Joint image reconstruction of multi-channel X-ray computed tomography data for material science

Daniil Kazantsev[1,2]

Joint work with: Jakob S. Jørgensen[1,3] and Martin S. Andersen[4]

[1] School of Materials, The University of Manchester, UK
[2] X-ray Imaging Facility, Research Complex at Harwell, UK
[3] School of Mathematics, The University of Manchester, UK
[4] Department of Applied Mathematics and Computer Science, DTU, Denmark

SIAM Conference on Imaging Science, 5-8 June, 2018
Bologna, Italy

MANCHESTER 1824
The University of Manchester

manchester X-Ray Imaging Facility

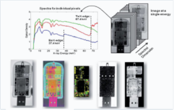
Project partners and applications

Working together with mathematicans, software developers and imaging scientists

A LARGE USER COMMUNITY

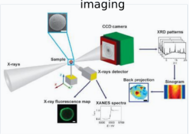
A LARGE USER COMMUNITY

MANCHESTER 1824 The University of Manchester



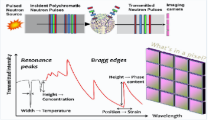
HEXITEC detectors based hyperspectral imaging modality
The Colour Bay

i18, i14 X-ray spectroscopy, difraction and fluorescence imaging



diamond

The neutron imaging facility IMAT
Hyperspectral imaging



ISIS

Reconstruction Toolkit for Multichannel CT (RT-MCT)

- Advanced iterative solutions
- Novel multi-channel correlative priors
- Compressed sensing tools
- Fast, efficient and parallelised software

MANCHESTER 1824 The University of Manchester

Research Complex at Harwell
Working across the life and physical sciences

NVIDIA CUDA C/C++ python

To be embedded into existing framework (Savu, CCPi)

CCPi Tomographic Imaging savu

Academic Impact
Journal publications
Conferences
Workshops
Seminars

Industrial Impact
Important contributions and solutions to materials manufacturing

A LARGE USER COMMUNITY

A LARGE USER COMMUNITY

1. The challenges of multi-energetic computed tomography

2. A generic multi-channel inverse problem

3. Multi-channel regularization approaches

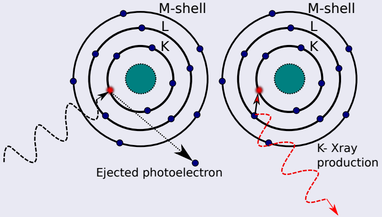
4. Numerical Experiments

5. Some software references

The challenges of multi-energetic computed tomography

X-ray matter interactions

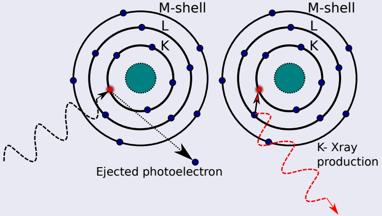
Photoelectric effect



- Inner shell electrons interactions
- Whole of the photon energy is transferred
- To eject, the photon energy must be $>=$ the binding energy

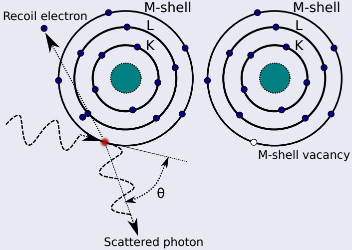
X-ray matter interactions

Photoelectric effect



- Inner shell electrons interactions
- Whole of the photon energy is transferred
- To eject, the photon energy must be >= the binding energy

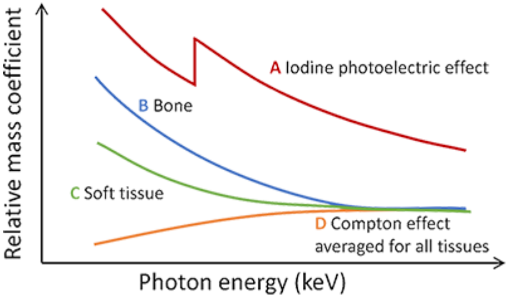
Compton effect



- Outer shell electrons interactions
- Part of the electron energy is transferred
- M-shell is vacant

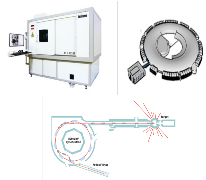
Search for absorption edges (K-edge imaging)

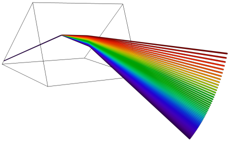
manchester X-Ray Imaging Facility

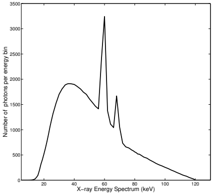


- When the photon energy as strong as the binding energy of K-shell Iodine - the **K-edge** appears (a sharp increase in attenuation)
- Specific energy (33keV for Iodine) can be used for a greater tissue contrast

Polychromaticity of sources



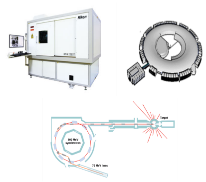


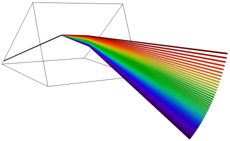


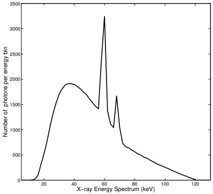
Challenges

- X-ray sources are polychromatic while detectors still being **energy-integrating** (results in nonlinearity, beam-hardening)
- How one can find the K-edge of an unknown material?

