`MiMeS`: Misalignment Mechanism Solver

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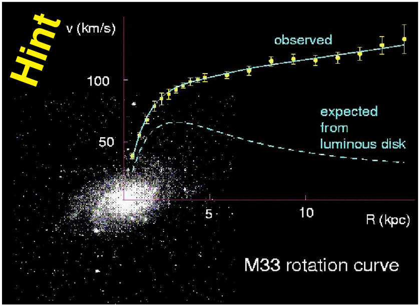
El Journal Club más Sabroso

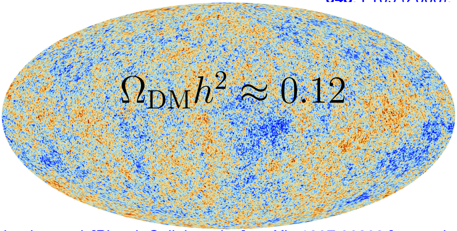
Outline

Why particle dark matter



E. Corbelli and P. Salucci, *Mon. Not. Roy. Astron. Soc.* **311** *441* (2000), arXiv:astro-ph/9909252.

M. Markevitch, ESA Spec. Publ. **604** (2006) 723, astro-ph/0511345.Clowe, Bradac, *et. al.* Astrophys. J. **648** L109 (2006), astro-ph/0608407



N. Aghanim *et al.* [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO].

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"I know one thing, that I know nothing."

–Socrates

The dark matter particle

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- Gravitational interactions.
- Mostly electrically neutral.
- Stable or very slow decay rate.
- Non-Baryonic.
- Cold/Warm and non-relativistic today.

The axion (like) particle

Notably, the original Axion was originally introduced in order to solve the *strong-CP problem* of the SM. Axion-Like-Particles (ALPs) arise in a number of new physics models, beyond the SM.

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Maybe DM has Axionic nature!

Calculating the Relic Abundance

The axion EOM

Axions and ALPs follow a similar equation of motion (EOM):

$$\left(\frac{d^2}{dt^2} + 3H(t)\,\frac{d}{dt}\right)\theta(t) + \tilde{m}_a{}^2(t)\;\sin\theta(t) = 0\;,$$

where $\theta = A\,f_a$, with A the axion filed, and f_a some energy scale that characterises the potential (Peccei-Quinn breaking scale).

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`MiMeS` *simulates the evolution of the Axion/ALP, for (virtually) any cosmological scenario and Axion/ALP (thermal) mass.*

Initial conditions

Some time at the very early Universe, $\tilde{m}_a \ll H(T)$, [1] with

$$\ddot{\theta} + 3H\,\dot{\theta} \approx 0\;.$$

The solution is

$$\theta = \theta_{\rm ini} + C \int_0^t dt' \, \left(\frac{a(t'=0)}{a(t')} \right)^3 \;.$$

So, $\dot{\theta} \sim a^{-3}$. Since we are interested in θ at much later times (once the potential becomes relevant), $\dot{\theta} \approx 0$. [2] Therefore, we can start integration at some point ($t = t_{\rm ini}$) with $3H \gg \tilde{m}_a$, and set $\theta(t = t_{\rm ini}) = \theta_{\rm ini}$ and $\dot{\theta}(t = t_{\rm ini}) = 0$.

---

[1] This is an assumption that `MiMeS` has to make, for the sake of generality.

[2] Standard misalignment mechanism. For the kinetic one see R. T. Co, L. J. Hall and K. Harigaya, Phys. Rev. Lett. **124** (2020) no.25, 251802 [arXiv:1910.14152 [hep-ph]], C. F. Chang and Y. Cui, Phys. Rev. D **102** (2020) no.1, 015003 [arXiv:1911.11885 [hep-ph]], or B. Barman, N. Bernal, N. Ramberg and L. Visinelli, [arXiv:2111.03677 [hep-ph]] .

(Bad) Analytical approximations

Once we agree on the initial conditions, we move to the next important things:

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Then, we get the “WKB"-approximate solution

$$\theta(t) \approx \theta_{\rm ini} \left(\frac{3}{4}\right)^{1/4} \sqrt{\frac{\tilde{m}_a(T_{\rm osc})}{\tilde{m}_a(T)}} \left(\frac{a}{a_{\rm osc}}\right)^{-3/2} \cos\left(\int_{t_{\rm osc}}^{t} dt' \; \tilde{m}_a(t')\right) \; .$$

The advantage of this approximation is that we get an easy formula for the axion/ALP energy density today:

$$\rho_{a,0} = \gamma^{-1} \frac{s_0}{s_{\rm osc}} \; \frac{1}{2} \; {f_a}^2 \; m_a \; \tilde{m}_{a,{\rm osc}} \; {\theta_{\rm ini}}^2 \; ,$$

where γ the amount of entropy injection between $T_{\rm osc}$ and today. [3]

[3] Defined from $s(T_0) = \gamma \; {a_{\rm osc}}^3 \; s_{\rm osc}$.

