Nonequilibrium processes 2018
Homework 2: Boltzmann equation

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Due: Saturday April 21 13:00

1 Thermo-current (9*p*)

Consider a classical dilute gas of charged particles with charge q on each, obeying the local Maxwell-Boltzmann distribution in equilibrium. Use the Boltzmann equation in a single relaxation time approximation to find the stationary heat current. Obtain the thermal conductivity κ from

$$j_Q = -\kappa \nabla T. \tag{1}$$

The calculation has to be done for the presence of constant gradient both in electrochemical potential ($\nabla\Phi = \text{const}$) and temperature ($\nabla T = \text{const}$), but for a system with no electric current ($j_e = 0$).
Note: Eq. (45) of the notes, not (46), contains the correct definition of $\tilde{L}$.

2 Hall effect (9*p*)

A thick metal sheet is subject to time-independent and $\vec{x}$-independent electric $\vec{E} = E_x\vec{e}_x$ and magnetic $\vec{H} = H\vec{e}_z$ fields, where $\vec{e}_{x,z}$ are unit vectors pointing in x/z direction. Therefore, the electrons are subject to the Lorentz force

$$\vec{F} = e(\vec{E} + \vec{v} \times \vec{H}), \quad e = -|e|. \tag{2}$$

After some time, the charge build-up at the edges of the conductor generates a transverse electric field $\vec{E}_H = E_y\vec{e}_y$ pointing in y direction.

(a) Consider the case of weak electric and possibly strong magnetic fields. Solve the Boltzmann equation in the relaxation time approximation. Calculate the induced stationary ($\neq$ equilibrium) charge current and obtain the conductivity tensor as

$$j_i = \sum_k \sigma_{ik} E_k. \tag{3}$$

A few important considerations: Assume an infinite size of the metallic plate. Since it is metallic, the equilibrium distribution is the Fermi distribution. Furthermore, you can assume that the anisotropy in the velocity dependence of the non-equilibrium part of the phase space distribution is dipolar: $\vec{C}(\epsilon) \cdot \vec{v}$. **However**, to get full points, you must argue why this assumption is reasonable.

(b) Set the y-component of the current to zero to apply the finite-size constraint on the system, i.e. no stationary current can flow in the y-direction. Calculate the Hall coefficient

$$R = \frac{E_y}{H j_x}, \quad j_y = 0. \tag{4}$$