`MiMeS`: Misalignment Mechanism Solver

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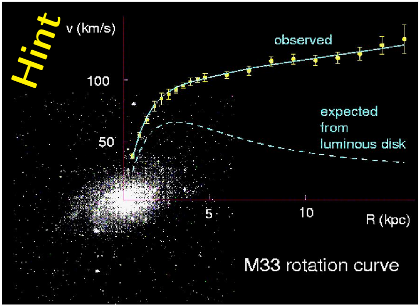
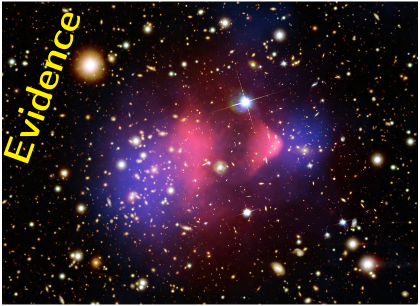
El Journal Club más Sabroso

Outline

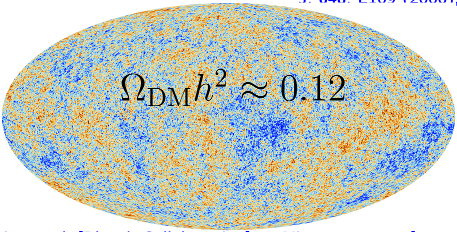
Why particle dark matter



E. Corbelli and P. Salucci, *Mon. Not. Roy. Astron. Soc.* **311** *441* (2000), arXiv:astro-ph/9909252.



M. Markevitch, ESA Spec. Publ. **604** (2006) 723, astro-ph/0511345.Clowe, Bradac, *et. al.* Astrophys. J. **648**, L109 (2006), astro-ph/0608407



N. Aghanim *et al.* [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO].

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"I know one thing, that I know nothing."

–Socrates

The dark matter particle

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- Gravitational interactions.
- Mostly electrically neutral.
- Stable or very slow decay rate.
- Non-Baryonic.
- Cold/Warm and non-relativistic today.

The axion (like) particle

Notably, the original Axion was originally introduced in order to solve the *strong-CP problem* of the SM. Axion-Like-Particles (ALPs) arise in a number of new physics models, beyond the SM.

Axions and ALPs generally:

- Have suppressed interactions with photons.
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Maybe DM has Axionic nature!

Calculating the Relic Abundance

The axion EOM

Axions and ALPs follow a similar equation of motion (EOM):

$$\left(\frac{d^2}{dt^2} + 3H(t)\ \frac{d}{dt}\right)\theta(t) + \tilde{m}_a{}^2(t)\ \sin\theta(t) = 0\ ,$$

where $\theta = A\,f_a$, with A the axion filed, and f_a some energy scale that characterises the potential (Peccei-Quinn breaking scale).

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`MiMeS` *simulates the evolution of the Axion/ALP, for (virtually) any cosmological scenario and Axion/ALP (thermal) mass.*

Initial conditions

Some time at the very early Universe, $\tilde{m}_a \ll H(T)$, [1] with

$$\ddot{\theta} + 3H\ \dot{\theta} \approx 0\ .$$

The solution is

$$\theta = \theta_{\rm ini} + C \int_0^t dt' \ \left(\frac{a(t'=0)}{a(t')} \right)^3 \ .$$

So, $\dot{\theta} \sim a^{-3}$. Since we are interested in θ at much later times (once the potential becomes relevant), $\dot{\theta} \approx 0$. [2] Therefore, we can start integration at some point $(t = t_{\rm ini})$ with $3H \gg \tilde{m}_a$, and set $\theta(t = t_{\rm ini}) = \theta_{\rm ini}$ and $\dot{\theta}(t = t_{\rm ini}) = 0$.

---

[1] This is an assumption that `MiMeS` has to make, for the sake of generality.