Polychromaticity of sources







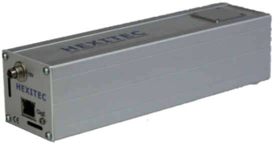
Challenges

- X-ray sources are polychromatic while detectors still being **energy-integrating** (results in nonlinearity, beam-hardening)
- How one can find the K-edge of an unknown material?

What is required in order to benefit on poly-beam?

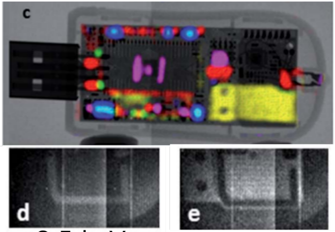
- **Energy-discriminating** detecting systems supporting finer spectral resolution (more than a hundred of energy channels)
- Novel reconstruction approaches employing spectral redundancy
- Efficient, well-written and documented software

On energy-discriminating detectors



HEXITEC[1] spectroscopic hard X-ray imaging detector for K-Edge Enhanced Imaging[2]

- Pixel size: 250 μm $\times$ 250 μm
- Number of pixels: $80 \times 80 = 6400$
- Energy Range: 2-200 keV
- Data Rate: 5M photons/second
- Dimensions: $21 \times 5 \times 5$ cm



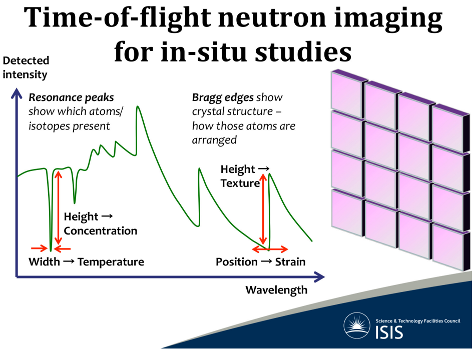
8.5 keV copper

11.9 keV bromine

[1] http://quantumdetectors.com/hexitec/

[2] S. Jacques et al. A laboratory system for element specific hyperspectral X-ray imaging

Time-of-flight Neutron Imaging



Hyper-spectral X-ray imaging on IMAT ISIS[3]

- Using the time-of-flight (TOF) technique applied to neutrons, one can get information about the energy (wavelength)
- 4D (x,y,z + energy) imaging with neutrons is possible

[3] W. Kockelmann, J. Kelleher, T. Minniti, G. Burca and others

Poly-energetic X-ray CT measurement model

Given a poly-energetic X-ray source, the spectral version of Beer's law can be expressed as:

$$\Lambda_i(E) = \sigma_i(E) \exp\left(-\int_{L_i} \mu(\boldsymbol{r}, E)\, \mathrm{d}l\right), \quad i = 1, \ldots, M.$$

Poly-energetic X-ray CT measurement model

manchester X-Ray Imaging Facility

Given a poly-energetic X-ray source, the spectral version of Beer's law can be expressed as:

$$\Lambda_i(E) = \sigma_i(E) \exp\left(- \int_{L_i} \mu(\boldsymbol{r}, E) \, \mathrm{d}l \right), \quad i = 1, \ldots, M.$$

With necessary parametrization we write the discrete model:

$$\Lambda_i(E_k) = \sigma_i(E_k) \exp\left(- \sum_{j=1}^{N} A_{ij} \mu_j(E_k) \right), \quad i = 1, \ldots, M, \quad k = 1, \ldots, K.$$

This model is an approximation, it tends to be more accurate with finer spectral discretization.

Poly-energetic X-ray CT measurement model

If we define $X_{jk} = \mu_j(E_k)$ and $S_{ik} = \sigma_i(E_k)$, we arrive at the discrete linear model:

$$B_{ik} = -\ln\left(\frac{Y_{ik}}{S_{ik}}\right) \approx \sum_{j=1}^{N} A_{ij} X_{jk}.$$

To simplify notation, we now define a matrix $\boldsymbol{X} \in \mathbb{R}^{N\times K}$ with elements X_{jk}, and $\boldsymbol{x} = \mathbf{vec}(\boldsymbol{X}) \in \mathbb{R}^{NK}$ denotes the vectorized image (i.e., $\boldsymbol{x}$ is obtained by stacking the columns of $\boldsymbol{X}$). Moreover, $\boldsymbol{X}_k$ denotes the kth column of $\boldsymbol{X}$.

Using new notation for $\boldsymbol{X}, \boldsymbol{B}$ we re-write more compactly

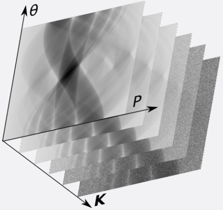
$$\boldsymbol{b} \approx \bar{\boldsymbol{A}}\boldsymbol{x},$$

where $\bar{\boldsymbol{A}} = \boldsymbol{A} \otimes \boldsymbol{I}_{K\times K}$ and $\boldsymbol{A} \in \mathbb{R}^{M\times N}$ is a sparse projection matrix, $\otimes$ is the Kronecker product, and $\boldsymbol{I}_{K\times K}$ is the identity matrix of order K.

