Project 1 - Martingale:

David Kassin

dkassin3@gatech.edu

Abstract—Two experiments were conducted to investigate the viability of using a martingale betting strategy, betting black on an american standard roulette wheel. The American standard roulette wheel consists of the following: 2 green spaces, 18 black and 18 red spaces (Caesars Entertainment, n.d.). Given the explained roulette wheel, we determined the probability of winning to be (18/38) or roughly 47.37% per spin. Each experiment consisted of 1000 individual spins of a roulette wheel, termed an episode, with the results recorded utilizing the martingale strategy, the results were recorded and analyzed.

EXPERIMENT 1

The first experiment executed the martingale strategy with only an upper bound in terms of winnings. If the winnings during any episode reach or exceed $80, then the betting was stopped and the current winnings were recorded for any subsequent spins left.

1.1 QUESTION SET 1

Upon investigating the results of experiment one, where we ran 1000 simulations and stopped once we reached winnings of at least $80, the probability of winning $80 within 1000 spins in 100%. The results did not produce a single instance where 80 was not reached within 1000 spins. Due to the constraints of experiment 1, there is no penalty for having a significant draw down. An example of an avoided negative outcome is reflected in the recorded minimum winnings value of -$65,532 across the simulations. We also see in the statistics the standard deviation goes to zero. This is caused by all episode winnings converging on $80, which is also indicated in both figure 1 and the mean in figure 2.

1.2 Question Set 2

The expected value of experiment one is 80. This can be determined by taking the results of the simulation with 1000 episodes, and taking the mean for the final column of winning results.[1] Again we can reference figure 2, to see that the mean reaches 80 and then stabilizes roughly around spin 200. Also our analysis of the data indicated that over the course of all simulations, 80 was reached 100% of the time.

1.3 Question Set 3

The standard deviation values show significant volatility until they begin to converge near the 200 spin area. We investigated further into the percentage of episodes which had reached the $80 threshold at given spin intervals, and noticed that even after 150 spins, only 7.08% percent of episodes had reached the $80 threshold.[2] It was also interesting to see that at the 50 and 100 spin intervals, there were no episodes that had reached the $80 threshold. This can explain the increasing movement of the standard deviations during those spin intervals. The standard deviations do begin to converge around the 200 spin time interval, which coincides with our investigation into the percentage of episodes which had reached the $80 threshold by 200 spins. The evidence indicates that by the 200th spin, 99.2% percent of episodes had reached the $80 winnings threshold. It makes sense that the standard deviations converge and stabilize, because 99.2% of the sample size is reporting a winnings value of 80. The actual standard deviation value becomes 0 because every single value in the sample set becomes the same.

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[1] We summarize expected value as the mean of values of possible outcomes(Wikimedia Foundation, 2024). It can mathematically be explained by the following equation: $E[X] = \sum_x xP[X = x]$

[2] The results of our analysis of the percentage of episodes that reached $80 at the 50,100,150, and 200 intervals were as follows: 0.0, 0.0, 7.81, 99.20.

Experiment 2

The second experiment is similar to the first, with the added constraint that it introduces the concept of a bankroll. Now there is a max drawdown that can be reached, -$256, which upon reaching that amount episode has run out of money and can no longer bet. For episodes that reach the -$256 threshold, they report -$256 for the remainder of their spins much the same way that this is done for the upper bound of $80.

2.1 Question Set 4

We determine that the estimated probability of winning $80 within 1000 sequential bets is 63.16%. We were able to ascertain this by expanding our investigation of episodes which had reached the $80 threshold for the full 1000 spins. Our results indicate that 63.16% reach the $80 winnings threshold. The nature of the experiment we are running, allows us to make an inference that because we have run so many simulations, the percentage of episode that reached $80 allow us to infer that about the strategy as a whole.

2.2 Question Set 5

The expected value of experiment 2 was -43.38. We were able to arrive at this number by taking the mean value of the final outcome for all 1000 simulations. Because expected value is the value of a random sample of our strategy, the 1000 simulations provide a fair estimate of what to expect utilizing the strategy with the given constraints. This is a majorly negative expected value, and would indicate that using this strategy over the long term would lose significantly. We see as a result that the martingale strategy has a negative expectancy when there is a lower bound that must be observed.

2.3 Question Set 6

The upper and lower standard deviations differ significantly from experiment 1. In experiment 2, we see an initial steeper divergence from the mean that begins to flatten as the spin intervals increase. It takes a more logarithmic shape. We see a similar phenomena to experiment 1, where in the early time intervals there is significantly more variability in winnings values, but as episodes get locked into either the higher or lower threshold by either winning $80 or losing $256, the standard deviations stabilize. The standard deviations here do not converge, this is because we now have an upper and lower bound that will lock some of the episode values, at either -$256 or $80. As a result there will always be some variability in the numbers. However we do see a decrease in the rate of change of the standard deviation, as the spin time interval increases. This occurs because over time fewer and fewer episodes are not locked at the upper or lower bounds.

