# Velocity Matching for Nonholonomic Blimp Robots

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## I. Graph Description

let $G = (V,E)$ be an undirected graph representing the a network of non-holonomic robots system. $V \in R^n$ is the number of nodes where each node represents a robot. $E$ is the set of all the edges between any two nodes. Each edge present between any two nodes means that there is two way communication between the two agents. Since an agent in the network will have access to its own states we also consider that there exists self loops. Let $A \in R^{n \times n}$ be the adjacency matrix of the graph $G$. The elements in the matrix $A$ is determined from the graph $G$ as given below.

$$A_{ij} = \begin{cases} 1 & \text{if } i,j \in E(i,j), \forall (i,j) \\ 0 & \text{if } i,j \notin E(i,j), \forall (i,j) \end{cases}$$

Let $\mathcal{N}_i$ be the set of all the neighbors agent $i$ in the network has and $|\mathcal{N}_i|$ gives the number of neighbors. In this network we consider that all the agents are non holonomic and have same dynamics. An example of an undirected graph with 5 nodes is given in figure(1) and follows the graph description given above.

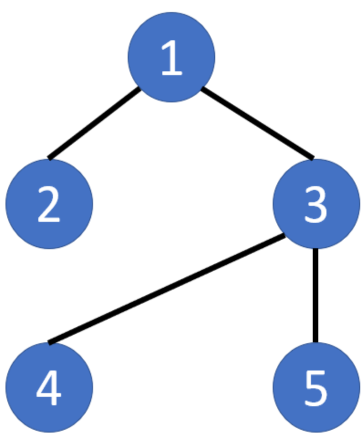


Fig. 1. Undirected 5 Node Graph.

## II. Agent Dynamics Description

Let $X_i = [v_{xi}, v_{zi}, \omega_i, x_i, y_i, z_i, \theta_i]^T$ be the states of a non-holonomic robot, where $i = \{1, ..., n\}$ gives the number of agents. $x_i, y_i, z_i$ and $v_{xi}, v_{zi}$ are the linear displacements and velocities in the respective X,Y,Z axis, $\omega$ is the angular velocity about Z axis and $\theta$ is the heading angle in XY plane.

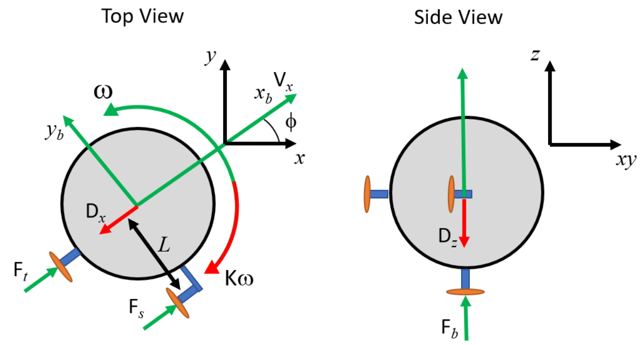


Fig. 2. Free body diagram for the robot in two views. Top view (left). Side view (Right).

The dynamics of the robot is as following.

$$\begin{aligned}
\dot{v_x} &= (F_t - D_x)/m && (1)\\
\dot{v_z} &= (F_b - D_z)/m && (2)\\
\dot{\omega} &= (L.F_s - K\omega)/J && (3)\\
\dot{\theta} &= \omega && (4)\\
\dot{x} &= v_x cos(\theta) && (5)\\
\dot{y} &= v_x sin(\theta) && (6)\\
\dot{z} &= v_z && (7)\\
D_x &= \rho v_x^2 C_d A/2 && (8)\\
D_z &= \rho v_z^2 C_d A/2 && (9)
\end{aligned}$$

Where $D_x, D_z, K$ are the drag in XY plane, drag in Z direction, and damping coefficients and $c_d$ is the coefficient of the drag. Since the robot is spherical, the coefficient of drag and the area of cross-section of the robot given by $A$ is same in any direction. $\rho$ is the density of the air. $L, m, J$ are the distance from the center of gravity of the blimp to the tail, mass of the blimp, moment of inertia respectively. The inputs are $F_t, F_b, F_s$ where they are tail thruster, bottom thruster and side thruster force. The free-body diagram of the blimp is given in figure(2).

