`MiMeS`: Misalignment Mechanism Solver

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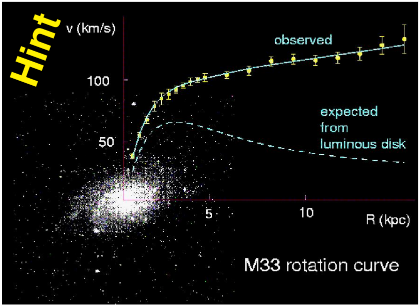
El Journal Club más Sabroso

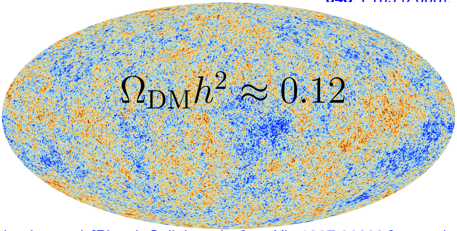
Outline

Why particle dark matter



E. Corbelli and P. Salucci, *Mon. Not. Roy. Astron. Soc.* **311** *441* (2000), arXiv:astro-ph/9909252.

M. Markevitch, ESA Spec. Publ. **604** (2006) 723, astro-ph/0511345.Clowe, Bradac, *et. al.* Astrophys. J. **648** L109 (2006), astro-ph/0608407



N. Aghanim *et al.* [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO].

The dark matter particle

"Ἓν οἶδα, ὅτι οὐδὲν οἶδα."
"I know one thing, that I know nothing."

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- Gravitational interactions.
- Mostly electrically neutral.
- Stable or very slow decay rate.
- Non-Baryonic.
- Cold/Warm and non-relativistic today.

The axion (like) particle

Notably, the original Axion was originally introduced in order to solve the *strong-CP problem* of the SM. Axion-Like-Particles (ALPs) arise in a number of new physics models, beyond the SM.

Axions and ALPs generally:

- Have suppressed interactions with photons.
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Maybe DM has Axionic nature!

The axion EOM

Axions and ALPs follow a similar equation of motion (EOM):

$$\left(\frac{d^2}{dt^2} + 3H(t)\,\frac{d}{dt}\right)\theta(t) + \tilde{m}_a{}^2(t)\,\sin\theta(t) = 0\,,$$

where $\theta = A\,f_a$, with A the axion filed, and f_a some energy scale that characterises the potential (Peccei-Quinn breaking scale).

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`MiMeS` *simulates the evolution of the Axion/ALP, for (virtually) any cosmological scenario and Axion/ALP (thermal) mass.*

Initial conditions

Some time at the very early Universe, $\tilde{m}_a \ll H(T)$, [1] with

$$\ddot{\theta} + 3H\,\dot{\theta} \approx 0\;.$$

The solution is

$$\theta = \theta_{\rm ini} + C \int_0^t dt' \, \left(\frac{a(t'=0)}{a(t')} \right)^3 \;.$$

So, $\dot{\theta} \sim a^{-3}$. Since we are interested in θ at much later times (once the potential becomes relevant), $\dot{\theta} \approx 0$. [2] Therefore, we can start integration at some point ($t = t_{\rm ini}$) with $3H \gg \tilde{m}_a$, and set $\theta(t = t_{\rm ini}) = \theta_{\rm ini}$ and $\dot{\theta}(t = t_{\rm ini}) = 0$.

[1] This is an assumption that `MiMeS` has to make, for the sake of generality.

[2] Standard misalignment mechanism. For the kinetic one see R. T. Co, L. J. Hall and K. Harigaya, Phys. Rev. Lett. **124** (2020) no.25, 251802 [arXiv:1910.14152 [hep-ph]], C. F. Chang and Y. Cui, Phys. Rev. D **102** (2020) no.1, 015003 [arXiv:1911.11885 [hep-ph]], or B. Barman, N. Bernal, N. Ramberg and L. Visinelli, [arXiv:2111.03677 [hep-ph]] .