A generic multi-channel inverse problem

The variety of multi-channel problems

3D spectral dataset (**K**-channels) consisting of **λ** materials



Reconstruction of all **K** energy channels *without* cross-correlation, i.e. ***channel-wise***

+ simple implementation, computationally efficient

− not effective to deal with low signal SNR, lots of data to process/analyze

Reconstruction of all **K** energy channels *with* cross-correlation, i.e. ***inter-correlated***

+ much improved SNR of reconstructed images

− complicated reconstruction algorithms, computational challenges

Basis materials decomposition

+ drastic dimensionality reduction from **K** to **λ**, when normally **λ** << **K**

− complex and expensive PCA-based methods or ML-based estimations

Reconstruction of only **λ** channels *without* cross-correlation, i.e. ***material-wise***

+ simple implementation, computationally efficient

− better than channel-wise but still not effective against poor SNR data

Reconstruction of only **λ** channels *with* cross-correlation, i.e. ***material-correlated***

+ much improved SNR of reconstructed images, less computationally difficult

− complicated reconstructon algorithms

General multi-channel reconstruction problem

The generic objective function for $\boldsymbol{x} \in \mathbb{R}^{NK}$ is:

$$\boldsymbol{x}^\star = \arg\min_{\boldsymbol{x}} \left\{ \mathcal{F}(\boldsymbol{x}) + \beta\, \mathcal{G}(\boldsymbol{x}) \right\},$$

where $\mathcal{F} : \mathbb{R}^{NK} \to \mathbb{R}_+$ is a smooth convex the **data misfit** term and the **penalty term** $\mathcal{G} : \mathbb{R}^{NK} \to \mathbb{R}_+$ defines a regularization penalty with $\beta > 0$ as a trade-off parameter.

General multi-channel reconstruction problem

manchester X-Ray Imaging Facility

The generic objective function for $\boldsymbol{x} \in \mathbb{R}^{NK}$ is:

$$\boldsymbol{x}^{\star} = \arg\min_{\boldsymbol{x}} \left\{ \mathcal{F}(\boldsymbol{x}) + \beta\, \mathcal{G}(\boldsymbol{x}) \right\},$$

where $\mathcal{F} : \mathbb{R}^{NK} \to \mathbb{R}_{+}$ is a smooth convex the **data misfit** term and the **penalty term** $\mathcal{G} : \mathbb{R}^{NK} \to \mathbb{R}_{+}$ defines a regularization penalty with $\beta > 0$ as a trade-off parameter.

Choosing the data fidelity term $\mathcal{F}(\bar{\boldsymbol{A}}\boldsymbol{x}, \boldsymbol{b})$

- $\mathcal{F}(\cdot) = \|\cdot\|_2^2$ - the Least-Squares misfit (Gaussian noise in data $\boldsymbol{b}$)

General multi-channel reconstruction problem

manchester X-Ray Imaging Facility

The generic objective function for $\boldsymbol{x} \in \mathbb{R}^{NK}$ is:

$$\boldsymbol{x}^\star = \arg\min_{\boldsymbol{x}} \left\{ \mathcal{F}(\boldsymbol{x}) + \beta\, \mathcal{G}(\boldsymbol{x}) \right\},$$

where $\mathcal{F} : \mathbb{R}^{NK} \to \mathbb{R}_+$ is a smooth convex the **data misfit** term and the **penalty term** $\mathcal{G} : \mathbb{R}^{NK} \to \mathbb{R}_+$ defines a regularization penalty with $\beta > 0$ as a trade-off parameter.

Choosing the data fidelity term $\mathcal{F}(\bar{\boldsymbol{A}}\boldsymbol{x}, \boldsymbol{b})$

- $\mathcal{F}(\cdot) = \| \cdot \|_2^2$ - the Least-Squares misfit (Gaussian noise in data $\boldsymbol{b}$)
- $\mathcal{F}(\cdot) = \| \cdot \|_{\boldsymbol{W}}^2$ - PWLS misfit (Poisson noise approximation of $\boldsymbol{b}$ by raw-data $\boldsymbol{y}$ diagonal weighting $\boldsymbol{W} = \mathrm{diag}(\boldsymbol{y})$)

General multi-channel reconstruction problem

The generic objective function for $\boldsymbol{x} \in \mathbb{R}^{NK}$ is:

$$\boldsymbol{x}^\star = \arg\min_{\boldsymbol{x}} \{\mathcal{F}(\boldsymbol{x}) + \beta\, \mathcal{G}(\boldsymbol{x})\},$$

where $\mathcal{F} : \mathbb{R}^{NK} \to \mathbb{R}_+$ is a smooth convex the **data misfit** term and the **penalty term** $\mathcal{G} : \mathbb{R}^{NK} \to \mathbb{R}_+$ defines a regularization penalty with $\beta > 0$ as a trade-off parameter.

Choosing the data fidelity term $\mathcal{F}(\bar{\boldsymbol{A}}\boldsymbol{x}, \boldsymbol{b})$

- $\mathcal{F}(\cdot) = \|\cdot\|_2^2$ - the Least-Squares misfit (Gaussian noise in data $\boldsymbol{b}$)
- $\mathcal{F}(\cdot) = \|\cdot\|_{\boldsymbol{W}}^2$ - PWLS misfit (Poisson noise approximation of $\boldsymbol{b}$ by raw-data $\boldsymbol{y}$ diagonal weighting $\boldsymbol{W} = \mathrm{diag}(\boldsymbol{y})$)
- Other nonlinear/nonconvex terms can be employed, such as, ℓ_1, Huber, Student's t misfit[a], and others.

