\boxed{n < 0, \quad y[n] = 0}$ 3

$\Rightarrow$ 1, 2, 3

$$y[n] = \begin{cases} \left(\frac{1}{2}\right)^n (2^L - 1) & , n \geq L \\ 2 - 2^{-n} & , 0 \leq n < L \\ 0 & , n < 0 \end{cases}$$

ii.)





2)(a) Since impulse response is the derivative of unit step response,

$$\underset{\substack{\uparrow \\ \text{impulse response}}}{h(t)} = \frac{d}{dt}(u(t-5)) = \delta(t-5)$$

, which fully characterizes the LTI system.

<u>Input-Output relation:</u>

$$y(t) = h(t) * x(t) = x(t-5)$$

ii.) Let $x(t) = \delta(t) \rightarrow y(t) = h(t)$

iii) Proof by counterexample, Let $h_1(t) = \delta(t)$, $h_2(t) = \delta(t-5)$

$x(t) = 1 \Rightarrow x(t) * h_1(t) = 1$

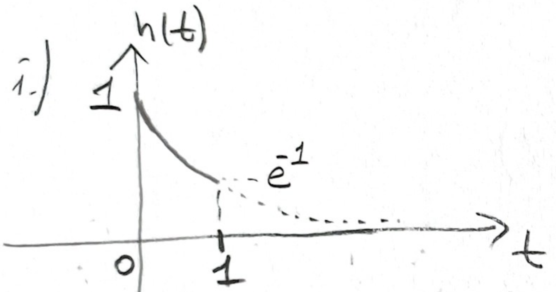
$x(t) * h_2(t) = 1$

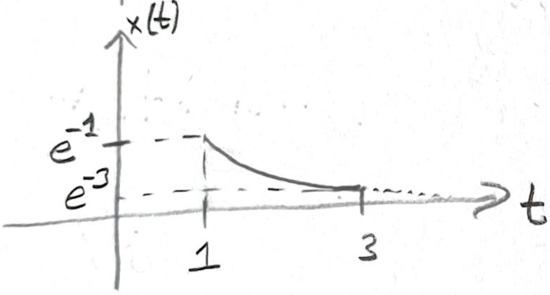
Same input applied to different systems leading to same outputs.

$\Downarrow$

$x(t)=1$ input cannot be used as a system identifier for any system

(b) i.)





ii) $y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau$

$= \int_{1}^{3} e^{-t} [u(t-\tau) - u(t-\tau-1)] d\tau \quad \dots \quad (1)$

If $t < 1$, $u(t-\tau) = u(t-\tau-1) = 0$ for $\boxed{1 \leq \tau \leq 3}$

If $1 \leq t \leq 2$, $u(t-\tau) = 0$ for $\tau \geq t$, $u(t-\tau) = 1$ for $\tau < t$

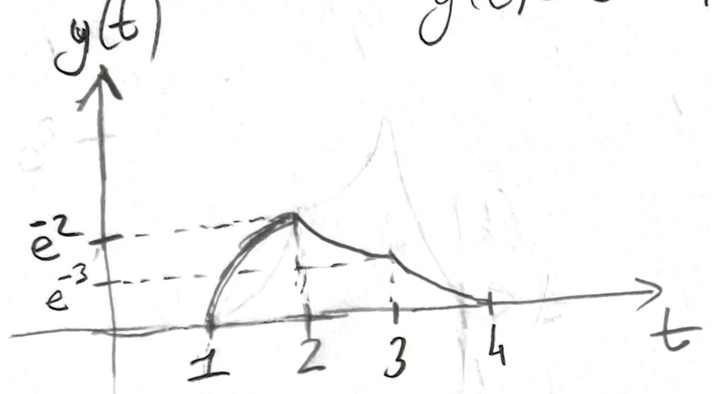
$u(t-\tau-1) = 0$ for $\tau \geq 1$ t

$\boxed{\Rightarrow y(t) = e^{-t} \int_{1} 1 \, d\tau = (t-1) e^{-t} \quad \text{for } 1 \leq t \leq 2}$