(Bad) Analytical approximations

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- Assume that $\theta_{\rm osc} \approx \theta_{\rm ini}$. *Generally quite bad.*

Then, we get the “WKB"-approximate solution

$$\theta(t) \approx \theta_{\rm ini} \left(\frac{3}{4}\right)^{1/4} \sqrt{\frac{\tilde{m}_a(T_{\rm osc})}{\tilde{m}_a(T)}} \left(\frac{a}{a_{\rm osc}}\right)^{-3/2} \cos\left(\int_{t_{\rm osc}}^{t} dt' \; \tilde{m}_a(t')\right) \; .$$

The advantage of this approximation is that we get an easy formula for the axion/ALP energy density today:

$$\rho_{a,0} = \gamma^{-1} \frac{s_0}{s_{\rm osc}} \; \frac{1}{2} \; {f_a}^2 \; m_a \; \tilde{m}_{a,{\rm osc}} \; {\theta_{\rm ini}}^2 \; ,$$

where γ the amount of entropy injection between $T_{\rm osc}$ and today. [3]

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Need for accuracy, speed, and automation

Serious disadvantages of the approximate results:

- The approximations can be tested against numerical results in a case-by-case basis; there is no way to tell if they will work in new models and cosmological scenarios.
- There is no available tool that can help us reproduce published results obtained by numerical integration; people use their own ***private*** code.
- If someone wants to simply see if an ALP model is compatible with a cosmological scenario, they have to develop their own private code; the overall effort of the community increases.

`MiMeS`:

- *Easy* to use; anyone can run it and see if their model can work.
- Reasonably fast; less than $0.05\ s$ for the scenarios tested.
- Tools that can help determine if the algorithm is accurate enough.
- The user provides too much input. This helps the user determine whether convergence of the algorithm is consistent.

`MiMeS`: General

- `MiMeS` is a `C++` header-only library that contains various templated classes; there is no “installation" and no special procedures, just include the header files.

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MiMeS: Under the hood

MiMeS relies on NaBBODES [4] for the numerical integration, and SimpleSplines [5] for the various interpolations.

Advantages:

- You only need to have the standard C++ library.
- The two libraries are developed by myself, so their integration with MiMeS is seamless.
- There is always going to be a compatible version of these libraries that works with MiMeS.

Disadvantages:

- These are not well tested libraries.
- No community of contributors; if it doesn't work, I have to fix it.
- Slow development.

[4] https://github.com/dkaramit/NaBBODES.

[5] https://github.com/dkaramit/SimpleSplines.

MiMeS: Notation

MiMeS uses a notation suitable (any) underlying cosmology, since it is up to the user to define the cosmological evolution. First, we define

$$u \equiv \log \left(a/a_{\rm ini} \right) \;,$$

with $a_{\rm ini}$ some initial value of the scale factor. [6] in order to express the time derivatives as

$$\frac{d}{dt} \to H \frac{d}{du} \;, \quad \frac{d^2}{dt^2} \to H^2 \; \left(\frac{d^2}{du^2} + \frac{1}{2} \frac{d \log H^2}{du} \frac{d}{du} \right) \;.$$

Then, we express the EOM as a system of first order ordinary differential equations

$$\frac{d\zeta}{du} + \left[\frac{1}{2} \frac{d \log H^2}{du} + 3 \right] \zeta + \; \left(\frac{\tilde{m}_a}{H} \right)^2 \; \sin\theta = 0 \;.$$
$$\frac{d\theta}{du} - \zeta = 0 \;.$$

Observe that, by definition, $\zeta = d\theta/du$. The initial conditions are $\zeta(0) = 0$ and $\theta(0) = \theta_{\rm ini}$.

[6] Only the ratios $a/a_{\rm ini}$ appear in the calculations. So, the choice does not matter as long as it is consistent.

MiMeS: When do you start and stop integrating?

The choice of a good starting point is important, as need to start at a temperature where $\zeta = 0$ is a good approximation. So you can start at some $T_{\rm ini}$ with a given ratio $3H(T_{\rm ini})/\tilde{m}_a(T_{\rm ini}) \gg 1$. This needs to be chosen carefully, as low values of $3H(T_{\rm ini})/\tilde{m}_a(T_{\rm ini})$ result in inaccurate result, while high values may result in a slow calculation. *Advice*: Use various values of that ratio, and find where the relic abundance becomes $T_{\rm ini}$-independent.