## III. Control Objective for Velocity Matching

For the network to match velocities, the difference between the velocities should be zero. That means the relative velocity is zero. Let $e_{xij}, e_{zij}$ be the error in the velocities in $X, Z$ direction and $e_{\omega ij}$ is the error in the angular velocity about

$Z$ axis of agent $i$ from agent $j$. The error for the self loop is always zero. This is given by the following equations.

$$e_{xij} = V_{xi} - V_{xj}, \forall(i,j) \tag{10}$$

$$e_{zij} = V_{zi} - V_{zj}, \forall(i,j) \tag{11}$$

$$e_{\theta ij} = \theta_i - \theta_j, \forall(i,j) \tag{12}$$

Using equations (10),(11),(12) we can create a positive semi-definite objective function given in equation (13).

$$H = \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}(e_{xij}^2 + e_{xij}^2 + e_{\omega ij}^2) \tag{13}$$

The lie derivative of the objective function is given below.

$$\dot{H} = \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}(e_{xij}\dot{e_{xij}} + e_{xij}\dot{e_{xij}} + e_{\omega ij}\dot{e_{\omega ij}}) \tag{14}$$

Substituting equations (10),(11),(12) in (14) we get the derivative of the objective function which includes the dynamics of the robots.

$$\dot{H} = \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}((v_{xi}-v_{xj})(\dot{v_{xi}}-\dot{v_{xj}})+(v_{zi}-v_{zi})(\dot{v_{zi}}-\dot{v_{zj}}) + (\omega_i - \omega_j)(\dot{\omega_i} - \dot{\omega_j})) \tag{15}$$

In equation (15) we can now substitute dynamics from (1)(2)(3) which gives us access to the inputs $F_t, F_b, F_s$ which controls the each single agent. The objective now is to find a control law for these inputs such that $\dot{H} < 0$ when $(e_{xij}, e_{zij}, e_{\omega ij}) \neq (0,0,0)$.

## IV. Finding the Control Law for the Inputs

Lets consider the first term in equation (15) *i.e.* $\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}(v_{xi} - v_{xj})(\dot{v_{xi}} - \dot{v_{xj}})$. Now expanding this out would give,

$$\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}((v_{xi} - v_{xj})(\dot{v_{xi}} - \dot{v_{xj}})) = 2\sum_{i=1}^{n}\dot{v_{xi}}(|\mathcal{N}_i|v_{xi} - \sum_{j\in\mathcal{N}_i}v_{xj}) \tag{16}$$

similarly for the second and third term on the right in (15), we get

$$\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}((v_{zi} - v_{zj})(\dot{v_{zi}} - \dot{v_{zj}})) = 2\sum_{i=1}^{n}\dot{v_{zi}}(|\mathcal{N}_i|v_{zi} - \sum_{j\in\mathcal{N}_i}v_{zj}) \tag{17}$$

$$\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}((\omega_i - \omega_j)(\dot{\omega_i} - \dot{\omega_j})) = 2\sum_{i=1}^{n}\dot{\omega}_i(|\mathcal{N}_i|\omega_i - \sum_{j\in\mathcal{N}_i}\omega_j) \tag{18}$$

Rewriting (15) using (16),(17) and (18), we transform $\dot{H}$ in terms of agent $i$.

$$\dot{H} = 2\sum_{i=1}^{n}(\dot{v_{xi}}(|\mathcal{N}_i|v_{xi} - \sum_{j\in\mathcal{N}_i}v_{xj}) + \dot{v_{zi}}(|\mathcal{N}_i|v_{zi} - \sum_{j\in\mathcal{N}_i}v_{zj}) + \dot{\omega}_i(|\mathcal{N}_i|\omega_i - \sum_{j\in\mathcal{N}_i}\omega_j)) \tag{19}$$

Consider equation first term of equation (19)

$$2\sum_{i=1}^{n}\dot{v_{xi}}(|\mathcal{N}_i|v_{xi} - \sum_{j\in\mathcal{N}_i}v_{xj}) \tag{20}$$

substituting (1) in $\dot{v_{xi}}$ in (20), we can rewrite (20) as

$$2\sum_{i=1}^{n}(F_t - D_x)(|\mathcal{N}_i|v_{xi} - \sum_{j\in\mathcal{N}_i}v_{xj})/m \tag{21}$$

for the first term in $\dot{H}$ to be negative semi-definite, $F_t$ needs to be as follows

$$F_{ti} = -((|\mathcal{N}_i|v_{xi} - \sum_{j\in\mathcal{N}_i}v_{xj})m - D_x) \tag{22}$$

Similarly, by substituting (2),(3) in $\dot{v_{zi}}$,$\dot{\omega_i}$ in the expansion of second term and third of (19) and $\dot{H}$ to be negative semi-definite, $F_b$,$F_s$ needs to be as follows

$$F_{bi} = -((|\mathcal{N}_i|v_{zi} - \sum_{j\in\mathcal{N}_i}v_{zj})m - D_z) \tag{23}$$

$$F_{si} = -((|\mathcal{N}_i|\omega_i - \sum_{j\in\mathcal{N}_i}\omega_j)J - K\omega)/L \tag{24}$$

substituting (22),(23) and (24) in (19)

$$\dot{H} = -\frac{2}{m}\sum_{i=1}^{n}((|\mathcal{N}_i|v_{xi} - \sum_{j\in\mathcal{N}_i}v_{xj})^2 + (|\mathcal{N}_i|v_{zi} - \sum_{j\in\mathcal{N}_i}v_{zj})^2 + (|\mathcal{N}_i|\omega_i - \sum_{j\in\mathcal{N}_i}\omega_j)^2) \tag{25}$$