If $2<t\le 3$, $y(t)=\int_{t-1}^{t} e^{-t}\,d\tau = \boxed{e^{-t} \text{ for } 2<t\le 3}$

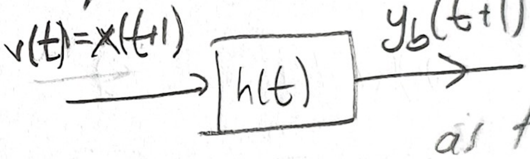
If $3<t\le 4$ $\quad y(t)=\int_{t-1}^{3} e^{-t}\,d\tau = \boxed{(4-t)e^{-t} \text{ for } 4\ge t>3}$

$y(t)=0$ if $t>4$.

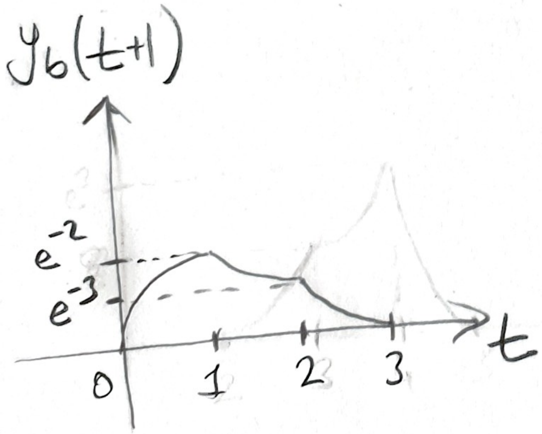


(c) Let $y_b(t)$ is the system output found in part b).

Since $v(t)=x(t+1)$,



as the system is time invariant.



3) a) $h_1[n] = u[n+2]$, $h_2[n] = u[n-2]$

b) i) Both of them not memoryless as outputs depend previous values of input.

ii) $y_1[n]$ depends on future input values → Not causal

$y_2[n]$ depends only on previous input values ↳ causal.

iii) Both of them not stable

Proof Consider $x_1[k] = x_2[k] = 1$ for $k < n-2$

$y[n] < \infty$ cannot be satisfied.

c) i) $h[n] = h_1[n] - h_2[n] = u[n+2] - u[n-2]$

ii) a) b) $h[-1] = 1 \neq 0 \Rightarrow$ Not memoryless, not causal

c) $\sum_{n=-\infty}^{+\infty} |h[n]| = 4 < \infty \Rightarrow$ Stable.

4) a) For the plots in part a), we set $N = 10$ and $L = 2$.

i)

