Velocity Matching for Nonholonomic Blimp Robots

Dinesh Kumar Karri

I. Graph Description

let $G = (V,E)$ be an undirected graph representing the a network of non-holonomic robots system. $V \in R^n$ is the number of nodes where each node represents a robot. E is the set of all the edges between any two nodes. Each edge present between any two nodes means that there is two way communication between the two agents. Since an agent in the network will have access to its own states we also consider that there exists self loops. Let $A \in R^{n \times n}$ be the adjacency matrix of the graph G. The elements in the matrix A is determined from the graph G as given below.

$$A_{ij} = \begin{cases} 1 & \text{if } i,j \in E(i,j), \forall (i,j) \\ 0 & \text{if } i,j \notin E(i,j), \forall (i,j) \end{cases}$$

Let $\mathcal{N}_i$ be the set of all the neighbors agent i in the network has and $|\mathcal{N}_i|$ gives the number of neighbors. In this network we consider that all the agents are non holonomic and have same dynamics. An example of an undirected graph with 5 nodes is given in figure(1) and follows the graph description given above.

II. Agent Dynamics Description

Let $X_i = [v_{xi}, v_{zi}, \omega_i, x_i, y_i, z_i, \theta_i]^T$ be the states of a non-holonomic robot, where $i = \{1, ..., n\}$ gives the number of agents. x_i, y_i, z_i and v_{xi}, v_{zi} are the linear displacements and velocities in the respective X,Y,Z axis, ω is the angular velocity about Z axis and θ is the heading angle in XY plane. The dynamics of the robot is as following.

$$\dot{v_r} = (F_t - D_r)/m \tag{1}$$
$$\dot{v_z} = (F_b - D_z)/m \tag{2}$$
$$\dot{\omega} = (L.F_s - K\omega)/J \tag{3}$$
$$\dot{\theta} = \omega \tag{4}$$
$$\dot{r_x} = v_r cos(\theta) \tag{5}$$
$$\dot{r_y} = v_r sin(\theta) \tag{6}$$
$$\dot{z} = v_z \tag{7}$$
$$D_r = \rho v_r^2 C_d A/2 \tag{8}$$
$$D_z = \rho v_z^2 C_d A/2 \tag{9}$$

Where D_x, D_z, K are the drag in XY plane, drag in Z direction, and damping coefficients and c_d is the coefficient of the drag. Since the robot is spherical, the coefficient of drag and the area of cross-section of the robot given by A is same in any direction. ρ is the density of the air. L, m, J are the distance from the center of gravity of the blimp to the tail, mass of the blimp, moment of inertia respectively. The inputs are F_t, F_b, F_s where they are tail thruster, bottom thruster and side thruster force. The free-body diagram of the blimp is given in figure(2).

III. XY Planar Velocity Matching

let e_1 the sum relative differences between the velocities in XY plane of all agents.

$$e_1 = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} (v_{ri} - v_{rj}) \tag{10}$$

let V_1 be the Lyapunov function with respect to e_1

$$V_1 = \frac{1}{2} e_1^2 \tag{11}$$

$$\dot{V_1} = e_1 \dot{e_1} \tag{12}$$

so,

$$\dot{V_1} = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} ((v_{ri} - v_{rj})(\dot{v_{ri}} - \dot{v_{rj}})) \tag{13}$$

We can rewrite (13) as following

$$\dot{V_1} = 2 \sum_{i=1}^{n} \dot{v_{ri}} (|\mathcal{N}_i| v_{ri} - \sum_{j \in \mathcal{N}_i} v_{rj}) \tag{14}$$

Substituting (1) in (14)

$$\dot{V_1} = 2 \sum_{i=1}^{n} (F_{ti} - D_r)(|\mathcal{N}_i| v_{ri} - \sum_{j \in \mathcal{N}_i} v_{rj})/m \tag{15}$$

if,

$$F_{ti} = -k_{1i}((|\mathcal{N}_i| v_{ri} - \sum_{j \in \mathcal{N}_i} v_{rj})m - D_r) \tag{16}$$

then,

$$\dot{V_1} = -2 \sum_{i=1}^{n} k_{1i} (|\mathcal{N}_i| v_{ri} - \sum_{j \in \mathcal{N}_i} v_{rj})^2 \tag{17}$$

so, using F_{ti} in (16) in (15), we can say that,

$$\dot{V_1} < 0 \tag{18}$$

IV. Z Velocity Matching

let e_2 the sum relative differences between the velocities in Z direction of all agents.

$$e_2 = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} (v_{zi} - v_{zj}) \tag{19}$$

let V_2 be the Lyapunov function with respect to e_2

$$V_2 = \frac{1}{2} e_2^2 \tag{20}$$

$$\dot{V_2} = e_2 \dot{e_2} \tag{21}$$

so,

$$\dot{V_2} = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} ((v_{zi} - v_{zj})(\dot{v_{zi}} - \dot{v_{zj}})) \tag{22}$$

We can rewrite (22) as following

$$\dot{V_2} = 2 \sum_{i=1}^{n} \dot{v_{zi}} (|\mathcal{N}_i| v_{zi} - \sum_{j \in \mathcal{N}_i} v_{zj}) \tag{23}$$

Substituting (2) in (23)

$$\dot{V_2} = 2 \sum_{i=1}^{n} (F_{bi} - D_z)(|\mathcal{N}_i| v_{zi} - \sum_{j \in \mathcal{N}_i} v_{zj})/m \tag{24}$$

if,

$$F_{bi} = -k_{2i}((|\mathcal{N}_i| v_{zi} - \sum_{j \in \mathcal{N}_i} v_{zj})m - D_z) \tag{25}$$

then,

$$\dot{V_2} = -2 \sum_{i=1}^{n} k_{2i} (|\mathcal{N}_i| v_{zi} - \sum_{j \in \mathcal{N}_i} v_{zj})^2 \tag{26}$$

so, using F_{bi} in (25) in (24), we can say that,

$$\dot{V_2} < 0 \tag{27}$$

V. Heading Matching Using Backstepping

Let e_3 be the sum of relative difference between the heading between the agents.