[a] **K.** et al. *A novel tomographic reconstruction method based on the robust Student's t function for suppressing data outliers*, IEEE TCI, 2017

Multi-channel regularization approaches

Channel-wise regularization strategies

manchester X-Ray Imaging Facility

The total variation (TV) penalty[4] for a (single-channel) image $\boldsymbol{v} \in \mathbb{R}^N$:

$$\mathsf{TV}(\boldsymbol{v}) = \sum_{j=1}^{N} \|\boldsymbol{D}_j \boldsymbol{v}\|_2,$$

where $\boldsymbol{D}_j$ is a $2 \times N$ matrix such that $\boldsymbol{D}_j \boldsymbol{v}$ is a finite-difference approximation of the gradient of $\boldsymbol{v}$ at pixel j.

[4] L. Rudin et al., *Nonlinear total variation based noise removal algorithms*, 1992

Channel-wise regularization strategies

The total variation (TV) penalty[4] for a (single-channel) image $\boldsymbol{v} \in \mathbb{R}^N$:

$$\mathsf{TV}(\boldsymbol{v}) = \sum_{j=1}^{N} \|\boldsymbol{D}_j \boldsymbol{v}\|_2,$$

where $\boldsymbol{D}_j$ is a $2 \times N$ matrix such that $\boldsymbol{D}_j \boldsymbol{v}$ is a finite-difference approximation of the gradient of $\boldsymbol{v}$ at pixel j.

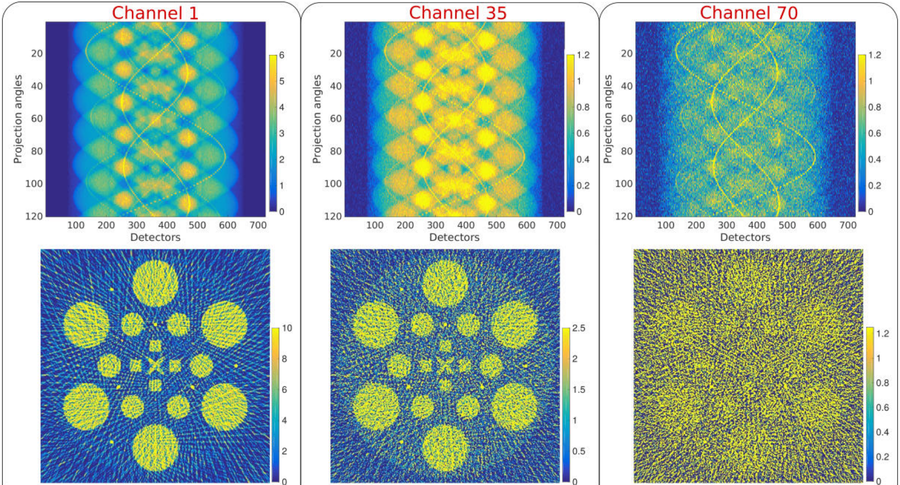
For multi-channel CT, one can consider using **channel-wise** TV:

$$\mathcal{G}_{\mathsf{TV}}(\boldsymbol{x}) = \sum_{k=1}^{K} \mathsf{TV}(\boldsymbol{X}_k), k = 1, \cdots, K$$

The significant disadvantage of channel-wise TV is that there is no correlation between channels k.

[4] L. Rudin et al., *Nonlinear total variation based noise removal algorithms*, 1992

Why one needs correlative regularization?



(a) FBP, $\Delta_1 = 0.017$ (b) FBP, $\Delta_{35} = 0.025$ (c) FBP, $\Delta_{70} = 0.032$

Image reconstruction using Filtered-Backprojection (FBP) method

Introducing vector-valued correlative strategies

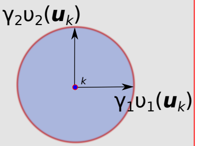
Uniform (constant) area

Vectorial (level sets) framework

$[D\boldsymbol{u}]_k = 0$

All singular values of Jacobian (J) are zero

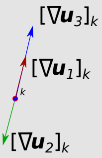
Tensor-based framework



Mixture of isotropic tensors

Correlated channels

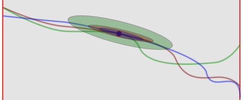
Vectorial (level sets) framework



Parallel level sets at point k
One singular value is non-zero ($\boldsymbol{u}_2$)

Tensor-based framework

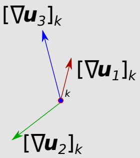
Three anisotropic (diffusion) tensors with the same eigenvectors, yet different eigenvalues



eigenvectors (υ): direction of diffusion
eigenvalues (γ): strength of diffusion

Uncorrelated channels

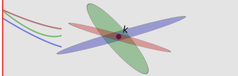
Vectorial (level sets) framework



Unique gradient directions
Nonzero singular values

Tensor-based framework

Unique diffusion directions and diffisive forces



One can interpolate between tensors to obtained an averaged tensor

?

Total Nuclear Variation functional

One can consider of penalizing Jacobian which is obtained by applying the finite difference operator at pixel j to all channels simultaneously, i.e., $\boldsymbol{D}_j\boldsymbol{X}$ for $j = 1, \ldots, N$. In particular the Total Nuclear Variation (TNV)[5] is defined as

$$\mathcal{G}_{\mathsf{TNV}}(\boldsymbol{x}) = \sum_{j=1}^{N} \|\boldsymbol{D}_j\boldsymbol{X}\|_*,$$

where $\|\cdot\|_*$ denotes the nuclear norm, i.e., the sum of the singular values. Singular value analysis shows that TNV encourages the gradient vectors of each energy channels to be aligned which is equivalent to a low rank Jacobian.