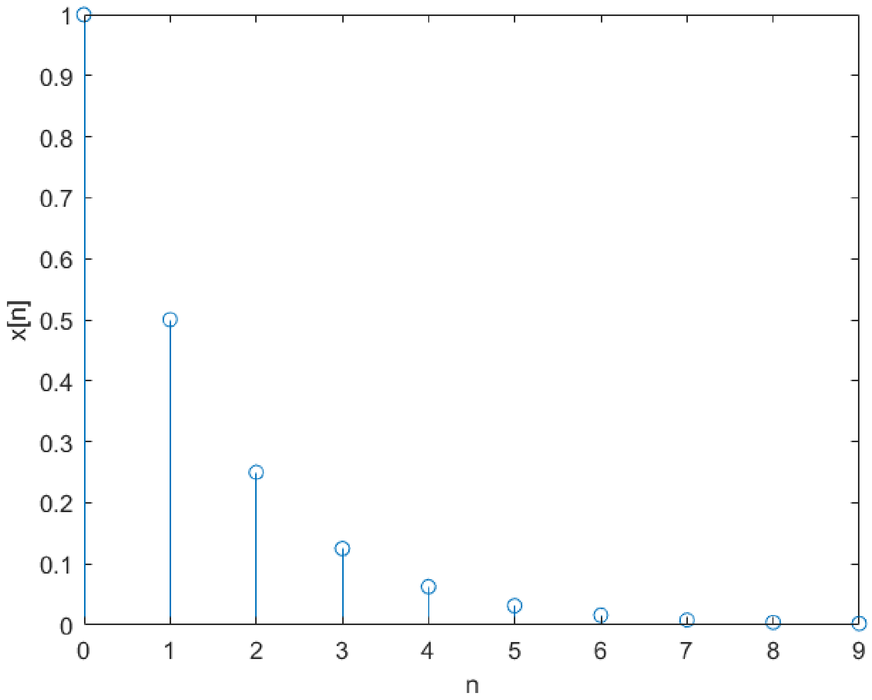


Figure 1

a) ii)

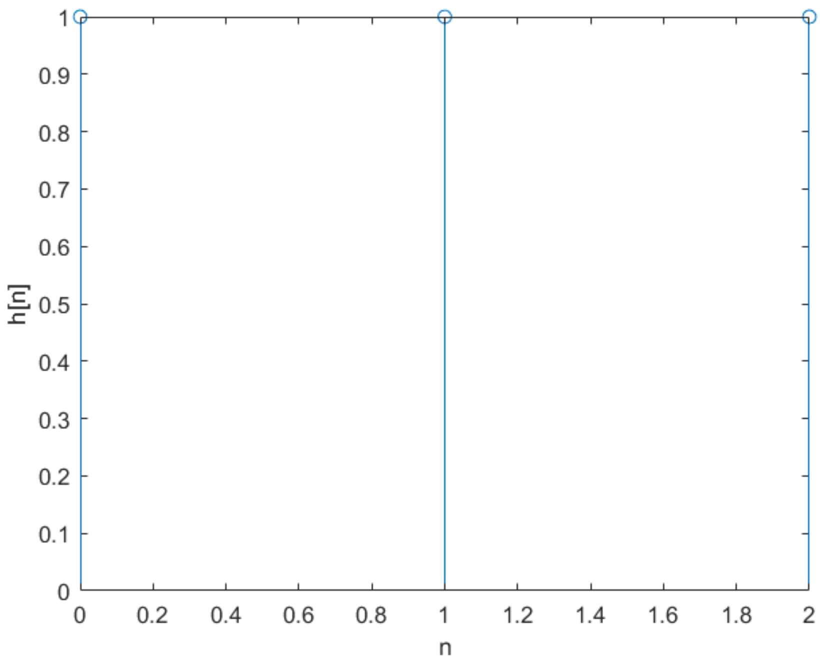


Figure 2

iii)

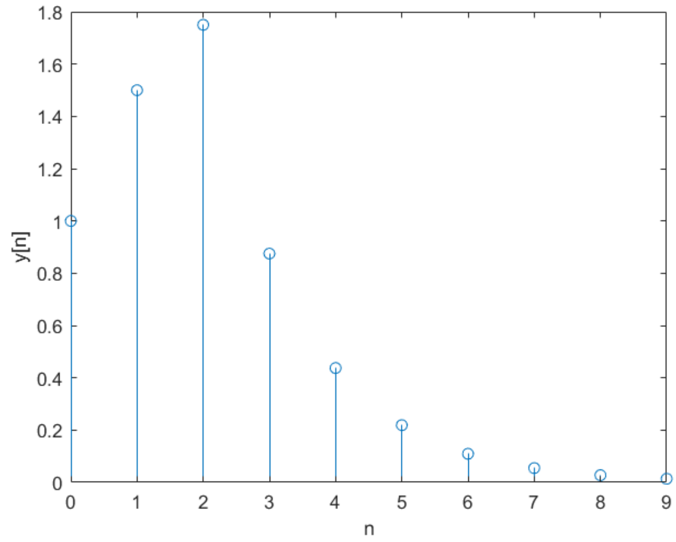


Figure 3

iv) When we plot y[n] and the theoretical result in Question 1(b) using the following code segment

```
n_1=0:L-1;
n_2=L:N-1;
y_real=[(2-2.^(-n_1)) 0.5.^(n_2)*(2^L-1)];
```