The stopping condition is more difficult. You should stop at some point where the axion/ALP evolves "adiabatically". Find a quantity that becomes constant as the system relaxes, and use this to determine when adiabaticity is reached. Once θ starts to evolve adiabatically, the amplitude of its oscillation is known at later times!

If a system exhibits closed orbits, the quantity

$$J \equiv C \oint p \, d\theta \; ,$$

is the adiabatic invariant. In this case, it becomes

$$J = a^3 \; \tilde{m}_a \; {\theta_{\rm peak}}^2 \, f(\theta_{\rm peak}) \; ,$$

with

$$f(\theta_{\rm peak}) = \frac{2\sqrt{2}}{\pi {\theta_{\rm peak}}^2} \int_{-\theta_{\rm peak}}^{\theta_{\rm peak}} d\theta \sqrt{\cos\theta - \cos\theta_{\rm peak}} \; ,$$

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Important: $\theta_{\rm peak}$ is the peak of the oscillation. So, J can be used to determine how $\theta_{\rm peak}$ changes with time. By definition, at $\theta = \theta_{\rm peak}$, $p \sim \dot{\theta} = 0$. This means that we can find $\rho_{a,0}$ on the peak of today's θ, as

$$\rho_{a,0} = \gamma^{-1} \; \frac{s_0}{s_*} \; m_a \; \tilde{m}_{a,*} \; \frac{1}{2} \; {f_a}^2 \; \theta_{{\rm peak},*}^2 \;\; f(\theta_{{\rm peak},*}) \; ,$$

where T_* the temperature at which adiabaticity was reached, and γ the entropy injection between T_* and today (*i.e.* $s_0 = \gamma a_*^3 s_*$).

Using MiMeS

How to get MiMeS

There are several ways you can get a stable version of MiMeS:

1. `git clone -b stable https://github.com/dkaramit/MiMeS.git`.
   This is the preferred way, as it is guaranteed to be the latest stable version.
2. Go to mimes.hepforge.org/downloads, and download it.
3. Go to github.com/dkaramit/MiMeS/releases, and download a released version.

You can get the most up-to-date code – not always the most stable one – including the latest version of NaBBODES and SimpleSplines, by running

```
git clone https://github.com/dkaramit/MiMeS.git
cd MiMeS
git submodule init
git submodule update --remote
```

Configure (and make)

There is no need to install anything if you are going to use `MiMeS` in a `C++` program. The only thing you *must* do is run

```
bash configure.sh
```

Alter that, you can include the header file `MiMeS/MiMeS.hpp`, and you are good to go.
However, you can also run

- `make lib`, in order to produce the (shared) libraries. This is needed in order to run the `python` interface.
- `make examples`, in order to compile the examples in `MiMeS/UserSpace/Cpp`.
- `make exec`, in order to produce some test executables (in `MiMeS/exec`). You just need to run then in order to see if you get any segfaults.

Classes

There are three classes useful to the user. [7]

- `mimes::Cosmo<LD>`: interpolation of relativistic degrees of freedom of the plasma. By default it uses the EOS2020 [8] data. The user can choose another file easily.

---

[7] There are various arguments that need to be passed to the constructors, and the are all listed and explained in the Appendix of the documentation.

[8] K. Saikawa and S. Shirai, JCAP **08** (2020), 011 [arXiv:2005.03544 [hep-ph]].

[9] S. Borsanyi, Z. Fodor, J. Guenther, K. H. Kampert, S. D. Katz, T. Kawanai, T. G. Kovacs, S. W. Mages, A. Pasztor and F. Pittler, *et al.* Nature **539** (2016) no.7627, 69-71 [arXiv:1606.07494 [hep-lat]].

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- `mimes::Axion<LD,Solver,Method>`: This is responsible for actually solving the EOM.

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2. $\zeta(0) = 0$. This will be changed in the future.
3. The energy density of the axion/ALP is always subdominant.
4. Only the EOM determines the energy density (no annihilations, no strings, etc.).