[2] Standard misalignment mechanism. For the kinetic one see R. T. Co, L. J. Hall and K. Harigaya, Phys. Rev. Lett. 124 (2020) no.25, 251802 [arXiv:1910.14152 [hep-ph]], C. F. Chang and Y. Cui, Phys. Rev. D 102 (2020) no.1, 015003 [arXiv:1911.11885 [hep-ph]], or B. Barman, N. Bernal, N. Ramberg and L. Visinelli, [arXiv:2111.03677 [hep-ph]] .

(Bad) Analytical approximations

Once we agree on the initial conditions, we move to the next important things:

[3] Defined from $s(T_0) = \gamma \ {a_{\rm osc}}^3 \ s_{\rm osc}$.

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Then, we get the "WKB"-approximate solution

$$\theta(t) \approx \theta_{\rm ini} \left(\frac{3}{4}\right)^{1/4} \sqrt{\frac{\tilde{m}_a(T_{\rm osc})}{\tilde{m}_a(T)}} \left(\frac{a}{a_{\rm osc}}\right)^{-3/2} \cos\left(\int_{t_{\rm osc}}^{t} dt' \; \tilde{m}_a(t')\right) .$$

The advantage of this approximation is that we get an easy formula for the axion/ALP energy density today:

$$\rho_{a,0} = \gamma^{-1} \frac{s_0}{s_{\rm osc}} \, \frac{1}{2} \, {f_a}^2 \, m_a \, \tilde{m}_{a,{\rm osc}} \, {\theta_{\rm ini}}^2 \, ,$$

where γ the amount of entropy injection between $T_{\rm osc}$ and today. [3]

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Need for accuracy, speed, and automation

Serious disadvantages of the approximate results:

- The approximations can be tested against numerical results in a case-by-case basis; there is no way to tell if they will work in new models and cosmological scenarios.
- There is no available tool that can help us reproduce published results obtained by numerical integration; people use their own ***private*** code.
- If someone wants to simply see if an ALP model is compatible with a cosmological scenario, they have to develop their own private code; the overall effort of the community increases.

`MiMeS`:

- *Easy* to use; anyone can run it and see if their model can work.
- Reasonably fast; less than $0.05\ s$ for the scenarios tested.
- Tools that can help determine if the algorithm is accurate enough.
- The user provides too much input. This helps the user determine is the convergence of the algorithm is consistent.

MiMeS

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MiMeS: under the hood

MiMeS relies on NaBBODES [4] for the numerical integration, and SimpleSplines [5] for the various interpolations.

Advantages:

- You only need to have the standard C++ library.
- The two libraries are developed by myself, so their integration with MiMeS is seamless.
- There is always going to be a compatible version of these libraries that works with MiMeS.

Disadvantages:

- These are not well tested libraries.
- No community of contributors; if it doesn't work, I have to fix it.
- Slow development.

[4] https://github.com/dkaramit/NaBBODES.
[5] https://github.com/dkaramit/SimpleSplines.

MiMeS: Notation

`MiMeS` uses a notation suitable (any) underlying cosmology, since it is up to the user to define the cosmological evolution. First, we define

$$u \equiv \log\left(a/a_{\rm ini}\right) \;,$$

with $a_{\rm ini}$ some initial value of the scale factor. [6] in order to express the time derivatives as

$$\frac{d}{dt} \to H \frac{d}{du} \;, \quad \frac{d^2}{dt^2} \to H^2 \; \left(\frac{d^2}{du^2} + \frac{1}{2} \frac{d \log H^2}{du} \frac{d}{du} \right) \;.$$

Then, we express the EOM as a system of first order ordinary differential equations

$$\frac{d\zeta}{du} + \left[\frac{1}{2} \frac{d \log H^2}{du} + 3 \right] \zeta + \; \left(\frac{\tilde{m}_a}{H} \right)^2 \; \sin\theta = 0 \;.$$
$$\frac{d\theta}{du} - \zeta = 0 \;.$$

Observe that, by definition, $\zeta = d\theta/du$. The initial conditions are $\zeta(0) = 0$ and $\theta(0) = \theta_{\rm ini}$.

[6] Only the ratios $a/a_{\rm ini}$ appear in the calculations. So, the choice does not matter as long as it is consistent.

MiMeS: When do you start and stop integrating?