[5]D.S. Rigie et al. *Joint reconstruction of multi-channel, spectral CT data via constrained total nuclear variation minimization*, PMB, 2015

K. Holt, *Total nuclear variation and Jacobian extensions of total variation for vector fields*, IEEE, 2014

Diffusion-based interpretation of TNV model

Regularization functional for vectorial image $\boldsymbol{u}$:

$$E(\boldsymbol{u}) = \int_{\Omega} \psi(\lambda^{+}, \lambda^{-}) d\Omega,$$

where $\lambda^{+,-}$ are the larger and smaller eigenvalues of the structure tensor $k_{\rho} * (\nabla \boldsymbol{u}_{\sigma}^{\top} \nabla \boldsymbol{u}_{\sigma})$, respectively.

[6] S. Lefkimmiatis et al. *Structure tensor total variation*, SIAM Imaging Sciences, 2015
J. Duran et al. *Collaborative Total Variation*, SIAM Imaging Sciences, 2016

Diffusion-based interpretation of TNV model

Regularization functional for vectorial image $\boldsymbol{u}$:

$$E(\boldsymbol{u}) = \int_{\Omega} \psi(\lambda^+, \lambda^-) d\Omega,$$

where $\lambda^{+,-}$ are the larger and smaller eigenvalues of the structure tensor $k_\rho * (\nabla \boldsymbol{u}_\sigma^\top \nabla \boldsymbol{u}_\sigma)$, respectively.
One chooses: $\psi(\lambda^+, \lambda^-) = \|(\sqrt{\lambda^+}, \sqrt{\lambda^-})\|_p$ and $p = 1$ for the nuclear norm[6].
The E-L equations then given as:

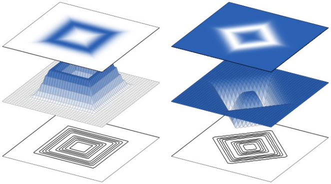
$$\frac{\partial \boldsymbol{u}}{\partial t} = \nabla \cdot (\boldsymbol{D} \nabla \boldsymbol{u}), \ \textit{s.t. BCS},$$

with the diffusion tensor: $\boldsymbol{D} = K_\sigma * \left(\frac{1}{\sqrt{\lambda^+}} \theta^+ \otimes \theta^+ + \frac{1}{\sqrt{\lambda^-}} \theta^- \otimes \theta^- \right)$

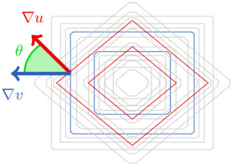
[6] S. Lefkimmiatis et al. *Structure tensor total variation*, SIAM Imaging Sciences, 2015
J. Duran et al. *Collaborative Total Variation*, SIAM Imaging Sciences, 2016

Parallel level sets to drive diffusivity

One can also consider the level sets of images



and introduce correlation between channels through parallel level sets[7]



[7] M.J. Ehrhardt, Joint Reconstruction for Multi-Modality Imaging with Common Structure, Ph.D. Thesis, UCL, UK

M.J. Ehrhardt, S.R. Arridge, *Vector-valued image processing by parallel level sets*. IEEE TIP, 2014

dTV regularization with known reference

The directional TV regularizer (dTV)[8] is introduced as:

$$\mathrm{dTV}(\boldsymbol{v}, \boldsymbol{z}) = \sum_{j=1}^{N} \|\boldsymbol{P}_{\boldsymbol{z}} \boldsymbol{D}_j \boldsymbol{v}\|_2, \tag{1}$$

where:

$$\boldsymbol{P}_{\boldsymbol{z}} = \begin{cases} \boldsymbol{I}_{2\times 2} - \dfrac{\boldsymbol{D}_j \boldsymbol{z} \boldsymbol{z}^\top \boldsymbol{D}_j^\top}{\boldsymbol{z}^\top \boldsymbol{D}_j^\top \boldsymbol{D}_j \boldsymbol{z}} & \boldsymbol{D}_j \boldsymbol{z} \neq 0 \\ \boldsymbol{I}_{2\times 2} & \boldsymbol{D}_j \boldsymbol{z} = 0. \end{cases}$$

The regularization function (1) is the *directional diffusion* of the channel $\boldsymbol{v}$, given a known reference $\boldsymbol{z}$.