What `MiMeS` expects from you

Apart from $\theta_{\rm ini}$ and f_a, keep in mind that `MiMeS` needs:

1. The mass of the axion/ALP. A data file or an actual function.
2. Data file with $\log a/a_i$ (a_i is some arbitrary value; `MiMeS` rescales it appropriately), T, and $\log H$ of the underlying cosmology.

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3. Value for $3H/\tilde{m}_a \gg 1$, which defines the point where integration begins.
4. Relative difference of J between a given number of peaks at which we consider adiabaticity to have been reached.
5. Other input, related to the algorithm, that might confuse you; *e.g.* temperature at which integration stops no *matter what*!

Template arguments

You need to choose what numeric type to use. This is done by the template argument `LD` which should be `double` (fast) or `long double` (accurate). [10]

You also need to tell `MiMeS` which integration strategy to use. This is done by choosing template arguments:

- `Solver` can be set to `1` for Rosenbrock (semi-implicit Runge-Kutta). The `Method` argument in this case can be:
  - `RODASPR2<LD>` (4th order).
  - `ROS34PW2<LD>` (3rd order).
  - `ROS3W<LD>` (2rd order, *very* bad).
- `Solver` can be set to `2` for explicit RK. The `Method` argument can be:
  - `DormandPrinceRK45<LD>` (7th order)
  - `CashKarpRK45<LD>` (5th order, *very* bad).
  - `RK45<LD>` (5th order, *very* bad).

[10] You could choose `float`, but we live in 2021.

MiMeS from python

In order to call the python interface of MiMeS, we need to first call make lib in the root directory of MiMeS.

Before that, we can take some time to decide what the template arguments and compilation options should be. In the file MiMeS/Definitions.mk, you can change the variables:

- LONG=long will compile the library with long double numeric types. LONG= will compile the library with double numeric types.
- SOLVER and METHOD, as in the template arguments.

Also, in the same file, you can change compilation options:

- Compiler:
  - CC=g++ in order to use the GNU C++ compiler.
  - CC=clang -lstdc++ in order to use the clang C++ compiler.
- Optimization level:
  - OPT=O0: No optimization.
  - O=O1, O2, or O3 (marginally faster, default): some optimization occurs (read the compiler documentation for more information).
  - OPT=Ofast: full optimization (fast, but dangerous).

Examples

Define everything and solve in just a few lines of code!

```cpp
// use chi_PATH to interpolate the axion mass.
mimes::AxionMass<long double> axionMass(chi_PATH,0,mimes::Cosmo<long double>::mP);

/*set $\tilde{m}_a{}^2$ for $T \geq T_{\rm max}$*/
long double TMax=axionMass.getTMax(), chiMax=axionMass.getChiMax();

axionMass.set_ma2_MAX(
    [&chiMax,&TMax](long double T, long double fa){
        return chiMax/fa/fa*std::pow(T/TMax,-8.16);
    }
);

/*set $\tilde{m}_a{}^2$ for $T \leq T_{\rm min}$*/
long double TMin=axionMass.getTMin(), chiMin=axionMass.getChiMin();

axionMass.set_ma2_MIN(
    [&chiMin,&TMin](long double T, long double fa){
        return chiMin/fa/fa;
    }
);

/*this path contains the cosmology*/
std::string inputFile = std::string(rootDir)+
    std::string("/UserSpace/InputExamples/MatterInput.dat");

/*declare an instance of Axion*/
mimes::Axion<long double, 1, RODASPR2<long double> > ax(
    0.1, 1e16, 500, 1e-4, 1e3, 15, 1e-2, inputFile, &axionMass,
    1e-1, 1e-8, 1e-1, 1e-11, 1e-11, 0.9, 1.2, 0.8, int(1e7)
    );

/*solve the EOM!*/
ax.solveAxion();
```

Outlook

Summing up

Commend from D: blah blah, what you did

Future

Commend from D: Freeze-out/in, kinematic misalignment mechanism, automatically compare with searches.

Thank you!

Language	files	blank	comment	code
C/C++ Header	35	677	444	1595
C++	20	483	198	1106
Python	22	483	367	1000
SUM:	77	1643	1009	3701

Backup

(equations, derivations, tables)

WKB

Adiabatic invariant

Input – mimes::AxionMass<LD>

Input – mimes::Axion<LD,Solver,Method>