The choice of a good starting point is important, as need to start at a temperature where $\zeta = 0$ is a good approximation. So you can start at some $T_{\rm ini}$ with a given ratio $3H(T_{\rm ini})/\tilde{m}_a(T_{\rm ini}) \gg 1$. This needs to be chosen carefully, as low values of $3H(T_{\rm ini})/\tilde{m}_a(T_{\rm ini})$ result in inaccurate result, while high values may result in a slow calculation. *Advice*: Use various values of that ratio, and find where the relic abundance becomes $T_{\rm ini}$-independent.

The stopping condition is more difficult. You should stop at some point where the axion/ALP evolves "adiabatically". Find a quantity that becomes constant as the system relaxes, and use this to determine when adiabaticity is reached. Once θ starts to evolve adiabatically, the amplitude of its oscillation is known at later times!

If a system exhibits closed orbits, the quantity

$$J \equiv C \oint p \, d\theta \; ,$$

is the adiabatic invariant. In this case, it becomes

$$J = a^3 \; \tilde{m}_a \; {\theta_{\rm peak}}^2 \, f(\theta_{\rm peak}) \; ,$$

with

$$f(\theta_{\rm peak}) = \frac{2\sqrt{2}}{\pi {\theta_{\rm peak}}^2} \int_{-\theta_{\rm peak}}^{\theta_{\rm peak}} d\theta \sqrt{\cos\theta - \cos\theta_{\rm peak}} \; ,$$

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Important: $\theta_{\rm peak}$ is the peak of the oscillation. So, J can be used to determine how $\theta_{\rm peak}$ changes with time. By definition, at $\theta = \theta_{\rm peak}$, $p \sim \dot{\theta} = 0$. This means that we can find $\rho_{a,0}$ on the peak of today's θ, as

$$\rho_{a,0} = \gamma^{-1} \; \frac{s_0}{s_*} \; m_a \; \tilde{m}_{a,*} \; \frac{1}{2} \; {f_a}^2 \; \theta_{{\rm peak},*}^2 \;\; f(\theta_{{\rm peak},*}) \,,$$

where T_* the temperature at which adiabaticity was reached, and γ the entropy injection between T_* and today (*i.e.* $s_0 = \gamma a_*^3 s_*$).

Using MiMeS

How to get MiMeS

There are several ways you can get a stable version of `MiMeS`:

1. `git clone -b stable https://github.com/dkaramit/MiMeS.git`. This is the preferred way, as it is guaranteed to be the latest stable version.
2. Go to mimes.hepforge.org/downloads, and download it.
3. Go to github.com/dkaramit/MiMeS/releases, and download a released version.

You can get the most up-to-date code – not always the most stable one – including the latest version of `NaBBODES` and `SimpleSplines`, by running

```
git clone https://github.com/dkaramit/MiMeS.git
cd MiMeS
git submodule init
git submodule update --remote
```

Configure (and make)

There is no need to install anything if you are going to use `MiMeS` in a `C++` program. The only thing you *must* do is run

```
bash configure.sh
```

However, you can run Commend from D: List all `make` ... that make useful things.

Know your constructors: `mimes::AxionMass<LD>`

Commend from D: Show the constructor for `mimes::AxionMass`, and discuss the input.

Know your constructors: `mimes::Axion<LD,Solver,Method>`

Commend from D: Show the constructor for `mimes::Axion`, and discuss the input.

What `MiMeS` expects from you

Commend from D: Discuss template arguments

MiMeS from python

Commend from D: Discuss `Definitions.mk` and show that the constructors are the same as in `C++`

Examples

Commend from D: can you show example code?

Commend from D: can you do a live presentation of `MiMeS` in jupyter?

Outlook

Summing up

Commend from D: blah blah, what you did

Future

Commend from D: Freeze-out/in, kinematic misalignment mechanism, automatically compare with searches.

Thank you!

Backup

(equations, derivations, tables)

Adiabatic invariant

Input – mimes::AxionMass<LD>

Input – `mimes::Axion<LD,Solver,Method>`