on the same plot, we obtain

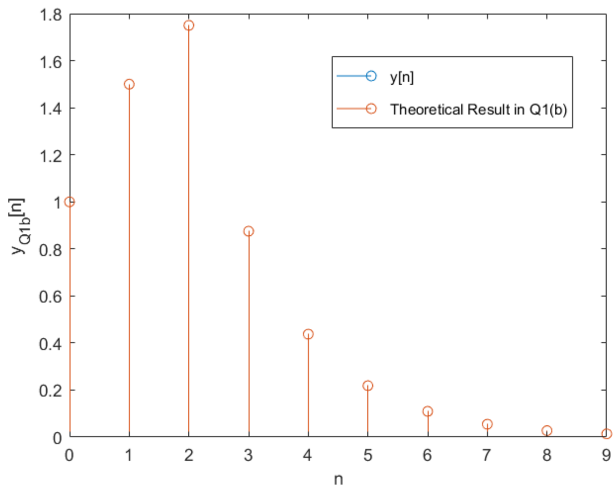


Figure 4

Since the theoretical result in Question 1(b) and y[n] obtained in iii) have the same values for n=0:1:N-1, when we plot them on the same plot, we see a single plot in Figure 4 as they cannot be differentiated. However, when we plot y[n] obtained in iii) and the theoretical result in Question 1(b) for n=0:1:N+L-1 on the same plot, we obtain the following figure:

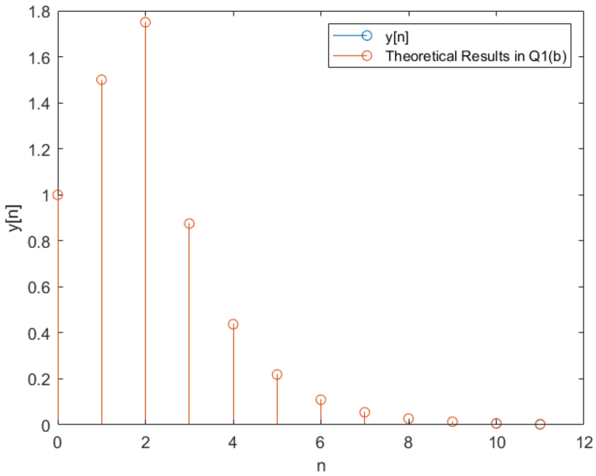


Figure 5

Although the two plots cannot be differentiated in Figure 5, when we zoom in the output values for n=N,N+1, we obtain the following plot:

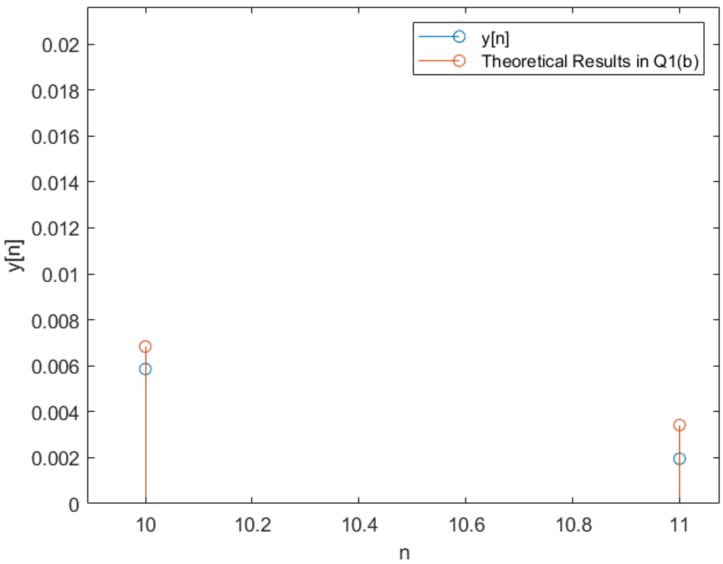


Figure 6

We see that the theoretical result deviate from the y[n] value since we equated x[n]=0 for $n \geq N$ with a zero-padding operation, which is not equal to the actual $x[n] = \left(\frac{1}{2}\right)^n u[n]$.