2.4 Question Set 7

The benefits of using expected value over simply using the results of a singular episode, is that by calculating and understanding the expected value, you are able to minimize randomness or "luck" from a probability problem. The probability of winning in both experiments was 47.37%. In figure 4 the final value for the standard deviation was $161.866. That gives a range of $323.732 between the upper and lower bound. That is a huge range of possible outcomes. As such, taking the expected value by running 1000 simulations provides a strong estimate of what the most likely outcome of the given strategy will be.

3.1 Figures

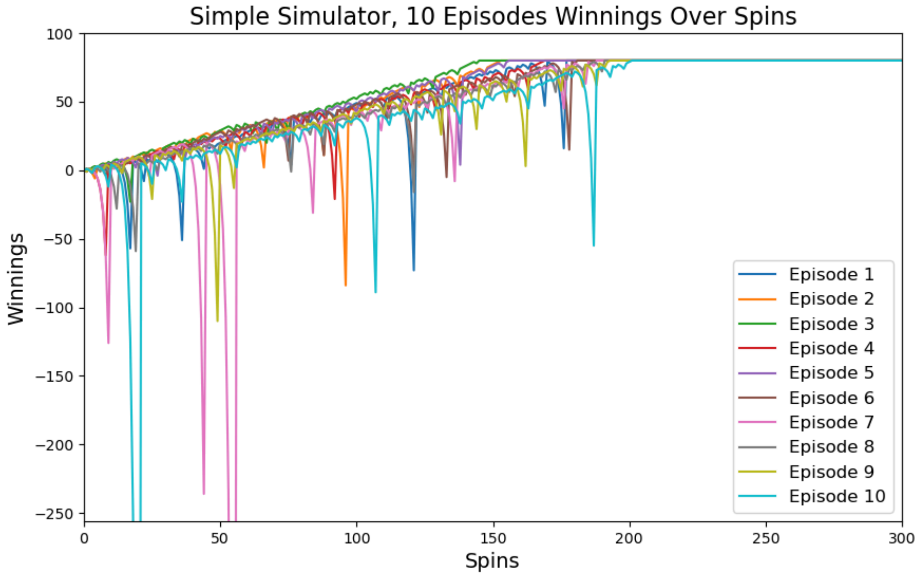

Figure 1— Episode winnings for 10 episodes from experiment one.

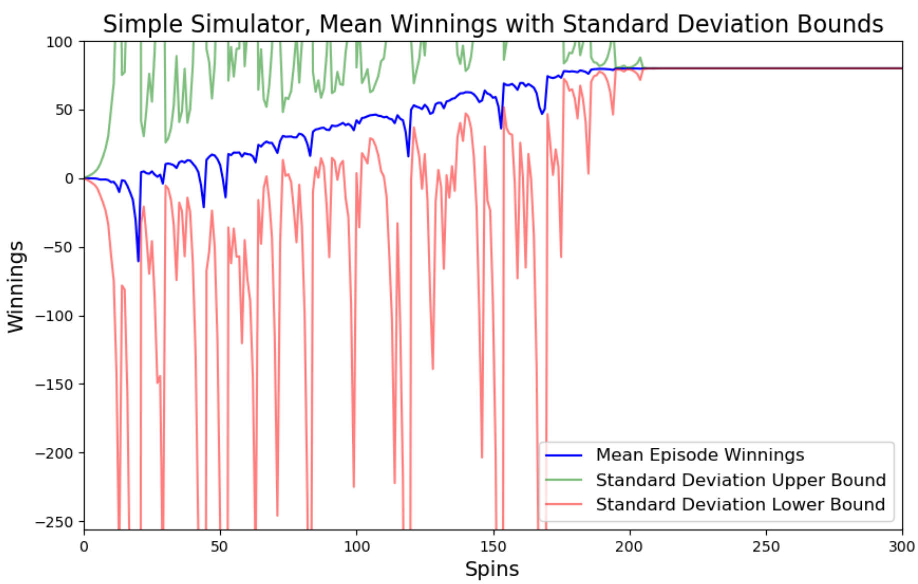

Figure 2— Mean of experiment 1 with upper and lower bounds of standard deviation.