[8] M.J. Ehrhardt M.M. Betcke, *Multicontrast MRI reconstruction with structure-guided total variation* SIAM Journal on Imaging Sciences, 2016

Multiple channels with unknown reference

The proposed[9] dTV-type probabilistic regularizer for multi-channel case:

$$\mathcal{G}_{\text{dTV-p}}(\boldsymbol{x}^{[s]}) = \sum_{k=1}^{K} \text{dTV}\left(\boldsymbol{X}_k^{[s]},\ \boldsymbol{X}_{k^\star(s,k)}^{[s-1]}\right), \tag{2}$$

where the *reference* channel (previous estimate) is chosen according to the probability mass function:

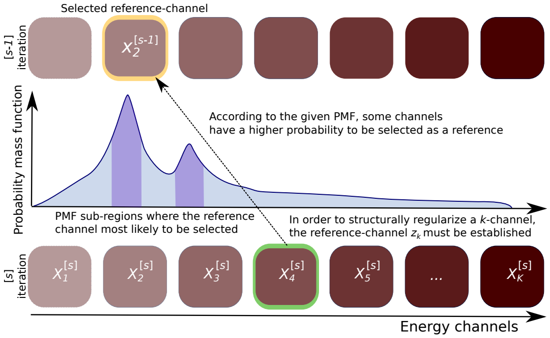
$$\bar{\rho}_{k^\star} = \frac{\rho_{k^\star}}{\sum_{k=1}^{K} \rho_k}, \tag{3}$$

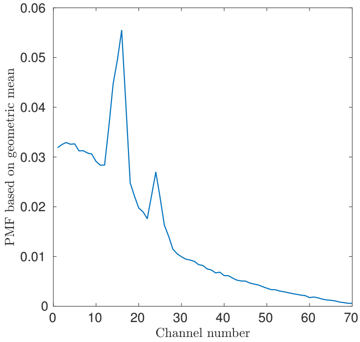
where channel-wise geometric mean of the approximate SNR is

$$\rho_k = \left(\prod_{i=1}^{M} B_{ik}\sqrt{Y_{ik}}\right)^{1/M}.$$

[9] **K.** et al. *Joint image reconstruction method with correlative multi-channel prior for x-ray spectral computed tomography*, Inverse Problems, 2018

Multiple channels with unknown reference





The reference-channel $\boldsymbol{z}_k^{[s]} = \boldsymbol{X}_{k^\star}^{[s-1]}$ is selected by randomly drawing a sample (index $k^\star \in \{1, \ldots, K\}$ of a channel) from the given probability mass function (PMF). The PMF specific to our experiment is depicted in the right figure.

Multi-channel reconstruction algorithm

Algorithm 1 Multi-channel FISTA-based reconstruction scheme

Input: $\boldsymbol{b}$, $\boldsymbol{x}^{[0]}$, β, S, L
Output: $\boldsymbol{x}^{[S]}$

$\boldsymbol{y}^{[1]} = \boldsymbol{x}^{[0]}$, $t^{[1]} = 1$

for all $s = 1, \dots, S$ **do** # FISTA outer iterations

1: $\boldsymbol{u}^{[s]} = \boldsymbol{y}^{[s]} - L^{-1}\nabla\mathcal{F}(\boldsymbol{y}^{[s]})$ # PWLS gradient step

2: $\boldsymbol{x}^{[s]} = \mathrm{prox}_{\beta/L}[\mathcal{G}](\boldsymbol{u}^{[s]})$ # Regularization proximal step

3: $t^{[s+1]} = \left(1 + \sqrt{1 + 4(t^{[s]})^2}\right)/2$

4: $\boldsymbol{y}^{[s+1]} = \boldsymbol{x}^{[s]} + (t^{[s]} - 1)/t^{[s+1]} \cdot (\boldsymbol{x}^{[s]} - \boldsymbol{x}^{[s-1]})$

end for

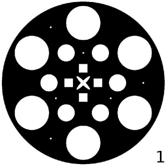
Where the data fidelity is used $\nabla\mathcal{F}(\boldsymbol{v}) = \bar{\boldsymbol{A}}^\top \boldsymbol{W}(\bar{\boldsymbol{A}}\boldsymbol{v} - \boldsymbol{b})$ with the Lipschitz constant of $\nabla\mathcal{F}$, $L = \|\bar{\boldsymbol{A}}^\top \boldsymbol{W}\bar{\boldsymbol{A}}\|_2$. The proximal operator[10] in step 2 is given generally as

$$\mathrm{prox}_{\beta/L}[\mathcal{G}](\boldsymbol{v}) = \arg\min_{\boldsymbol{u}} \left\{ \frac{\beta}{L}\mathcal{G}(\boldsymbol{u}) + \frac{1}{2}\|\boldsymbol{u} - \boldsymbol{v}\|_2^2 \right\}.$$

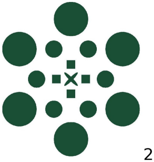
[10] A. Beck, *First-Order Methods in Optimization*, SIAM, 2017

Numerical Experiments

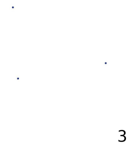
Four materials synthetic phantom



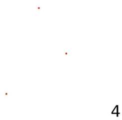
(a) **Quartz**



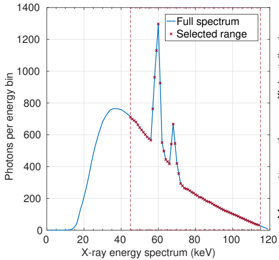
(b) **Pyrite**



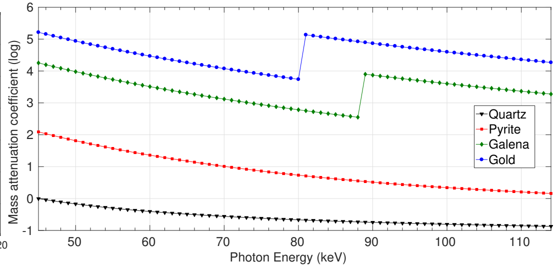
(c) **Galena**



(d) **Gold**



(e) Energy spectr.