v)

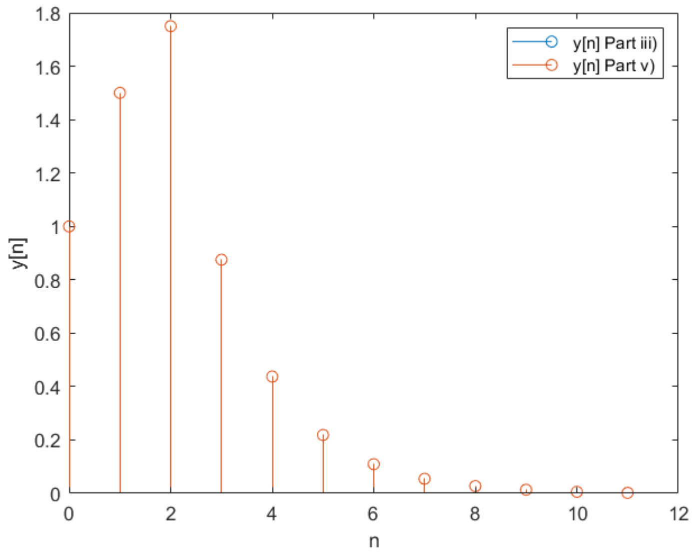


Figure 7

As can be noted in Figure 7, the results obtained in part iii) and v) are the same (two plots cannot be differentiated as they have the same value).

b) Random $x[n]$ for $N = 40$:

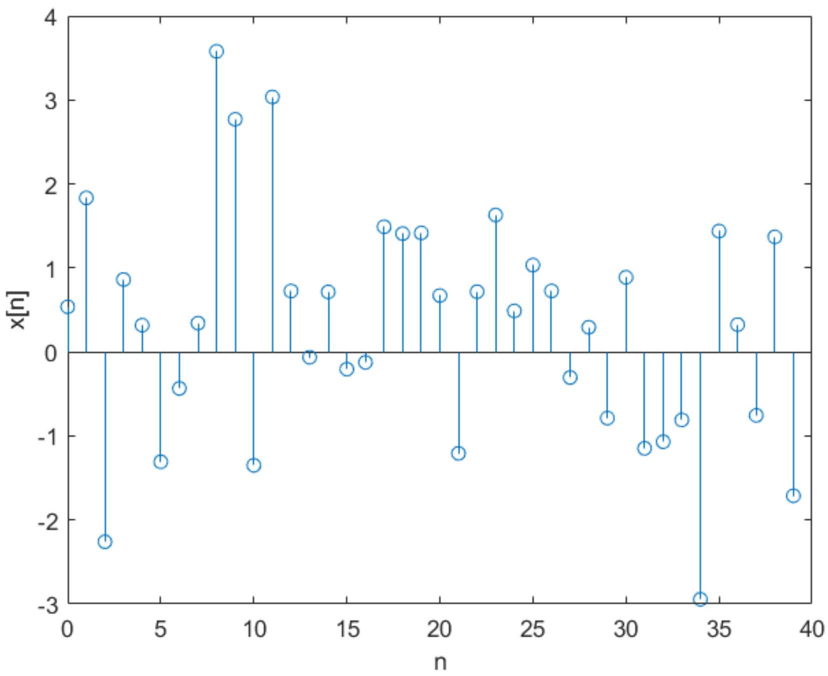


Figure 8

For $L = 2$, output $y[n]$:

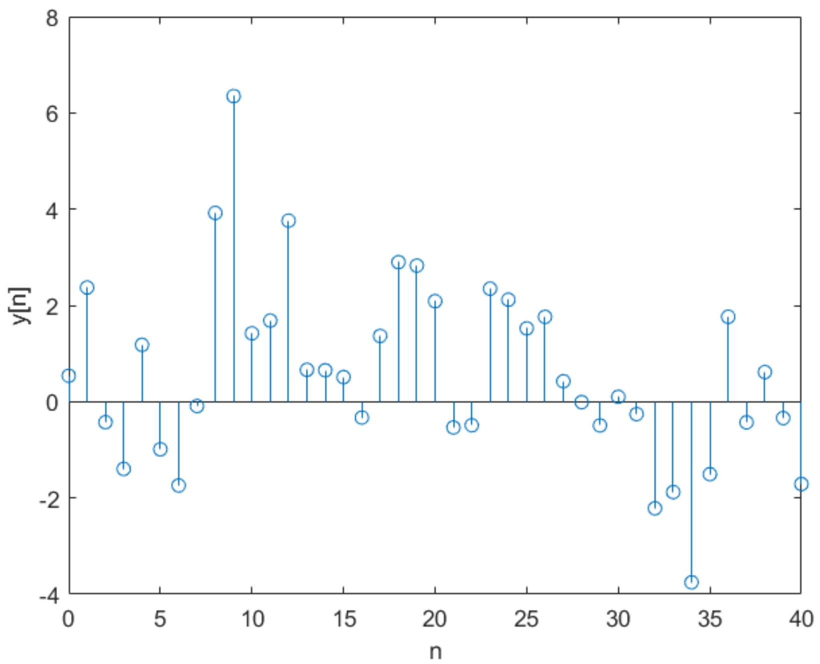


Figure 9

For $L = 15$, output $y[n]$:

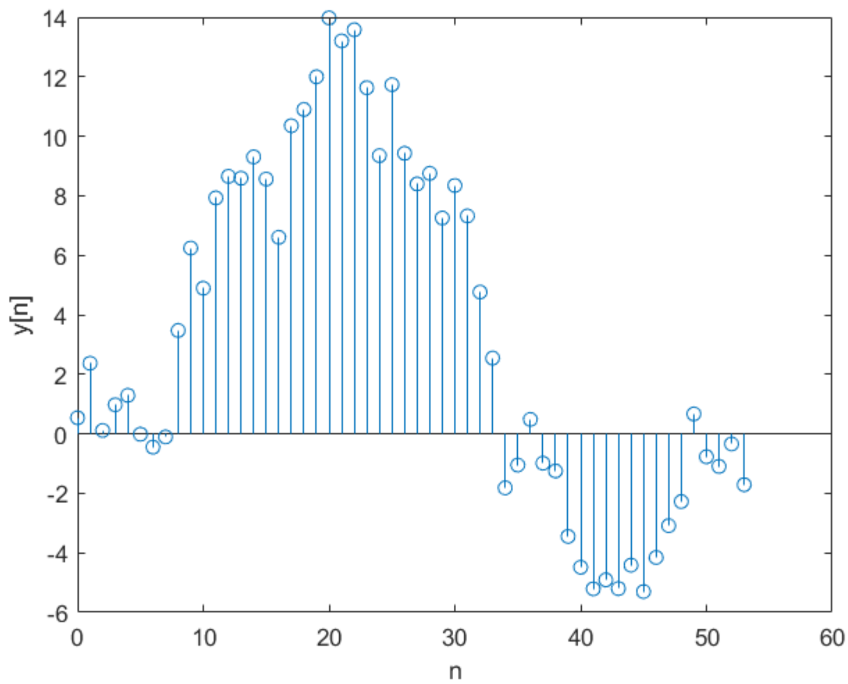


Figure 10

As can be noted in Figure 9 and Figure 10, we obtain a much smoother output. This is expected as more samples are included in the average, the fluctuation in the average value becomes less, which implies a much smoother output.