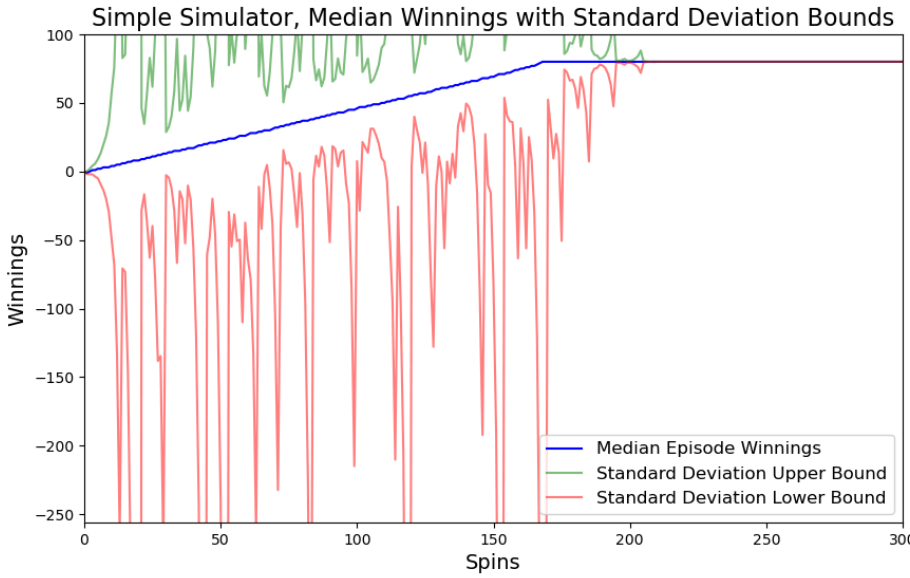

Figure 3—Median of experiment 1 with upper and lower bounds of standard deviation.

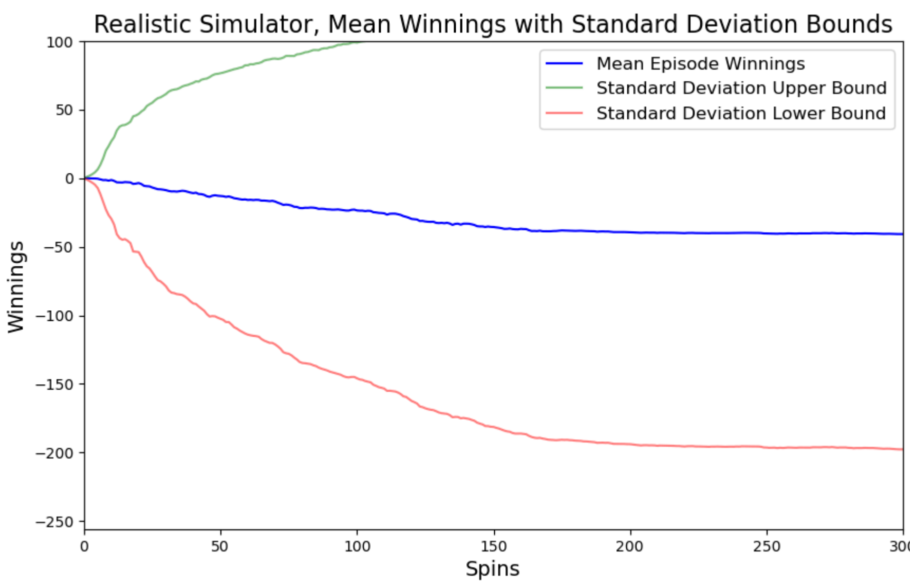

Figure 4—Mean of experiment 2 with upper and lower bounds of standard deviation.

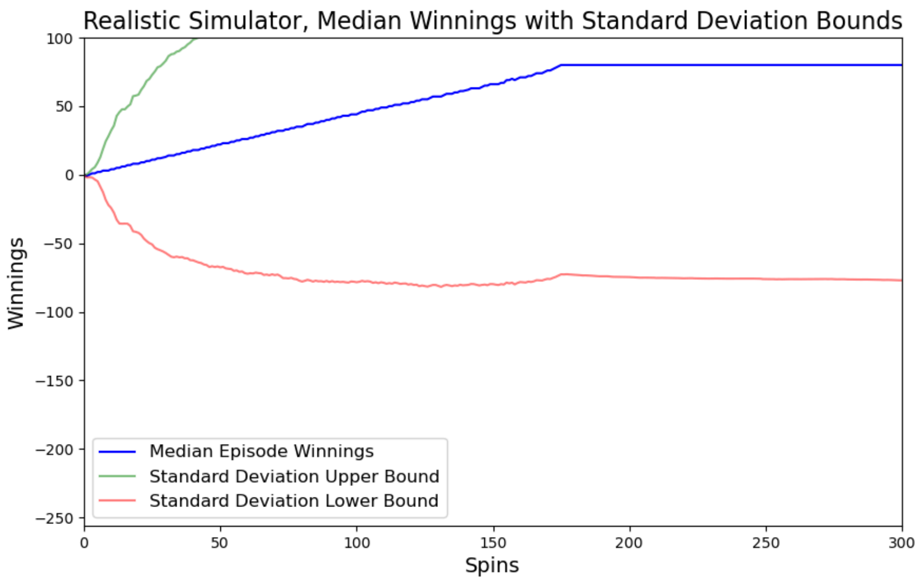

Figure 5—Median of experiment 2 with upper and lower bounds of standard deviation

4 REFERENCES

1. Caesars_Entertainment.(n.d.).*Roulette*. https://www.caesars.com/content/dam/uba/Gaming/updated-rack--cards/roulette.pdf

2. Wikimedia Foundation. (2024, July 23). *Expected value*. Wikipedia. https://en.wikipedia.org/wiki/Expected_value