



**STATISTICS AND DISCRETE MATHEMATICS**

(Course Code: 23MA3BSSDM)

**UNIT-5: COMBINATORICS**

**I. Binomial Theorem and multinomial theorem:**

1. What is the coefficient of  $x^{12}y^{13}$  in the expansion of  $(2x-3y)^{25}$ .
2. Find the coefficient of
  - (i)  $x^9y^3$  in the expansion of  $(2x-3y)^{12}$
  - (ii)  $x^0$  in the expansion of  $\left(3x^2 - \frac{2}{x}\right)^{15}$
  - (iii)  $x^{12}$  in the expansion of  $x^3(1-2x)^{10}$
  - (iv)  $x^k$  in the expansion of  $(1+x+x^2)(1+x)^n$ ,  $0 \leq k \leq n+2$ .
3. Determine the coefficient of
  - (i)  $xyz^2$  in the expansion of  $(2x-y-z)^4$
  - (ii)  $x^2y^2z^3$  in the expansion of  $(3x-2y-4z)^7$
  - (iii)  $a^2b^3c^2d^5$  in the expansion of  $(a+2b-3c+2d+5)^{16}$

**II. The Principle of Inclusion Exclusion and Generalizations of The Principles:**

1. In a survey of 200 musicians, it was found that 40 wore gloves on the left hand and 39 wore gloves on the right hand. If 160 wore no gloves at all, how many wore a glove on only the right hand? Only the left hand? On both hands?
2. Out of 1200 students at a college, 582 took economics, 627 took English, 543 to mathematics 217 took both economics and English, 307 took both economics and mathematics, 250 took both mathematics and English and 222 took all three courses. How many took none of the three?
3. Among the first 500 positive integers, determine the integers which are not divisible by 2, nor by 3 nor by 5.
4. Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics, 10 study biology and physics and 30 do not study any of the subjects. Find the number of students studying all three subjects?
5. How many integers between 1 and 300 (inclusive) are
  - a. Divisible by at least one of 5, 6, 8?
  - b. Divisible by none of 5, 6, 8?

6. In how many ways 5 number of a's , 4 number of b's and 3 number of c's can be arranged so that all the identical letters are not in a single block?
7. In how many ways 5 number of a's, 4 number of b's and 3 number of c's can be arranged so that all the identical letters are not in a single block?
8. In how many ways can the 26 letters of the English alphabet be permuted so that none of the pattern's CAR, DOG, PUN or BYTE occurs?

**III. Catalan Numbers:**

1. Define Catalan number. Obtain the number of paths from  $(2,1)$  to  $(7,6)$  and not rise above the line  $y = x - 1$  using the moves  $R: (x, y) \rightarrow (x + 1, y)$  and  $U: (x, y) \rightarrow (x, y + 1)$ .
2. Using the moves  $R: (x, y) \rightarrow (x + 1, y)$  and  $U: (x, y) \rightarrow (x, y + 1)$ . Find in how many ways can one go from  $(2,6)$  to  $(6,10)$  and not rise above the line  $y = x + 4$ .
3. Using the moves  $R: (x, y) \rightarrow (x + 1, y)$  and  $U: (x, y) \rightarrow (x, y + 1)$ . Find in how many ways can one go from  $(7,3)$  to  $(10,6)$  and not rise above the line  $y = x - 4$ .
4. Using the moves  $R: (x, y) \rightarrow (x + 1, y)$  and  $U: (x, y) \rightarrow (x, y + 1)$ . Find in how many ways can one go from  $(3,8)$  to  $(11,16)$  and not rise above the line  $y = x + 5$ .

**IV. Derangements :(Nothing is in its Right Place)**

1. Find the number of derangements of 1,2,3,4.
2. Evaluate  $d_5, d_6, d_7, d_8$ .
3. While at race track, a person bets on each of the ten horses in a race to come in accordance to how they are favoured. In how many ways can they reach the finish line so that he losses all his bets?
4. For the positive integers 1, 2, 3,..., n there are 11660 derangements where 1,2,3,4,5 appear in the first five positions. What is the value of 'n'?
5. In how many ways can the integers 1, 2, 3,...,10 be arranged in a line so that no even integer is in its natural place?
6. There are  $n$  pairs of children's gloves in a box. Each pair is of a different colour. Suppose the right gloves are distributed at random to  $n$  children, and thereafter the left gloves are also distributed to them at random. Find the probability that
  - (i) No child gets a matching pair,
  - (ii) Every child gets a matching pair,
  - (iii) Exactly one child gets a matching pair,
  - (iv) At least 2 children get matching pairs.
7. Thirty students take a quiz. Then for the purpose of grading, the teacher asks the students to exchange papers. Find the probability that
  - (i) No one is grading his own paper.
  - (ii) Every student gets his own paper.
  - (iii) Exactly one student gets his own paper.
8. At a restaurant, 10 men hand over their umbrellas to the receptionist. In how

many ways can their umbrellas be returned so that

- (i) No man receives his own umbrella?
- (ii) At least one of the men receives his own umbrella?
- (iii) At least two of the men receives his own umbrella?

**V. Rook Polynomials:**

1. Find the Rook polynomial for the following boards for the non-shaded squares.

a.

1
---

b.

1	2
3	4

c.

2	3	4
1		5

d.

1	2	
		3
4	5	6

e.

	1	2
	3	4
5	6	7

f.

1	2	3
4		5
6	7	8

2. Find the Rook polynomial for the  $2 \times 2$  board by using the expansion formula.
3. By using the expansion formula, find the Rook polynomial for the board 'C' shown below.

		1
	2	3
4	5	6
7	8	

4. By using the expansion formula, find the Rook polynomial for the board ‘C’ shown below.

1		2
3	4	5
	6	

5. Find the Rook polynomial for the board ‘C’ shown below.

1	2			
3	4			
			5	6
			7	8
		9	10	11

6. Find the Rook polynomial for the board ‘C’ shown below.

	1	2		
3		4		
	5		6	7
			8	

\*\*\*\*\*