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September 28, 2019

18.701 Problem Set 4

This pset is due Wednesday, October 3

1. Chapter 3, Exercise M.3. (*polynomial paths*)

This exercise is designed to teach two simple things:
(a) If V is a vectgor space of dimension d, a set $(v_1, ..., v_n)$ of elements of V with $n > d$ is dependent.
(b) Two functions u, v satisfy a polynomial equation of degree $\leq d$ if and only if
the functions $u^i v^j$ with $i+j \leq d$ are dependent.

2. Chapter 4, Exercise 1.4 (*rank one matrices*)

3. Chapter 4, Exercise 1.5. (*about the dimension formula*)

4. Chapter 4, Exercise 6.11 (*eigenvector of a 2×2 matrix*)

5. Chapter 4, Exercise M.1 (*permuting entries of a vector*)

6. Determine the finite-dimensional spaces W of differentiable functions $f(x)$ with this property:
If f is in W, then $\frac{df}{dx}$ is in W.

Before attempting this problem, review constant coefficient differential equations of the form

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_{n-1} \frac{dy}{dx} + a_n = 0$$

It is most convenient, though not essential, to allow complex coefficients