(f) Mass attenuation curves material-wise

Image reconstruction with TV

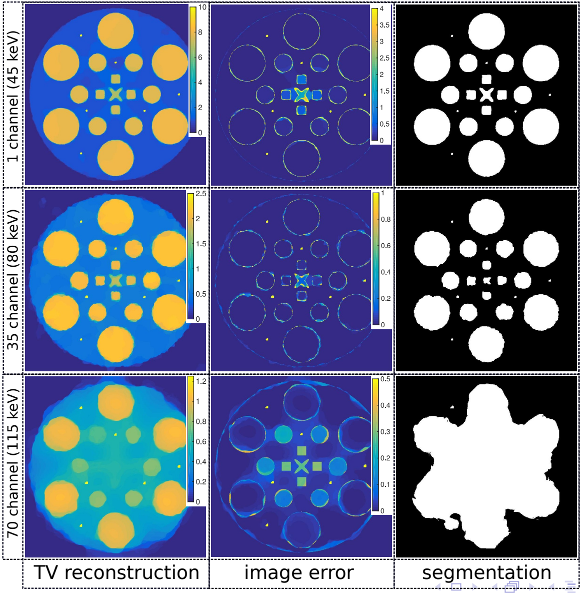


Image reconstruction with TNV

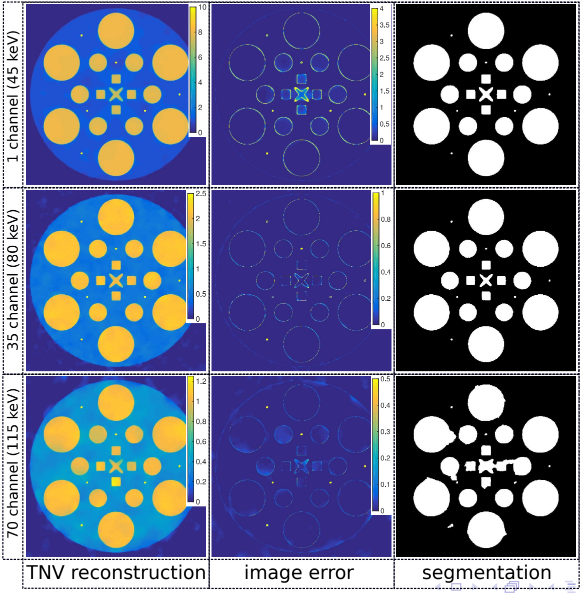
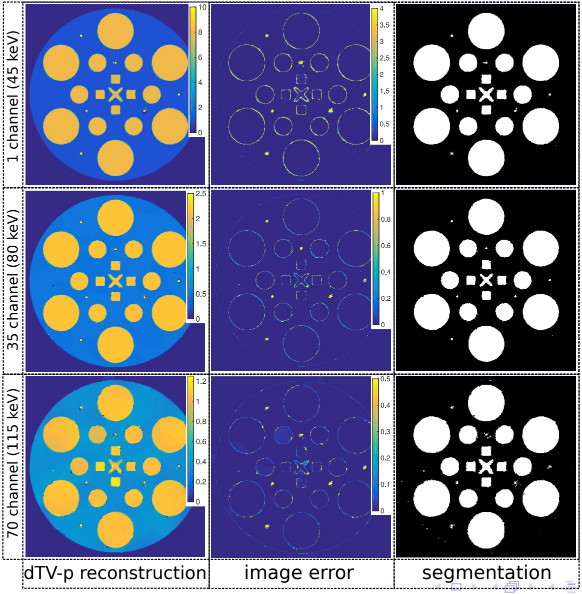


Image reconstruction with dTV-p



Reconstructed energy profiles

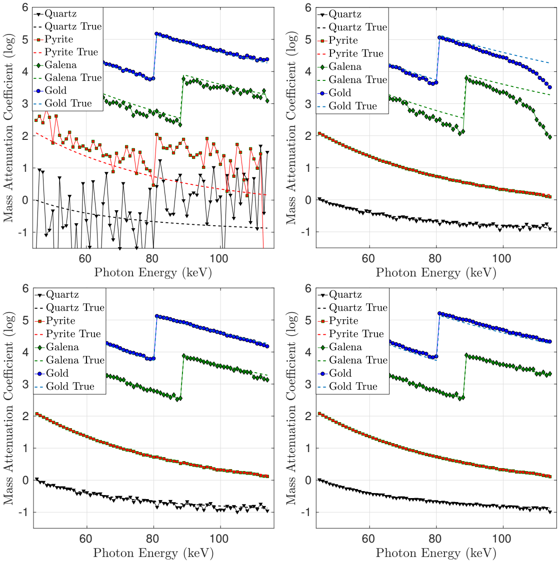


Figure: MACs plots for FBP, TV, TNV, and dTV-p reconstruction methods.

SNR data invariance possible solutions

Proposed: Choose a reference-channel based on SNR of data. Other parameters kept fixed to simplify comparisons.

SNR data invariance possible solutions

Proposed: Choose a reference-channel based on SNR of data. Other parameters kept fixed to simplify comparisons.