Need for accuracy, speed, and automation

Serious disadvantages of the approximate results:

- The approximations can be tested against numerical results in a case-by-case basis; there is no way to tell if they will work in new models and cosmological scenarios.
- There is no available tool that can help us reproduce published results obtained by numerical integration; people use their own ***private*** code.
- If someone wants to simply see if an ALP model is compatible with a cosmological scenario, they have to develop their own private code; the overall effort of the community increases.

`MiMeS`:

- *Easy* to use; anyone can run it and see if their model can work.
- Reasonably fast; less than $0.05\ s$ for the scenarios tested.
- Tools that can help determine if the algorithm is accurate enough.
- The user provides too much input. This helps the user determine whether convergence of the algorithm is consistent.

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MiMeS: Under the hood

MiMeS relies on NaBBODES [4] for the numerical integration, and SimpleSplines [5] for the various interpolations.

Advantages:

- You only need to have the standard C++ library.
- The two libraries are developed by myself, so their integration with MiMeS is seamless.
- There is always going to be a compatible version of these libraries that works with MiMeS.

Disadvantages:

- These are not well tested libraries.
- No community of contributors; if it doesn't work, I have to fix it.
- Slow development.

4 https://github.com/dkaramit/NaBBODES.
5 https://github.com/dkaramit/SimpleSplines.

MiMeS: Notation

MiMeS uses a notation suitable (any) underlying cosmology, since it is up to the user to define the cosmological evolution. First, we define

$$u \equiv \log\left(a/a_{\rm ini}\right) \;,$$

with $a_{\rm ini}$ some initial value of the scale factor. [6] in order to express the time derivatives as

$$\frac{d}{dt} \to H \frac{d}{du} \;, \quad \frac{d^2}{dt^2} \to H^2 \left(\frac{d^2}{du^2} + \frac{1}{2}\frac{d \log H^2}{du}\frac{d}{du} \right) \;.$$

Then, we express the EOM as a system of first order ordinary differential equations

$$\frac{d\zeta}{du} + \left[\frac{1}{2}\frac{d\log H^2}{du} + 3\right]\zeta + \left(\frac{\tilde{m}_a}{H}\right)^2 \sin\theta = 0 \;.$$
$$\frac{d\theta}{du} - \zeta = 0 \;.$$

Observe that, by definition, $\zeta = d\theta/du$. The initial conditions are $\zeta(0) = 0$ and $\theta(0) = \theta_{\rm ini}$.

[6] Only the ratios $a/a_{\rm ini}$ appear in the calculations. So, the choice does not matter as long as it is consistent.

MiMeS: When do you start and stop integrating?

The choice of a good starting point is important, as need to start at a temperature where $\zeta = 0$ is a good approximation. So you can start at some $T_{\rm ini}$ with a given ratio $3H(T_{\rm ini})/\tilde{m}_a(T_{\rm ini}) \gg 1$. This needs to be chosen carefully, as low values of $3H(T_{\rm ini})/\tilde{m}_a(T_{\rm ini})$ result in inaccurate result, while high values may result in a slow calculation. *Advice*: Use various values of that ratio, and find where the relic abundance becomes $T_{\rm ini}$-independent.

The stopping condition is more difficult. You should stop at some point where the axion/ALP evolves "adiabatically". Find a quantity that becomes constant as the system relaxes, and use this to determine when adiabaticity is reached. Once θ starts to evolve adiabatically, the amplitude of its oscillation is known at later times!

If a system exhibits closed orbits, the quantity

$$J \equiv C \oint p \, d\theta \,,$$

is the adiabatic invariant. In this case, it becomes

$$J = a^3 \, \tilde{m}_a \, {\theta_{\rm peak}}^2 \, f(\theta_{\rm peak}) \,,$$

with

$$f(\theta_{\rm peak}) = \frac{2\sqrt{2}}{\pi {\theta_{\rm peak}}^2} \int_{-\theta_{\rm peak}}^{\theta_{\rm peak}} d\theta \sqrt{\cos\theta - \cos\theta_{\rm peak}} \,,$$

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Important: $\theta_{\rm peak}$ is the peak of the oscillation. So, J can be used to determine how $\theta_{\rm peak}$ changes with time. By definition, at $\theta = \theta_{\rm peak}$, $p \sim \dot{\theta} = 0$. This means that we can find $\rho_{a,0}$ on the peak of today's θ, as

$$\rho_{a,0} = \gamma^{-1} \; \frac{s_0}{s_*} \; m_a \; \tilde{m}_{a,*} \; \frac{1}{2} \; {f_a}^2 \; \theta_{{\rm peak},*}^2 \;\; f(\theta_{{\rm peak},*}) \;,$$

where T_* the temperature at which adiabaticity was reached, and γ the entropy injection between T_* and today (*i.e.* $s_0 = \gamma a_*^3 s_*$).