$\therefore \dot{H} < 0$

## V. Heading Matching instead of Angular Velocity

Replacing (12) $E_{\theta ij} = \theta_i - \theta_j, \forall(i,j)$ and using this in the third term of (13) we will still have $H > 0$. Then the third term in (15) will be as forlloving

$$\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}(\theta_i - \theta_j)(\dot{\theta}_i - \dot{\theta}_j) \tag{26}$$

Similar to (16) we can write (26) as

$$\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}((\theta_i-\theta_j)(\dot{\theta}_i-\dot{\theta}_j)) = 2\sum_{i=1}^{n}\dot{\theta}_i(|\mathcal{N}_i|\theta_i-\sum_{j\in\mathcal{N}_i}\theta_j) \quad (27)$$

Using (4) in (27) we get,

$$\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i}((\theta_i-\theta_j)(\omega_i-\omega_j)) = 2\sum_{i=1}^{n}\omega_i(|\mathcal{N}_i|\theta_i-\sum_{j\in\mathcal{N}_i}\theta_j) \quad (28)$$

for LHS to be always negative,

$$\omega_i = -(|\mathcal{N}_i|\theta_i-\sum_{j\in\mathcal{N}_i}\theta_j) \quad (29)$$

Since our input for lies in $\dot{\omega_i}$, We differentiate (29). we get,

$$\dot{\omega_i} = -(|\mathcal{N}_i|\dot{\theta_i}-\sum_{j\in\mathcal{N}_i}\dot{\theta_j}) \quad (30)$$

substituting (3) in (30)

$$LF_{si}-K\omega_i = -(|\mathcal{N}_i|\omega_i-\sum_{j\in\mathcal{N}_i}\omega_j)J \quad (31)$$

then,

$$F_{si} = -((|\mathcal{N}_i|\omega_i-\sum_{j\in\mathcal{N}_i}\omega_j)J-K\omega_i)/L \quad (32)$$

Where (32) is equal to (24)

## VI. Modifying Dynamics with inputs

Substituting (22),(23) and (24) in (1),(2) and (3) respectively we get,

$$\dot{v_{xi}} = (-(|\mathcal{N}_i|v_{xi}-\sum_{j\in\mathcal{N}_i}v_{xj})) \quad (33)$$

$$\dot{v_{zi}} = (-(|\mathcal{N}_i|v_{zi}-\sum_{j\in\mathcal{N}_i}v_{zj})) \quad (34)$$

$$\dot{\omega_i} = (-(|\mathcal{N}_i|\omega_i-\sum_{j\in\mathcal{N}_i}\omega_j)) \quad (35)$$

Replacing (33),(34),(35) in place of (1),(2),(3) respectively and we rewrite the dynamics as,

$$\dot{v_{xi}} = (-(|\mathcal{N}_i|v_{xi}-\sum_{j\in\mathcal{N}_i}v_{xj})) \quad (36)$$

$$\dot{v_{zi}} = (-(|\mathcal{N}_i|v_{zi}-\sum_{j\in\mathcal{N}_i}v_{zj})) \quad (37)$$

$$\dot{\omega_i} = (-(|\mathcal{N}_i|\omega_i-\sum_{j\in\mathcal{N}_i}\omega_j)) \quad (38)$$

$$\dot{\theta_i} = \omega_i \quad (39)$$

$$\dot{xi} = v_{xi}cos(\theta_i) \quad (40)$$

$$\dot{yi} = v_{xi}sin(\theta_i) \quad (41)$$

$$\dot{zi} = v_{zi} \quad (42)$$

$$D_{xi} = \rho v_{xi}^2 C_d A/2 \quad (43)$$

$$D_{zi} = \rho v_{zi}^2 C_d A/2 \quad (44)$$

$$\dot{H} = 2\sum_{i=1}^{n}(F_t-D_x)(|\mathcal{N}_i|v_{xi}-\sum_{j\in\mathcal{N}_i}v_{xj})/m + 2\sum_{i=1}^{n}(F_b-D_z)(|\mathcal{N}_i|v_{zi}-\sum_{j\in\mathcal{N}_i}v_{zj})/m + 2\sum_{i=1}^{n}\omega_i(|\mathcal{N}_i|\theta_i-\sum_{j\in\mathcal{N}_i}\theta_j)/J \quad (45)$$