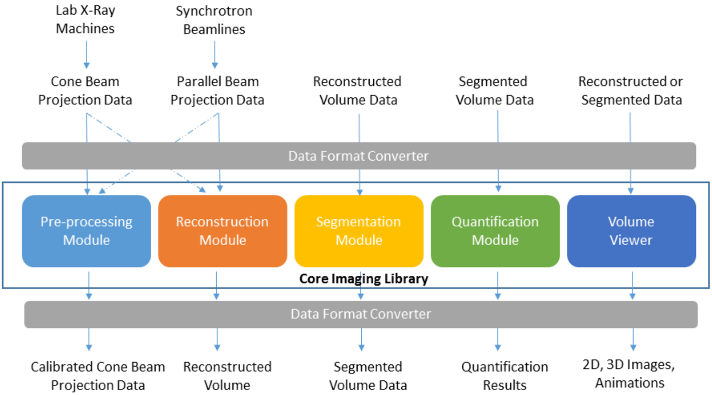
Other approaches to compensate for variable SNR

- Data normalization (D. Rigie, Phys. Med. Biol. 2015)
- Variable regularization parameters (A. Sawatzky et al, IEEE TMI 2014)
- Variable smoothing constant to establish gradient of the reference (preliminary tested to improve dTV-p method)
- Variable convergence parameters (methods normally converge with different rates)

Some software references

CCPi Core Imaging Library (CIL)

manchester X-Ray Imaging Facility



- CCPi website: https://www.ccpi.ac.uk/
- Open-source software: https://github.com/vais-ral
- Software supporting publication: https://github.com/dkazanc

CCPi Regularisation Toolkit

CCPi - Regularisation Toolkit (2D/3D CPU/GPU)

Regularisers (single-channel)

- ROF Total Variation (TV)
- Fast Gradient Projection TV
- Split Bregman TV
- Total Generilised Variation
- LLT-ROF (Higher order)
- Nonlinear Diffusion
- Anisotropic Diffusion (HO)
- ....

Regularisers (multi-channel)

- FGP directional TV
- Total Nuclear Variation

Regularisers (time-lapse)

- to be added...

Inpainters

- Linear/Nonlinear diffusion
- Nonlocal vertical marching

Python

Cython

Access to

High-level platforms

MATLAB

C-MEX

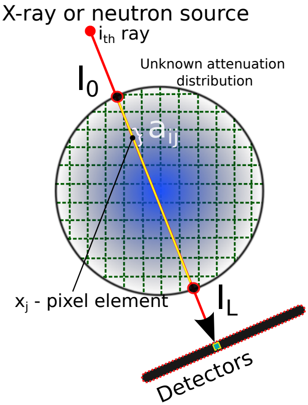
- https://github.com/vais-ral/CCPi-Regularisation-Toolkit
- Multi-threaded software for CPU/GPU architectures.

Questions

Thank you!

Questions

Collecting tomographic projection data



The basic CT measurement model: A **line** integral through the dense media

Forward (acquisition) model assumptions:

1. No refraction or diffraction
2. The X-rays are **monochromatic**
3. The linearity of Beer-Lamberts law

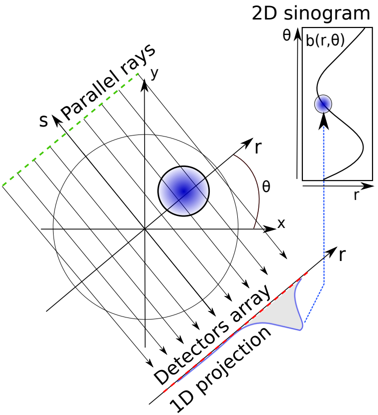
The exit beam intensity measured as:

$$\Lambda_i = I_{0,i} \exp\left(-\int_{L_i} \mu(\mathbf{r})dl\right), \; i = 1, \ldots, m,$$

where $\mathbf{r} \in \mathbb{R}^2$ is the spatial position. Then:

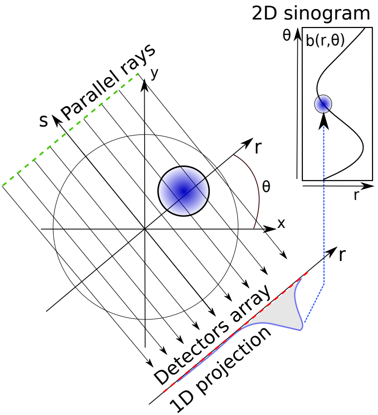
$$\int_{L_i} \mu(\mathbf{r})dl = -\ln\left(\frac{\Lambda_i}{I_{0,i}}\right).$$

Collecting tomographic projection data

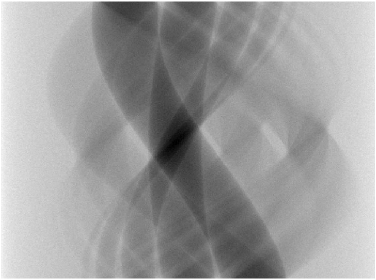


Sinogram $b(r, \theta)$ is a function of the detector's position r and the angle of acquisition θ.

Collecting tomographic projection data



Sinogram $b(r, \theta)$ is a function of the detector's position r and the angle of acquisition θ.



The measured data Y_i is modelled as a Poisson random variable with parameter Λ_i:

$$Y_i \sim \text{Poiss}\{\bar{\Lambda}_i\} = \text{Poiss}\{I_{0,i} e^{-[\mathbf{A}\mu]_i}\},$$

where attenuation coefficient on a square grid of n pixels: $\mu(\mathbf{r}) = \sum_{j=1}^{n} x_j \chi_j(\mathbf{r})$.