Using MiMeS

How to get MiMeS

There are several ways you can get a stable version of MiMeS:

1. `git clone -b stable https://github.com/dkaramit/MiMeS.git`.
   This is the preferred way, as it is guaranteed to be the latest stable version.
2. Go to mimes.hepforge.org/downloads, and download it.
3. Go to github.com/dkaramit/MiMeS/releases, and download a released version.

You can get the most up-to-date code – not always the most stable one – including the latest version of NaBBODES and SimpleSplines, by running

```
git clone https://github.com/dkaramit/MiMeS.git
cd MiMeS
git submodule init
git submodule update --remote
```

Configure (and make)

There is no need to install anything if you are going to use `MiMeS` in a `C++` program. The only thing you *must* do is run

```
bash configure.sh
```

Alter that, you can include the header file `MiMeS/MiMeS.hpp`, and you are good to go.
However, you can also run

- `make lib`, in order to produce the (shared) libraries. This is needed in order to run the `python` interface.
- `make examples`, in order to compile the examples in `MiMeS/UserSpace/Cpp`.
- `make exec`, in order to produce some test executables (in `MiMeS/exec`). You just need to run then in order to see if you get any segfaults.

Classes

There are three classes useful to the user. [7]

- `mimes::Cosmo<LD>`: interpolation of relativistic degrees of freedom of the plasma. By default it uses the EOS2020 [8] data. The user can choose another file easily.

---

7 There are various arguments that need to be passed to the constructors, and the are all listed and explained in the Appendix of the documentation.

8 K. Saikawa and S. Shirai, JCAP **08** (2020), 011 [arXiv:2005.03544 [hep-ph]].

9 S. Borsanyi, Z. Fodor, J. Guenther, K. H. Kampert, S. D. Katz, T. Kawanai, T. G. Kovacs, S. W. Mages, A. Pasztor and F. Pittler, *et al.* Nature **539** (2016) no.7627, 69-71 [arXiv:1606.07494 [hep-lat]].

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- `mimes::Axion<LD,Solver,Method>`: This is responsible for actually solving the EOM.

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2. $\zeta(0) = 0$. This will be changed in the future.
3. The energy density of the axion/ALP is always subdominant.
4. Only the EOM determines the energy density (no annihilations, no strings, etc.).

What MiMeS expects from you

Apart from $\theta_{\rm ini}$ and f_a, keep in mind that MiMeS needs:

1. The mass of the axion/ALP. A data file or an actual function.
2. Data file with $\log a/a_i$ (a_i is some arbitrary value; MiMeS rescales it appropriately), T, and $\log H$ of the underlying cosmology.

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4. Relative difference of J between a given number of peaks at which we consider adiabaticity to have been reached.
5. Other input, related to the algorithm, that might confuse you; *e.g.* temperature at which integration stops no *matter what*!

Template arguments

You need to choose what numeric type to use. This is done by the template argument `LD` which should be `double` (fast) or `long double` (accurate). [10]

You also need to tell `MiMeS` which integration strategy to use. This is done by choosing template arguments:

- `Solver` can be set to `1` for Rosenbrock (semi-implicit Runge-Kutta). The `Method` argument in this case can be:
  - `RODASPR2<LD>` (4th order).
  - `ROS34PW2<LD>` (3rd order).
  - `ROS3W<LD>` (2rd order, *very* bad).
- `Solver` can be set to `2` for explicit RK. The `Method` argument can be:
  - `DormandPrinceRK45<LD>` (7th order)
  - `CashKarpRK45<LD>` (5th order, *very* bad).
  - `RK45<LD>` (5th order, *very* bad).

[10] You could choose `float`, but we live in 2021.

MiMeS from python

In order to call the `python` interface of `MiMeS`, we need to first call `make lib` in the root directory of `MiMeS`.

Before that, we can take some time to decide what the template arguments and compilation options should be. In the file `MiMeS/Definitions.mk`, you can change the variables:

- `LONG=long` will compile the library with `long double` numeric types. `LONG=` will compile the library with `double` numeric types.
- `SOLVER` and `METHOD`, as in the template arguments.

Also, in the same file, you can change compilation options:

- Compiler:
  - `CC=g++` in order to use the `GNU C++` compiler.
  - `CC=clang -lstdc++` in order to use the `clang C++` compiler.
- Optimization level:
  - `OPT=O0`: No optimization.
  - `O=O1`, `O2`, or `O3` (marginally faster, default): some optimization occurs (read the compiler documentation for more information).
  - `OPT=Ofast`: full optimization (fast, but dangerous).