$$e_3 = \sum_{i=1}^{n} \sum_{j \in \mathcal{N}_i} (\theta_i - \theta_j) \tag{28}$$

Let V_3 be the Lyapunov function with respect to e_3.

$$V_3 = \frac{1}{2} e_3^2 \tag{29}$$

$$\dot{V_3} = e_3 \dot{e_3} \tag{30}$$

$$\dot{V_3} = 2 \sum_{i=1}^{n} \omega_i (|\mathcal{N}_i| \theta_i - \sum_{j \in |\mathcal{N}_i|} \theta_j) \tag{31}$$

let α_1 be the desired value for ω_i which would make $\dot{V_3}$ negative.

$$\alpha_1 = -(|\mathcal{N}_i| \theta_i - \sum_{j \in |\mathcal{N}_i|} \theta_j) \tag{32}$$

So we need

$$\omega_i \to \alpha_1 \tag{33}$$

if $\omega_i = \alpha_1$

$$\dot{V_3} = -2 \sum_{i=1}^{n} (|\mathcal{N}_i| \theta_i - \sum_{j \in |\mathcal{N}_i|} \theta_j)^2 \tag{34}$$

$$\dot{V_3} < 0 \tag{35}$$

now that we have a desired value for ω_i we want to drive the difference between the actual ω_i and α_1 to zero. This difference is given by e_4.

$$e_4 = \omega_i - \alpha_1 \tag{36}$$

$$e_4 = \omega_i + (|\mathcal{N}_i| \theta_i - \sum_{j \in |\mathcal{N}_i|} \theta_j) \tag{37}$$

then

$$\dot{e_4} = \dot{\omega_i} + (|\mathcal{N}_i| \omega_i - \sum_{j \in |\mathcal{N}_i|} \omega_j) \tag{38}$$

V_4 is the Lyapunov function used to drive e_4 to zero.

$$V_4 = \frac{1}{2} e_4^2 \tag{39}$$

$$\dot{V_4} = e_4 \dot{e_4} \tag{40}$$

$$\dot{V_4} = (\omega_i + (|\mathcal{N}_i| \theta_i - \sum_{j \in |\mathcal{N}_i|} \theta_j)) \\ (\dot{\omega_i} + (|\mathcal{N}_i| \omega_i - \sum_{j \in |\mathcal{N}_i|} \omega_j)) \tag{41}$$

substituting (3) in (41) in $\dot{\omega_i}$.

$$\dot{V_4} = (\omega_i + (|\mathcal{N}_i| \theta_i - \sum_{j \in |\mathcal{N}_i|} \theta_j)) \\ (((L.F_{si} - K\omega)/J) + (|\mathcal{N}_i| \omega_i - \sum_{j \in |\mathcal{N}_i|} \omega_j)) \tag{42}$$

if,

$$F_{si} = (J(-(\omega_i + (|\mathcal{N}_i| \theta_i - \sum_{j \in |\mathcal{N}_i|} \theta_j)) - (|\mathcal{N}_i| \omega_i - \sum_{j \in |\mathcal{N}_i|} \omega_j)) \\ + K\omega_i) \frac{1}{L} \tag{43}$$

Then,

$$\dot{V_4} = -(\omega_i + (|\mathcal{N}_i| \theta_i - \sum_{j \in |\mathcal{N}_i|} \theta_j))^2 \tag{44}$$

$$\dot{V_4} < 0 \tag{45}$$

let

$$V_5 = V_3 + V_4 \tag{46}$$

$$\dot{V_5} = \dot{V_3} + \dot{V_4} \tag{47}$$

So,

$$\dot{V_5} < 0 \tag{48}$$

So,

$$V = V_1 + V_2 + V_5 \tag{49}$$

then

$$\dot{V} = \dot{V_1} + \dot{V_2} + \dot{V_5} \tag{50}$$

since $\dot{V_1} < 0$,$\dot{V_2} < 0$ and $\dot{V_5} < 0$,

$$\dot{V} < 0 \tag{51}$$

VI. Modified Agent Dynamics in 2D Description

$$\dot{v_x} = (F_t \cos\delta - D_x \cos\beta - S \sin\beta)/m \tag{52}$$

$$\dot{v_y} = (F_t \sin\delta - D_x \sin\beta + S \cos\beta)/m \tag{53}$$

$$\dot{\omega} = (L.F_t \sin\delta + M_z - K\omega)/J \tag{54}$$

$$\dot{\theta} = \omega \tag{55}$$

$$\dot{x} = v_x \cos\theta - v_y \sin\theta \tag{56}$$

$$\dot{x} = v_x \sin\theta + v_y \cos\theta\beta = v_y/v_x; \tag{57}$$

Where,

$$D_x = \rho V^2 C_D(\beta) A/2 \tag{58}$$

$$C_D(\beta) = C_D^0 + C_D^\beta \beta^2 \tag{59}$$

$$S = \rho V^2 C_S(\beta) A/2 \tag{60}$$

$$C_S(\beta) = C_S^0 + C_S^\beta \beta \tag{61}$$

$$M_z = \rho V^2 C_M(\beta) A/2 \tag{62}$$

$$C_M(\beta) = C_M^0 + C_M^\beta \beta \tag{63}$$