Examples

Define everything and solve in just a few lines of code!

```python
from time import time; from sys import stderr #you need thes in order to print the time in stderr

#add the relative path for MiMeS/src
from sys import path as sysPath; sysPath.append('../src')

from interfacePy.AxionMass import AxionMass #import the AxionMass class
from interfacePy.Axion import Axion #import the Axion class
from interfacePy.Cosmo import mP #import the Planck mass

def main():
    # AxionMass instance
    axionMass = AxionMass(r'../src/data/chi.dat',0,mP)

    # define $\tilde{m}_a^2$ for $T \leq T_{\rm min}$
    TMin, chiMin=axionMass.getTMin(), axionMass.getChiMin()

    axionMass.set_ma2_MIN( lambda T,fa: chiMin/fa/fa )

    # define $\tilde{m}_a^2$ for $T \geq T_{\rm max}$
    TMax, chiMax=axionMass.getTMax(), axionMass.getChiMax()

    axionMass.set_ma2_MAX( lambda T,fa: chiMax/fa/fa*pow(TMax/T,8.16))

    #in python it is more convenient to use relative paths
    inputFile="../UserSpace/InputExamples/MatterInput.dat"

    ax = Axion(0.1, 1e16, 500, 1e-4, 1e3, 10, 1e-2, inputFile, axionMass,
        1e-2, 1e-8, 1e-2, 1e-9, 1e-9, 0.9, 1.2, 0.8, int(1e7))

    ax.solveAxion()

    print(ax.relic)

    #once we are done we should run the destructor
    del ax
    del axionMass

if __name__ == '__main__':
    _=time()
    main()
    print(round(time()-_,3),file=stderr)
```

Define everything and solve in just a few lines of code!

```cpp
#include"MiMeS.hpp"
#define numeric long double //makes life easier if you define a macro for this

int main(){
    mimes::util::Timer _timer_;//use this to time it!

    // use chi_PATH to interpolate the axion mass.
    mimes::AxionMass<numeric> axionMass(chi_PATH,0,mimes::Cosmo<numeric>::mP);

    /*set $\tilde{m}_a^2$ for $T \geq T_{\rm max}$*/
    numeric TMax=axionMass.getTMax(), chiMax=axionMass.getChiMax();

    axionMass.set_ma2_MAX(
        [&chiMax,&TMax](numeric T, numeric fa){ return chiMax/fa/fa*std::pow(T/TMax,-8.16);}
    );

    /*set $\tilde{m}_a^2$ for $T \leq T_{\rm min}$*/
    numeric TMin=axionMass.getTMin(), chiMin=axionMass.getChiMin();

    axionMass.set_ma2_MIN(
        [&chiMin,&TMin](numeric T, numeric fa){ return chiMin/fa/fa;}
    );

    /*this path contains the cosmology*/
    std::string inputFile = std::string(rootDir)+
        std::string("/UserSpace/InputExamples/MatterInput.dat");

    /*declare an instance of Axion*/
    mimes::Axion<numeric, 1, RODASPR2<numeric> > ax(0.1, 1e16, 500, 1e-4, 1e3, 10, 1e-2,
        inputFile, &axionMass, 1e-2, 1e-8, 1e-2, 1e-9, 1e-9, 0.9, 1.2, 0.8, int(1e7) );

    /*solve the EOM!*/
    ax.solveAxion();

    std::cout<<ax.relic<<"\n";
    return 0;
}
```

Notice: `C++` and `python` are quite similar!

Outlook

1. Axion Dark Matter
   - Why particle dark matter
   - The dark matter particle
   - The axion (like) particle
2. Calculating the Relic Abundance
   - The axion EOM
   - How hard can it be?
   - Initial conditions
   - (Bad) Analytical approximations
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3. `MiMeS`
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   - `MiMeS`: Under the hood
   - `MiMeS`: Notation
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   - `MiMeS`: Some (classical) physics
4. Using `MiMeS`
   - How to get `MiMeS`
   - Configure (and make)
   - Classes
   - Assumptions
   - What `MiMeS` expects from you
   - Template arguments
   - `MiMeS` from `python`
5. Examples
   - `python`
   - `C++`
6. Outlook
   - Summing up
   - Future

Summing up

Commend from D: blah blah, what you did

Future

Commend from D: Freeze-out/in, kinematic misalignment mechanism, automatically compare with searches.

Thank you!

Language	files	blank	comment	code
C/C++ Header	35	677	444	1595
C++	20	483	198	1106
Python	22	483	367	1000
SUM:	77	1643	1009	3701

Backup

(equations, derivations, tables)

Adiabatic invariant

Input – mimes::AxionMass<LD>

Input – `mimes::Axion<LD,Solver,Method>`